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A simple optimization approach for the insulation thickness distribution in household refrigerators

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Highlights

- New analytical method to find the optimal insulation thickness in refrigerators.
- The method is suitable for both single and dual compartment configurations.
- The method can provide the optimal configuration for a given energy efficiency index.
- Vacuum insulation panels can be easily included in the optimization strategy.
- Method extended considering the ventilation channel clearance behind the unit.

A simple optimization approach for the insulation thickness distribution in household refrigerators

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Abstract

Determination of the optimal insulation thickness is of great relevance in many thermal engineering applications. In this paper, a simple optimization strategy based on the Lagrange multipliers is presented. The optimal set of thicknesses is analytically found for different constraints and objective functions of interest for the refrigeration industry. Namely, the minimization of heat losses in a single compartment with fixed internal and external volumes and the optimal configuration for a prescribed energy efficiency index. Then, these two basic problems are extended for configurations with two compartments, *e.g.* domestic refrigerators-freezers, and for configurations with vacuum insulation panels. Optimization problems for realistic configurations show the great potential of the proposed methodology for industrial refrigeration applications.

Keywords: Insulation, Optimization, Lagrange multipliers, Domestic refrigerator, Energy labeling

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Nomenclature

A	area $[m^2]$
$\mathbf{A}$	set of areas $[m^2]$
AV	corrected volume $[L]$
BI	built-in volume correction factor $[-]$
c	condenser clearance $[mm]$
CC	climate class volume correction factor $[-]$
CH	volume correction factor $[kWh\ year^{-1}]$
COP	Coefficient of Performance
d	insulation thickness $[mm]$
$\mathbf{d}$	set of insulation thicknesses $[mm]$
E_i	energy efficiency index $[-]$
FF	frost-free volume correction factor $[-]$
k	thermal conductivity $[Wm^{-1}K^{-1}]$
$\mathbf{k}$	set of thermal conductivities $[Wm^{-1}K^{-1}]$
M	volume correction factor $[kWh\ year^{-1}L^{-1}]$
N	volume correction factor $[kWh\ year^{-1}]$
$\dot{Q}$	heat losses $[W]$
S	total number of surfaces $[-]$
T	temperature $[K]$
V	volume $[L]$
VIP	vacuum-insulation panel

Greek symbols

α	fraction of internal volume corresponding to the refrigerator $[-]$
ΔT	thermal gradient $[K]$
$\mathbf{\Delta T}$	set of thermal gradients $[K]$
λ	Lagrange multiplier $[Wm^3]$

Subscripts

ext	external
i	number of surface
int	internal
opt	optimal

Supercripts

$\widetilde{(\cdot)}$	properties associated with the vacuum-insulation panels
$\overline{(\cdot)}$	equivalent properties associated with the vacuum-insulation panels

1. Introduction

The worldwide challenge to address global warming threat is affecting all human kind activities, in particular those with a relative importance in terms of energy consumption. Among them we can identify buildings as a key consumer in terms of energy [13], and within them the vapor compression systems in general [2] and the domestic refrigerator [12] in particular as dominant electricity consumers.

As in other appliances, the competitive race to obtain the most energy efficient refrigerator has been driven by public awareness on the environmental issues, but at the same time articulated by the use of Energy Labeling measures. As reported in Ref. [7], the energy labeling measures are steadily increasing around the world, covering new appliances, but keeping the domestic refrigerator as the widest covered device. As a consequence of this social/market framework, the domestic refrigerator manufacturers need to adapt to the situation, developing new products with higher efficiencies, while keeping a portfolio of products with different cost-efficiency level.

A full re-design of a refrigerator should consider all its components (compressor, evaporator, condenser, expansion device, insulation), as having their relative importance in reaching the desired efficiency [11, 1], or even analyze possible alternatives in its layout [16, 17]. However, the level of insulation is a key aspect in terms of energy consumption in a refrigerator [1, 16, 15], as being a device with an all year long high temperature difference between external and internal environments. From the market and manufacturing point of view, the change in thickness insulation has also an additional relevance, as affecting all the internal components design (shelves, drawers, etc.).

The concern about refrigerator energy consumption and its relation with the insulation panels thickness distribution has been attracting the interest of researchers within the refrigeration field. In his early work, Christensen [3] analyzed the effective impact of the thermal insulation on the heat gains for a single compartment unit, concluding that about 20% energy savings can be obtained increasing the thickness from 55 to 100 mm for a freezer, or 30% savings increasing from 30 to 110 mm for a cooler. Dmitriyev [5] complemented the work of Christensen by studying the impact of the insulation thickness on the overall costs (running + manufacturing) of a refrigerator-freezer, suggesting an optimum thickness around 100 to 120 mm, which also contributes to longer compressor life extension by its lower operation time. After these initial studies, other authors have tackled the panel thickness optimization problem. Solëymez and Ünsal [15] applied the P1-P2 method of Duffie-Beckman [6] to provide a thermoeconomic optimization of the thickness in a single compartment refrigerator. Recently, Sevindir *et al.* [14] have presented an optimization procedure for a single compartment based on equalizing the heat transfer derivatives with panel thickness, while also introducing a cost based optimization study that includes the heating/cooling costs on the neighbor ambients. Yoon *et al.* [16] focused the optimization study for the dual compartment refrigerator case, fixing a single thickness for each compartment, and a thickness for the common mullion in a side-by-side configuration. They obtained the minimum cost thickness distribution while keeping constant the internal volume. Regarding model-based optimization, Mitishita *et al.* [11] presented the use of a genetic algorithm optimization procedure engined by a thermodynamic model of a household refrigerator with dual compartment, finding the lowest consumption solution for a given cost, in this case using a single panel thickness for each compartment, but including the design parameters of the rest of the system. These studies have been also completed by the analysis of the heat leakage through the gasket region [8, 9], confirming its relative low share of the total heat gains (13% to 17%), thus the dominant role of the insulation.

In this context, a novel analytical approach to determine the optimal wall thickness insulation distribution (with an individual wall approach) for household refrigerators is presented in this paper. It is based on the Lagrange multipliers method and it is suitable for both single compartment and dual compartment layouts, while also considering the introduction of vacuum insulation panels (VIP). Considering the previous context regarding the environmental labeling, special attention is given to link the optimization to the energy efficiency index that categorize the refrigerator in terms of energy consumption (function of cooling load and volume). This model provides to the manufacturer a tool to devise the limits of a given refrigeration system (set of compressor, evaporator, compressor) by changing the insulation, and also a method to generate a portfolio of optimum insulation solutions for each particular labeling level, keeping the refrigeration system with minimal changes.

The rest of the paper is organized as follows. Firstly, the mathematical model of a refrigerator is presented in Section 2 together with the expressions to compute the heat losses and the internal volume (the external volume is considered fixed). Then, on the basis of this mathematical model, the optimal set of thicknesses is analytically found for different constraints and objective functions of interest. In Section 4, the newly proposed optimization approach is applied to a domestic refrigerator-freezer for a given coefficient of performance (COP) either using only conventional insulation materials or combining them with vacuum-insulation panels. The final test-case includes the effects of the condenser clearance to both the COP and the external refrigerator volume, keeping the space for the refrigerator fixed. Finally, relevant results are summarized and conclusions are given.

2. Mathematical model

In this work we aim to find the optimal set of thicknesses of a refrigerator for different constraints and objective functions of interest for the industry. To study this, we consider that a refrigerator basically consists on a set of S surfaces with their associated areas, $A_i,\ i = 1, \ldots, S$. Each of these surfaces is characterized by its thermal conductivity, k_i, thickness, d_i, and temperature gradient, ΔT_i. The external volume, V_{ext}, is considered fixed. Then, the heat losses are given by

$$\dot{Q} = \sum_{i=1}^{S} \frac{k_i A_i \Delta T_i}{d_i}, \tag{1}$$

and the internal volume is given by

$$V_{int} = V_{ext} - \sum_{i=1}^{S} A_i d_i. \tag{2}$$

In the forthcoming optimization strategy the set of areas, $\mathbf{A} = \{A_i\}$, are considered constant. However, in general, they depend on the set of thicknesses, $\mathbf{d} = \{d_i\}$, *i.e.* $\mathbf{A}(\mathbf{d})$, being $\partial A_i/\partial d_j \sim \sqrt{A_i}$. Nevertheless, for practical problems with small variations of $\mathbf{d}$, *i.e.* $\Delta d_j \ll \sqrt{A_i}$, relative variations of $\mathbf{A}$ are expected to be very small:

$$\frac{\Delta A_i}{A_i} \sim \frac{\Delta d_j}{\sqrt{A_i}}. \tag{3}$$

This partially justifies to keep $\mathbf{A}$ constant. In any case, it is straightforward to update the set of areas, $\mathbf{A}(\mathbf{d}) \longrightarrow \mathbf{A}(\mathbf{d}+\Delta\mathbf{d})$ and apply the optimization again.

3. Optimization strategy

The optimization strategy is presented in this section. Using the mathematical model presented in the previous section, the optimal set of thicknesses, $\mathbf{d}$, is analytically found for different constraints and objective functions of interest for the refrigeration industry. In doing so, two basic problems are firstly analyzed for a single compartment: namely, (i) minimizing the heat losses given the internal and external volumes of the compartment, and (ii) finding the optimal configuration for a prescribed energy efficiency index. Then, these approaches are extended for configurations with two compartments, *e.g.* domestic refrigerator-freezers, and for configurations with vacuum insulation panels.

The forthcoming optimization strategy is only limited by the assumptions of the mathematical model given in Eqs.(1) and (2). Namely, there is a finite number of walls having different thermal resistances which are approximated by the conductive heat transfer resistance, d_i/k_i, which are in contact with different environments at different temperatures. Using *"the conductive heat transfer resistance assumption, which typically accounted for 86% or more of the total thermal resistance"* [16] is therefore a very common approach [12, 16]. In any case, the limitation of this assumption can be easily overcame by using an equivalent thermal conductivity that takes into account the total thermal resistance. Apart from this, the optimization strategy presented here relies on the heat transfer areas, A_i, the approximations on the internal volume, V_{int}, and an accurate estimation of the COP.

3.1. Optimization of a single compartment with fixed internal and external volumes

Given a set of areas, $\mathbf{A}$, temperature gradients, $\mathbf{\Delta T}$, thermal conductivity, $\mathbf{k}$, an internal volume, V_{int}, and an external volume, V_{ext}, $V_{int} < V_{ext}$, we aim to find the optimal set of thicknesses, $\mathbf{d} = \{d_1, d_2, ..., d_S\}$, for which the heat losses, $\dot{Q}$, are minimal. To solve this, we use the method of Lagrange multipliers [10]. In this case, the Lagrange function is defined as follows

$$L(d_1, d_2, \cdots, d_S, \lambda) = \dot{Q} + \lambda(V_{ext} - V_{int}), \tag{4}$$

where the λ is the Lagrange multiplier. Then, to find the optimal solution we need to solve the following linear system of equations

$$\frac{\partial L}{\partial d_i} = -\frac{k_i A_i \Delta T_i}{d_i^2} + \lambda A_i = 0 \qquad \forall i = 1, \cdots, S \tag{5}$$

$$\frac{\partial L}{\partial \lambda} = V_{ext} - V_{int} = \sum_{i=1}^{S} A_i d_i. \tag{6}$$

From Eq.(5) we can express d_i in terms of λ

$$d_i = \sqrt{k_i \Delta T_i / \lambda}. \tag{7}$$

Here, no summation over i is implied. Then, plugging Eq.(7) into Eq.(6) leads to

$$\sqrt{\lambda} = \frac{\sum_{i=1}^{S} A_i \sqrt{k_i \Delta T_i}}{V_{ext} - V_{int}}. \tag{8}$$

Finally, substituting this expression into Eq.(7) we get an analytical expression for d_i

$$d_{i,opt} = \frac{\sqrt{k_i \Delta T_i}(V_{ext} - V_{int})}{\sum_{j=1}^{S} A_j \sqrt{k_j \Delta T_j}}, \tag{9}$$

where the heat losses, $\dot{Q}$, are given by

$$\dot{Q}_{opt} = \frac{\left(\sum_{i=1}^{S} A_i \sqrt{k_i \Delta T_i}\right)^2}{V_{ext} - V_{int}}. \tag{10}$$

In summary, given the characteristics of the set of surfaces together with the internal, V_{int}, and external, V_{ext}, volumes, the set of thicknesses given in Eq.(9) provides the minimal heat losses, $\dot{Q}_{opt}$, given in Eq.(10).

3.2. Finding the optimal configuration for a given energy efficiency index

Notice that the expression for $\dot{Q}_{opt}$ given in Eq.(10) has the following form

$$\dot{Q}_{opt} = C_1^2 / (V_{ext} - V_{int}), \tag{11}$$

where $C_1 = \sum_{i=1}^{S} A_i \sqrt{k_i \Delta T_i}$. Following the European Union (EU) labeling policy [4], the energy efficiency index, E_i, is defined as follows

$$E_i = E_a / E_{st}, \tag{12}$$

where $E_{st} = M \cdot (AV) + N + CH$ and E_a is related with the heat losses, $\dot{Q}$, via the coefficient of performance

$$E_a = \dot{Q}/COP. \tag{13}$$

On the other hand, the corrected volume, AV, is proportional to the internal volume, *i.e.* $AV = K_1 V_{int}$. At this point, the question is whether is possible to find the optimal refrigerator for a given E_i. To do so, we need to solve the following equation

$$E_i = \frac{C_1^2/COP}{(V_{ext} - V_{int})(MK_1 V_{int} + N + CH)}, \tag{14}$$

where now the unknown is V_{int}. This equation is obtained by plugging Eqs.(11) and (13) into Eq.(12). This results into a quadratic equation for V_{int}

$$AV_{int}^2 + BV_{int} + C = 0, \tag{15}$$

where $A = MK_1$, $B = N + CH - MK_1 V_{ext}$ and $C = C_1^2/(E_i \cdot COP) - (N + CH)V_{ext}$. Hence, the optimal solution is given by

$$V_{int} = \frac{-B + \sqrt{B^2 - 4AC}}{2A}. \tag{16}$$

Then, the set of optimal thicknesses follows from Eq.(9). It is important to notice that hereafter we consider that proper unit conversion is applied when necessary accordingly to the EU labeling policy [4].

3.3. Extension to configurations with two compartments

The optimization strategy presented above is extended to problems with two compartments, typically domestic refrigerators-freezers. In this case, the problem can be more cumbersome because the corrected volume, AV, is given by $AV = K_2 \alpha V_{int} + K_3(1 - \alpha)V_{int}$, where $0 < \alpha < 1$ is the fraction of volume corresponding to the refrigerator [4]. Notice that the value of α can depend on the set of thicknesses, *i.e.* $\alpha(\mathbf{d})$. However, to make the problem more tractable, we propose to consider a fixed value of α. Doing so, the above-described solution procedure remains exactly the same, except that K_1 is now given by $K_1 =$

Algorithm 1 Determination of the optimal configuration for a given internal volume, V_{int}.

Input: $\mathbf{\Delta T}$, $\mathbf{k}$, $\mathbf{A}$, α, V_{ext}, V_{int}, $*\{\tilde{\mathbf{d}}, \tilde{\mathbf{k}}\}$. *Output:* $\mathbf{d}_{opt}$, $\dot{Q}_{opt}$, and its corresponding E_i.

Note: steps/data marked with * are only necessary for problems with VIPs.

1. * The set of equivalent thicknesses, $\overline{\mathbf{d}}$, is computed with Eq.(17).
2. * Compute the equivalent volume $\overline{V}$ associated to the VIPs with Eq.(20).
3. The set of optimal thicknesses, $\mathbf{d}_{opt}$, is computed with Eq.(21).
4. The heat losses, $\dot{Q}_{opt}$ are given by Eq.(22).
5. Compute the energy efficiency index, E_i, with Eq.(14).
6. Recompute the set of areas, $\mathbf{A}(\mathbf{d})$, and the fraction of volume corresponding to the refrigerator, $\alpha(\mathbf{d})$, with the new set of thicknesses, $\mathbf{d}_{opt}$.
7. Go back to step 3. until solution converges.

$\alpha K_2 + (1-\alpha)K_3$. Then, once the new set of thicknesses, $\mathbf{d}$, is computed, the value of α must be necessarily recomputed. Likewise the set of areas, $\mathbf{A}$, very small variations are also expected for α; therefore, the overall algorithm should converge in few iterations.

3.4. Extension to configurations with vacuum-insulation panels

Highly efficient refrigerators cannot only rely on conventional insulation materials such as polyurethane foam. The set of thicknesses would reduce the internal space, V_{int}, in a significant manner leading to impractical refrigerator designs. Alternatively, vacuum-insulation panels (VIP) can be embedded to the sidewalls and doors of refrigerators. They offer outstanding insulation properties compared with conventional materials offering the required energy savings with reasonable wall thicknesses. In this context, the above-explained optimization strategy is adapted to consider VIP panels. The main difficulty arises from the

Algorithm 2 Determination of the optimal configuration for a prescribed energy efficiency index, E_i.

Input: $\mathbf{\Delta T}$, $\mathbf{k}$, $\mathbf{A}$, α, V_{ext}, E_i, $*\{\tilde{\mathbf{d}}, \tilde{\mathbf{k}}\}$. *Output:* $\mathbf{d}_{opt}$, $\dot{Q}_{opt}$, and its corresponding internal volume, V_{int}.

Note: steps/data marked with * are only necessary for problems with VIPs.

1. * The set of equivalent thicknesses, $\overline{\mathbf{d}}$, is computed with Eq.(17).
2. * Compute the equivalent volume $\overline{V}$ associated to the VIPs with Eq.(20).
3. Compute the internal volume, V_{int}, with Eq.(18).
4. The set of optimal thicknesses, $\mathbf{d}_{opt}$, is computed with Eq.(21).
5. The heat losses, $\dot{Q}_{opt}$ are given by Eq.(22).
6. Recompute the set of areas, $\mathbf{A}(\mathbf{d})$, and the fraction of volume corresponding to the refrigerator, $\alpha(\mathbf{d})$, with the new set of thicknesses, $\mathbf{d}_{opt}$.
7. Go back to step 3. until solution converges.

fact that the thickness of the VIP is given by the manufacturer; therefore, the only degree of freedom is the thickness of the conventional insulation material. To model this, we simply consider that an additional insulation material with thermal conductivity $\tilde{k}_i$ and thickness $\tilde{d}_i$ is added to the surface A_i. In this case, the heat losses and the internal volume are given by

$$\dot{Q} = \sum_{i=1}^{S} k_i A_i \Delta T_i / (\overline{d}_i + d_i) \quad \text{where} \quad \overline{d}_i = \tilde{d}_i k_i / \tilde{k}_i \tag{17}$$

$$V_{int} = V_{ext} - \sum_{i=1}^{S} A_i (\tilde{d}_i + d_i). \tag{18}$$

Then, applying the same reasonings than in Section 3.1 it yields

$$d_i = \sqrt{k_i \Delta T_i / \lambda} - \overline{d}_i, \tag{19}$$

instead of Eq.(7), and the expressions

$$\sqrt{\lambda} = \frac{\sum_{i=1}^{S} A_i \sqrt{k_i \Delta T_i}}{V_{ext} - V_{int} + \overline{V}} \quad \text{where} \quad \overline{V} = \sum_{i=1}^{S} A_i(\overline{d}_i - \tilde{d}_i), \tag{20}$$

instead of Eq.(9). Finally, plugging the previous expression into Eq.(19) the analytical expression for the optimal set of thicknesses follows

$$d_{i,opt} = \frac{\sqrt{k_i \Delta T_i}(V_{ext} - V_{int} + \overline{V})}{\sum_{j=1}^{S} A_j \sqrt{k_j \Delta T_j}} - \overline{d}_i, \tag{21}$$

and the heat losses are then given by

$$\dot{Q}_{opt} = \frac{\left(\sum_{i=1}^{S} A_i \sqrt{k_i \Delta T_i}\right)^2}{V_{ext} - V_{int} + \overline{V}}. \tag{22}$$

It must be noted that heat losses have the same form than in Eq.(10). Hence, the calculations presented in Sections 3.2 and 3.3 can be re-used by simply replacing V_{ext} by $\overline{V_{ext}} = V_{ext} + \overline{V}$. Then, the optimal solution for a given energy efficiency index, E_i, is given by Eq.(16)

$$V_{int} = \frac{-B + \sqrt{B^2 - 4AC}}{2A}, \tag{23}$$

where $A = MK_1$, $B = N + CH - MK_1\overline{V_{ext}}$ and $C = C_1^2/(E_i \cdot COP) - (N + CH)\overline{V_{ext}}$. Then, the optimal set of thicknesses follows from Eq.(21). Refrigerators with two compartments and VIPs are solved following the strategy presented in Section 3.3.

In summary, the proposed approach allows to find the optimal configuration for refrigerators with two compartments and with VIPs embedded to (some) walls. Simpler configurations such as refrigerators with one single compartment or/and without VIPs can be viewed as particular cases. The steps of the general algorithm are detailed for two problems of interests in Algorithms 1 and 2. Namely, the steps to determine the optimal configuration for a given internal volume, V_{int}, are given in Algorithm 1. In this case, the target is to compute the optimal set of thicknesses, $\mathbf{d}_{opt}$, and its corresponding $\dot{Q}_{opt}$, and energy efficiency index, E_i. The second problem of interest is outlined in Algorithm 2. In this case, the energy efficiency index, E_i, is prescribed and the target is to compute $\mathbf{d}_{opt}$, $\dot{Q}_{opt}$, and its corresponding internal volume, V_{int}.

			Initial	Optimal configurations			
			C0	**C1**	**C2**	**C3**	**C4**
Wall number (i)	ΔT_i [^oC]	A_i [m^2]	d_i [mm]	d_i [mm]	d_i [mm]	d_i [mm]	d_i [mm]
1	45	0.1669	83	46.2	78.5	75.3	113.5
2	45	0.0677	72	46.2	78.5	75.3	113.5
3	45	0.0677	72	46.2	78.5	75.3	113.5
4	50	0.1131	83	48.7	82.8	79.4	119.6
5	53	0.1113	51.4	50.1	85.2	81.7	123.1
6	45	0.2219	72	46.2	78.5	75.3	113.5
7	45	0.2219	72	46.2	78.5	75.3	113.5
8	53	0.2439	70	50.1	85.2	81.7	123.1
9	45	0.3685	80	46.2	78.5	75.3	113.5
10	20	0.5896	56	30.8	52.4	50.2	75.7
11	20	0.5896	56	30.8	52.4	50.2	75.7
12	28	0.6556	58	36.4	61.9	59.4	89.5
13	20	0.2636	72	30.8	52.4	50.2	75.7
14	20	0.6556	57	30.8	52.4	50.2	75.7
		$\dot{Q}[W]$	47.57	79.47	46.59	48.66	32.25
		$Vint$ [L]	**328.4**	442.7	**328.4**	340.2	340.2
		E_i	**30.62**	42	29.97	**30.62**	27.07

Table 1: From left to right: wall number, temperature gradient ΔT_i, area A_i, initial set of thicknesses and optimal set of thicknesses for different values of the energy efficiency index, E_i. A simplified schema showing the location of the most relevant walls is displayed in Figure 1.

4. Results and discussion

The optimization of an existing domestic refrigerator-freezer has been chosen to test the proposed approach. Namely, it consists of 14 walls, all of them with thermal conductivity $k_i = 0.023\ Wm^{-1}K^{-1}$ (polyurethane foam). A simplified schema showing the location of the most relevant walls is displayed in Figure 1.

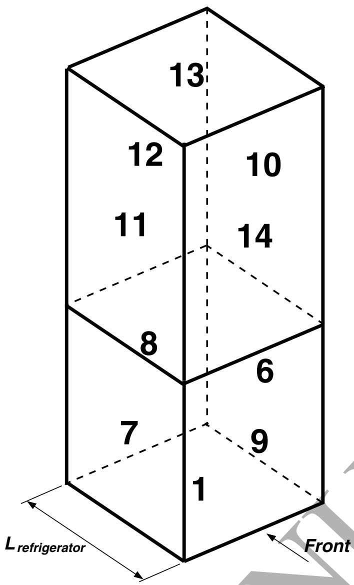


Figure 1: Simplified schema of the domestic refrigerator-freezer used to test the proposed approach. Wall numbers are placed at the center of their corresponding wall. The complete list of walls with their properties is given in Table 1.

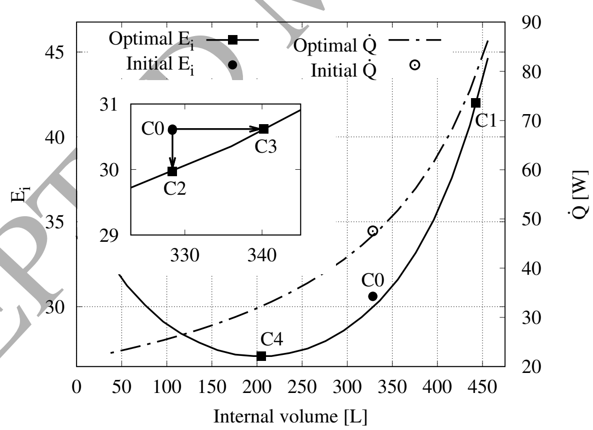
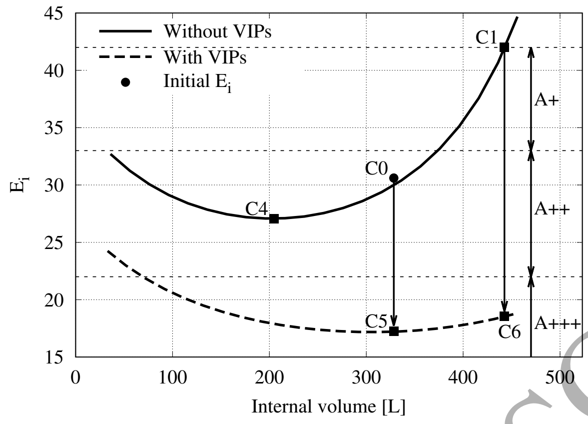


Figure 2: Energy efficiency index, E_i, and heat losses, $\dot{Q}$, for the optimal configuration for a given internal volume. Details about the initial configuration, **C0**, are given in Table 1.

The temperature gradients, ΔT_i, are given in Table 1 together with the set of areas, A_i, and thicknesses, d_i, of the initial configuration, **C0**. This data



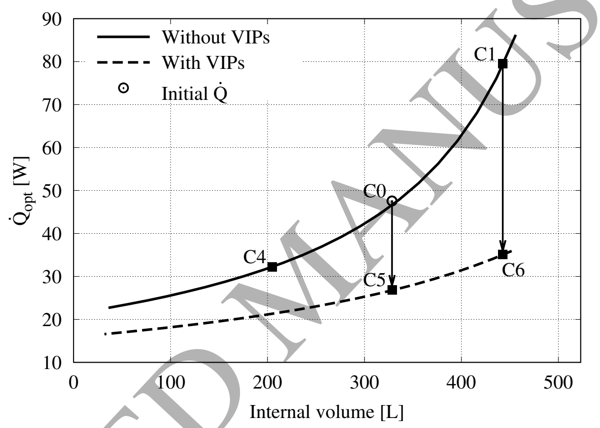


Figure 3: Results for the optimal configuration with and without vacuum-insulation panels (VIP). Top: energy efficiency index, E_i. Bottom: heat losses, $\dot{Q}$. Details about the initial configuration, **C0**, are given in Table 1.

is enough to apply the proposed optimization approach. The detailed schema or the names of the walls are not given to preserve confidentiality. The external volume, the fraction of volume corresponding to the refrigerator and the coefficient of performance are $V_{ext} = 0.606\ m^3$, $\alpha = 0.7155$ and $COP = 1.7$, respectively. The temperatures of the refrigerator and freezer compartments are 5^oC and -20^oC, respectively. Following the EU labeling policy [4] the rest of parameters are: $N = 303\ kWh\ year^{-1}$, $M = 0.707\ kWh\ year^{-1}$, $FF = 1.2$, $CH = 0$ and $BI = 1$.

The optimization strategy described in the previous section has been applied to this particular case for a wide range of internal volumes, V_{int}. Results for the energy efficiency index, E_i, and heat losses, $\dot{Q}$, are displayed in Figure 2. Among all these configurations, detailed results for four optimal configurations of interest are shown in Table 1: namely, the configuration **C1** corresponding with an energy efficiency index $E_i = 42$ (the threshold for A+ category), the optimal configuration **C2** with the same internal volume ($V = 328.4\ L$) than the initial configuration **C0**, the optimal configuration **C3** with the same energy efficiency index ($E_i = 30.62$) than the initial configuration **C0** and the optimal configuration **C4** with the minimal energy efficiency index ($E_i = 27.07$). It is observed that keeping the same internal volume (**C0** $\rightarrow$ **C2**), the energy efficiency index improves from $E_i = 30.62$ to $E_i = 29.97$, whereas keeping the same energy efficiency index (**C0** $\rightarrow$ **C3**) the internal volume increases almost $12\ L$, *i.e.* from $328.4\ L$ to $340.2\ L$. Although it is not a case of practical interest, it is interesting to notice that there is a minimal (configuration **C4**) for the energy efficiency index, E_i (corresponding to an internal volume of $V = 204.8\ L$) regardless to the fact that heat losses, $\dot{Q}$, can always be reduced by increasing the insulation thickness (see Figure 2) up to the point to reach a degenerate solution.

The energy efficiency index, E_i, and heat losses, $\dot{Q}$, corresponding to the optimal configuration are shown again in Figure 3 indicating the boundaries between different categories: *i.e.* A+ ($33 \leq E_i < 42$), A++ ($22 \leq E_i < 33$) and A+++ ($E_i < 22$). Although there is a range of A++ configurations (*e.g.* configurations **C2**, **C3** and **C4**), there is an important range of practical configurations that fall within the range of A+ category (*e.g.* **C1** configuration). This is an intrinsic limitation of conventional insulation materials. As explained in Section 3.4, VIPs are necessary to built highly efficient refrigerators (A++ and A+++), however, they impose an additional restriction since the thickness of the VIP is given by the manufacturer. Here, we consider a set of VIPs of thickness $\tilde{d}_i = 20\ mm$ and thermal conductivity $\tilde{k}_i = 0.005\ Wm^{-1}K^{-1}$ embedded

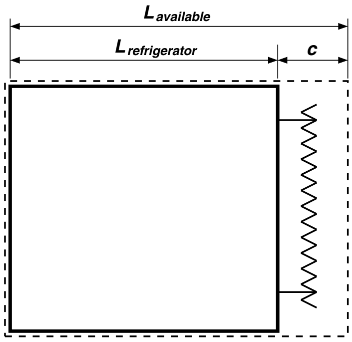


Figure 4: Schema showing the geometrical effect of the condenser clearance, c.

to 5 walls: 9, 10, 11, 12 and 14 (see Table 1). Results obtained using the optimization strategy for problems with VIPs (see Section 3.4) are displayed in Figure 3. Compared with the optimal solutions without VIP panels, both energy efficiency index, E_i, and heat losses, $\dot{Q}$, improve in a significant manner. Actually, in this case, practical configurations fall within the range of A+++ category. In particular, keeping the same internal volume than the initial configuration (**C0** $\rightarrow$ **C5**), the energy efficiency index improves from $E_i = 30.62$ to $E_i = 17.24$. Even more interesting, keeping the same internal volume than the optimal configuration without VIP panels in the threshold for A+ category (**C1** $\rightarrow$ **C6**), the energy efficiency index improves from $E_i = 42$ to $E_i = 18.54$.

Finally, we consider the same optimization problem but including the effects of the condenser clearance, c. This parameter has two opposite effects: the external volume, V_{ext}, decreases with c (see Figure 4) whereas the COP tends to increase with c. In this regard, the following expression has been used to model the dependency of the COP respect to c $[mm]$:

$$COP(c) = \frac{1}{A^{(B-c)} + C} \qquad \text{where} \qquad A = 1.04221, B = -32.0016, C = 0.54681 \tag{24}$$

This corresponds to a least-square regression of a set of energy consumption experiments (see Figure 5) hold by the industrial partner in its experimental facilities, following their standard procedures. Results for the optimal config-

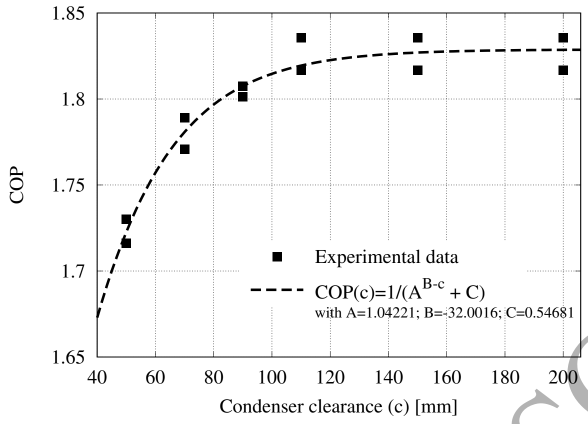
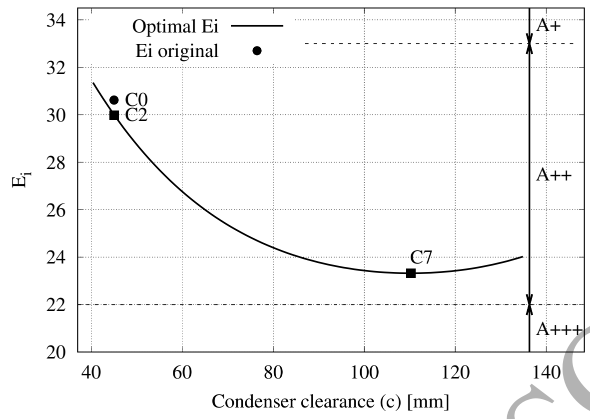


Figure 5: Quadratic least square regression of the COP respect to the condenser clearance, c, given in Eq.(24). See schema displayed in Figure 4.

uration respect to the condenser clearance, c, are displayed in Figure 6. As mentioned above, the initial configuration **C0** has a $COP = 1.7$ which corresponds with a condenser clearance of $c \approx 45\ mm$ (see Figure 5). Moreover, as seen before, the optimization approach outlined in Algorithm 1 has allowed to improve the energy efficiency index from $E_i = 30.62$ to $E_i = 29.97$ keeping the same internal volume (**C0** $\rightarrow$ **C2**). However, it is possible to improve it further by increasing the condenser clearance (see Figure 6, top) and, therefore, clearly falling within the range of A++ category (**C0** and **C2** are close to the upper limit). Furthermore, the energy efficiency index reaches a minimum of $E_i = 23.32$ for $c = 110.3\ mm$ (configuration **C7**). On the other hand Figure 6 (bottom) shows results obtained keeping the same energy efficiency index, E_i. In this case, as seen before, the Algorithm 2 has allowed to increase the internal volume from $328.4\ L$ to $340.2\ L$ (**C0** $\rightarrow$ **C3**). Nevertheless, the condenser clearance can have a more significant effect reaching a maximum internal volume of $V_{int} = 402.1\ L$ (configuration **C8**) for $c = 92mm$.



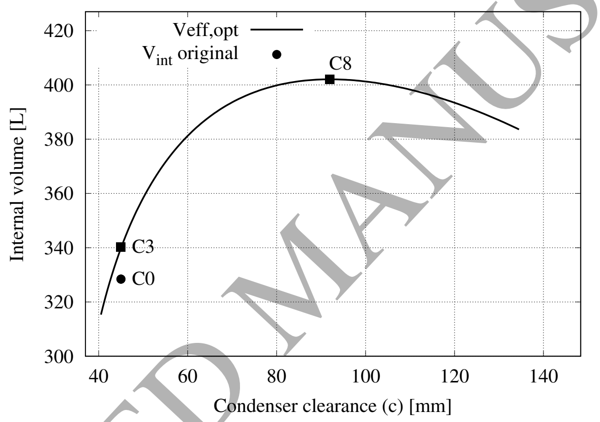


Figure 6: Results for the optimal configuration respect to the condenser clearance, c. Top: energy efficiency index, E_i, keeping the internal volume equal to the initial configuration **C0**, *i.e.* $V_{int} = 328.4\ L$. Bottom: internal volume, V_{int}, keeping the energy efficiency index equal to the original configuration **C0**, *i.e.* $E_i = 30.62$. The quadratic regression given in Eq.(24) and displayed in Figure 5 has been used to compute the COP.

5. Concluding remarks

The household refrigerator market is being dominated by a competitive race to obtain highly efficient devices to comply with the public awareness on environmental issues, currently articulated through Energy Labeling measures. Therefore, the manufacturers need to adapt their products to obtain the highest efficiencies, but also keeping a portfolio of products with different cost-efficiency/labeling levels.

Considering this context, this paper has presented an analytical approach based on the Lagrange multipliers method, to determine the optimal wall thickness insulation distribution for household refrigerators. Single compartment and dual compartment layouts are considered, while introducing as an option the integration of VIPs. The optimization strategy is only limited by the assumptions of the mathematical model described in Section 2.

The optimization results are focused on the identification of possible improvements from a baseline case, reducing the energy consumption or increasing the available volume. Special attention is given to link the optimization to the energy efficiency index that categorize the refrigerator in terms of energy labeling (function of cooling load and volume), then identifying for a given scenario the classes that can be achieved. For example, the case that introduces VIPs shows the strong impact of this technology on the energy class, upgrading an A++ solution to an A+++ device.

As a variant of the model, an optimization case that takes into account the geometry of the whole refrigerator space (refrigerator + ventilation channel) has also been analyzed, thus selecting not only the best panel thickness distribution but also the best clearance. The effect of the ventilation clearance on the system performance is introduced by means of fitting a set of energy consumption experiments, thus determining its impact on the COP.

Summarizing, this model provides to the manufacturer a tool to devise the limits of a given refrigeration system (set of compressor, evaporator, compressor) by changing the insulation, and also a method to generate a portfolio of optimum insulation solutions for each particular labeling level, keeping the refrigeration system with minimal changes.

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References

[1] Belman-Flores, J. M., Barroso-Maldonado, J. M., Rodríguez-Muñoz, A. P., and Camacho-Vázquez, G. (2015). Enhancements in domestic refrigeration, approaching a sustainable refrigerator - A review. *Renewable and Sustainable Energy Reviews*, 51:955–968.

[2] Buzelin, L. O. S., Amico, S. C., Vargas, J. V. C., and Parise, J. A. R. (2005). Experimental development of an intelligent refrigeration system. *International Journal of Refrigeration*, 28(2):165 – 175.

[3] Christensen, L. (1981). The insulation of freezers and refrigerators - how thick should it be? *International Journal of Refrigeration*, 4:7376.

[4] Council of European Union (2011). Directive 2010/30/EU of the European Parliament and of the Council of 19 May 2010 on the indication by labelling and standard product information of the consumption of energy and other resources by energy-related products. `http://eur-lex.europa.eu/eli/dir/2010/30/oj`.

[5] Dmitriyev, V. (1984). Optimum insulation thicknesses for domestic refrigerators and freezers. *International Journal of Refrigeration*, 7:72–73.

[6] Duffie, J. A. and Beckman, W. A. (2013). *Solar Energy Thermal Processes*. A Wiley-Interscience Publication. 4th edition.

[7] Energy Efficient Strategies and Amaia Consulting (2013). Energy Standards and Labelling Programs Throughout the World in 2013. https://www.iea-4e.org/document/343/energy-standards-labelling-programs-throughout-the-world-in-2013.

[8] Gaoa, F., Naini, S. S., Wagner, J., and Miller, R. (2017). An experimental and numerical study of refrigerator heat leakage at the gasket region. *International Journal of Refrigeration*, 73:99110.

[9] Hessami, M. A. and Hilligweg, A. (2003). Energy efficient refrigerators: The effect of door gasket and wall insulation on heat transfer. In *ASME 2003 International Mechanical Engineering Congress and Exposition*, Washington D.C., USA.

[10] Lasdon, L. (2002). *Optimization Theory for Large Systems*. Dover Publication, second edition.

[11] Mitishita, R. S., Barreira, E. M., Negrão, C. O. R., and Hermes, C. J. L. (2013). Thermoeconomic design and optimization of frost-free refrigerators. *Applied Thermal Engineering*, 50(1):1376–1385.

[12] Negrão, C. O. R. and Hermes, C. J. L. (2011). Energy and cost savings in household refrigerating appliances: A simulation-based design approach. *Applied Energy*, 88(9):3051 – 3060.

[13] Pérez-Lombard, L., Ortiz, J., and Pout, C. (2008). A review on buildings energy consumption information. *Energy and Buildings*, 40(3):394 – 398.

[14] Sevindir, M. K., Demir, H., Ağra, Ö., Atayilmaz, Ş. Ö., and Teke, İ. (2017). Modelling the optimum distribution of insulation material. *Renewable Energy*, 113:74–84.

[15] Söylemez, M. S. and Ünsal, M. (1999). Optimum insulation thickness for refrigeration applications. *Energy Conversion and Management*, 40(1):13–21.

[16] Yoon, W. J., Seo, K., and Kim, Y. (2013). Development of an optimization strategy for insulation thickness of a domestic refrigerator-freezer. *International Journal of Refrigeration*, 36(3):1162–1172.

[17] Zhang, L., Fujinawa, T., and Saikawa, M. (2016). Theoretical study on a frost-free household refrigerator-freezer. *International Journal of Refrigeration*, 62:60–71.