Indian J Phys. **80 (9)**, 879-885 (2006)

IJP

Thermal stability of radiant black holes

Parthasarathi Majumdar
Theory Group, Saha Institute of Nuclear Physics, 1/AF Bidhannagar, Kolkata-700 064, India

E-mail parthasarathi majumdar@saha ac in

Abstract Beginning with a brief sketch of the derivation of Hawking's theorem of horizon area increase, based on theRaychaudhuri equation, we go on to discuss the issue as to whether generic black holes, undergoing Hawking radiation, can ever remain in stable thermal equilibrium with that radiation We derive a universal criterion for such a stability, which relates the black hole mass and microcanonical entropy, both of which are well-defined within the context of the Isolated Horizon, and in principle calculable within Loop Quantum Gravity The criterion is argued to hold even when thermal fluctuations of electric charge are considered, within a *grand* canonical ensemble

Keywords : Black hole, entropy, Hawking radiation, thermal stability

PACS Nos. : 04 60 Ds, 04.60 Pp, 04 70 Dy

1. Introduction

the equation for the rate of change of expansion (Raychaudhuri's equation) plays a central role in the proofs of the singularity theorems [1].

Even in its fifty-first year, the Raychaudhuri equation [2] retains its prime position as a tool to analyze spacetime singularities (for a review for non-specialists, see [3]) However, lest one should feel that the equation has only one specialized role in general relativity, namely that of delineating singularities, we remark that it has had far wider applications, as for example in establishing the so-called laws of black hole mechanics. The earliest of these laws or theorems is called Hawking's theorem of increase of horizon area of a black hole It states that [4].

The area of the event horizon of an isolated stationary black hole can never decrease in any physical process.

We recall first that a black hole spacetime $\mathcal{B}$ is the set of events that lie in the *complement* of the choronological past of events infinitely distant and infinitely far in the future (the future asymptotic null infinity $\mathcal{I}^+$). The event horizon of the black hole is the boundary $\partial\mathcal{B}$ of events in spacetime accessible to such observers. It is a null 2+1 dimensional surface hiding the black hole singularity.

A sketch of the proof of Hawking's theorem may be given as follows [4] · Recall that the Raychaudhuri equation [2]

$$l^a \Delta_a \theta \equiv \frac{d\theta}{d\tau} = = -\frac{1}{3}\theta^2 - \sigma^2 + \omega^2 - R_{ab} l^a l^b \qquad (1)$$

relates the evolution of the (volume) expansion $\theta(\tau)$ of (timelike and null) geodesics, to the 'shear' σ which quantifies relative stretching of geodesics in a congruence, the rotation or vorticity ω which exhibits how the geodesics twist around each other as they evolve and the spacetime curvature responsible for geodesic deviation.

Consider any null surface generated by null geodesics which we assume complete A fixed time slice of such a surface is a spacelike 2-sphere of area A. The Raychaudhuri equation (1) now implies that

$$\theta^{-1}(\tau) \geq \frac{1}{2}\tau + \theta^{-1}(0). \qquad (2)$$

The expansion $\theta = d\log A/d\tau$. Assume $\theta(0) > 0$ in contrast to the case for studying singularities; the area A then

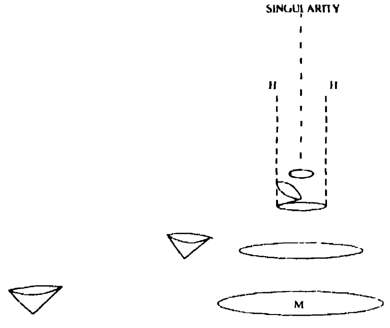

Figure 1. Black hole spacetime emerging from spherically symmetric collapse of a massive star M, ellipses denote successive sizes of the collapsing star, and the null surface H is the event horizon

increases locally Consider fixed time slices S_1, S_2 of event horizon at τ_1, $\tau_2 > \tau_1$. The number of null geodesic generators intersecting S_2 is at least the same or bigger than those intersecting S_1 At any instant, $A \propto$ *no of* Null geodesic generators, it follows then that $\Rightarrow \quad A_2 \geq A_1$ which is what the theorem states

Hawking's theorem brings to mind the law of entropy increase (for isolated systems) that follows from the second law of thermodynamics Yet, for a classical black hole spacetime, the complete absence of 'microstates' involving atoms, electrons, photons or other discrete entities, stymies pushing the analogy very far. The situation turns curiouser because of other 'laws of (stationary) black hole mechanics' derived on the basis of general relativity [6], wherein the geometrical quantity known as surface gravity (κ) appears as an analogue of temperature

This thermodynamic analogy of the theorems on stationary black hole mechanics, especially, Hawking's theorem, led Bekenstein [5] to *postulate* that *black holes must have must an entropy proportional to the area of their horizons*

$$S_{bh} = \varsigma k_B \frac{A}{l_P^2} \tag{3}$$

Here, ς is a dimensionless constant of $O(1)$, and l_P is the Planck length $l_P \equiv (G\hbar/c^3)^{1/2} \sim 10^{-33}$ cm, characterizing the length scale at which spacetime can no longer be thought of in terms of classical Riemannian geometry It follows that black hole *thermodynamics* must have to do with *quantum*, rather than classical, general relativity The microstates from which S_{bh} originates must then correspond to the states of the quantum spacetime associated with the black hole, especially the event horizon.

Similarly, black hole temperature also has quantum origins, being associated with the mysterious Hawking radiation [6] Every generic black hole radiates particles in a thermal (black body) spectrum, at a temperature $T_{bh} = \hbar \kappa_{horizon}$, a surprising result, if one remembers that the surface gravity κ is purely a geometric quantity Hawking radiation thus corresponds to *transformation* of quantum spacetime states describing the black hole, into thermal particle states In a sense, it is an inversion of gravitational collapse whereby particle states collapse into spacetime states The extra feature here is the thermalization of the particle states produced through Hawking radiation, which eventually leads to the Information Loss conundrum

The issue we address here is that of thermal stability of generic radiant black holes in the heat bath made up of their own radiation The asymptotically flat Schwarzschild spacetime is well-known [7] to have a thermal *instability* the Hawking temperature for a Schwarzschild black hole of mass M is given by $T \sim 1/M$ which implies that the specific heat $C \equiv \partial M/\partial T < 0$ [1] The instability is attributed within a standard canonical ensemble approach, to the superexponential growth of the density of states $\rho(M) \sim \exp M^2$ which results in the canonical partition function diverging for large M

The problems with an approach based on an equilibrium canonical ensemble do not exist, at least for isolated spherically symmetric black holes, formulated as *isolated horizons* [8] of fixed horizon area, these can be consistently described in terms of an equilibrium *microcanonical* ensemble with fixed A (and hence disallowing thermal fluctuations of the energy M). For $A >> l_{Planck}^2$, it has been shown using Loop Quantum Gravity [9], that all spherically symmetric four dimensional isolated horizons possess a microcanonical entropy obeying the Bekenstein-Hawking Area Law (BHAL) [5,6]. Further, the microcanonical entropy has corrections to the BHAL due to quantum spacetime fluctuations at fixed horizon area. These arise, in the limit of large A, as an infinite series in inverse powers of horizon area beginning with a term logarithmic in the area [10], with completely finite coefficients,

$$S_{MC} = S_{BH} - \frac{3}{2}\log S_{BH} + const + O(S_{BH}^{-1}), \tag{4}$$

where $S_{BH} \equiv A/4l_{Planck}^2$.

On the other hand, asymptotically anti-de Sitter (adS) black holes with spherical symmetry are known [7] to be describable in terms of an equilibrium canonical ensemble, so long as the cosmological constant is large in magnitude. For this range of black hole parameters, to leading order in Λ the canonical entropy obeys the BHAL. As the magnitude of the cosmological constant is reduced, one approaches the so-called Hawking-Page phase transition to a 'phase' which exhibits the same thermal instability as mentioned above.

In this article, we focus on the following

(i) Is an understanding of the foregoing features of black hole entropy and thermal stability on some sort of a 'unified' basis possible ? We shall argue, following [11–15] that it is indeed so, at least in the case of non-rotating black holes.

(ii) In addition to corrections (to the area law) due to fixed area quantum spacetime fluctuations computed using a microcanonical approach, can one compute corrections due to *thermal* fluctuations of horizon area within the canonical ensemble ? Once again, the answer is in the affirmative. The result found in [11-15], at least for the leading log *area* corrections, turns out to be *universal* in the sense that just like the BHAL, it holds for all black holes independent of their parameters.

2 Canonical partition function : holography ?

Following [11], we start with the canonical partition function in the quantum case

$$Z_C(\beta) = Tr\exp-\beta\hat{H} \tag{5}$$

Recall that in classical general relativity in the Hamiltonian formulation, the bulk Hamiltonian is a first class constraint, so that the entire Hamiltonian consists of the boundary contribution H_S on the constraint surface. In the quantum domain, the Hamiltonian operator can be written as

$$\hat{H} = \hat{H}_V + \hat{H}_S, \tag{6}$$

with the subscripts V and S signifying bulk and boundary terms respectively. The Hamiltonian constraint is then implemented by requiring

$$\hat{H}_V|\psi\rangle_V = 0 \tag{7}$$

for every physical state $|\psi\rangle_V$ in the bulk. Choose as basis for the Hamiltonian in (6) the states $|\psi\rangle_V \otimes |\chi\rangle_S$. This implies that the partition function may be factorized as

$$Z_C \equiv Tr\exp-\beta\hat{H}$$

$$= \underbrace{\dim\mathcal{H}_V}_{\text{indep of}\,\beta}\underbrace{Tr_S\exp-\beta\hat{H}_S}_{\text{boundary}}. \tag{8}$$

Thus, the relevance of the bulk physics seems rather limited due to the constraint (7). The partition function further reduces to

$$Z_C(\beta) = \dim\mathcal{H}_V Z_S(\beta), \tag{9}$$

where $\mathcal{H}_V$ is the space of bulk states $|\psi\rangle$ and Z_S is the 'boundary' partition function given by

$$Z_S(\beta) = Tr_S\exp-\beta\hat{H}_S \tag{10}$$

Since we are considering situations where, in addition to the boundary at asymptopia, there is also an inner boundary at the black hole horizon, quantum fluctuations of this boundary lead to black hole thermodynamics. The factorization in eq (9) manifests in the canonical entropy as the appearance of an additive constant proportional to dim $\mathcal{H}_V$. Since thermodynamic entropy is defined only upto an additive constant, we may argue that the bulk states do not play any role in black hole thermodynamics. This may be thought of as the origin of a weaker version of the holographic hypothesis [16].

For our purpose, it is more convenient to rewrite (10) as

$$Z_C(\beta) = \sum_{n\in Z}\underbrace{g\left(E_S\left(A(n)\right)\right)}_{\text{degeneracy}}\exp-\beta E_S\left(A(n)\right), \tag{11}$$

where, we have made the assumptions that (a) the energy is a function of the area of the horizon A and (b) this area is quantized. The first assumption (a) basically originates in the idea in the last paragraph of that black hole thermodynamics ensues solely from the boundary states whose energy ought to be a function of some property of the boundary like area. The second assumption (b) is actually explicitly provable in theories like NCQGR (nonperturbative cononical quantum general relativity) as we now briefly digress to explain.

3. Spin network basis in NCQGR

The basic canonical degrees of freedom in NCQGR are holonomies of a distributional $SU(2)$ connection and fluxes of the densitized triad conjugate to this connection. The Gauss law (local $SU(2)$ invariance) and momentum (spatial diffeomorphism) constraints are realized as self-adjoint operators constructed out of these variables. States annihilated by these constraint operators span the kinematical Hilbert space. Particularly convenient bases for this kinematical Hilbert space are the spin network bases. In any of these bases, a ('spinet') state is described in terms of *links* $l_1, .., l_n$ carrying spins ($SU(2)$ irreducible representations) $j_1, .., j_n$ and *vertices* carrying invariant $SU(2)$ tensors ('intertwiners') A particularly important property of such bases is that geometrical observables like area operator are diagonal in this basis with *discrete spectrum* An internal boundary of a spacetime like a horizon appears in this kinematical description as a punctured S^2 with each puncture having a deficit angle $\theta = \theta(j_i)$, $i = 1, \ldots, p$, as shown in Figure 2

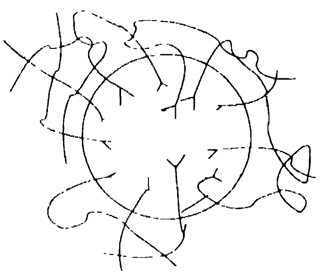

Figure 2 Internal boundary (horizon) pierced by spinet links

For macroscopically large boundary areas $A >> l^2_{\mathrm{Planck}}$, the area spectrum is dominated by $j_i = 1/2$, $\forall_i = 1, \ldots, p$, $p >> 1$. This is the situation when the deficit angles at each puncture takes its smallest nontrivial value, so that a classical horizon emerges. That implies that

$$A(p) \sim p l^2_{\mathrm{Planck}}, p >> 1. \tag{12}$$

This completes our digression on NCQGR

4. Fluctuation effects on canonical entropy

We now move on to discuss the effect of inclusion of Gaussian thermal fluctuations of the horizon area The canonical entropy is expected to receive additional corrections due to such fluctuations over and above those due to quantum spacetime fluctuations already included in the microcanononcal entropy Going back to eq (11), we can now rewrite the partition function as an integral, using the Poisson resummation formula

$$\sum_{n=-\infty}^{\infty} f(n) = \sum_{m=-\infty}^{\infty} \int_{-\infty}^{\infty} dx \exp(-2\pi i m x) f(x). \tag{13}$$

For macroscopically large horizon areas $A(p)$, $x >> 1$, so that the summation on the *rhs* of (13) is dominated by the contribution of the $m = 0$ term. In this approximation, we have

$$Z_C = \int_{-\infty}^{\infty} dx g\left(E(A(x))\right) \exp - \beta E(A(x))$$

$$= \int dE \exp\left[S_{MC}(E) - \log\left|\frac{dE}{dx}\right| - \beta E \right] \tag{14}$$

where $S_{MC} = \log g(E)$ is the microcanonical entropy Now, in equilibrium statistical mechanics, there is an inherent ambiguity in the def-inition of the microcanonical entropy, since it may also be defined as $\bar{S}_{MC} \equiv \log \rho(E)$ where $\rho(E)$ is the density of states. The relation between these two definitions involves the 'Jacobian' factor $|dE/dx|^{-1}$

$$\bar{S}_{MC} = S_{MC} - \log\left|\frac{dE}{dx}\right|. \tag{15}$$

Clearly, this ambiguity is irrelevant if all one is interested in is the leading order BHAL. However, if one is interested in logarithmic corrections to BHAL as we are, this difference is crucial and must be taken into account

We next proceed to evaluate the partition function in eq. (14) using the saddle point approximation around the point $E = M$ where M is to be identified with the (classical) mass of the boundary (horizon). Integrating over the Gaussian fluctuations around the saddle point, and dropping higher order terms, we get

$$Z_C = \exp\left\{ S_{MC}(M) - \beta M - \log\left[\left|\frac{dE}{dx}\right|_{E=M}\right]\right\}$$

$$\times \left[\frac{\pi}{-S''_{MC}(M)}\right]^{1/2}. \tag{16}$$

Using $S_C = \log Z_C + \beta M$, we obtain for the canonical entropy S_C

$$S_C = S_{MC}(M) \underbrace{- \frac{1}{2}\log(\Delta),}_{\delta_{fl} S_c} \tag{17}$$

where

$$\Delta \equiv \left[A'(x)\right]^2 \left[S'_{MC}(A) \frac{M''(A)}{M'(A)} - S''_{MC}(A) \right]. \tag{18}$$

Thus, the canonical entropy is expressed in terms of the microcanonical entropy for an average large horizon area, and the mass which is also a function of the area. Clearly, stable equilibrium ensues so long as $\Delta > 0$.

Additional support for this condition can be gleaned by considering the thermal capacity of the system, using the standard relation

$$C(A) = \frac{dM}{dT} = \frac{M'(A)}{T'(A)}, \tag{19}$$

with T being derived from the microcanonical entropy $S_{MC}(A)$, and hence a function of A. One obtains for the heat capacity the relation

$$C(A) = \left[\frac{M'(A)}{T(A)A'(x)}\right]^2 \Delta^{-1}, \tag{20}$$

so that $C > 0$ if only if $\Delta > 0$ Since the positivity of the heat capacity is certainly a necessary condition for stable thermal equilibrium, it is gratifying that an identical criterion emerges for Δ as found from the canonical entropy (17).

Using now eq. (18) for the expression for Δ, the criterion for thermal stability of non-rotating macroscopic black holes is then easily seen to be

$$M(A) > S_{MC}(A) \tag{21}$$

as already mentioned in the summary. We have been using units in which $G = \hbar = c = k_B = 1$. If we revert back to units where these constants are not set to unity, the lower bound eq (21) can be re-expressed as

$$M(A) > \left(\frac{\hbar c}{G k_B^2}\right)^{1/2} S_{MC}(A) \tag{22}$$

We remind the reader that in contrast to semiclassical approaches based on specific properties of classical metrics, our approach incorporates crucially the microcanonical entropy generated by quantum spacetime fluctuations that leave the horizon area constant. Apart from the plausible assumption of the black hole mass being dependent only on the horizon area, no other assumption has been made to arrive at the result. Even so, it subsumes most results based on the semiclassical approach.

As a byproduct of the above analysis, the canonical entropy for stable black holes can be expressed in terms of the Bekenstein-Hawking entropy S_{BH} as

$$S_C = S_{BH} - \frac{1}{2}(\xi - 1)\log S_{BH} - \frac{1}{2}\log\left[\frac{S'_{MC}(A)M''(A)}{S''_{MC}(A)M'(A)}\right] \tag{23}$$

For any smooth $M(A)$, one can truncate its power series expansion in A at some large order and show that the quantity in square brackets in eq. (23) does not contribute to the $\log(area)$ term, so that

$$S_C = S_{BH} - \frac{1}{2}(\xi - 1)\log S_{BH} + \text{const.} + O(S_{BH}^{-1}), \tag{24}$$

where $\xi = 3$ in eq. (4). Note that this is the result for an isolated horizon described by an $SU(2)$ Chern Simons theory. For a $U(1)$ Chern Simons theory, $\xi = 1$ [17,13]. The interplay between constant area quantum spacetime fluctuations and thermal fluctuations is obvious in the coefficient of the $\log(area)$ term where the contribution due to each appears with a specific sign It is not surprising that the thermal fluctuation contribution increases the canonical entropy The cancellation occurring for horizons on which a residual $U(1)$ subgroup of $SU(2)$ survives, because of additional gauge fixing by the boundary conditions describing an isolated horizon [8], may indicate a possible non-renormalization theorem, although no special symmetry like supersymmetry has been employed anywhere above. It is thus generic for all non-rotating black holes, including those with electric or dilatonic charge.

While so far we have restricted our attention to thermal fluctuations of area due to energy fluctuations alone, the stability criterion (21) can be shown to hold when in addition thermal fluctuations of electric charge are incorporated within a grand canonical ensemble [14]. As in [12], we assume that energy spectrum is a function of the discrete area spectrum (well-known in LQG [18]) and a discrete charge spectrum. The charge spectrum is of course equally spaced in general; for large macroscopic black holes the area spectrum is equally spaced as well.

In a basis in which both the area and charge operators are simultaneously diagonal, the grand canonical partition function can be expressed as

$$Z_G \sum g(m,n)\exp -\beta\left[E(A_m, Q_n) - \Phi Q_n\right], \tag{25}$$

where, $g(m,n)$ is the degeneracy corresponding to the area eigenvalue A_m and charge eigenvalue Q_n. Using a generalization of the Poisson resummation formula

$$\sum_{m,n} f(m,n) = \sum_{k,l}\int dxdy \exp\{-i(kx+ly)\} f(x,y) \tag{26}$$

and assuming that the partition sum is dominated by the large eigenvalues A_m, Q_n, it can be expressed as a double integral

$$Z_G = \int dxdy \exp - \beta\{E(A(x), Q(y)) - \Phi Q(y)\} g(A(x), Q(y)). \tag{27}$$

Note that the transition from the discrete sum to the integral for Z_G requires only that the dominant eigenvalues are large compared to the fundamental units of discreteness which for the area is the Planck area and for the charge is the electronic charge These conditions are of course fulfilled for all astrophysical black holes.

Changing variables in eq. (27) form x, y to E, Q

$$Z_G = \int dEdQ\mathcal{J}(EQ)g(E,Q)\exp\{-\beta(E-\Phi Q)\}$$

$$= \int dEdQ\rho(EQ)\exp\{-\beta(E-\Phi Q)\}, \tag{28}$$

where, the Jacobian $\mathcal{J} = |E_{,x}|^{-1}|Q_{,y}|^{-1}$, and $\rho = \mathcal{J}(E,Q)\, g(E,Q)$ is the density of states Employing the saddlepoint approximation and using eq (36) one obtains

$$S_G(M,Q_0) = \tilde{S}_{MC}(M,Q_0) - \frac{1}{2}\log\Delta + \text{const} \tag{29}$$

where using $S_{MC} = \log\rho = \tilde{S}_{MC}(M,Q_0) + \log\mathcal{J}$, we have defined

$$\Delta \equiv \det\Omega\mathcal{J}^2 = \det\Omega(E_{,x})^2(Q_{,y})^2\Big|_{m,Q_0}$$

$$= \det\Omega(A_{,E})^2(A_{,x})^2(Q_{,y})^2\Big|_{m,Q_0}, \tag{30}$$

where the Hessian matrix

$$\Omega = \begin{pmatrix} S_{MC,EE} & S_{MC,EQ} \\ S_{MC,EQ} & S_{MC,QQ} \end{pmatrix}_{M,Q_0}. \tag{31}$$

Since the microcanonical entropy is known to be only a function of the horizon area even for *charged* non-rotating black holes [8,10], one can express $\det\Omega$ as

$$\det\Omega = (S_{MC,A})^2\left[(A_{,EE})(A_{,QQ}) - (A_{,EQ})^2\right]\Big|_{M,Q_0}$$

$$+ S_{MC,AA}S_{MC,A}\left[(A_{,E})^2A_{,QQ} + (A_{,Q})^2\right.$$

$$\left.\times A_{,EE} - 2A_{,E}A_{,Q}A_{,EQ}\right]\Big|_{M,Q_0} \tag{32}$$

The necessary and sufficient conditions for thermal stability are

$$Tr\Omega = S_{MC,EE}\,|_{M,Q_0} + S_{MC,QQ}\,|_{M,Q_0} < 0$$

$$\det\Omega = \left[S_{MC,EE}\,S_{MC,QQ} - S^2_{MC,EQ}\right]_{M,Q_0} > 0 \tag{33}$$

which necessarily imply

$$S_{MC,EE}\Big|_{M,Q_0} < 0, \text{ and } S_{MC,QQ}\Big|_{M,Q_0} < 0. \tag{34}$$

Note that while these conditions together imply the first of the necessary and sufficient conditions (33) for stability of Ω, they are not sufficient to guarantee the second one

Using the microcanonical relations for temperature and potential, we may expess the necessary conditions for stability in terms of the heat capacity $C_Q \equiv (\partial E = \partial T)_Q$ and the capacitance $C \equiv (\partial Q/\partial\Phi)_E$ in the following way

$$C_Q > 0 \text{ and } C\Phi < T\left(\frac{\partial Q}{\partial T}\right)_E. \tag{35}$$

The more stringent necessary and sufficient conditions can also be similarly expressed in terms of C_Q and C

The grand partition function, evaluated in the saddle point approximation, can now be substituted in the standard thermodynamic relation in the presence of a chemical (electrostatic) potential

$$S_G = \beta M - \beta Q\Phi + \log Z_G, \tag{36}$$

so as to yield the grand canonical entropy

$$S_G = S_{MC} - \frac{1}{2}\log\det\Omega + \text{const.} \tag{37}$$

We now make use of eq. (4) to observe that both $S_{MC,A}$ and $S_{MC,AA}$ are positive denite for macroscopically large areas The necessary and sufficient conditions for thermal stability, under both energy and charge fluctuations, can now be transcribed into the three simple inequalities

$$\frac{A_{MM}}{A_M^2} + \frac{S_{MC,AA}}{S_{MC,A}} < 0,$$

$$\frac{A_{QQ}}{A_Q^2} + \frac{S_{MC,AA}}{S_{MC,A}} < 0,$$

$$\frac{A_{MM}}{A_M^2} + \frac{S_{QQ}}{S_Q^2} > \left(\frac{S_{MC,AA}}{S_{MC,A}}\right)^2, \tag{38}$$

where, we have used the notation $A_{,EE}\big|_{M,Q_0} \equiv A_{MM}$ *etc* We have assumed of course that the horizon area $A = A(M,Q)$ Since we are identifying the black hole mass M with the mass defined for the isolated non-radiating non-accreting horizon, we can recall our earlier assumption that $M = M(A)$, so that $A = A(M(A),Q)$. In other words, the mass associated with the isolated horizon ought to relate to the horizon area just as the bulk Hamiltonian relates to the volume operator (as shown in Ref [19]). The quantum mass spectrum thus should be related to the area spectrum On the other hand, electric charge has no such geometric origin, and is independent of the area, as far as one can make out. This implies that

$$A_M M'(A) = 1$$

$$A_{MM}M'^2(A) + A_M M''(A) = 0. \tag{39}$$

Substituting eq. (39) into the inequality (38), the stability criterion eq (21) emerges once again. It is not difficult to that this, when substituted back into (38), merely reproduces (38), thereby establishing consistency between the inequalities

Is it possible to derive as a bonus, as in the case of non-fluctuating charge, a general formula for the thermal fluctuation correction to the canonical entropy, logarithmic in the horizon area ? We believe it is indeed the case, but do not include that discussion here

Can this criterion continue to hold if the black hole is characterized by a number of $U(1)$ charges $Q_1 ..., Q_{n-1}$, all independently quantized ? This situation typically ensues in black holes arising in the low energy supergravity limit of string theories. The matrix is then an $n \times n$ matrix, and the analogues of eq (38) now become more complicated, involving sums of products of derivatives of S_{MC}. Despite this so long as one stays away from extremality, it is not inconceivable that the mass alone decides on the stability

5 Concluding remarks

The laws of black hole mechanics derived from the Raychaudhuri equation have inevitably led to worldwide attempts to seek quantum formulations of general relativity. A reasonably satisfactory understanding of black hole entropy has been achieved . the entropy of radiant black holes is completely described in terms of the microcanonical opy of isolated horizons which, in turn, is more or less entirely understood within the scheme of Loop Quantum Gravity, at least for the cases without rotation. Inclusion of rotation into the LQG approach to microcanonical entropy remains now foremost on the agenda, since, like he mass of the isolated horizon, and unlike $U(1)$ charges, the angular momentum is also likely to be determined by the horizon area in the isolated case.

Acknowledgment

I thank A Chatterjee for several illuminating discussions.

References

[1] S W Hawking and G F R Ellis *The Large Scale Structure of Space Time* (Cambridge . Cambrige Univ Press) (1973)

[2] A K Raychaudhuri *Phys Rev* **90** 1123 (1955)

[3] P Majumdar *Phys Teacher* **47** 41 (2005)

[4] R Wald *General Relativity* (Chicago) (1984), *Quantum Field Theory in Curved Spacetime and Black Hole Thermodynamics* (Chicago Chicago Univ Press) (1991)

[5] J D Bekenstein *Phys Rev* **D7** 2333 (1973)

[6] S W Hawking *Nature* **248** 30 (1974), *Commu Math Phys* **43** 199 (1975)

[7] S W Hawking and D N Page *Commu Math Phys* **87** 577 (1983)

[8] A Ashtekar and J Lewandowski *Class Quant Grav* **21** R53 (2004)

[9] A Ashtekar, J C Baez and K Krasnov *Adv Theor Math Phys* **4** 1 (2000)

[10] R K Kaul and P Majumdar *Phys Rev Lett* **84** 5255 (2000)

[11] A Chatterjee and P Majumdar *Black hole entropy, quantum versus thermal fluctuations*, *ArXiv gr-qc/0303030*, S Das, P Majumdar and R K Bhaduri *Quantum Grav* **19** 2355 (2002)

[12] A Chatterjee and P Majumdar *Phys Rev. Lett* **92** 141301 (2004)

[13] A Chatterjee and P Majumdar *Pramana-J Phys* **63** 851 (2004)

[14] A Chatterjee and P Majumdar *Phys Rev* **D71** 024003 (2005)

[15] A Chatterjee and P Majumdar *Phys Rev* **D72** 044005 (2005)

[16] G 't Hooft *Dimensional reduction in quantum gravity, ArXiv gr-qc/9310006. in Salam Festschrift* (eds) A Alo, J Ellis and S Randjbar-Daemi (Singapore World Scientic) (1993)

[17] S Das, R K Kaul and P Majumdar *Class Quant Grav.* **14** A55 (1997)

[18] A Ashtekar and J Lewandowski *Class Quant Grav* **14** A55 (1997)

[19] T Thiemann *Phys Lett* **B380** 257 (1996)