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# Understanding Electromigration in Cu-CNT Composite Interconnects: A Multiscale Electro-Thermal Simulation Study

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**Abstract**—In this paper, we report a hierarchical simulation study of the electromigration problem in Cu-CNT composite interconnects. Our work is based on the investigation of the activation energy and self-heating temperature using a multiscale electro-thermal simulation framework. We first investigate the electrical and thermal properties of Cu-CNT composites, including contact resistances, using the Density Functional Theory and Reactive Force Field approaches, respectively. The corresponding results are employed in macroscopic electro-thermal simulations taking into account the self-heating phenomenon. Our simulations show that although Cu atoms have similar activation energies in both bulk Cu and Cu-CNT composites, Cu-CNT composite interconnects are more resistant to electromigration thanks to the large Lorenz number of the CNTs. Moreover, we found that a large and homogenous conductivity along the transport direction in interconnects is one of the most important design rules to minimize the electromigration.

**Index Terms**—Cu-CNT composites, Interconnects, Electromigration, Self-heating, Electro-thermal, DFT, Multi-scale simulation

## I. INTRODUCTION

**D**UAL damascene process technology with proper barrier metals, such as TaN, has been successfully used to fabricate Cu interconnects for the past 20 years [1]–[3]. However, aggressive scaling of the dimensions of microchips resulted in an increase of the resistivity in Cu interconnects due to grain boundary scattering (GBS) and surface roughness scattering (SRS) [4]–[6]. The increased resistance of the interconnect causes increased interconnect delay [7]. Moreover, many types

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of reliability issues arise during both the manufacturing process, such as dislocations and voids, and the operation of the device, such as electromigration (EM) [8]–[12]. When scaling down the interconnect, the power consumption per unit chip area also increases. [13] This is related mainly to the low layers of interconnects and is due to the increase in the capacitance between the taller interconnect lines and the increased specific resistance of the narrower interconnects. Nowadays, these issues must be considered carefully, since the overall performance and reliability of chips are dominated by interconnects [14], [15].

For more than a decade, carbon-based interconnects have attracted attention as a potential future interconnect technology to overcome some of these problems – GBS, SRS, and electromigration [16]–[20]. Indeed, carbon nanotubes (CNTs) and graphene show high ampacity (maximum current-carrying-capacity) and thermal conductivity thanks to their strong C-C bonding [21]–[23]. Ampacity is one of the key figures used to quantify EM. In particular, to prevent capacitance build-up by the barrier metal, Li *et al.* used single- or bi-layer graphene as the Cu diffusion barrier in Cu interconnects [24]. In addition, they successfully demonstrated their smaller resistivity and longer EM lifetime than as-deposited Cu.

More promising results were shown by Subramaniam *et al.* in 2013 with CNTs [18]. They reported that Cu-CNT composite interconnects without the diffusion barrier can have not only a similar conductivity to Cu interconnects but also a 100 times better ampacity than Cu interconnects. However, an in-depth analysis of the migration reduction of Cu atoms in Cu-CNT composite interconnects is still lacking.

In this paper, we present a multi-scale electro-thermal simulation study to understand the electromigration phenomenon in Cu-CNT composite interconnects. In general, EM in interconnects can be characterized by the Time-To-Fail (*TTF*) figure of merit, whose expression is given by Black's equation [25]:

$$TTF = A^* J^{-r} e^{E_a/(k_B T)} \quad (1)$$

where  $A^*$  is an empirical constant,  $J$  is the current density,  $r$  is a positive dimensionless exponent,  $k_B$  is the Boltzmann constant,  $E_a$  is the activation energy, and  $T$  is the temperature of the interconnect. According to Eq. 1, there are two ways to obtain a larger current density while maintaining the same *TTF*. The first option is to increase the activation energy.

To do this, the diffusion barrier with appropriate properties is adopted in the standard Cu interconnect [3]. The second way is to decrease  $T$ , which can be achieved by suppressing the self-heating effects. Unfortunately, decreasing  $T$  is difficult because the self-heating temperature  $\tilde{T}$  is caused by high  $J$  or high applied voltage ( $V$ ) during the chip operation. For this reason, we implemented the heat diffusion equation [26] in our simulation framework to capture the self-heating effects. Finally, from all simulation results, we provide design guidelines to optimize the performance of Cu-CNT composite interconnects.

This paper is organized as follow. Section II shows the first principle simulation results of the activation energy and the contact resistance between Cu and CNT. In Section III, we examine the self-heating temperature of Cu and Cu-CNT composite interconnects using the first principle calculation results. In Section IV, we provide the optimal design guidelines for Cu-CNT composite interconnects. Finally, conclusions are drawn in Section V.

## II. FIRST PRINCIPLE SIMULATIONS

We investigated the electrical and thermal properties of Cu-CNT composite interconnects using the Density Functional Theory (DFT) and the Reactive Force Field (ReaxFF) methods, respectively [27]–[31]. Both methods are implemented in the Atomistix Tool Kit (ATK) from QuantumWise [32]. The atomistic structures are optimized until the maximum force on each atom becomes less than  $10^{-2}$  eV/Å for electrical (DFT) and  $10^{-4}$  eV/Å for thermal (ReaxFF) analysis, respectively. For the DFT calculations, we use the Generalized Gradient Approximation (GGA) as the exchange-correlation functional, as proposed by Perdew, Burke, and Ernzerhof [33]. The norm-conserving pseudopotentials, as generated with the Fritz-Haber Institute pseudopotential code, are used with the double- $\zeta$  singly-polarized pseudoatomic orbital basis set. The tolerance of the self-consistent field loop calculation is set to  $10^{-4}$ .

Fig. 1 illustrates examples of optimized atomistic structures of Cu-CNT(4,4) and Cu-CNT(6,0) composites. The lattice orientation of the Cu part is selected to reduce the lattice mismatch with CNT along the transport direction. We assume that CNT is not strained by Cu in this study. The lattice parameters of bulk Cu, zigzag CNT(6,0), and armchair CNT(4,4) in DFT-GGA calculations are 3.670, 4.280, 2.474 Å, respectively. In addition, Cu atoms were placed in positions that prevent the CNT from being distorted severely.

### A. Activation energy

The activation energy  $E_a$  in Eq. 1, which is the energy required for a thermo-dynamic reaction to occur, is an important parameter to describe the EM phenomenon. In general, the structure with low value of the activation energy is vulnerable to the EM. In 2013, measurements of the  $TTF$  by Subramaniam *et al.* [18] showed that Cu-CNT composite interconnects exhibit much better EM properties than their Cu counterpart. They attributed this outperformance to the enhancement of Cu's activation energy thanks to the presence of C atoms. Nevertheless, considering the weak van der Waals interaction

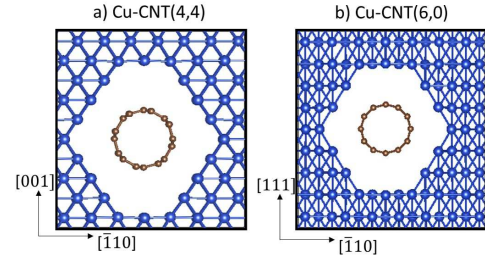


Fig. 1. Atomistic structures of a) Cu-CNT(4,4) and b) Cu-CNT(6,0) composites. The blue and brown spheres are Cu and C atoms, respectively.

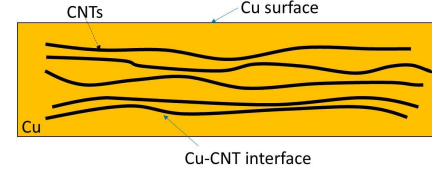


Fig. 2. The schematic diagram of a Cu-CNT composite interconnect [18].

at the interface between Cu and CNT, it is difficult to see how the presence of the CNT could enhance the activation energy of Cu atoms at its interface. Moreover, in a Cu-CNT composite, there is not only a Cu-CNT interface but also a Cu surface, which is the most vulnerable part to the EM in a Cu interconnect as shown in Fig. 2. Consequently, a further theoretical investigation of this phenomenon is needed.

Since the Cu surface in a Cu-CNT composite interconnect is the same as that in a Cu interconnect, we focused on investigating the activation energy at the interface between Cu and CNT. To do this, we constructed a large unit cell by repeating the primitive unit cell 5 times in the transport direction for a Cu-CNT composite interconnect. For bulk Cu,  $4 \times 4 \times 4$  face-centered cubic unit cells are created. Moreover, we used the basis set superposition error (BSSE) correction which is usually caused by a linear combination of atomic orbital basis (LCAO).

Fig. 3a) shows the three pathways (see red arrows) of the selected Cu atom – marked in yellow – in the Cu-CNT composite structure. This Cu atom is the closest to C atoms; the Cu-C distance (2.178 Å) at the interface in the Cu-CNT composite is shorter than Cu-Cu distance (2.595 Å) in bulk Cu. Fig. 3b) illustrates the potential profile, which is calculated by the total energy difference obtained from DFT-GGA calculations. The potential barriers in this figure indicate the activation energies for Cu atom migration. In the same manner, we calculated the activation energies using the DFT-GGA-D3 approach to correct van der Waals interactions by Grimme D3 [34]. In addition, we estimated the activation energies from the DFT-GGA simulations after removing CNT from a Cu-CNT composite structure. All corresponding results are summarized in Table I.

From all simulation results, it was found that the calculated activation energies at the Cu-CNT interface are not large compared to the activation energies at the surface in bulk Cu (0.567 eV). Moreover, we note again that the surface of a Cu-

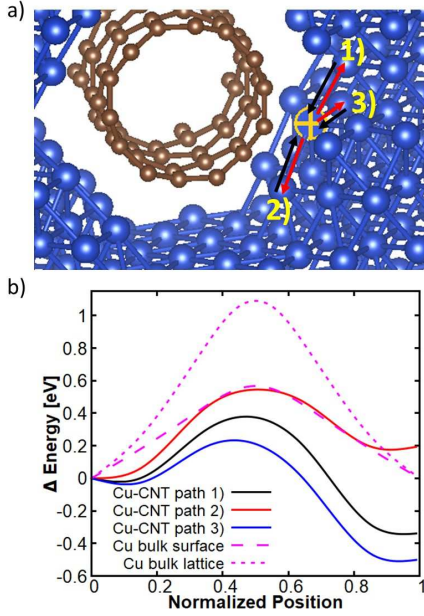


Fig. 3. a) Three pathways where the selected Cu atom (marked in yellow) moves. b) Calculated potential profiles seen by a Cu atom in bulk Cu and a Cu-CNT composite using the DFT-GGA method. For bulk Cu, we calculated the activation energies at the surface and lattice, respectively. The activation energy for each case is the barrier height for the corresponding curves.

CNT composite interconnect is still vulnerable to EM like a Cu interconnect. It can, therefore, be concluded that the CNT does not contribute significantly to increasing the activation energies of Cu atoms in a Cu-CNT composite interconnect. This is why we need to consider the other parameter in Eq. 1: the temperature.

### B. Contact resistance

Fig. 4 shows the schematic diagram of the Cu-CNT composite interconnect used in this study for the macroscopic simulations. As it can be seen from this figure, there are two kinds of contact resistances in the Cu-CNT composite interconnect; end ( $R_c^{end}$ ) and side contact resistances ( $R_c^{side}$ ). Due to the low density-of-states (DOS) of CNTs near the Fermi-level, metal-metallic CNT junctions have large contact resistances ( $R_c$ ), unlike other metal-metal junctions [19], [35]. Therefore, the accurate self-heating simulation of Cu-CNT composite interconnects requires a good understanding of the electrical and thermal contact resistances between Cu and CNTs.

In this study, we used the Non-Equilibrium Green's Function (NEGF) method assuming ballistic transport to calculate the electrical and thermal contact resistances of Cu-CNT junctions. For the sake of computational efficiency, end and side contact resistances are considered separately. The atomistic structures used in this study are shown in Fig. 5. The left end of the CNT in Fig. 5b) is passivated with hydrogen atoms. We used a  $k$ -point sampling of  $1 \times 1 \times 90$  in the DFT-NEGF simulations.

The end contact resistance  $R_c^{end}$  can be extracted from NEGF simulations. Since the number of conducting channels

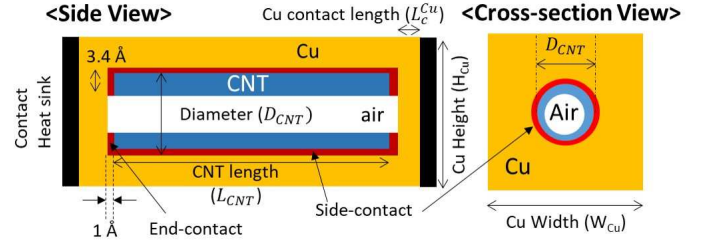


Fig. 4. The schematic diagram of Cu-CNT composite interconnects for a macroscopic electro-thermal simulation. CNT is perfectly surrounded by Cu, and left and right contacts are assumed to be perfect reservoir and heat sink. In the default structure in this study,  $L_{CNT}$ ,  $L_c^{Cu}$ ,  $D_{CNT}$ ,  $H_{Cu}$ , and  $W_{Cu}$  are  $10 \mu\text{m}$ ,  $20 \text{ nm}$ ,  $1 \text{ nm}$ ,  $3 \text{ nm}$ , and  $3 \text{ nm}$ , respectively.

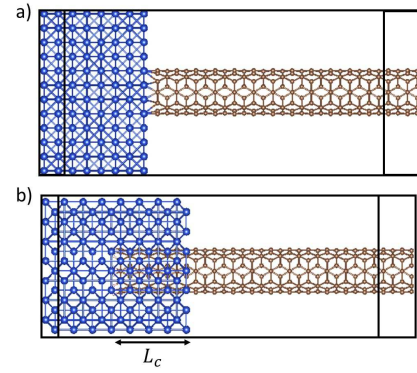


Fig. 5. Atomistic structures to calculate a) the end contact and b) the side contact resistances. Dangling bonds of CNT in the side contact is passivated by hydrogen atoms.

of bulk Cu near the Fermi-level is much larger than that of single-walled CNT (SWCNT), the total resistance  $R_{total}^{end}$  of the structure shown in Fig. 5a) reads:

$$R_{total}^{end} = R_{CNT} + R_c^{end}, \quad (2)$$

where  $R_{CNT}$  is the ballistic resistance of CNT that we calculated separately. From Eq. 2, we can calculate  $R_c^{end}$ . The total resistance of the side contact  $R_{total}^{side}$  (see Fig. 5b)) can be expressed as [36]:

$$\begin{aligned} R_{total}^{side} &= R_{CNT} + R_c^{int} + R_c^{side} \\ &= R_{CNT} + R_c^{int} + \frac{\rho_c^{side}}{\pi D_{CNT} L_c}, \end{aligned} \quad (3)$$

where  $R_c^{int}$  is the resistance at the interfaces between Cu and Cu-CNT composite and between Cu-CNT composite and CNT, and  $\rho_c^{side}$ ,  $D_{CNT}$  and  $L_c$  are respectively the resistivity of the side contact, the diameter of the CNT, and the overlap length between CNT and Cu as shown in Fig. 5b).  $R_{total}^{side}$  with different  $L_c$  should be calculated to extract  $\rho_c^{side}$ .

The thermal contact resistances have been computed from Eqs. 2 and 4 using the ReaxFF method instead of the DFT approach [31]. The calculated electrical ( $\sigma$ ) and thermal ( $\kappa$ ) conductivities of the end and side contacts for the CNT(4,4) are summarized in Table II with the corresponding experimental results from literature [36]–[40]. The calculated results are larger than the experimental results because we considered ideal structures in this simulation study. However,

TABLE I

SUMMARY OF ACTIVATION ENERGIES AT THE CU-CNT INTERFACE USING THE DFT-GGA AND THE DFT-GGA-D3 APPROACHES. ACTIVATION ENERGIES OF THE CU ATOM WITHOUT CNT ARE ALSO CALCULATED. FOR REFERENCE, WE GOT ACTIVATION ENERGIES OF 1.090 AND 0.567 eV AT THE LATTICE AND THE SURFACE IN BULK CU RESPECTIVELY FROM DFT-GGA CALCULATIONS. THE NUMBERS IN PARENTHESES ARE THE DIFFERENCE BETWEEN THE ACTIVATION ENERGY AT THE CU-CNT INTERFACE AND THE ACTIVATION ENERGY AT THE SURFACE IN BULK CU.

	Activation energy (eV) at the Cu-CNT interface					
	DFT-GGA calculation				DFT-GGA-D3 calculation	
	With CNT		Without CNT		With CNT	
	Red arrow	Black arrow	Red arrow	Black arrow	Red arrow	Black arrow
the 1st pathway	0.374 (-0.193)	0.712 (0.145)	0.205 (-0.362)	0.763 (0.196)	0.320 (-0.247)	0.697 (0.130)
the 2nd pathway	0.545 (-0.022)	0.370 (-0.197)	0.577 (0.010)	0.471 (-0.096)	0.513 (-0.054)	0.315 (-0.252)
the 3rd pathway	0.211 (-0.356)	0.710 (0.143)	0.092 (-0.475)	0.666 (0.099)	0.168 (-0.399)	0.718 (0.151)

TABLE II

OUR CALCULATED ELECTRICAL AND THERMAL CONDUCTIVITIES ARE COMPARED TO EXPERIMENTAL RESULTS. IN THIS SIMULATION, ELECTRICAL AND THERMAL CONDUCTIVITIES AT THE INTERFACE BETWEEN CU AND CNT(4,4) ARE CALCULATED AT 300K.

	$\sigma$ S/cm		$\kappa$ (mW/cm/K)	
	Simulation	Experiment	Simulation	Experiment
End contact	890	33.3 [37], [38]	0.58	0.001-0.045
Side contact	38.2	2.9 [36]	0.29	[39], [40]

it is noteworthy that such discrepancies do not affect the device simulation results significantly, as discussed later and demonstrated in Fig. 9a). Resistance values obtained for CNTs with other diameters are of the same order of magnitude. Because of the small interaction at the CNT surface, electrical and thermal conductivities at the end contact part are larger than those at the side contact part.

### III. MACROSCOPIC SIMULATION

To understand why a Cu-CNT composite interconnect is much more resistant to EM than a Cu interconnect, the self-heating temperature caused by Joule heating should be calculated. The methodology used to perform a self-heating simulation is described in the flowchart shown in Fig. 6. Based on the finite volume discretization scheme, the heat diffusion and current continuity equations are solved self-consistently [26]. The current continuity and heat diffusion equations in steady state are given by:

$$\nabla \cdot J = \nabla \cdot (\sigma \nabla \varphi) = 0, \quad (5)$$

$$\nabla \cdot (\kappa \nabla T) = -q, \quad (6)$$

where  $\varphi$  is the potential. The heat source  $q$  can be written as:

$$q = J \cdot E, \quad (7)$$

where  $E$  is the electric field.

The electrical conductivity of the CNT ( $\sigma_{CNT}$ ) was calculated using first principle simulations with the mean free path approximation [19], [20]. The conductivity including phonon scattering effects is obtained by introducing the electron mean free path  $\lambda$ :

$$\sigma_{CNT} = \frac{G_{bal} L_{CNT}}{A} \left( 1 + \frac{L_{CNT}}{\lambda} \right)^{-1}, \quad (8)$$

where  $G_{bal}$  is the ballistic conductance, and  $L_{CNT}$  and  $A$  are the length and cross-sectional area of the CNT, respectively.

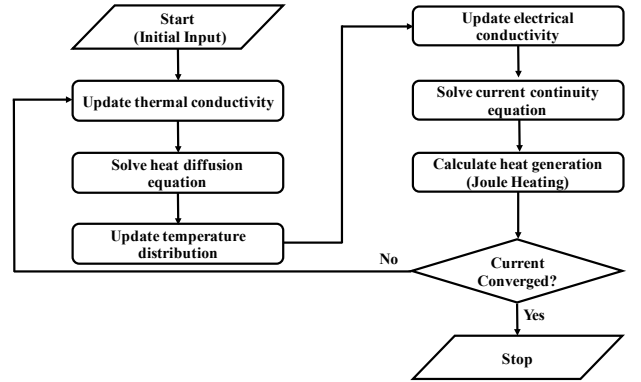


Fig. 6. Flow chart for the electro-thermal steady-state simulation.

In addition,  $\lambda$  of CNT can be given by Matthiessen's Rule [26], [41], [42]:

$$\lambda = \left( \frac{1}{\lambda_{ac}} + \frac{1}{\lambda_{op,ems}^{fld}} + \frac{1}{\lambda_{op,ems}^{abs}} + \frac{1}{\lambda_{op,abs}} \right)^{-1}, \quad (9)$$

where  $\lambda_{ac}$ ,  $\lambda_{op,ems}^{fld}$ ,  $\lambda_{op,ems}^{abs}$ , and  $\lambda_{op,abs}$  are the electron mean free paths from acoustic phonon, optical phonon emission after electrons gained energy from the electric field, optical phonon emission after absorption, and optical phonon absorption, respectively. These parameters can be calculated following references [41], [42] as:

$$\lambda_{ac} = 400460 \times \frac{D_{CNT}}{T}$$

$$\lambda_{op,abs} = 56.4 \times D_{CNT} \frac{N_{op}(300) + 1}{N_{op}(T)}$$

$$\lambda_{op,ems}^{abs} = \lambda_{op,abs} + 56.4 \times D_{CNT} \frac{N_{op}(300) + 1}{N_{op}(T) + 1}$$

$$\lambda_{op,ems}^{fld} = \frac{\hbar\omega_{op} - k_B T}{qV/L_{CNT}} + 56.4 \times D_{CNT} \frac{N_{op}(300) + 1}{N_{op}(T) + 1}$$

$N_{OP}(T) = \{1/\exp((\hbar\omega_{op})/(k_B T))\} - 1$  is the average optical phonons number defined by the Bose-Einstein distribution, and  $\hbar\omega_{op}$  is the optical phonon energy. We set  $\hbar\omega_{op}$  to 0.18 eV in this study [42]. The mean free path of CNTs is thus a function of  $T$ ,  $L_{CNT}$ ,  $D_{CNT}$ , and  $V$ .

Using the linear approximation for resistivity, the electrical conductivity of Cu ( $\sigma_{Cu}$ ) is given by:

$$\sigma_{Cu}(T) = \frac{\sigma_0}{1 + \alpha(T - T_0)} \quad (10)$$

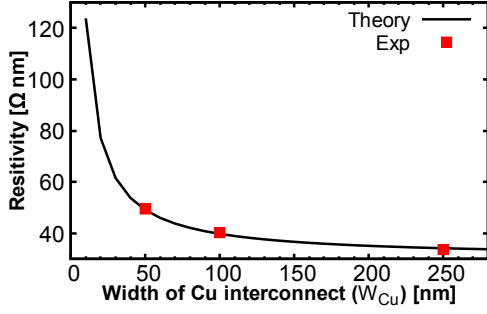


Fig. 7. Experimental and theoretical Cu resistivities versus  $W_{Cu}$  when  $H_{Cu} = 90$  nm. The corresponding parameters we extracted are:  $\rho_0$ ,  $p$ ,  $S$  and  $R$ , which are 30.42  $\Omega$  nm, 0.816, 3.762 and 0.253, respectively.

where  $T_0$  is a fixed reference temperature, 293 K,  $\alpha = 0.003862$   $\text{K}^{-1}$  is the temperature coefficient of resistivity and  $\sigma_0$  is the conductivity at  $T_0$ .  $\sigma_0$  for Cu will be discussed in the next subsection.

The measured thermal conductivity of CNT (1750-5800 W/m/K) [21] is larger than that of Cu (401 W/m/K) [43]. In this study, we compute the thermal conductivity  $\kappa$  for both Cu and CNT using Wiedemann-Franz law [44]:

$$\kappa = \sigma \tilde{L} T, \quad (11)$$

where  $\tilde{L}$  is the Lorenz number. Instead of the typical value of  $\tilde{L}$  ( $\tilde{L}_0 = 2.44 \times 10^{-8}$   $\text{W}\Omega\text{K}^{-2}$ ), we used the experimentally derived  $\tilde{L}$ 's,  $2.23 \times 10^{-8}$  and  $4.0 \times 10^{-6}$   $\text{W}\Omega\text{K}^{-2}$  for Cu and CNT, respectively [21], [45].  $\tilde{L}$  of the CNT is larger than  $\tilde{L}_0$ , meaning that the thermal conductivity ( $\kappa$ ) of the CNT is mostly dominated by phonons (as opposed to Cu where  $\kappa$  is dominated by electrons).

In this study we have used simplified structures and simulated scenarios to highlight specifically the self-heating differences between Cu and Cu-CNT interconnects and the underlying physical phenomena. Therefore, in the simulations we do not consider heat dissipation through the environment of the interconnects, such as heat dissipation through the silicon substrate and other metal layers. As a result the calculated  $\tilde{T}$  is overestimated as compared to experimental results. However, this is a global effect affecting both the Cu and Cu-CNT interconnects and therefore does not invalidate the conclusions of our simulation study.

#### A. Scattering effects in nanoscale Cu interconnect

Fig. 7 shows the experimental and theoretical dependence of Cu resistivity  $\rho_{Cu}$  on the width of the interconnect  $W_{Cu}$  when its height  $H_{Cu}$  is 90 nm. As expected, the resistivity increases as  $W_{Cu}$  decreases due to the SRS and GBS effects. To describe these effects, we used the empirical model introduced by Rossnagel *et al.* [6]:

$$\frac{\rho_{Cu}}{\rho_0} = \left\{ 1 + \frac{0.375(1-p)S\lambda}{W_{Cu}} + \frac{1.5 \frac{R}{1-R} \lambda}{g} \right\}, \quad (12)$$

where  $\rho_0$  is the bulk Cu resistivity,  $p$  is the surface scattering parameter,  $S$  is the roughness factor,  $R$  is the grain boundary scattering parameter, and  $g$  is the average value of grains size.

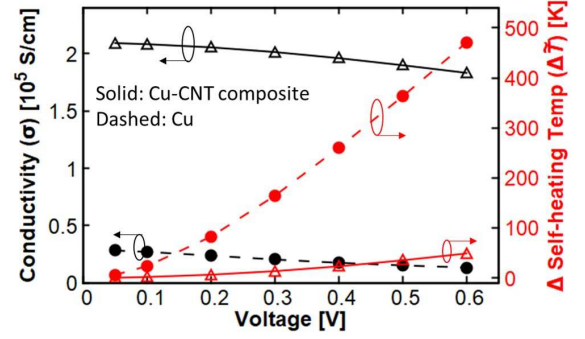


Fig. 8. Calculated conductivity and self-heating temperature difference  $\Delta\tilde{T}$  of Cu and Cu-CNT composite interconnects with different applied voltages.  $L_{CNT}$ ,  $L_{Cu}^{CNT}$ ,  $D_{CNT}$ ,  $H_{Cu}$ , and  $W_{Cu}$  are 10  $\mu\text{m}$ , 20 nm, 1 nm, 3 nm, and 3 nm, respectively. The length of Cu interconnect ( $L_{Cu}$ ) of 10  $\mu\text{m}$  is the same as  $L_{CNT}$ .

From our experimental data, we successfully extracted the values of these parameters assuming that  $g$  is equal to  $W_{Cu}$  and the mean free path  $\lambda$  is 40 nm.

#### B. Self-heating effects in Cu and Cu-CNT composite interconnects

We calculated  $\Delta\tilde{T}$  as the difference between the maximum self-heating temperature in the device and the temperature of the heat sink ( $T_{sink} = 300$  K). The conductivity and  $\Delta\tilde{T}$  resulting from Joule heating with respect to the applied voltage to the Cu and Cu-CNT composite interconnects are depicted in Fig. 8. It shows that self-heating temperature increases more rapidly with the applied bias in the Cu interconnect as compared to the Cu-CNT composite interconnect. We also found that the conductivity of both interconnects is slightly degraded at higher operating temperatures. This behavior results from an increased phonon scattering at higher  $T$ . It is important to note that an increase in  $T$  and a decrease in  $\sigma$  of interconnects exacerbate the EM. Indeed, the reduction in  $\sigma$  requires a larger electric field to deliver the same current density, which in turn causes the temperature rise in the interconnects; i.e. there is a negative feedback between the self-heating effect and EM. From this point of view, it is clear that Cu-CNT composite interconnects can be stronger against EM than Cu interconnects thanks to its lower self-heating temperature and the significantly larger thermal conductivity of CNT.

#### C. Variability simulations of Cu-CNT composite interconnect

To gain a deeper insight into self-heating effects in Cu-CNT composite interconnects, we performed variability simulations of  $\sigma$  and  $\Delta\tilde{T}$  in terms of the electrical and thermal contact resistances,  $L_{CNT}$ , and the Cu ratio. The results are shown in Fig. 9. We found that there is a very weak dependence on the electrical and thermal contact resistances. Since the electrical and thermal conductivities at both end and side contacts – calculated by DFT and ReaxFF, respectively – are much smaller than the electrical and thermal conductivities of Cu and CNT, their influence appears to be negligible as shown in Fig. 9a).

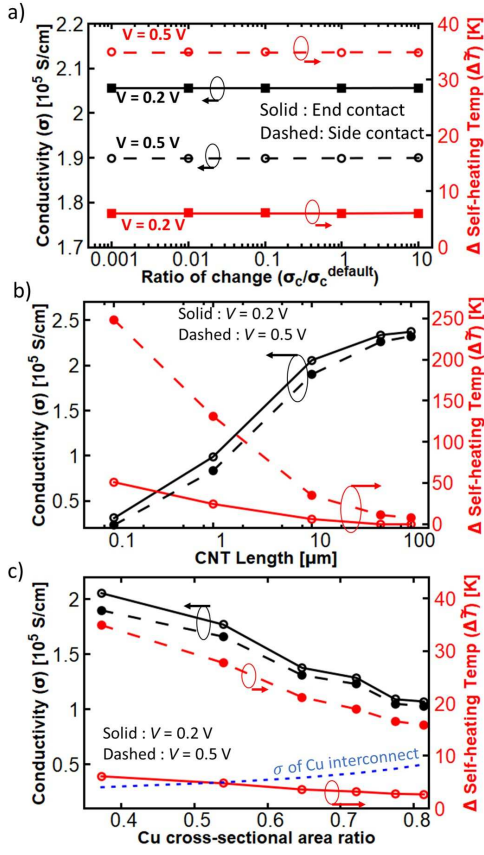


Fig. 9. Variability simulations of Cu-CNT composite interconnects when applying the different voltage (0.2 and 0.5 V). We investigated dependence on a) the electrical and thermal conductivities at the end and side contacts, b) the CNT length, and c) the Cu ratio, respectively. All non-displayed structure parameters are default values;  $L_{CNT} = 10 \mu\text{m}$ ,  $L_c^{Cu} = 20 \text{nm}$ ,  $D_{CNT} = 1 \text{nm}$ ,  $H_{Cu} = 3 \text{nm}$ , and  $W_{Cu} = 3 \text{nm}$ .

The dependence on  $L_{CNT}$  is shown in Fig. 9b). As  $L_{CNT}$  decreases,  $\sigma$  decreases and  $\Delta \tilde{T}$  increases. It is well-known that  $\sigma_{CNT}$  decreases as  $L_{CNT}$  decreases due to its large  $\lambda$  [19], [20]. Moreover, as  $L_{CNT}$  decreases, the electric field increases. This is the cause of the increase in the self-heating temperature. We also found that Cu-CNT composite interconnects outperform Cu interconnects for short lengths. Indeed,  $\Delta \tilde{T}$  of  $0.1 \mu\text{m}$  length Cu-CNT composite interconnect ( $\Delta \tilde{T} = 248 \text{K}$ , see Fig. 9b)) is smaller than that of  $10 \mu\text{m}$  length Cu interconnects ( $\Delta \tilde{T} = 364 \text{K}$ , see Fig. 8)), when  $V = 0.5 \text{V}$ .

Fig. 9c) shows the self-heating simulation results of  $\sigma$  and  $\Delta \tilde{T}$  when  $D_{CNT}$  is fixed and  $H_{Cu}$  and  $W_{Cu}$  are varied from 3 to 5.5 nm. As the Cu ratio increases, both  $\sigma$  and  $\Delta \tilde{T}$  decrease. For  $\sigma$ , it is consistent with the general expectations because as Cu becomes increasingly dominant,  $\sigma$  of a Cu-CNT composite interconnect becomes closer to that of a Cu interconnect. However, the calculated  $\Delta \tilde{T}$  shows an unexpected behavior. It is highlighted that although a Cu interconnect is suffering more from the self-heating effects as shown in Fig. 8,  $\Delta \tilde{T}$  decreases when the Cu ratio becomes large. This behaviour is explained by the build-up of an electric field difference at the interface between the Cu contact and the Cu-CNT composite (see Fig. 4). Fig. 10 depicts the electric field and temperature

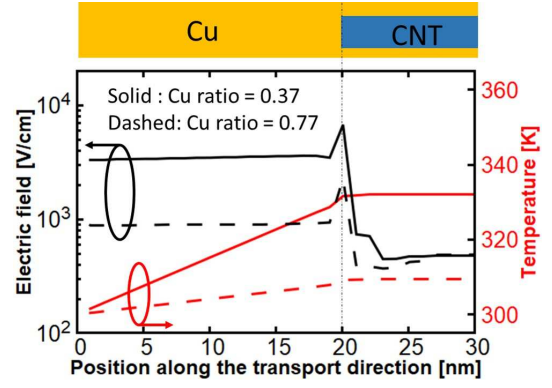


Fig. 10. Calculated electric field and temperature as a function of position along the transport direction near the left contact when Cu ratios are 0.37 and 0.77, respectively at  $V = 0.5 \text{V}$ .  $L_{CNT} = 10 \mu\text{m}$ ,  $L_c^{Cu} = 20 \text{nm}$ , and  $D_{CNT} = 1 \text{nm}$ .

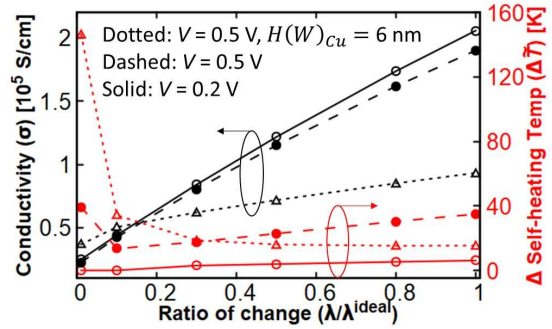


Fig. 11. Calculated conductivity and self-heating temperature difference  $\Delta \tilde{T}$  as the mean free path of CNTs is degraded. All non-displayed structure parameters are default values;  $L_{CNT} = 10 \mu\text{m}$ ,  $L_c^{Cu} = 20 \text{nm}$ ,  $D_{CNT} = 1 \text{nm}$ ,  $H_{Cu} = 3 \text{nm}$ , and  $W_{Cu} = 3 \text{nm}$ .

along the transport direction. Two important conclusions can be drawn from this figure. Firstly, the temperature rise in both interconnects occurs in the Cu contact part. Secondly, when the Cu ratio is higher, the electric field at the Cu contact part is smaller because  $\sigma$  of Cu-CNT composite is similar to that of Cu. Therefore, to decrease the self-heating temperature in interconnects, it is very important to ensure a small variation of the electrical conductivity along the transport direction. This will prevent the build-up of a large electric field at the regions where  $\sigma$  is small.

Only ideal CNTs were considered so far. Since measured conductivities in CNTs are usually lower than the ideal values, we also investigated the self-heating effect when the mean free path  $\lambda$  of CNTs is degraded. The dependence of  $\sigma$  and  $\Delta \tilde{T}$  on degraded  $\lambda$  is shown in Fig. 11, where  $\lambda^{\text{ideal}}$  is given by Eq. 9. As the ratio of change of  $\lambda$  decreases, both  $\sigma$  and  $\Delta \tilde{T}$  decrease. Despite the poor CNT characteristics, the reason behind the decrease of  $\Delta \tilde{T}$  is related to the smaller difference of conductivity  $\sigma$  between the Cu contact and the degraded Cu-CNT composite. However,  $\Delta \tilde{T}$  increases again at a certain point because the CNT conductivity becomes too different (smaller) from the Cu conductivity. We also found that  $\Delta \tilde{T}$  with  $H_{Cu} = W_{Cu} = 6 \text{nm}$  starts to increase at a larger  $\lambda/\lambda^{\text{ideal}}$  ratio because of the small contribution from CNTs.

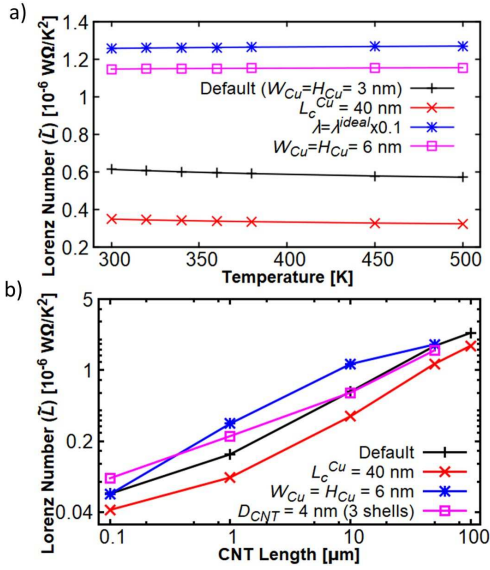


Fig. 12. Simulation study of dependence of the Lorenz number on a) the temperature and b) CNT length. All non-displayed structure parameters are default values;  $L_{CNT} = 10 \mu\text{m}$ ,  $L_c^{Cu} = 20 \text{ nm}$ ,  $D_{CNT} = 1 \text{ nm}$ ,  $H_{Cu} = 3 \text{ nm}$ , and  $W_{Cu} = 3 \text{ nm}$ .

#### IV. DESIGN GUIDELINES FOR CU-CNT COMPOSITE INTERCONNECTS

All simulation results suggest that a Cu-CNT composite interconnect without the barrier metal is highly resistant to EM thanks to the reduced self-heating, and not due to increased activation energy. We propose the Lorenz number  $\tilde{L}$  as a new parameter to assess self-heating and electromigration. The dependence of  $\tilde{L}$  on various parameters is shown in Fig. 12. The linear relationship between  $\tilde{L}$  and  $T$  in Cu-CNT composite interconnects is consistent with experimental results [18]. The correlation of Figs. 12a) and b) with all previous simulation results shows that  $\Delta\tilde{T}$  is always higher for the interconnect with the lower  $\tilde{L}$ . For example, as  $L_{CNT}$  decreases,  $\Delta\tilde{T}$  increases (see Fig. 9b)).  $\tilde{L}$  follows an inverse trend as it decreases when decreasing the CNT length (Fig. 12b)). Furthermore,  $\Delta\tilde{T}$  decreases for  $L_{CNT} = 10 \mu\text{m}$  when the Cu ratio or  $W(H)_{Cu}$  increases (see Fig. 9c). We observe that the Lorenz number also increases in this case (Fig. 12a)).

To see clearly the relationship between  $\sigma$ ,  $\Delta\tilde{T}$ , and  $\tilde{L}$ , we investigated the self-heating effect with and without the GBS and the SRS effects while artificially varying  $\tilde{L}$  in the Cu interconnect, as shown in Fig. 13. Although  $\sigma$  is degraded significantly due to scatterings,  $\Delta\tilde{T}$  does not depend on  $\sigma$ . We note that  $\Delta\tilde{T}$  is inversely proportional to  $\tilde{L}$ . We thus rewrite Eq. 1 that will serve as a model to design Cu-CNT composite interconnects:

$$TTF = A^* \{J(\sigma)\}^{-n} e^{\frac{E_a}{k_B(T_{sink} + C/L(\sigma, \kappa))}}, \quad (13)$$

where  $C$  is a constant related to the structural information including the homogeneous conductivity.

Taking all the above results together, we can provide three important rules applicable to the design of Cu-CNT composite interconnects that show good self-heating properties and have

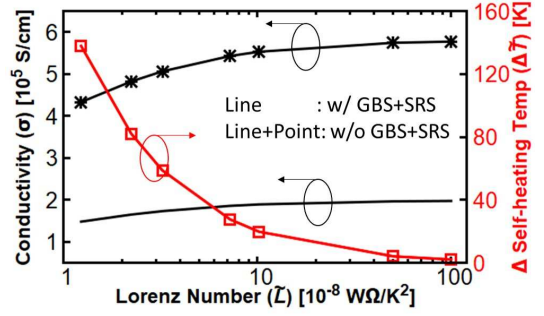


Fig. 13. Calculated conductivity and self-heating temperature difference  $\Delta\tilde{T}$  with artificially changed Lorenz number in Cu interconnects with and without grain boundary and surface roughness scattering effects. The interconnect length, width, and height are  $10 \mu\text{m}$ ,  $50 \text{ nm}$ , and  $90 \text{ nm}$ , respectively. The applied voltage is  $0.2 \text{ V}$ .

high resistance to EM. Firstly, interconnects should have a large electrical conductivity. To achieve this, CNTs without defects are desirable in long interconnects. Secondly, interconnects must also possess a high Lorenz number to evacuate the heat more efficiently. Thus, the Cu ratio must be carefully chosen, especially when CNTs are defective. Finally, it is very important that Cu-CNT composite interconnects have a homogenous conductivity to avoid large electric fields in the regions where  $\sigma$  is discontinuous. Therefore,  $L_c^{Cu}$  (as illustrated in Fig. 4) should be minimized.

#### V. CONCLUSIONS

In this work, by comparing experimental data with multi-scale electro-thermal simulations results, we have provided an explanation why Cu-CNT composite interconnects outperform Cu interconnects in terms of electromigration. Our results show that the electromigration is governed by the self-heating related temperature raise, rather than the activation energy of Cu-CNT composites. Also, we have conducted a systematic study which shows that a good Cu-CNT composite interconnect should be designed by co-optimizing the electrical conductivity, the Lorenz number, and the homogeneous conductivity. Based on the results obtained from our study, some important modeling and design guidelines have been suggested as follows: large electrical conductivity, large Lorenz number, and homogeneous electrical conductivity are desirable in the design of efficient highly reliable Cu-CNT composite interconnects for aggressively scaled chips. These guidelines could be generalized to other interconnect materials.

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