Beyond the Fold: The Math, History, and Technology behind Origami

Natalie Hook, Miami University Kristin Paul, Miami University

Abstract: The ancient art of origami can be used to strengthen mathematical subjects in the classroom. By integrating origami into the curriculum, teachers of mathematics provide their students with new opportunities to solidify their understanding of diverse concepts, such as regular polygons, symmetry, and transformation. Using one of many available and free online origami applets, teachers can introduce this ancient craft to their students from a technology-oriented perspective.

Keywords. geometry, history, origami

1 Introduction

In the following paper, we discuss several aspects of origami. First we explain the history of origami, where it originated, and how it has evolved into what it is today. Next, we look at ways one can use origami to strengthen student understanding of mathematical subjects, such as geometry. For example, the use of origami can be used to communicate a specific concept such as symmetry. We also explore ways in which origami is connected to technology. Through discussions of technology, history, and concepts of origami, we provide a well-rounded understanding of this ancient art.

2 History

Depending on the source, origami can be defined in several ways; however, in its purest form, origami is the folding of a single sheet of paper into a figure (Lang, 1996). It is often debated as to where origami originated. This is due to the fact that paper is very difficult to preserve, leaving archeologists with little evidence from the past. What we do know is that the Chinese were the first to create paper around 105 A.D. However, there is no evidence of paper folding at this time. It is possible that the Chinese were creating paper figures, but it wasn't until the sixth century that paper finally reached Japan and became a definitive art

form. Origami is often classified as an ancient Japanese craft because origami actually got its name from the Japanese language. "Oru" is the Japanese word for folding, and "kami" is the Japanese word for paper (UltimateOrigami, 2010).

In the beginning, only the rich and elite could afford paper, and when paper was used, it was used for ceremonial purposes only (UltimateOrigami, 2010). As time progressed, methods of paper-making became much less expensive, and origami slowly became a form of art that was no longer limited to the wealthy. As trade routes became more universal, people all over the world soon began to experiment with origami through trial and error, creating more and more complex figures as years went by (Lang, 1996). By the 19th century, origami had made its way around the globe (UltimateOrigami, 2010). Today, it is a popular art for people of all ages and is explored by many mathematicians.

3 Symmetry

One important aspect of origami is symmetry. It is actually common for various types of origami constructions to be grouped or categorized according to the types of symmetry they exhibit once fully created. Translations, rotations, and reflections are all connected to symmetry because they are each isometries. Isometries are created when one figure is transformed to create several other identical figures that preserve area, angle measure, and distance. Classifying origami constructions by symmetry is also a way to view the art from a mathematical point of view and could easily be used in a classroom when discussing symmetrical shapes (Robichaux and Rodrigue, 2003). Figures 1 through 3 illustrate three origami figures, which are labeled with the types of symmetry they represent.



Fig. 1: Origami panda bear (Embroidery & Origami, 2011, p. 5)

The folded paper panda bear illustrated in Figure 1 can be divided into two identical portions, hence it is an example of reflectional symmetry. Reflectional symmetry, sometimes referred to as "mirror symmetry," is often the easiest of the three types of symmetries to recognize. As one can see, the left side of this figure is the mirror image of the right. If one

were to fold the panda bear on the red line, also known as the line of symmetry, one side would match the other identically.



Fig. 2: *Origami star (Torres, 2009, p. 1)*

The origami star shown in Figure 2 can be subdivided into six symmetrical shapes. This figure provides an example of rotational symmetry because the resulting image, when rotated in either a clockwise or counterclockwise position in 60° increments, is identical to the original shape. Each one of the six sections labeled in Figure 1 occupies 60° of the figure. By adding up all six sections, one finds that the section angles sum to a total of 360°, which is the amount of degrees necessary for a figure to make a full rotation. So, by rotating the figure 60° clockwise or counterclockwise, the image will remap onto itself.

The origami bracelet illustrated in Figure 3 is neither an example of reflectional symmetry nor rotational symmetry, but of translational symmetry. In mathematical terms, translational symmetry is the "sliding" of a shape in any direction as long as the shape remains identical. We know this figure falls into this category because one can simply slide the selected portion of the figure above to the right and it repeats itself over and over until making a full rotation.

4 Technology

Computers can be used to explore origami on a deeper level, allowing folders to create intricate figures that otherwise would be impossible to construct. There are several computer programs, such as TreeMaker, that assist in the generation of sophisticated origami designs using geometric algorithms. They do not actually create origami figures but assist the artist in preparing their mental blueprints. These programs bring the mathematical thoughts of artists to life and physical form. In reality, they only compute the extensive algorithms needed to create a figure; however, it's up to the artist to put forth the creativity and hands-on folding (Lang, 1996; Jensen, 2007). These origami computer programs can



Fig. 3: Origami bracelet (Dorman, 2008, p. 1)

complete a design in less than a minute, when the same task would take a human anywhere from hours to years to create and complete.

4.1 TreeMaker

TreeMaker is a computer program that was created by Robert Lang in the 1990s that is capable of designing "non-trivial origami figures based on a description of the number, lengths, and connectedness of the flaps" (Lang, 2012, p. 1). The program is called TreeMaker because all of the designs that can be created on the program start as simple stick figures that are referred to as trees in graph theory (Lang, 2012). TreeMaker is not only free, but is also extremely easy to access and download on your own personal computer, where you can then experiment on your own time and create your own designs. The download link can be found at http://www.langorigami.com/science/computational/treemaker/treemaker.php. Figure 4 illustrates the TreeMaker program showing the fold lines necessary to create a scorpion figure. Again, this program doesn't create the entire figure for the artist; it simply determines the sequence and number of folds needed to create the desired object, in this case, a scorpion.

Another option that TreeMaker provides involves symmetry. Lang created the program so that its users would have the option of whether or not they want their figures to be symmetrical. Figure 5 illustrates how symmetry can be achieved by the simple checking of a box. By selecting the symmetry option, you indicate that you want your figure to have a strictly symmetrical relationship.

5 Summary

Origami can be many things, especially now that we have technological programs and applets, such as TreeMaker, that allow us to advance our ideas even further and create



Fig. 4: Fold lines for scorpion design in TreeMaker program (Lang, 2012, p. 1)



Fig. 5: Formatting box in TreeMaker applet showing symmetry option

figures that we otherwise would have never considered possible. Origami can be viewed through a mathematical lens and categorized as an example of reflectional, rotational, and even translational symmetry. Origami can be seen and used as a sacred means of decoration at a spiritual event, a symbol to promote peace, or even as a hobby to pass time. Figures can be complex folds or very simple folds taking anywhere from several minutes to several months to create. From the Chinese, to the Japanese, to our very own computers, origami has evolved into the complex art that it is today.

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Natalie Hook, hooknm@muohio.edu, is an undergraduate preservice teacher currently enrolled at Miami University. She is working on completing her Bachelor's degree in Speech Pathology and Audiology and plans to further her education after graduating in May of 2014.



Kristin Paul, paulkn@muohio.edu, is a student from Midwest Ohio who is a junior at Miami University. She is currently working towards her Bachelor's degree in Middle Childhood Education with a focus on Language Arts and mathematics education.