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A Reduced Order Kalman Filter for Computational Fluid-Dynamics Applications

Carolina Introini, Antonio Cammi, Stefano Lorenzi, Davide Baroli
Nuclear Reactors Group, Nuclear Engineering Division (CeSNEF),
Department of Energy, Politecnico di Milano, Milan, Italy

RATIONALE

- **Objective**: reducing the shortcomings of numerical simulations in the analysis of complex engineering systems, namely the **epistemic uncertainties** associated with the modelling procedure itself and the **computational requirements** of real, industrial cases.
- In the **nuclear power plants field, t**he development and improvement of simulations tools for the real-time control of the nuclear reactor plays a fundamental role. Ideally, these tools should be **fast-running** (for real-time applications) but at the same time **detailed enough** to improve the knowledge of the state of the system.
- **Question**: it is possible to provide the control simulation tools with relevant spatial information capabilities, enhancing the level of detail without a strong computational burden?

1D Modelling Lumped parameter approach	VS	3D Modelling CFD simulations
Control-oriented		Design-oriented
Main features Fast-running ODE-based Integral information		**Main features** High-detailed PDE-based Spatial information
Main drawback Lacks predictive capabilities		**Main drawback** Too expensive for most available analysis tools

AVAILABLE TOOLS

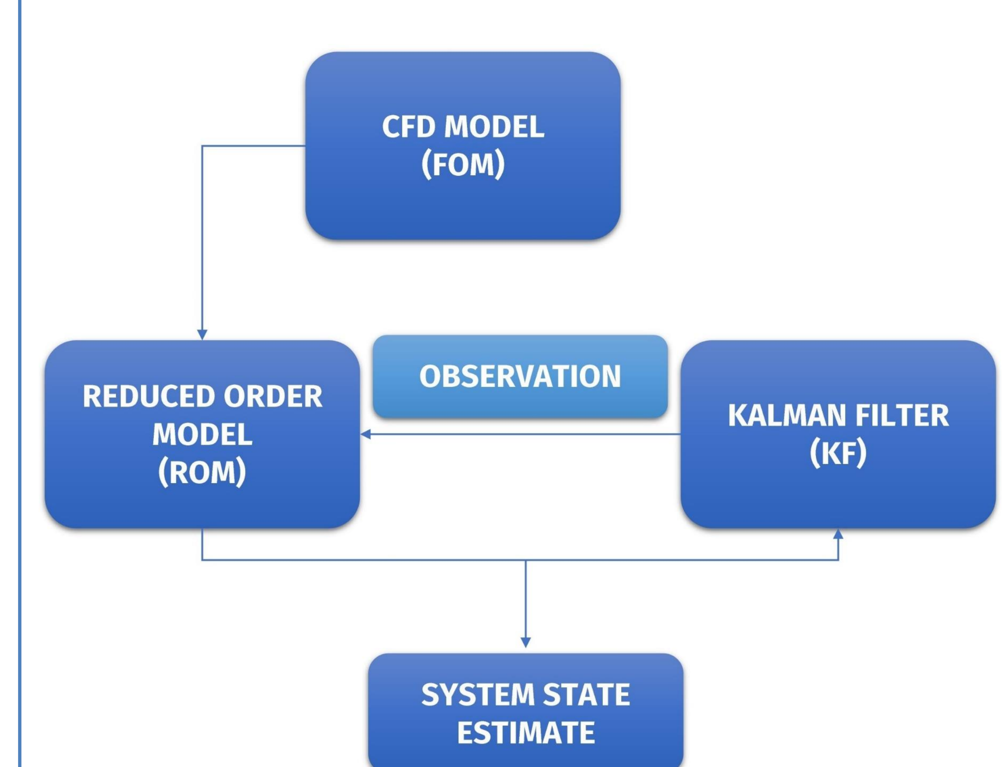
To reduce the computational burden	To improve the computational model accuracy
REDUCED ORDER MODELS (ROM)	DATA-DRIVEN ALGORITHMS (DDA)

REDUCED ORDER MODELS (ROM)
- Replace the high-fidelity (accurate) problem by one featuring much lower complexity
- Input-output relationships have to be preserved
- Must be stable, sufficiently accurate and within scope of the analysis and design tools
- Computationally efficient

DATA-DRIVEN ALGORITHMS (DDA)
- Real-time integration of experimental data within the numerical model, thus improving the efficiency of the latter
- Observations offers a local (spatial) but accurate information on the state of the system
- Feedback on the accuracy of both the model prediction and the experimental data itself

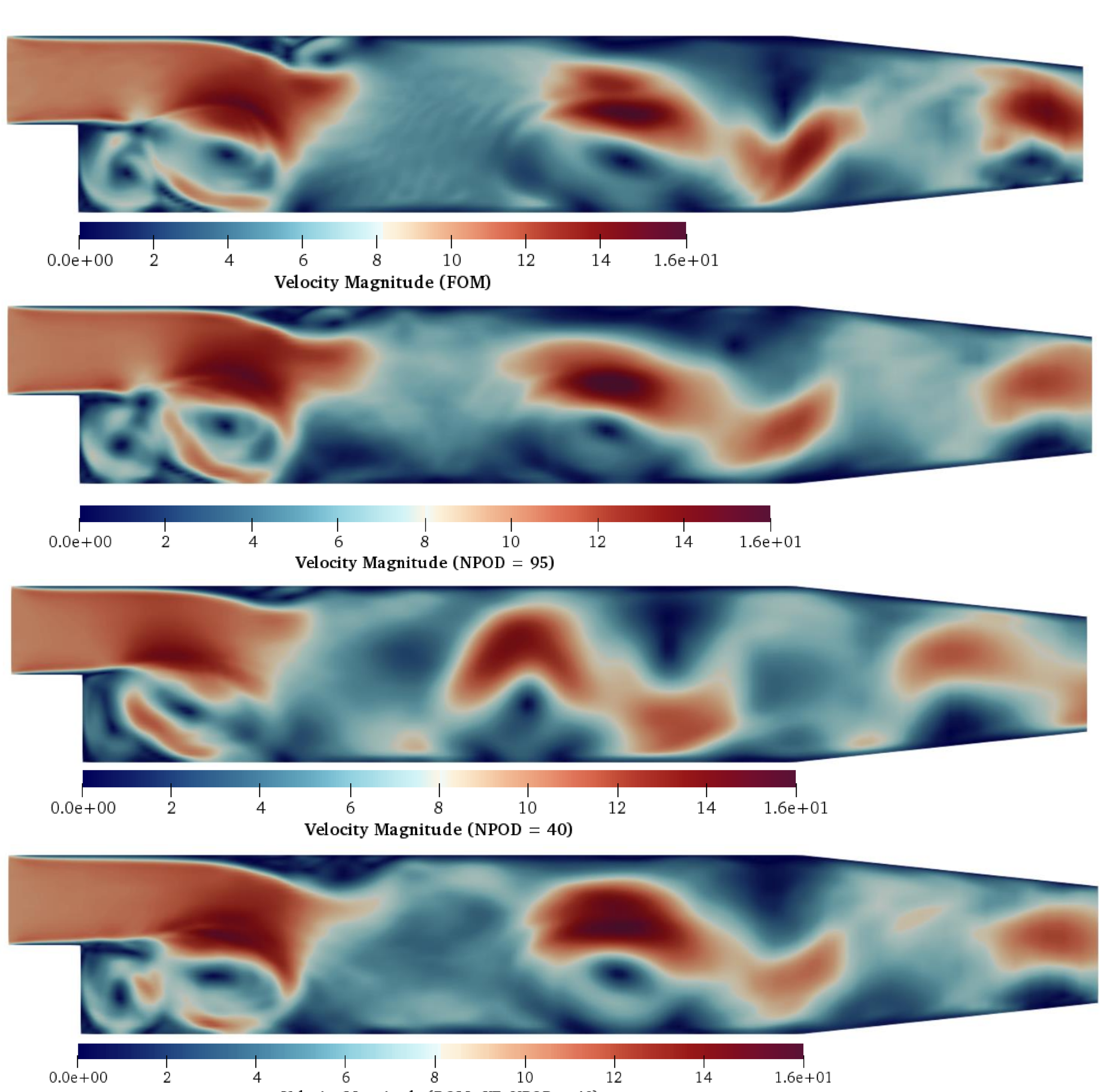
PROPOSED SOLUTION

Combine the reduced order model (POD-Galerkin) and the data-driven algorithm (Kalman filter) in order to develop an online control system with feedback from real-time experimental data



- The filter acts on the reduced variable (POD coefficients)
- Size of the error covariance matrix **P** equal to the number of reduced basis instead of the number of elements of the numerical mesh
- Sensible saving with respect to the Full-Order Model is expected

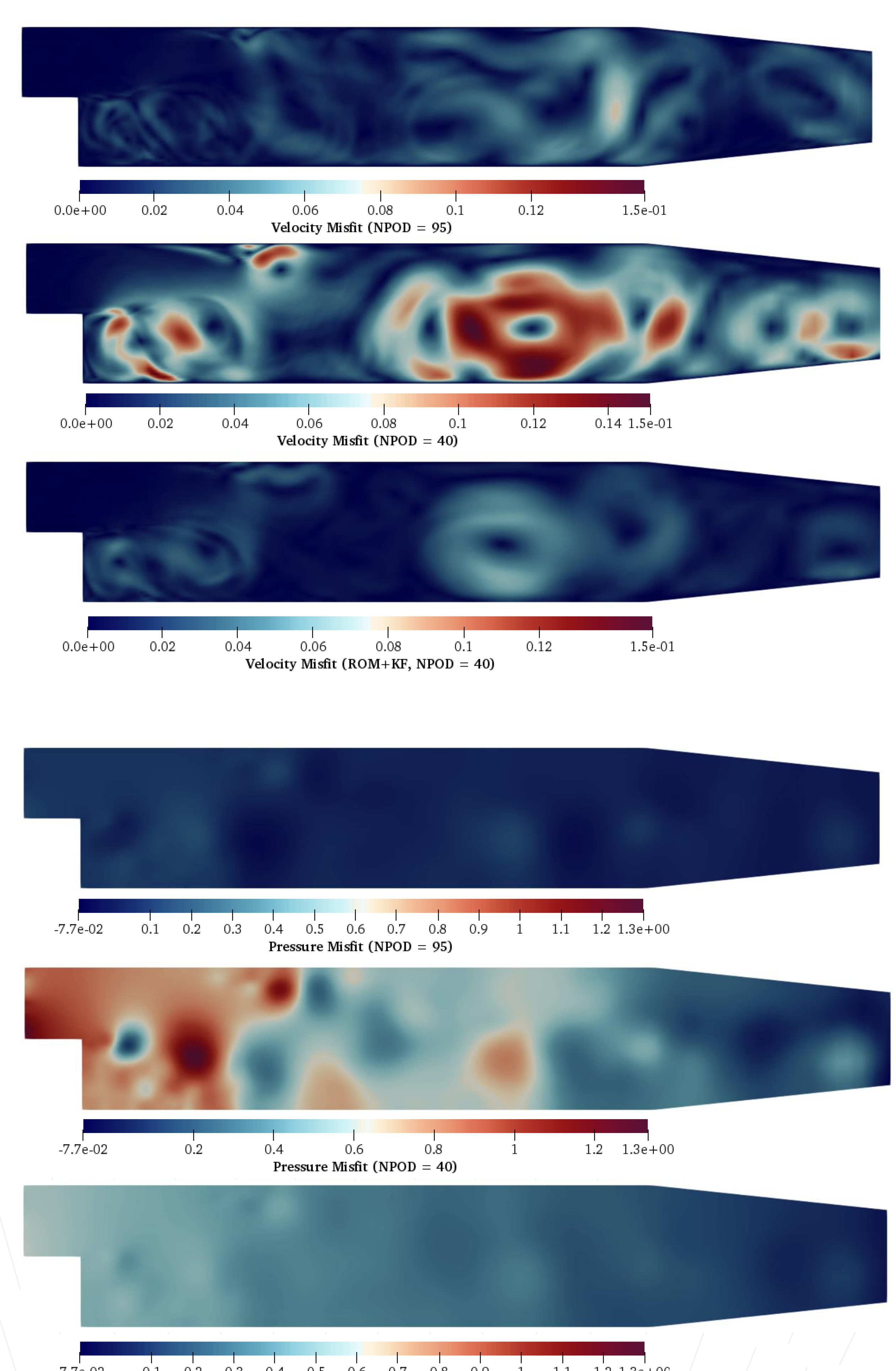
TEST CASE – BACKWARD FACING STEP



Comparison of the velocity magnitude [m/s] at a given time step for the various cases. From top to bottom: full-order model, reduced model with 95 basis, reduced model with 40 basis, and combined reduced model – Kalman filter with 40 basis.

	FOM	ROM (NPOD = 95)	ROM (NPOD = 40)	ROM + KF (NPOD = 40)
Offline	1260 s			
Online		428.57 s	75.00 s	230.70 s

Computational times for the various cases, evaluated as the CPU time needed in order to perform 5000 iterations with a constant time step equal to 0.00005 using the OpenFOAM software for CFD analysis and the PISO algorithm. A personal computer with 4GB RAM and CPU Intel Core i7-65000 @ 2.50 GHz



Comparison of the velocity (top) and pressure (bottom) relative error at a given time, computed as the normalised L2 norm between the full order model solution (taken as reference) and the reduced order ones. From top to bottom: error between FOM and ROM with 95 basis, error between FOM and ROM with 40 basis and error between FOM and ROM with Kalman filter with 40 basis.

CONCLUSIONS

- Given the same accuracy of the reconstruction (namely, the same number of basis), the integration between ROM and Kalman filter allows for better results, comparable to those obtained by a ROM with a greater number of basis (namely, greater accuracy)
- The increase in computational time due to the implementation of the Kalman filter is not negligible, however it remains lower than the one required by the more accurate ROM