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Magnetic symmetry, flux screening and colour confinement in dual QCD

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Abstract: Investigating the magnetic symmetry structure of QCD, a field-theoretical analysis for exploring the colour confining mechanism has been done in dual QCD. The topological structure associated with the field equations has been used to establish the screening currents in magnetically condensed QCD vacuum. The density effects and various confinement parameters along with their implications on the nature of QCD vacuum have been discussed in infrared regime of the dual QCD.

Keywords: Confinement, nonperturbative QCD and SU(2) symmetry

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1. Introduction

Quantum Chromodynamics (QCD) is the fundamental theory of the strong interaction [1,2] which describes the properties and the underlying structure of hadrons in terms of quarks and gluons. Due to the asymptotic degree of freedom, the gauge coupling constant of QCD becomes small in the high energy region where the perturbative QCD behaviour gives a nice description of the hadronic system in terms of quarks and gluons. On the other hand, in the low energy region, the strong coupling nature of QCD leads to the nonperturbative features like colour confinement, dynamical chiral symmetry breaking and the nontrivial topological effect [3,4], and it is hard to understand them by dealing the quarks and gluons in a simple perturbative manner. In mid seventies, Nambu, 't Hooft and Mandelstam proposed the interesting idea that quark confinement may be understand using the dual version of the microscopic theory of the conventional superconductivity [5-7]. In such formulations, the magnetic charges of QCD vacuum are expected to condense and lead to the dual Meissner effect, which constricts the colour electric fields into thin flux tubes and paves the way for quark confinement [8]. Furthermore, there has been an extensive study in the pure gauge sector of the lattice QCD [9,10] also, where the low-energy hadronic phenomena such as quark confinement and chiral symmetry breaking have been analysed by using the Abelian gauge fixing approach [6]. In view of the fact that the non-Abelian gauge theories exhibit an inherent built-in dual structure and the associated magnetic symmetry might play an important role in various low-energy hadronic phenomena, we have recently formulated [11] an effective theory of non-perturbative QCD by magnetic gauge fixing and have used it to establish the link between dual version of microscopic theory of superconductivity and the confinement of coloured charges in dual QCD vacuum. The purpose of this paper is to further explore the magnetic symmetry structure of the QCD vacuum and to investigate the flux screening conditions at microscopic scale to analyse the response of the dual QCD vacuum.

2. Magnetic symmetry and dual QCD Lagrangian

Let us first briefly review the magnetic symmetry [12] structure associated with the colour gauge theory. We define the magnetic symmetry as an additional isometry of the internal space described by a Killing vector field $\hat{m}$ which has the Cartan's subgroup of the gauge symmetry as its little group. For the simple case of gauge group $G = SU(2)$ and the little group $H = U(1)$, the Killing condition associated with the unified $(4+n)$ dimensional metric manifold, in fact, reduces to the gauge covariant condition given by,

$$D_\mu \hat{m} = \partial_\mu \hat{m} + g\mathbf{W}_\mu \times \hat{m} = 0, \qquad (1)$$

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where, $\hat{m}$ constitutes the adjoint representation of the gauge group G and $\boldsymbol{W}_\mu$ is the gauge potential associated with the gauge group G. This condition thus implies that the magnetic symmetry imposes a constraint on the metric as well as on connection and may therefore be regarded as the symmetry of the potential. The multiplet $\hat{m}$ may thus be viewed to identify the homotopy of the mapping $\pi_2(S^2)$ (S^2 being the group coset space) and the monopole then emerges as a topological object which is associated with the elements of the second homotopy group $\pi_2(G/H)$. The typical gauge potential which satisfies eq. (1), may then be identified as,

$$\boldsymbol{W}_\mu = A_\mu \hat{m} - g^{-1} \hat{m} \times \partial_\mu \hat{m}, \tag{2}$$

where, A_μ is the Abelian component of $\boldsymbol{W}_\mu$ and the second part on right hand side, which is completely determined by the magnetic symmetry, is the magnetic (dual) counterpart. With the choice of G and H as simple $SU(2)$ and $U(1)$ gauge groups, one can fix the magnetic gauge by rotating $\hat{m}$ to the third direction in isospace such that, $\hat{m} \xrightarrow{U} \xi_3 = [0, 0, 1]^T$. It then leads to the redefinition of the potential (eq. (2)) as given below,

$$\boldsymbol{W}_\mu \rightarrow \boldsymbol{W}'_\mu = \left(A_\mu + B_\mu\right)\hat{\xi}_3 \tag{3}$$

with the parameterization

$$\begin{pmatrix} \sin\alpha \ \cos\beta \\ \sin\alpha \ \sin\beta \\ \cos\alpha \end{pmatrix} \quad \text{and} \quad U = \exp\left(-\alpha\, t_2\right) \exp\left(-\beta\, t_3\right).$$

The part B_μ is the magnetic potential fixed completely by the magnetic symmetry. The corresponding field strength (in magnetic gauge) may then be identified as,

$$\boldsymbol{G}_{\mu\nu} = \boldsymbol{W}_{\nu,\mu} - \boldsymbol{W}_{\mu,\nu} + g\, \boldsymbol{W}_\mu \times \boldsymbol{W}_\nu$$
$$\equiv \left(F_{\mu\nu} + B^{(d)}_{\mu\nu}\right)\hat{\xi}_3, \tag{4}$$

where, $F_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu}$ and $B^{(d)}_{\mu\nu} = B_{\nu,\mu} - B_{\mu,\nu}$.

Hence, the topological properties of the magnetic symmetry may be brought into the dynamics explicitly. However, in order to avoid the problems due to the point-like structure and the singular behaviour of the potential associated with monopoles, we use the dual magnetic potential $B^{(d)}_\mu$ (with associated field strength as $B_{\mu\nu}$) for topological (magnetic) part of the formulation and at the same time introduce a complex scalar field ϕ for the monopole. Such considerations leads to the following gauge invariant QCD Lagrangian in presence of the quark doublet source $(\Psi(x))$, given by,

$$\pounds = -\frac{1}{4}G^2_{\mu\nu} + \bar{\psi}(x) i \gamma^\mu D_\mu \psi(x) - m_o \bar{\psi}(x)\psi(x) \tag{5}$$

In the magnetic gauge, it leads to the dual symmetric field equations given in the following form,

$$G^{\nu}_{\mu\nu,} = F^{\nu}_{\mu\nu,} = j_\mu \quad \text{and} \quad G^{(d)\nu}_{\mu\nu,} = B^{\nu}_{\mu\nu,} = k_\mu . \tag{6}$$

Such non-trivial dual structure has a close relationship with the confining properties of QCD vacuum. In order to explore the typical dual QCD vacuum properties and the dynamics of the system, let us express the Lagrangian given by eq. (5) in the quenched approximation (in absence of quarks) in the following form,

$$\pounds^{(m)}_d = \frac{1}{\cdot} B^2_{\mu\nu} \quad \partial_\mu + i\frac{4\pi}{\cdot} B^{(d)}_\mu \quad - V(\phi^*\phi),$$

where the effective potential $V(\phi^*\phi)$ is obtained by using the single-loop expansion technique [13] along with the requirement of the ultra-violet finiteness and infrared instability of the dual QCD Lagrangian and has the following form,

$$V(\phi^*\phi) = \frac{24\pi^2}{g^4} \left[\phi_0^4 + (\phi^*\phi)^2 \left(2 \ln\frac{\phi^*\phi}{\phi_0^2} - 1\right.\right.$$

where $\phi_0^2 = \langle\phi\phi^*\rangle_0$ is the vacuum expectation value of ϕ. The one-loop effective potential then induces magnetic symmetry breaking in a dynamical way which leads to the magnetic condensation of the QCD vacuum. The presence of the monopole field thus signals the cross-over to the condensed phase of QCD vacuum and therefore acts as an order parameter for such a phase transition. Further, the above Lagrangian has the form that exactly coincides with that of the Ginzberg and Landau type Lagrangian [14] of the microscopic theory of superconductivity and, therefore, essentially has the flux confining features associated with it. It, in fact, ensures the appearance of two phases in QCD vacuum. One is the deconfinement phase, where the magnetic symmetry is preserved and the other is the confinement phase, where the magnetic symmetry is indeed broken dynamically. In the first phase, not only the quarks but the monopoles also appear as the physical particle states. On the other hand, in the confinement phase, they disappear from the physical spectrum of the theory. The magnetic condensation of QCD vacuum, therefore, leads to the emergence of an effect which is dual to the well known Meissener effect of conventional superconductivity. As a result the confinement of any coloured (electric) source becomes unavoidable in such a vacuum in its confined phase. In addition the confined phase reflects itself in terms of the appearance of two different mass modes, scalar and vector ones, of the condensed vacuum. The vector mode which is essentially linked to the mass of the dual gauge field (m_B) determines the magnitude of the dual Meissner effect and the scalar mode (m_ϕ) corresponds to the threshold energy to excite the monopoles in dual QCD vacuum. In the magnetically condensed QCD vacuum with finite monopole density, the parameters specifying these mass modes of dual QCD vacuum are closely related to the density of the condensed monopoles $n_m(\phi)$ which is identified

$$n_m(\phi) = | \tag{9}$$

the numerical computation of which will be used to explain the confining properties of dual QCD vacuum in the next section.

3. Flux screening in magnetically condensed dual QCD vacuum and confinement mechanism

In order to visualise these relationships and their possible implications on the nature of dual QCD vacuum and colour confinement, let us try to analyse the supercurrent structure of dual QCD vacuum resulting from the condensation of monopole configurations. For this purpose, let us start with the field equations associated with the Lagrangian given by eq. (7) which are derived in the form as given below,

$$\left[\partial^\mu - i \frac{4\mu}{} B^{(d)\mu} \qquad + i \frac{4\pi}{g} B_\mu^{(d)} \right.$$

$$+ 24 \frac{\pi^2}{g^4} \left[4\phi\phi^* \ln\left\{ \phi_0^{-2} (\phi^* \phi) \right\} \right] \phi = 0. \tag{10}$$

$$B^{\nu}_{\mu\nu} + i \frac{4\pi}{g} \left(\phi^* \bar{\partial}_\mu \phi \right) - 32 \frac{\pi^2}{g^2} B_\mu^{(d)} \phi\phi^* = 0. \tag{11}$$

These field equations governing the magnetic condensation of QCD vacuum, involve the interaction between the macroscopic field ϕ and the dual gauge field $B_\mu^{(d)}$ and hence lead to typical flux screening currents due to strong correlations among topological charges. For small variations in the monopole field which are small enough in comparison with coherence length (in the confinement regime), the field satisfies, $\partial_\mu \phi' = 0 = \partial_\mu \phi$, and eq. (11) then takes the form given as,

$$\partial^\nu B_{\mu\nu} = {}^{32\pi} \quad B_\mu^{(d)} n_m(\phi) \tag{12}$$

In Lorentz gauge, this equation may be expressed as,

$$\Box + \frac{32\pi^2}{g^2} n_m(\phi) \; B_\mu^{(d)} = 0. \tag{13}$$

This equation appears as the equation of the massive vector type which may be identified with that of the condensed mode of the dual QCD vacuum and therefore leads to the mass of the vector mode (m_B) as given below,

$$m_B^2 = \frac{32\pi^2}{g^2} n_m(\phi) \tag{14}$$

The vector mode mass thus appears as a function of the density of QCD monopoles participating in the vacuum condensation. The QCD scale is thus expected to restrict the monopole density upto a finite critical value for the colour confinement of colour isocharges. The massive vector eq. (13) also demonstrates that the QCD vacuum, as a result of dynamical breaking of magnetic symmetry, acquires properties similar to that of a relativistic superconductor where the quantum fields generate the non-zero expectation value and induce the screening currents. The massless (dual) gauge quanta which propagates in this dual QCD (condensed) vacuum then satisfies a equation of the form,

$$\Box \; B_\mu^{(d)} = J_\mu^{Sc} \; (\hat{m}, \phi) \tag{15}$$

where, J_μ^{Sc} $(\hat{m}, \phi)$ is the current that resides in the vacuum and is generated as a result of the magnetic condensation of the QCD vacuum. Comparison of eq (13) and eq. (15) along with the use of eq. (14) then leads to

$$J_\mu^{Sc} (\hat{m}, \phi) = -m_B^2 \; B_\mu^{(d)} \tag{16}$$

which is the typical screening current condition established in dual QCD vacuum. In the static case, it reduces to the form given as,

$$\boldsymbol{J}^{Sc} (\hat{m}, \phi) = -m_B^2 \; \boldsymbol{B} \tag{17}$$

The setting-up of such condition for the screening currents in dual QCD vacuum then makes the confinement of any coloured source inevitable. In the present dual formalism, the field strength $\boldsymbol{B}_{\mu\nu}$ has its field content as $\boldsymbol{H}_m$ (colour magnetic field) and $-\boldsymbol{E}_m$ (colour electric field), with the colour electric field identified as,

$$\boldsymbol{E}_m = \bar{\nabla} \times \boldsymbol{B} \; . \tag{18}$$

In the static case, it satisfies the field equation given by,

$$\bar{\nabla} \times \boldsymbol{E}_m = \boldsymbol{J}^{Sc} \; (\hat{m}, \phi) \tag{19}$$

Using the screening current condition given by eq. (17) along with eq. (18) and taking the curl of eq. (19), one can immediately deduce,

$$\nabla^2 \boldsymbol{E}_n = m_B^2 \; \boldsymbol{E}_n \tag{20}$$

For the simplest case of one-dimensional variation in colour electric field in the half- plane $x \geq 0$, *eq.* (20) reduces to the equation given by

$$\frac{d^2 \mathbf{E}_m}{dx^2} = m_B^2 \mathbf{E}_m \tag{21}$$

which has its exponential solution given by,

$$E_m = \mathbf{E_m}(0) \exp\left(-m_B x\right). \tag{22}$$

This equation demonstrates that the colour electric field penetrates the dual QCD vacuum upto a depth of $\lambda_{QCD}^{(d)}$ given by,

$$\lambda_{QCD}^{(d)} = m_B^{-1} = \left[\frac{4\pi}{g}\sqrt{2n_m(\phi)}\right]^{-1}. \tag{23}$$

It clearly shows that the screening current in dual QCD vacuum as a result of monopole condensation, gives rise to a force with range inversely proportional to the mass of the condensed vector mode m_B. The above relation between penetration depth and the density of condensed monopoles plays an important role in further exploring the QCD vacuum properties. The characteristic mass and length scales in dual QCD depend on the strong coupling constant, $\alpha_s = g^2/4\pi$, and the coupling constant in QCD is known to have the running behaviour as confirmed by the various deep inelastic lepton-nucleon scattering and $e^+ - e^-$ annihilation experiments. In low energy regime GeV, the strong coupling rises $(\alpha_s > 0.2)$ and all the non-perturbative effects (like the colour confinement) start appearing there. As such, for the numerical computation of the dual QCD characteristic scales, let us use the different extreme values of $\alpha_s (> 0.2)$ in non-perturbative region and the associated parameters [11] as given by,

$$g = 1.66, \quad n_m(\phi) = 0.63 \ \text{for} \ \alpha_s = 0.22,$$

$$g = 3.47, \quad n_m(\phi) = 0.87 \ \text{for} \ \alpha_s = 0.96$$

Using the ratio of two characteristic mass scales as fixed by the effective potential and given by $m_\phi / m_B = \sqrt{3}\,(2\pi\alpha_s)^{-\frac{1}{2}}$, we can evaluate the mass of the vector and scalar modes for strong couplings in the non-perturbative regime as,

$$m_B = 1.66\,\text{GeV}, \ m_\phi = 2.44\,\text{GeV for} \ \alpha_s = 0.22;$$

$$m_B = 0.93\,\text{GeV}, \ m_\phi = 0.65\,\text{GeV for} \ \alpha_s = 0.96.$$

The inverse of these two mass scales straightforwardly leads to the penetration and coherence lengths respectively in dual (superconducting) QCD vacuum. The two length scales then leads the Ginzburg- Landau (GL) parameter as given by,

$$k_{QCD}^{(d)} = \frac{m_B^{-1}}{m_\phi^{-1}} \tag{24}$$

which plays an important role in identifying the behaviour of QCD vacuum. In the present scenario, it may then be shown that the GL -parameter takes the value, $k_{QCD}^{(d)} > 1$, for the case of low density of condensed monopoles, which belongs to the region of relatively lower couplings in infrared regime. More precisely, as an example for the case of $\alpha_s = 0.22$, we get $k_{QCD}^{(d)} = 1.47$, which guarantees the type-II superconducting behaviour of dual QCD vacuum. However, in case of sufficiently high density of condensed monopoles which belongs to the case of relatively higher couplings, the GL-parameters takes the values (*e.g.*, for $\alpha_s = 0.96$, $k_{QCD}^{(d)} = 1.47$), the GL-parameter is estimated as $k_{QCD}^{(d)} = 0.69$, which indicates that the QCD vacuum behaves like type-I superconducting vacuum. Moreover for the purpose of comparison and to see the effect of the density of condensed monopoles on the QCD vacuum structure, one can further estimate the various values of the characteristic length and mass scales discussed above in an identical way for analysing the nature of QCD vacuum. Such an estimate is summarized in Table 1 and has been graphically presented in Figure 1.

Table 1. Length scales and condensed monopole density estimate in QCD vacuum

			$n_m(\phi)$	$\lambda_{QCD}^{(d)}$	$\xi_{QCD}^{(d)}$	$k_{QCD}^{(d)}$
		GeV	(fm^{-2})	(fm)	(fm)	
0.22	1.66	0.156	0.634	0.118	0.080	1.47
0.24	1.73	0.159	0.654	0.121	0.086	1.40
0.47	2.42	0.170	0.75	0.157	0.157	1.00
0.96	3.47	0.183	0.87	0.209	0.300	0.69

The graphical plot of different values of penetration and coherence lengths for various values of density of the condensed monopoles demonstrates that the QCD vacuum changes its nature from type-I to type-II at a particular value of the density of the condensed monopoles. In other words, the phase change in QCD vacuum from type-I to type-II occurs for the critical value of the density of the condensed monopoles to induce the phase change in QCD vacuum from type-I to type-II behaviour, is computed as $n_m^c(\phi) = 0.75\ fm^{-2}$ as shown in Figure 1.

4. Conclusions

The above analysis shows that the dual superconducting model for QCD vacuum based on the dynamical breaking of the magnetic symmetry provides an attractive possibility for the explanation of quark confinement. The screening mass for the dual gauge bosons has a direct dependence on the density of condensed monopoles in QCD vacuum and leads to some interesting insights for the dual QCD vacuum. The graphical plot for the characteristic length scales clearly shows that in the deep infrared regime with considerably high density of the condensed monopoles, the QCD vacuum favours type-I superconducting behaviour whereas it switches over to type-II superconducting once the monopole density falls down considerably to its critical value.

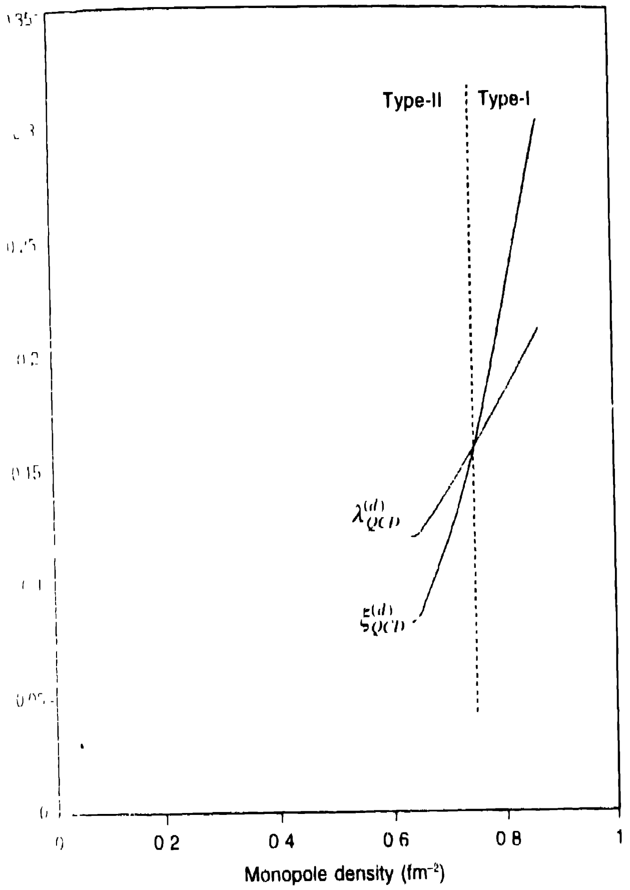

Figure 1. Plots for the characteristic lengths in dual QCD showing the different nature of the superconducting QCD vacuum in high and relatively low density sectors of the condensed monopoles

In addition, the characteristic length scales computed here, further lead to the estimation of the masses of the magnetic glueballs in dual QCD vacuum. As a specific case for, a typical value of strong coupling, $\alpha_s = 0.22$ the vector glueball assumes a mass close to the value of 1.66 GeV whereas the scalar glueball mass lie close to 2.44 GeV. Since the corresponding monopole density (0.63 fm^{-2}) falls much below the critical value, the QCD vacuum favours type-II behaviour in such case. However, beyond the critical coupling value of $\alpha_s = 0.47$ (at which the characteristic length scales lead to the vector and scalar glueball masses as 1.25 GeV each), which corresponds to the critical monopole density point (0.75 fm^{-2}), the QCD vacuum is pushed to the type-I superconducting phase which gurantees the absolute confinement of any colour electric source present. It is interesting that all the confinement parameters discussed above are in good agreement with our previous results based on flux tube model [11] of the QCD vacuum as well as with the results obtained by other authors [15] also.

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