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Self-similarity and a parameterization of proton structure function at small x

D K Choudhury* and Rupjyoti Gogoi

Department of Physics, Gauhati University, Guwahati – 781 014, Assam, India

E-mail : dkc_phys@yahoo.co.in

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Abstract : Sometimes back, the concept of self-similarity in the structure of the proton at small x has been introduced. We make reanalysis of HERA data modifying the formalism proposed by Lastovicka so that the measured fractal dimensions of proton are all positive. Further ...ment of the model is also suggested.

Keywords : Self similarity, fractal dimension, deep inelastic scattering, structure function, low x

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1. Introduction

...e complex irregular shapes of nature possess a hidden ...metry called self-similarity [1,2]. It is not translational or ...ational symmetry, rather it is a symmetry with respect to scale ...ize. Systems exhibiting self-similarity is defined through its ...similar dimension, which is in general fraction, hence called ...tal dimension. Cantor dust, Koch curve and Sierpinski gasket ...some classical fractals having fractal dimensions 0.63, 1.26 ...1.585 respectively, which lie between Euclidean point and ...faces.

The self-similar nature of hadron multiparticle production ...cesses has been studied since nineteen eighties [3-9]. ...wever, these ideas did not attract much attention in ...temporary physics of deep inelastic lepton-hadron scattering ...2002 when Lastovicka [10,11] proposed relevant formalism ...functional form of the structure function $F_2(x,Q^2)$ at small ...Specifically, a description of $F_2(x,Q^2)$ reflecting self-similarity ...eved with four unknown parameters D_0, D_1, D_2 and D_3 to ...determined from data. While one of them (D_0) is just the ...malization constant, the other three are identified as fractal ...ensions which are fitted to HERA collider data [12,13]. The ...ific parameterization as described in Ref. [10,11] provide an ...cellent description of the data which covers a region of four ...mentum transferred squared $0.045 \leq Q^2 \leq 120\,GeV^2$ with a cut ...0.01 to exclude the valence quark region.

*Corresponding Author

One apparent limitation of the above parameterization is that out of the four fitted parameters D_0, D_1, D_2 and D_3, one is negative ($D_3 = -1.3$). As D_3 is identified as the self-similarity dimension associated with the magnification factor $1 + Q^2/Q_0^2$, a positive value will be more reasonable.

In order to explore such a possibility, sometimes back, it was suggested that [14,15] the proton is described by a single fractal dimension D, characterizing its self-similar property in analogy with classical monofractals. More recently [16], it is shown that in this limit, monofractal dimension is closely related to more familiar x-slope [17] or Pomeron intercept [18-20]. Interestingly, such monofractal nature in hadrons is also advocated in references [21-23] within a variant of statistical quark model.

However, our recent analysis [16] suggests that only in a limited x, Q^2 range of HERA data, this oversimplified model appears to survive from the phenomenological point of view.

The aim of the present paper is to report an alternative analysis of HERA data by suitable modification of magnification factors as occurred in the formalism where the estimated parameters are all positive.

2. Formalism

As noted in Ref. [10], the self-similar objects are characterized by fractal dimension D and magnification factor M related by

$$D = \frac{\log M^D}{\log M} = \frac{\log(\text{number of self-similar objects})}{\log(\text{magnification factor})} \quad (1)$$

The dimension D should be, by definition, positive so that the number of self-similar objects increases as the length scale is decreased Magnification factors are expected to fulfil some criteria They should be positive, non-zero and have no physical dimension In Ref [10], it is argued that while $1/x$ is one of the unique magnification factors, in Q^2 space, two alternative forms $\left(Q_0^2+q^2\right)/Q_0^2$ and $Q_0^2/\left(Q_0^2+q^2\right)$ are possible

These two possibilities suggest two alternative forms of unintegrated parton densities $f_i(x, q^2)$

$$\log f_i\left(x, q^2\right) = D_1 \log \tfrac{1}{x} \log\left(\frac{Q_0^2+q^2}{Q_0^2}\right) + D_2 \log \tfrac{1}{x} + D_3 \log\left(\frac{Q_0^2+q^2}{Q_0^2}\right) + D_0' \quad (2)$$

and

$$\log f_i\left(x, q^2\right) = D_1 \log \tfrac{1}{x} \log\left(\frac{Q_0^2}{Q_0^2+q^2}\right) + D_2 \log \tfrac{1}{x} + D_3 \log\left(\frac{Q_0^2}{Q_0^2+q^2}\right) + D_0' \quad (3)$$

In eqs (2) and (3), D_1 is the dimensional correlation relating the x and q^2 factors in the unintegrated parton density while D_2 and D_3 are the self-similarity dimensions associated with x and q^2 factors respectively, D_0 being the normalization constant Since the magnification factors should be positive, non-zero and dimensionless, a choice $\left(Q_0^2+q^2\right)/Q_0^2$ and $Q_0^2/\left(Q_0^2+q^2\right)$, rather than q^2 has been made Integrating over q^2, one obtains the integrated parton densities $q_i(x,Q_2)$ as

$$q_i\left(x, Q^2\right) = \int_0^{Q^2} f_i\left(x, q^2\right) dq^2 \quad (4)$$

Using the definition of structure function as

$$F_2\left(x, Q^2\right) = x\sum\left(e_i^2\left(q_i\left(x, Q^2\right) + \bar{q}_i\left(x, Q^2\right)\right)\right), \quad (5)$$

we now get two alternative forms

$$F_2\left(x, Q^2\right) = \frac{(\exp D_0) Q_0^2 x^{-D_2+1}}{1 + D_3 + D_1 \log \frac{1}{x}} \times \left(x^{-D_1 \log\left(1+\frac{Q^2}{Q_0^2}\right)} \left(1+\frac{Q^2}{Q_0^2}\right)^{D_3+1} - 1 \right) \quad (6)$$

and

$$F_2\left(x, Q^2\right) = \frac{(\exp D_0) Q_0^2 x^{-D_2+1}}{1 - D_3 - D_1 \log \frac{1}{x}} \times \left(x^{-D_1 \log\left(1+\frac{Q^2}{Q_0^2}\right)} \left(1+\frac{Q^2}{Q_0^2}\right)^{D_3+1} - 1 \right).$$

While eq (6) is same as in Ref [10], eq (7) is the main resu[lt] of the present paper, with the change of the magnification fac[tor]. It is to be noted that eq. (7) is obtainable from (6) by [the] substitution of D_1 and D_3 instead of D_1 and D_3

3. Results and discussion

For the HERA data [12,13], we find two sets of fits one with D_1 = 0 (fit 1) corresponding to the absence of dimensional correlati[on] relating $1/x$ and q^2 factors in the unintegrated parton dens[ity] and the other with $D_1 \neq 0$ (fit 2) In Figures 1, 2, we have plott[ed]

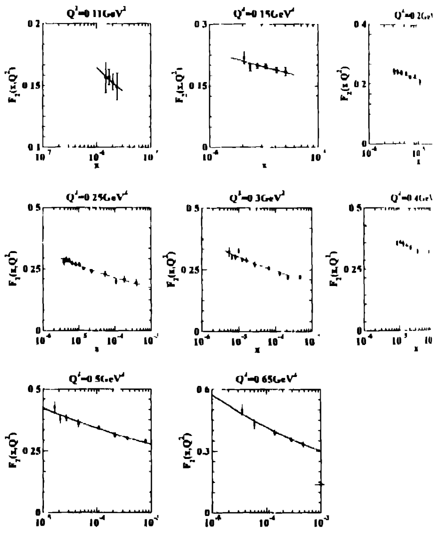
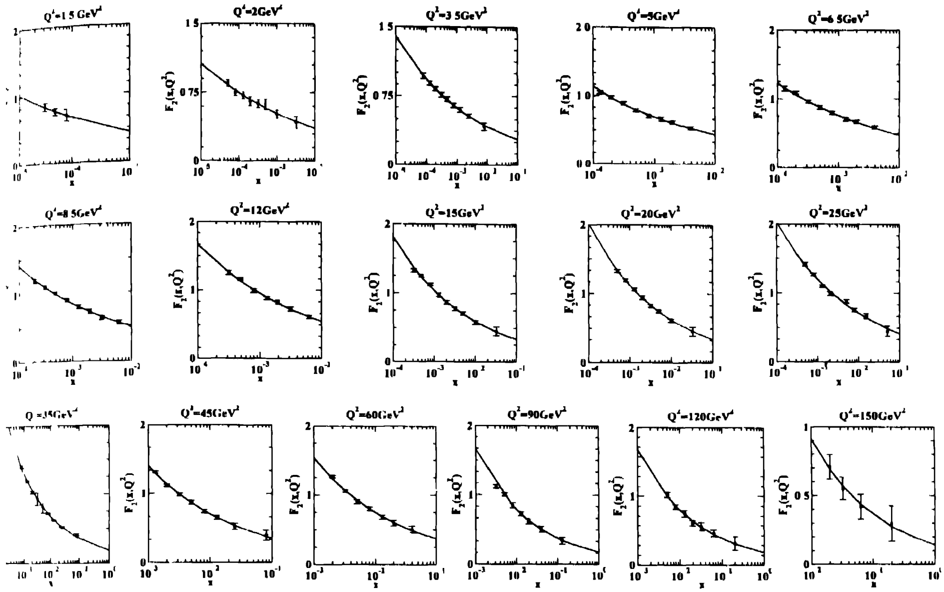


Figure 1. $F_2(x,Q^2)$ *versus* x in bins of Q^2 with D_1 = 0 (eq (8)) The bars represent total experimental error of ZEUS measurement which quadratic sum of statistical and systematic errors

$F_2(x, Q^2)$ *versus* x in bins of Q^2 as measured by Q^2 data of ZE[US] [13] and H1 [12], respectively using eq (7) and considering D 0 Results of the fit yields

$$D_3 = 1\ 4279 \pm 0\ 0584,\ Q_0^2 = 0\ 0427 \pm 0\ 0039\ \text{GeV}^2$$



2 $F_2(x,Q^2)$ *versus* x in bins of Q^2 with $D_1 = 0$ (eq (8)) The error bars represent total experimental error of H1 measurement which are quadratic of statistical and systematic errors

he other set corresponds to $D_1 \neq 0$ (fit 2) In Figures 3 we have shown the fitted curves using eq (7) for S [13] and H1 [12] data Results of the fit yields

$$D_0 - 0.6345 + 0.0145, D_1 = 0.2398 \pm 0.0125,$$
$$D \; 1.2581 \pm 0.0157, D_3 - 1.4352 \pm 0.0113,$$
$$Q_0^2 \; 0.0498 \pm 0.0013 \text{GeV}^2 \tag{9}$$

Table 1, we have recorded the estimated χ^2 For parison, we also record in Table 2, the set of parameters determined in Ref [10]

As is evident from figures, we are able to explain the A data of structure function at low x without abandoning positivity of any of the fractal dimensions This we have eved through suitable redefinition of one of the nification factors occurred in the formalism A study of able 1 also shows that the fit with parameter D_1 fixed to (Fit 1) has an almost twice better value of χ^2 than the with D_1 parameter relaxed (fit 2), suggesting nomenological preference of the former Further, due to metry of eqs (6) and (7), $D_1 \to -D_1$ and $D_3 \to -D_3$, expects that the obtained parameters should be identical pt for the change of sign of D_3 and D_1 A comparison of le 2 and fits of eqs (8) and (9) however, show that the

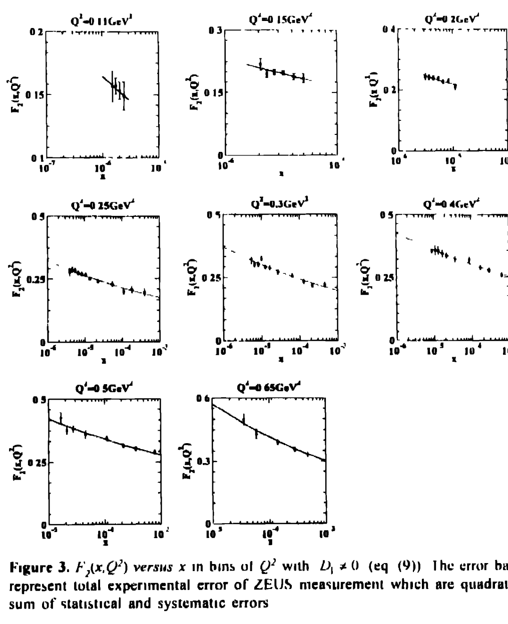


Figure 3. $F_2(x,Q^2)$ *versus* x in bins of Q^2 with $D_1 \neq 0$ (eq (9)) The error bars represent total experimental error of ZEUS measurement which are quadratic sum of statistical and systematic errors

Table 1 Results of the fit of our analysis

Fit	D_0	D_1	D_2	D_3	Q_0^2	χ^2	χ^2 /dof
All fit	0.6345±0.0145	0.2398±0.0125	1.2581±0.0157	1.4352±0.0113	0.0498±0.0013	272.743	1.356
D_1 fixed	0.4961±0.041	0	1.2009±0.039	1.4279±0.0584	0.0427±0.0039	142.281	0.071

Table 2 Results of the fit of Ref [10]

Fit	D_0	D_1	D_2	D_3	Q_0^2	χ^2	χ^2 /dof
All fit	0.339±0.145	0.073±0.001	1.013±0.01	-1.287±0.01	0.062±0.01	136.6	0.82
D_1 fixed	0.523±0.014	0.074±0.001	1(const)	-1.282±0.01	0.051±0.002	138.4	0.82

parameters differ significantly and D_1 even has opposite sign (*i.e.* D_1 is still positive). The difference in the measured values of the parameters is presumably due to the difference of choice of Q_0^2 compared with Refs [10,11].

Let us make a few comments on improving the model. This formalism developed in Ref [10,11] as well as in the present work is based on the relation (4) relating the unintegrated and integrated quark density. As the unintegrated quark densities are by definition, dimensionless (topological dimension $\tau = 0$ (eqs (2) and (3)), the integrated quark density $q(x, Q^2)$ will have dimension of GeV^2 or measure of inverse area as is evident from the occurrence of Q_0^2 in eqs (6) and (7). This can be avoided if we use the relation [24,25]

$$q_i\left(x, Q^2\right) = \int_0^{Q} \frac{f_i\left(x, q^2\right)}{q^2} dq^2$$

instead of (4), so that both the unintegrated and integrated q density are dimensionless. Such a possibility is currently u study [26].

Finally, we comment on negativity of D_3 itself dimensional analysis, it is argued [11] that it is not D_3 but combination $D_f = I + D_3 + D_1 \log(1/x)$ which should positive. For $D_3 \approx -1.2$ (Table 2), this condition in Ref [1 satisfied so long as $x < 0.01$. But this inference is strictly [26] only if the un-integrated quark density has dimen

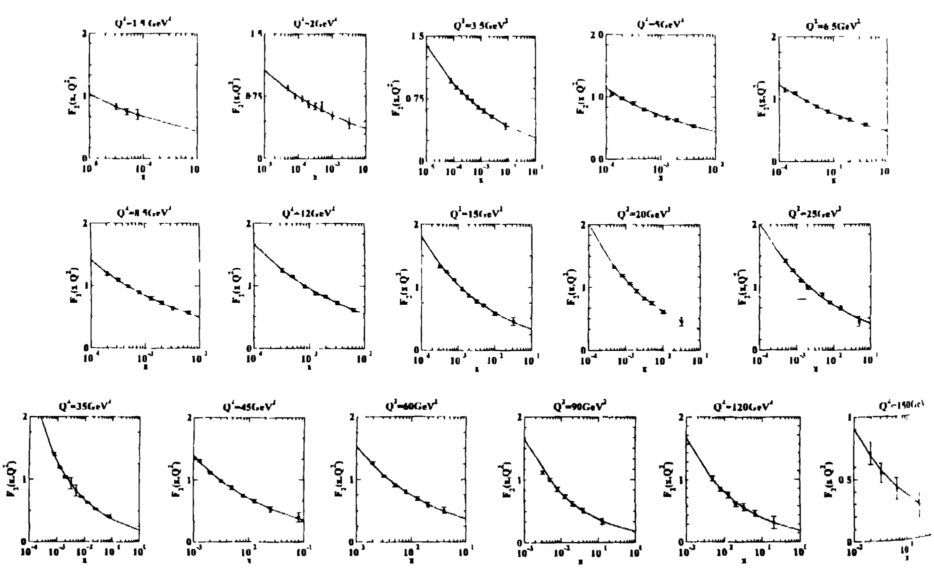


Figure 4 $F_2(x, Q^2)$ *versus* x in bins of Q^2 with $D_1 \neq 0$ (eq (9)). The error bars represent total experimental error of H1 measurement which is quadratic sum of statistical and systematic errors

... contrary to eqs (2) and (3) For dimensionless un-integrated quark density, the corresponding limit on x is 10^{-5} beyond the present HERA regime $6.2 \times 10^{-7} < x < 0.2$ A ... D_3 is free from such small x constraint as suggested in present paper

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