FLEXIBLE SUMMATIVITY: A TYPE-LOGICAL APPROACH TO PLURAL SEMANTICS

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Abstract

This paper explores some theoretical properties of summativity, a generalization of cumulativity. It presents an approach to plural semantics in which summativity can apply not only to lexical predicates, but also to partially saturated predicates. It is shown how this approach can be tied to an explicit type-logical syntax.

1 Introduction

The distinction between distributivity and collectivity has been taken as the starting point for many discussions of the semantics of plural noun phrases (hereafter PNPs). However, as noted early on in (Scha 1981) (also discussed in (Langendoen 1978)), it does not exhaust the possibilities. An example like (1) also exhibits a "neutral" construal, which says that the men can be divided up into groups, each of which lifted the piano, and which put together just add up to the men; but says nothing about how they are divided up. What is interesting about this construal is that it subsumes both the collective and distributive construals, as we shall see in section 4.

(1)   The men lifted the piano.

A common generalization for a semantics for PNPs extends the distributive/collective distinction to the various argument places of multivalent verbs which can be occupied by plural arguments. For instance, in the following example, we can interpret both arguments distributively (understanding that each teacher marked each exam), interpret the subject collectively and the object distributively (so that all the teachers as a group marked each exam), and so forth.

(2)   The teachers marked the exams.

Scha's original treatment in (Scha 1981) was similar, and also allowed the neutral construal as a third possibility for each argument (his C_2 reading). However, (2) also exhibits a construal that Scha called "cumulative": this commits us to all of the teachers marking and all the exams being marked, but to nothing about the "division of labor," i.e. how the teachers relate to the exams (what this exactly amounts to will be spelled out in section 3). What is interesting is that the cumulative construal can't be derived from any combination of distributivity, collectivity, and neutrality of the separate argument places. To see why, given a binary relation R and individuals x and y, say x is "R-involved" with y if x is part of some group g_1 and y is part of some group g_2 such that g_1 stands in the relation R to g_2. It is easy to prove that any reading of "the As R the Bs" derived from any of the nine combinations (of distributivity, collectivity, and neutrality for both of the arguments) requires that each individual in the As is R-involved with each individual in the Bs. The cumulative construal on the other hand breaks this tight connection; this is what makes it (properly) cumulative, by allowing various unconnected relations between parts of groups to "add up" to a relation between groups. This perspective also drives home the point that despite the complexity of some of the classic examples used to illustrate it, there's nothing "exotic" about the cumulative construal. For instance, distributivity and collectivity are too specific, but cumulativity is sufficiently general, to account for the truth of a sentence like *the men lifted the boxes* in the simple situation where each man lifts just one box.

Recall that in the single argument case, the neutral construal subsumed both the collective and distributive ones. The cumulative construal (more accurately, the "generalized cumulative" construal to be defined in section 3) exhibits parallel behavior in the two argument case: it is more general that any of the nine combinations discussed above. Furthermore, we will see in section 3 that when we generalize cumulativity to relations of arbitrary arity, neutrality falls out as just the special instance of it in the unary case.

Because of its generality, it seems evident that cumulativity ought to occupy a special place in any theory of plural semantics. First of all, one should be suspicious of any approach such as (Scha 1981), (Kamp & Reyle 1993) etc. that starts with a simple approach to distributivity, collectivity, and neutrality, but is then forced to adopt additional, complicated mechanisms to account for cumulativity. A more reasonable approach would start with the

most general construal and derive the more specific instances by further restrictions. In fact, it is arguable to what degree these restrictions have to be imposed semantically at all. It seems advisable to postulate ambiguity (as opposed to simply vagueness of reference) only where linguistic evidence motivates it, and some semanticists have questioned whether the different construals derivable from restrictions on cumulativity have any such justification to be treated as resolutions of true ambiguity (see especially (Schwarzschild 1991), (Schwarzschild 1996), (van der Does 1993), (van der Does & Verkuyl 1996), and (Verkuyl 1994)). This paper will not attempt to reiterate or evaluate these arguments, but will rather simply take the unambiguity hypothesis as its basis and study its semantic properties and syntactic implementation.[1]

The paper is organized as follows: in section 2, I outline the algebraic approach to plurality introduced in (Link 1983). Section 3 shows how cumulativity (or in a broader sense, "summativity") can be formalized and generalized within this framework and points out what some different approaches to the phenomenon have in common. Section 4 presents the main theoretical claim of this paper (which builds off of (Sternefeld 1998)): that summativity should not be taken as a lexical property of predicates, but rather as a combinatory operation that may apply at different levels of predicate saturation to yield different semantic results. A type-logical syntax/semantics interface is provided in sections 5 and 6, and the derivations of some desired readings are illustrated.

2 Modeling Structures

An obvious way to model groups is with sets. This approach was taken in early work on plural semantics (e.g. (Bennett 1975)) and in much subsequent research. However, this paper will use the algebraic approach introduced in (Link 1983). In this framework, both basic individuals and groups made out of them are of a single semantic type, which has a lattice structure imposed on it. In fact, this structure will be isomorphic to an appropriate kind of set-theoretic lattice, so the algebraic approach does not buy us much in principle. The primary justification that Link notes is the ease with which it can be adapted to the arguably non-well-founded domain of mass term denotations. For our purposes, the only real advantage of this approach is its perspicuity: it is convenient for e.g. the sets of individuals that model predicate or common noun denotations to be kept typographically distinct from the denotations of PNPs. Everything discussed here can be translated straightforwardly into purely set-theoretic terminology.

The algebraic structure that best mimics the part-whole structure the subset relation imposes on sets is a Boolean algebra. However, the operation we most clearly need for linguistic applications is a join which works like set-union to put objects together into groups, and it is unclear that meets and bottom have any role to play. Thus we will follow

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[1]For this reason, I will use the term *construal* in this paper whenever I am not committed to treating the relevant distinction as an ambiguity. The term *reading* is only used where the analysis presented predicts an ambiguity, i.e. where the syntax-semantics interface associates multiple, truth conditionally distinct semantic representations with a single string of words.

(Link 1998) (appendix) in using only semilattices. Nonetheless, the semilattices we want—what Link dubs "plural semilattices"—will be just those that are like the "top half" of a Boolean algebra. The following definitions set the stage:

1 Definition (Join Semilattice)
A *join semilattice* is a poset $L = \langle S, \sqsubseteq_L \rangle$ where any two elements x and y of S have a least upper bound w.r.t. $\sqsubseteq_L$ (written $x \sqcup_L y$).

In what follows, all our semilattices will be join semilattices by default. Semilattices can also be characterized algebraically by viewing join as a binary operation. The conditions on $\sqcup_L$ that ensure a semilattice structure are that it be commutative, associative, and idempotent. $\sqsubseteq_L$ and $\sqcup_L$ then become interdefinable: $x \sqsubseteq_L y \equiv x \sqcup_L y = y$. In what follows, we will leave off the subscripts when it is clear which semilattice is being discussed.

It is easy to prove that in a semilattice, not only every pair of elements but also every non-empty finite set of elements has a least upper bound. *Completeness* extends this property to arbitrary non-empty sets:

2 Definition (Complete Semilattice)
A semilattice $\langle S, \sqsubseteq_L \rangle$ is *complete* if for every non-empty subset S' of S, S' has a least upper bound in S w.r.t. $\sqsubseteq_L$ (written $\bigsqcup_L S'$).[2]

3 Definition (Atom)
Given a join semilattice $L = \langle S, \sqsubseteq_L \rangle$, an *L-atom* is any minimal element of S w.r.t. $\sqsubseteq_L$. We write $u \sqsubseteq_L^\circ x$ as an abbreviation for "u is an L-atom and $u \sqsubseteq_L x$" (omitting reference to L when convenient).

This gives us enough machinery to define plural semilattices; the following definition of plural semilattices is taken from (Link 1998) (appendix, p. 376):

4 Definition (Plural Semilattice)
A *plural semilattice* (written PSL) is a complete join semilattice $\langle S, \sqsubseteq \rangle$ that obeys the following conditions:

1. $S \neq \emptyset$. (Non-Emptiness)

2. S has no least element under $\sqsubseteq$. (No Bottom)

3. For every $x \in S$, there is a $u \in S$ s.t. $u \sqsubseteq^\circ x$. (Atomicity)

4. For each $x, y \in S$ s.t. $x \not\sqsubseteq y$, there is a $u \sqsubseteq^\circ x$ s.t. $u \not\sqsubseteq y$. (A-Separation)

5. For any $X \subseteq S$, for any $u \sqsubseteq^\circ \bigsqcup X$, there is a $b \in X$ s.t. $u \sqsubseteq b$.
(Sup-Primes)

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[2]Note that some authors use a stricter definition of completeness that requires the existence of $\bigsqcup \emptyset$, and thus of a least element.

As hinted at above, a plural semilattice is just like the "top half" of a Boolean algebra, which itself is like a subset lattice. The following theorem relates plural semilattices and set-theoretic lattices directly:

5 Proposition (PSL Representation Theorem)
For any PSL $\langle S, \sqsubseteq \rangle$ with A the set of its atoms, the function h which maps each $s \in S$ to $\{u \in A \mid u \sqsubseteq^\circ s\}$ is a complete join semilattice isomorphism from $\langle S, \sqsubseteq \rangle$ to $\langle \wp(A) \setminus \{\emptyset\}, \subseteq \rangle$. Equivalently, for any non-empty $X \subseteq S$, $h(\bigsqcup X) = \bigcup h[X]$. (For proof, see (Link 1998) appendix p. 381f.).

The last concept we discuss in this section can be used to model what (Quine 1960) calls "cumulative reference."[3] Basically, (the extension of) a predicate P is said to have cumulative reference if whenever P holds of two objects x and y, it holds of their join $x \sqcup y$. More generally, if a cumulatively referring P holds of all of the elements of some set S, it also holds of $\bigsqcup S$. *Algebraic closure* is just a way of minimally extending a predicate P so that it refers cumulatively.

6 Definition ($^\star P$)
Given a complete join semilattice $L = \langle S, \sqsubseteq \rangle$ and a $P \subseteq S$, the *algebraic closure of P w.r.t. L* (written $^\star P$) is defined as the smallest superset of P which is closed under arbitrary non-empty joins (i.e. $^\star P = \bigcap \{Q \supseteq P \mid \forall Q' \subseteq Q(Q' \neq \emptyset \rightarrow \bigsqcup Q' \in Q)\}$).[4]

Now we can put these concepts to use in analyzing some plural expressions, following (Link 1983). Assume a domain D of the objects we use to model individuals and groups. The former will be the atoms, the set of which we'll call A; the latter will be the non-atomic elements of D. $[\![-]\!]$ will be the interpretation function mapping expressions of the language to their model-theoretic interpretation.

One kind of PNP can be made from a plural common noun by adding the definite determiner. Let's assume that any singular common noun CN denotes a set of individuals, i.e. $[\![CN]\!] \subseteq A$. We want its plural counterpart CNs to be true of just those groups that are made up of individuals which of which CN holds; this can be expressed by setting $[\![CNs]\!] = {}^\star [\![CN]\!]$. Then, we can treat *the CNs* as "all the *CNs* put together," i.e. $[\![\text{the CNs}]\!] = \bigsqcup [\![CNs]\!]$.

Another way of making PNPs is by conjoining definite NPs, such as proper nouns, or NPs constructed from singular or plural common nouns with the definite determiner. We interpret conjunction as putting its conjuncts' denotations together in a group: for any definite NPs DNP_1 and DNP_2, $[\![DNP_1 \text{ and } DNP_2]\!] = [\![DNP_1]\!] \sqcup [\![DNP_2]\!]$.

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[3]One should not confuse cumulative reference with cumulativity in the sense used in this paper, though as we will see in the next section, the two are closely related.

[4]The algebraic closure of P in L coincides with the complete sub-semilattice of L generated by P, a more standard notation for which is $[|P|]$. We will use $^\star P$ to avoid confusion with the denotation function $[\![-]\!]$.

3 Summativity

In this section, we explore how predicates combine with their plural arguments. We assume that the predicate denotations encode the difference between individual and group action by whether individuals or groups are contained in them. n-place predicates in general denote appropriate subsets of $D^{(n)}$.[5]

With the tools from the previous section, we can formalize various possibilities for combining predicates with arguments. (3) gives the one-argument case with an intransitive verb V. The simplest option is (3a): the collective construal just amounts to the group denoted by the NP inhabiting the predicate. The distributive construal in (3b) on the other hand breaks the NP denotation up into its atomic parts and applies the predicate to each of them individually. The neutral construal can be expressed by (3c): the NP denotation is broken up into subgroups to which the predicate applies. The universal quantification over atomic parts guarantees that the subgroups exhaust the NP denotation, i.e. that their combined join is just $[\![NP]\!]$.

(3)   (a) Collective:
$$[\![NP]\!] \in [\![V]\!]$$
   (b) Distributive:
$$(\forall x \sqsubseteq^{\circ} [\![NP]\!])(x \in [\![V]\!])$$
   (c) Neutral:
$$(\forall x \sqsubseteq^{\circ} [\![NP]\!])(\exists g \sqsubseteq [\![NP]\!])(x \sqsubseteq g \wedge g \in [\![V]\!])$$

This formulation makes the relationships between the construals easy to understand. The neutral construal associates each atomic part x with some group g it belongs to and applies the predicate to g. The collective construal results from the neutral construal by associating each x with the same group, namely the whole $[\![NP]\!]$. The distributive construal is similarly just the special case of the neutral one where each atomic x is associated with the trivial group x containing just itself.

Now, one way to avoid a proliferation of different combinatoric options is to always combine a predicate with its NP argument by just checking to see if the latter is an element of the former, but first modifing the predicate denotation so that it has the desired properties. Recall that our methodologial assumption is that only the most general construal needs to be represented. Thus, the relevant question is how a predicate P can be modified to yield a new predicate P' which contains $[\![NP]\!]$ if and only if (3c) holds.

This problem has been approached a few different ways in the literature. One line of thought represented by Krifka in (Krifka 1989) and subsequent work links neutrality to cumulative reference — P' should contain just those groups which can be built up out of memebers of P, i.e. $^{\star}P$.

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[5] We write $S^{(n)}$ for $S \times \cdots \times S$ (n times).

Another tradition begins with Higginbotham's suggestion in (Higginbotham 1980) that P' contain just those groups that (speaking set-theoretically) can be divided into a partition each of whose cells is in P. For empirical reasons, Gillon found it necessary in (Gillon 1987) to relax the requirement imposed by a partition that its cells not overlap, and reformulated the Higginbotham semantics with the weaker notion of a *cover*. We can define an algebraic notion of cover as follows:

7 Definition (*L*-Cover)
Given a PSL $L = \langle S, \sqsubseteq \rangle$ and an $a \in S$, an *L-cover of a* is a non-empty set $C \subseteq S$ such that $\bigsqcup C = a$. (We drop reference to L when convenient).

What's interesting is that with the relaxation to covers, the Krifka and Higginbotham approaches become equivalent. The following proposition relates them to each other and to (3c):

8 Proposition
Given a PSL $L = \langle S, \sqsubseteq \rangle$, an $a \in S$, and a $P \subseteq S$, the following three statements are equivalent:

  (i) $(\forall x \sqsubseteq^\circ a)(\exists g \sqsubseteq a)(x \sqsubseteq g \land g \in P)$

  (ii) There is an *L*-cover C of a such that $C \subseteq P$.

  (iii) $a \in {}^\star P$.

Proof: This equivalence falls out as the special case of proposition 11 below, where $n = 1$.

$\square$

Next we discuss the combination of multivalent verbs with their NP arguments. It is at this point that the cumulative reading becomes important. Although this construal is usually associated with Scha, the general truth condition schema in (4a) for a transitive verb TV with plural arguments NP_1 an NP_2 is already to be found in (Langendoen 1978). It says in essence that every atom in the subject did the relation to some atom in the object, and that to every atom in the object was done the relation by some atom in the subject. This captures the general conditions on sentences like *the men talked to the women* in an appealing way. However, the restriction to atomicity in (4a) is rather arbitrary, and in fact only works when the relation is inherently distributive on both argument positions. The need for a generalization of this schema is exemplified by Langendoen's example *the men released the prisoners*. Here, releasing someone is a property a group can have without it's individual members having it (though this property holds less convincingly for being released by someone, *pace* Langendoen). The revision in (4b) requires that every subject atom belongs to some subject subgroup that does the relation to some object subgroup and vice versa.

(4)   (a) Cumulative:

$$(\forall x \sqsubseteq^\circ [\![NP_1]\!])(\exists y \sqsubseteq^\circ [\![NP_2]\!])(\langle x, y \rangle \in [\![TV]\!]) \land$$
$$(\forall z \sqsubseteq^\circ [\![NP_2]\!])(\exists w \sqsubseteq^\circ [\![NP_1]\!])(\langle w, z \rangle \in [\![TV]\!])$$

    (b) Generalized Cumulative:

$$(\forall x \sqsubseteq^\circ [\![NP_1]\!])(\exists g_1 \sqsubseteq [\![NP_1]\!])$$
$$(x \sqsubseteq g_1 \land (\exists g_2 \sqsubseteq [\![NP_2]\!])(\langle g_1, g_2 \rangle \in [\![TV]\!])) \land$$
$$(\forall z \sqsubseteq^\circ [\![NP_2]\!])(\exists g_3 \sqsubseteq [\![NP_2]\!])$$
$$(z \sqsubseteq g_3 \land (\exists g_4 \sqsubseteq [\![NP_1]\!])(\langle g_4, g_3 \rangle \in [\![TV]\!]))$$

Here we suppress the various construals that derive from the various combinations of distributive, collective, and neutral readings for the two argument places; it is straighforward but tedious to verify that they all entail (4b).

The advantages of an approach which derives the cumulative construal by modifying the relation and then applying it directly to its NP arguments are clearer in the multivalent case than for the derivation of the neutral construal with single argument predicates. It is rather difficult to devise combinatoric rules that put two NPs and a verb together to yield (4b). A DRT treatment of such rules is hinted at in (Kamp & Reyle 1993) and (Reyle 1996), but these rules involve copying of quantificational material into various DRS-boxes in a way that is essentially non-compositional. Referring to the simpler (4a), Sternefeld comments that "[i]t is fairly obvious that [it] cannot be derived from the syntactic structure…by compostional methods…[T]he paraphrase with four quantifiers will always exhibit a kind of 'cross-over effect,' so that the quantifiers get in one another's way, excluding a compositional analysis."

The problem is exacerbated if we make the plausible assumptions that predicates with more than two argument places combine with their arguments in a parallel way, so that e.g. *the men gave the presents to the children* means (5):

(5)   (i) Each man belongs to a subgroup of the men that gave some subgroup of the presents to some subgroup of the children, and

     (ii) each present belongs to a subgroup of the presents that was given by some subgroup of the men to some subgroup of the children, and

    (iii) each child belongs to some subgroup of the children that was given some subgroup of the presents by some subgroup of the men.

The problem is that the general schema for this kind of three-place cumulativity can't be gotten at by first deriving the two-place cumulative meaning for the combination of the predicate with two of its arguments via (4b) and then adding something further; for each argument A, existential quantification over subgroups of *every* other argument must occur within the scope of universal quantification over A's atomic parts. This difficulty increases the desirablity of deriving (4b) and its counterparts at higher arities by modifying the predicate somehow and then simply applying the result to the arguments.

One way to do this is what Krifka calls "summativity" (Krifka 1989). This operation closes an n-ary relation R under (binary) pairwise joins:

(6) $\mathsf{summate}(R) =_{def}$
$$\bigcap \{Q \mid R \subseteq Q \wedge (\forall x_1, \ldots, x_n)(\forall y_1, \ldots, y_n)$$
$$(\langle x_1, \ldots, x_n \rangle \in Q \wedge \langle y_1, \ldots, y_n \rangle \in Q . \rightarrow$$
$$\langle x_1 \sqcup y_1, \ldots, x_n \sqcup y_n \rangle \in Q)\}$$

The cover approach can get the same result by existentially quantifying over the right kind of pair-cover, in a way introduced in (Schwarzschild 1991).

These approaches are obviously related to the corresponding closure and cover approaches for the one argument place. In fact, one can view the multivalent versions as exactly the same as the one-argument versions if one thinks about the structure that the semilattice structure of the domain D imposes on the domain $D^{(n)}$ of n-tuples of which an n-ary relation is a subset:

9 Definition ($L^{(n)}$)
Given a complete join semilattice $L = \langle S, \sqsubseteq_L \rangle$ and a natural number $n > 0$, we define *the n-th product of L* to be the order $L^{(n)} = \langle S^{(n)}, \sqsubseteq_{L^{(n)}} \rangle$, where for each $\vec{x}, \vec{y} \in S^{(n)}$, $\vec{x} \sqsubseteq_{L^{(n)}} \vec{y}$ iff for all $i, 0 < i \le n, \pi_i(\vec{x}) \sqsubseteq_L \pi_i(\vec{y})$. We abbreviate $\sqsubseteq_{L^{(n)}}$ as $\sqsubseteq_n$ whenever L is fixed.[6]

It is easy to prove that the n-th product of any (complete) semilattice is itself a (complete) semilattice. In particular, if L is a PSL, then so is $L^{(n)}$. This follows from the easily proved lemmata in 10:

10 Lemma
For any PSL $\langle S, \sqsubseteq_L \rangle$, the following hold of any $\langle S^{(n)}, \sqsubseteq_n \rangle$:

(i) For all non-empty $R \subseteq S^{(n)}$, the least upper bound of R w.r.t. $\sqsubseteq_n$ (written $\bigsqcup_n R$) exists in $S^{(n)}$: it is the n-tuple $\langle \bigsqcup_L \pi_1[R], \ldots, \bigsqcup_L \pi_n[R] \rangle$.

(ii) For any $\vec{u}, \vec{x} \in S^{(n)}, \vec{u} \sqsubseteq_n^{\circ} \vec{x}$ iff for all i $(0 < i \le n), \pi_i(\vec{u}) \sqsubseteq_L^{\circ} \pi_i(\vec{x})$.

(iii) For all $\vec{x}, \vec{y} \in S^{(n)}$, if $\vec{x} \not\sqsubseteq_n \vec{y}$, then there is some $\vec{u} \sqsubseteq_L^{\circ} \vec{x}$ and some i $(0 < i \le n)$ s.t. $\pi_i(\vec{u}) \not\sqsubseteq_n \pi_i(\vec{y})$.

(iv) For any $\vec{a} \in S^{(n)}$, any $R \subseteq S^{(n)}$, if $\vec{a} = \bigsqcup_n R$, then there is a $\vec{b} \in R$ s.t. for all i $(0 < i \le n), \pi_i(\vec{a}) \sqsubseteq_L \pi_i(\vec{b})$.

Recall that Krifka's modification of a unary predicate P was just the algebraic closure of P; it's easy to see that his modification of a higher arity relation, $\mathsf{summate}(R)$, is

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[6] $\vec{x}$ abbreviates the n-tuple $\langle x_1, \ldots, x_n \rangle$, and we write $\pi_i(\vec{x})$ for the i-th projection of $\vec{x}$.

just the algebraic closure of R, but with respect to the product ordering $D^{(n)}$ (the only difference being that (6) above is insufficiently general in that **summate** only closes a relation under finite, but not arbitrary non-empty joins). In what follows, we use the notation $\overset{\star}{_n}R$ for the upper closure of R w.r.t. $\sqsubseteq_n$; then the general modification we make to an R of any arity n, including 1, is $\overset{\star}{_n}R$. The same kind of generalization can be made for the cover approach: for an R of any arity n, we want $\{\vec{a} \in D^{(n)} \mid$ there is an $L^{(n)}$-cover C of $\vec{a}$ s.t. $C \subseteq R\}$. This applies equally well for $n = 1$ if we equate the 1-tuple $\langle x \rangle$ with x.

The equivalence of the closure and cover approaches extends to the multivalent case as well. The following theorem shows this, also shows that and that they both are equivalent to the generalization of the cumulativity schema (4b) to arbitrary arities. The proof is given in the appendix.

11 Proposition

Given a PSL $\langle S, \sqsubseteq_L \rangle$, its n-product PSL $\langle S^{(n)}, \sqsubseteq_n \rangle$, an $\vec{a} \in S^{(n)}$, and an $R \subseteq S^{(n)}$, the following three statements are equivalent:

1.
$$\bigwedge_{0 < k \leq n} \left(\begin{array}{l} (\forall x_k \sqsubseteq^{\circ}_L \pi_k(\vec{a})) \\ \quad (\exists g_1 \sqsubseteq_L \pi_1(\vec{a})) \ldots (\exists g_n \sqsubseteq_L \pi_n(\vec{a})) : \\ \quad\quad (x_k \sqsubseteq g_k \\ \quad\quad \wedge \langle g_1, \ldots, g_n \rangle \in R) \end{array} \right)$$

2. There is an $L^{(n)}$-cover C of $\vec{a}$ such that $C \subseteq R$.

3. $\vec{a} \in \overset{\star}{_n}R$.

Despite the equivalence between the closure and cover approaches, their different formulations make them lend themselves differently to certain extensions. For instance, it has proved heuristically fruitful to begin with covers but then consider various stronger notions, such as "pseudo-partitions"; this path is followed in (Verkuyl 1994) and (van der Does & Verkuyl 1996). Another line of thinking introduced in (Schwarzschild 1991) replaces quantification over covers with pragmatic determination. This allows some of the distinctions the summative approach erases to be reintroduced, but in a parsimonious and context-dependent way that is consistent with a treatment of sentences with PNPs as semantically unambiguous. For the remainder of this paper however we will stick to the closure approach and refer to the general phenomenon of algebraic closure over relations of arbitrary arity as "summativity."

What this section has shown is that for any arity n, there exists a general method for associating the combination of an n-place predicate with n NP arguments yielding truth conditions that account for the cumulative construal (and its higher arity generalizations), and of which the distributive, collective, and neutral construals and their various combinations are special subcases. Furthermore, this method ties together two different strands of thought in the study of cumulativity and shows where they converge on the same result.

4 Flexible Summativity

In the previous section, we relied on intuitions about the "basic" denotations of predicates, and derived more complex summative denotations by means of algebraic closure or existential quantification over covers. In this section, we consider what linguistic status these "derived" predicates have.

Some researchers view it as a lexical property of verbs. For instance, (Lasersohn 1995) assumes that certain verbs are inherently algebraically closed. Notice that for these verbs, the intuitive distinction between the "basic" denotation and the derived one is lost, since nothing in the lexicon represents the basic sense. Lasersohn's motivation for lexicalizing summativity comes from his conviction that the cumulative construal is not available for all predicates. However, Bayer argues that although certain sentences seem to require construals stronger than the cumulative one, the same predicates occurring in them can be combined with different arguments in different contexts to allow the weaker construal (Bayer 1996). Thus, it seems possible to preserve the general hypothesis that all predicates allow summativity. Bayer makes this assumption, but still views summativity as a lexical property. It is not clear to me what the motivation for this is. It would seem that we could divorce specific predicate meanings from the general phenomenon of summativity by hanging on to basic predicate denotations and then applying a summativity operator to them. It might seem that there is no empirical difference between these two options. However, I will argue that the nonlexical approach gives us a kind of flexibility that allow us to account for a wider range of data.

Notice that the examples of summativity presented so far have all involved definite noun phrases. An important property that these have is that they are referentially independent, i.e. they don't enter into scope relations that can affect truth conditions. Compare (7a) with example (2), repeated here as (8a). Assume that predicate denotations are basic (not summative). Let's give an existential treatment to numerals as in (8b) following (Link 1983) (*six* being interpreted in a parallel fashion), where $|x|$ is interpreted as the cardinality of the set of atoms under x. Although we haven't discussed an explicit syntax-semantics interface, it should be clear that the only reading we get for (8a) from the machinery presented so far without further assumptions is something like (8c), a cumulative construal exactly parallel to (7b). Since permuting existential quantifiers preserves truth conditions, the two variables x and y are independent of each other, i.e. the choice of a value for one doesn't depend on the choice of a value for the other.

(7) (a) The teachers marked the exams.

(b) $\langle \bigsqcup {}^\star[\![\mathit{teacher}\,]\!], \bigsqcup {}^\star[\![\mathit{exam}\,]\!]\rangle \in {}^\star_2[\![\mathit{marked}\,]\!]$

(8) (a) Three teachers marked six exams.

(b) $[\![\mathit{three\ CNs}\,]\!] = \{Q \subseteq D \mid (\exists x \in [\![\mathit{CNs}\,]\!])(|x| = 3 \wedge x \in Q)\}$

(c) $(\exists x \in [\![\mathit{teachers}\,]\!])(\exists y \in [\![\mathit{exams}\,]\!])(|x| = 3 \wedge |y| = 6 \wedge \langle x, y\rangle \in {}^\star_2[\![\mathit{marked}\,]\!])$

145

However, (8c) does not cover all the ways (8a) can be understood. For instance, it can be read as making a claim about not six exams, but eighteen—six per teacher for each of three teachers. Now on this construal, it seems as if the predicate *marked six exams* is being applied to each of the individual teachers in turn, i.e. being interpreted distributively. The existence of such a construal with a concomitant difference in the total cardinality of the object is taken by e.g. (Roberts 1987) as prime evidence that such sentences are in fact ambiguous, not just vague, and that the distributive reading corresponds to one potential disambiguation. I accept the first conclusion, but deny that the data show anything about distributivity *per se*. I argue that the important difference between this construal and the cumulative one lies in the referential dependency of the exams on the teacher. A sentence can be ambiguous as to the dependencies between its NPs i.e. the scopes of its quantifiers, while still remaining vague about the divisions of labor involved. For instance, a sentence could have one reading where the choice of A depends on the choice of B and different one where the choice of B depends on A. At the same time, in the disambiguation where the choice of B depends on the choice of A, once we choose an x from the As, get the dependent choice of y from the Bs, and relate x and y, we still don't need to specify the division of labor that underlies this relation.

The reason this bears on the issue of the locus of summativity is that if we view the summativity operator as not tied to the completely unsaturated lexical predicate, but free to apply to a partially saturated, syntactically derived predicate, we can get just such a representation that specifies dependencies but underspecifies the division of labor. Consider (9a), which by proposition 8 is equivalent to (9b); this analysis of (8a) differs from (8c) in that the quantification over exams occurs within the scope of the closure, i.e. the closure applies not to the lexical predicate *marked* but to the derived one *marked six exams*. Such scope relationships encode the referential dependencies, while the use of summativity (instead of forcing a choice between more specific options like distributivity and collectivity) keeps the division of labor out of the semantics.

(9)　　(a)　$(\exists x \in [\![teachers]\!])(|x| = 3 \wedge x \in {}^{\star}\{g \in D \mid (\exists y \in [\![exams]\!])(|y| = 6 \wedge \langle g, y \rangle \in$ $[\![marked]\!])\})$

　　　　(b)　$(\exists x \in [\![teachers]\!])(|x| = 3 \wedge (\forall z \sqsubseteq^{\circ} x)(\exists g \sqsubseteq x)(z \sqsubseteq g \wedge (\exists y \in [\![exams]\!])(|y| = 6 \wedge \langle g, y \rangle \in [\![marked]\!]))))$

Lastly, note that summativity has no effect when the predicate it yields gets applied to only to singular arguments, so there is no harm in associating summativity with predication in general, not just predication of plurals. This allows us to have a reading for a sentence like *the lawyers hired a secretary* where the choice of secretary is dependent on the choice of lawyer:[7]

(10)　$\bigsqcup {}^{\star}[\![lawyer]\!] \in {}^{\star}\{x \in D \mid (\exists y \in [\![secretary]\!])(\langle x, y \rangle \in [\![hired]\!]\}$

---

[7]Unfortunately, nothing in the account I will give blocks the reverse dependency $(\exists y \in [\![secretary]\!])(y \in {}^{\star}\{g \in D \mid \langle \bigsqcup {}^{\star}[\![lawyer]\!], g \rangle \in [\![hired]\!]\})$. This yields a purely collective reading, while our aim is to avoid specifying readings stronger than necessary.

A similar proposal is developed by Sternefeld in (Sternefeld 1998) which allows operators to apply to partially saturated predicates. However, that account differs in both spirit and specifics from the proposal I sketched above and develop more fully in section 6. First of all, the operators Sternefeld applies to these predicates are not restricted to summativity; he also allows the option of using a more specific distributivity operator, which I avoid. Secondly, I will assume that only one summativity operator is used with each predicate, while Sternefeld allows no such restriction. This allows him to represent a neutral–neutral reading of (8a) with two instances of 1-place summativity as in (11), as well as a distributive–distributive one, a distributive–neutral one, etc. However, since all these construals can be derived as special cases of the use of summativity plus scope variation, I do not see the necessity of this option.

(11) $(\exists x \in [\![\textit{teachers}]\!])(|x| = 3 \wedge x \in {}^\star\{g \in D \mid (\exists y \in [\![\textit{exams}]\!])$
       $(|y| = 6 \wedge \in {}^\star\{g' \in D \mid \langle g, g' \rangle \in [\![\textit{marked} \}\ ]\!])\})$

Lastly, Sternefeld's syntax-semantics interface makes use of a syntactic level of logical form where these operators can be introduced non-deterministically as "semantic glue" in a way not related to the construction or lexical items used in a sentence. In the next two sections, I will develop a more tightly constrained interface in type-logical grammar that anchors summativity to predication.

5   Overview of Type-Logical Grammar

Type-logical grammar developed out of categorial grammar by focusing on the logical nature of categorial combinatorics. The account developed in this paper presupposes familiarity with the basics of the semantically annotated Lambek calculus, as it is presented in (Carpenter 1997). The sequent rules for this calculus are given in figure 1.

Following the presentation in (Carpenter 1997), this calculus can be extended with a binary connective '$\Uparrow$' to account for *in situ* binding phenomena. A category of the form $A \Uparrow B$ has type $(\mathsf{type}(A) \to \mathsf{type}(B)) \to \mathsf{type}(B)$. The application we will be concerned with here is the analysis of some NP expressions as of category $np \Uparrow s$, which occurs in an NP position but can be interpreted as combining with and "quantifying-into" a sentence. For example, assigning *every* the meaning and category $\lambda P \lambda Q. \forall x [P(x) \to Q(x)] : (np \Uparrow s)/n$ permits a standard Montogovian treatment of universal quantification. Sequent are given in figure 2.

The next extension we will need to make to the Lambek calculus for the purposes of this paper is to incorporate a limited kind of polymorphism. This allows us to manipulate not only constant categories, but also category variables. Polymorphic extensions to the Lambek calculus are discussed in e.g. (Emms 1993) and (Moortgat 1997). The motivation for such an extension is the desire to have a single, unambiguous lexical item be able to take on different categories in different contexts. For example, the word *and* can be used to

$$\frac{}{\alpha\colon A \vdash \alpha\colon A}\ (\text{Ax}) \qquad\qquad \frac{\Delta \vdash \alpha\colon A \quad \Gamma, \alpha\colon A, \Gamma' \vdash \beta\colon B}{\Gamma, \Delta, \Gamma' \vdash \beta\colon B}\ (\text{Cut})$$

$$\frac{\Gamma, \pi_1(\alpha)\colon A, \pi_2(\alpha)\colon B, \Gamma' \vdash \beta\colon C}{\Gamma, \alpha\colon A \bullet B, \Gamma' \vdash \beta\colon C}\ (\bullet\ \text{L}) \qquad\qquad \frac{\Gamma \vdash \alpha\colon A \quad \Delta \vdash \beta\colon B}{\Gamma, \Delta \vdash \langle \alpha, \beta \rangle\colon A \bullet B}\ (\bullet\ \text{R})$$

$$\frac{\Delta \vdash \beta\colon B \quad \Gamma, \alpha(\beta)\colon A, \Gamma' \vdash \gamma\colon C}{\Gamma, \Delta, \alpha\colon B\backslash A, \Gamma' \vdash \gamma\colon C}\ (\backslash\ \text{L}) \qquad\qquad \frac{x\colon A, \Gamma \vdash \alpha\colon B}{\Gamma \vdash \lambda x.\alpha\colon A\backslash B}\ (\backslash\ \text{R})$$

$$\frac{\Delta \vdash \beta\colon B \quad \Gamma, \alpha(\beta)\colon A, \Gamma' \vdash \gamma\colon C}{\Gamma, \alpha\colon A/B, \Delta, \Gamma' \vdash \gamma\colon C}\ (/\ \text{L}) \qquad\qquad \frac{\Gamma, x\colon A \vdash \alpha\colon B}{\Gamma \vdash \lambda x.\alpha\colon B/A}\ (/\ \text{R})$$

Figure 1: Sequent rules for the Lambek calculus with semantic annotations

$$\frac{\Delta_1, x\colon B, \Delta_2 \vdash \beta\colon A \quad \Gamma_1, \alpha(\lambda x.\beta)\colon A, \Gamma_2 \vdash \gamma\colon C}{\Gamma_1, \Delta_1, \alpha\colon B \Uparrow A, \Delta_2, \Gamma_2 \vdash \gamma\colon C}\ (\Uparrow\ \text{L})\ [x\ \text{fresh}]$$

$$\frac{\Gamma \vdash \alpha\colon A}{\Gamma \vdash \lambda x.x(\alpha)\colon A \Uparrow B}\ (\Uparrow\ \text{R})\ [x\ \text{fresh}]$$

Figure 2: Sequent rules for $\Uparrow$

conjoin to expressions of category X for (almost) any X. We can capture this by assigning it to the polymorphic category $(\forall X)(X \backslash X / X)$. This universal category can be instantiated to any constant category as needed.

In certain situations, we might want to limit this polymorphism so that the category variable can only be instantiated to certain categories. For instance, say we want a certain expression to be able to act as either a noun or noun phrase. In the system given here, we provide names for classes of categories, e.g. *nominal*, and define the class-membership relation "is a" in the metalanguage, by recursive definition or, as in this case, by simple enumeration:

(12)   • *np* is a *nominal*;

      • *n* is a *nominal*;

      • nothing else is a *nominal*.

We can now assign an expression to category $(\forall X \leq nominal)X$. The restriction that in such a category, X can be instantiated only to *np* or *n*, is implemented in the sequent rule by a side condition. Of course, in the case of a finite category class like this there are simpler ways to deal with category vagueness; the usefulness of bounded polymorphism is more apparent when the relevant class is infinite.

One question that needs to be asked about polymorphic categories is what type their semantic labels should have. This can be answered most satisfactorily in a richer type theory that countenances polymorphic types, such as that of (Cardelli & Wegner 1985). However, for our purposes here we will sidestep this problem by requiring that polymorphic categories always be labeled by syncategorematic semantic terms. For instance, our category for *and* can be labeled by the syncategorematic generalized conjunction symbol of (Partee & Rooth 1983), which has no type of its own, but when combining with two expressions of type τ is evaluated at type $\tau \rightarrow (\tau \rightarrow \tau)$. The rules of proof and use for $(\forall - \leq -)$ thus leave the semantic terms unchanged. Note the resulting calculus no longer exhibits a strict Curry-Howard correspondence between proofs and lambda-terms.

The details are as follows. We assume a denumerable set of category variables, which we write as X, Y, etc. The set of basic categories is now expanded to contain these alongside our atomic category constants. We also assume a set of category class symbols. We add to our recursive definition of complex category the clause (13), and extend our deductive calculus as in figure 3.

(13) If V is a category variable, c is a category class symbol, and A is a category, then $(\forall V \leq c)A$, is a category.

The last extension we will need is a way for selected expressions to escape the strict structural requirements of the Lambek calculus. We introduce a new unary category

$$\frac{\Gamma, \alpha\colon A^{\{B\backslash X\}}, \Delta \vdash \beta\colon B}{\Gamma, \alpha\colon (\forall X \leq T)A, \Delta \vdash \beta\colon B} \ (\forall L) \qquad [\text{where } B \text{ is a } T]$$

$$\frac{\Gamma \vdash \alpha\colon A}{\Gamma \vdash \alpha\colon (\forall X \leq T)A^{\{X\backslash B\}}} \ (\forall R) \qquad [\text{where } B \text{ is a } T]$$

Figure 3: Sequent rules for $\forall$

$$\frac{\Gamma_1, \alpha\colon A, \Gamma_2 \vdash \gamma\colon C}{\Gamma_1, \alpha\colon \triangle A, \Gamma_2 \vdash \gamma\colon C} \ (\triangle L) \qquad \frac{\Gamma_1, \alpha\colon \triangle A, \beta\colon B, \Gamma_2, \vdash \gamma\colon C}{\Gamma_1, \beta\colon B, \alpha\colon \triangle A, \Gamma_2 \vdash \gamma\colon C} \ (\triangle P)$$

Figure 4: Sequent rules for $\triangle$

constructor '$\triangle$', borrowed from (Morrill 1994), which allows a formula it annotates to undergo permutation. Since we will not need a rule of proof in our applications, only a left rule is shown in figure 4: all it does is eliminate the $\triangle$ from a formula, with no effect on the semantics.[8]

This concludes the presentation of the type-logical machinery used in this paper. The next section applies it to the syntax-semantic interface for flexible summativity.

6   A Syntax-Semantics Interface

Our goal is to analyze predications of PNPs in such a way that they are associated with a single summativity operator which can apply at any level of predicate saturation. This variability in the behavior of summativity will be modeled using bounded polymorphism and structural permutation.

First we need to add a summativity operator to our semantic representation language. It will be associated with a polymorphic category, since it can combine with a relation of any arity to yield a summative interpretation of that relation. For this reason, it will need to be syncategorematic. The following gives a meta-level definition of a semantic type-class *Erel*, which contains all types of (curried) relations over type *e*. The summativity operator Σ combines with any expression of such a type to yield a new expression of the same type, which is interpreted as the algebraic closure of the denotation of the original expression.

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[8]The double line in the $\triangle$P rule indicates that it should be read biconditionally.

$$\cfrac{\cfrac{\cfrac{\overline{x\colon np \vdash x\colon np}\ \text{Ax} \quad \overline{\phi(y)(x)\colon s \vdash \phi(y)(x)\colon s}\ \text{Ax}}{x\colon np, \phi(y)\colon np\backslash s \vdash \phi(y)(x)\colon s}(\backslash \text{L}) \quad \overline{\gamma\colon s \vdash \gamma\colon s}\ \text{Ax}}{Q\colon np \Uparrow s, \phi(y)\colon np\backslash s \vdash \gamma\colon s}(\Uparrow \text{L})}{\cfrac{\overline{y\colon np \vdash y\colon np}\ \text{Ax} \qquad Q\colon np \Uparrow s, \phi\colon (np\backslash s)/np, y\colon np \vdash \gamma\colon s}{Q\colon np \Uparrow s, \phi\colon (np\backslash s)/np \vdash \lambda y.\gamma\colon s/np}(/\text{R})}(/\text{L})}$$

Figure 5: Quantifier subject + transitive verb; γ abbreviates $Q(\lambda x.\phi(y)(x))$

(14)   • $e \to t$ is an *Erel*;

   • If α is an *Erel*, then $e \to \alpha$ is an *Erel*;

   • Nothing else is an *Erel*.

(15)   • The degree of $e \to t$ is 1.

   • For an *Erel* $e \to \alpha$, the degree of $e \to \alpha$ is 1+ the degree of α.

(16)   If ϕ is a wff of type τ and τ is an *Erel* with degree n, then $\Sigma(\phi)$ is a wff of type τ and
$[\![\Sigma(\phi)]\!] = \mathsf{curry}(^{\star}_{n}\mathsf{decurry}([\![\phi]\!]))$.[9]

Next, we define a category class *Everb* that picks out just those verbal categories whose semantic terms are of an *Erel* type; this class will form the restriction of Σ's polymorphic category.

(17)   • $np\backslash s$ is a *Everb*;

   • np/s is a *Everb*;

   • If α is a *Everb*, then $np\backslash\alpha$ is a *Everb*;

   • If α is a *Everb*, then α/np is a *Everb*;

   • Nothing else is a *Everb*.

Now, if we give the summativity operator the category of a polymorphic *Everb* modifier, it can combine with a lexical predicate, but also with a lower-arity partially saturated predicate. For instance, if we have a way to combine a subject directly with a transitive verb with category $(np\backslash s)/np$, we can wait till after we do this to apply closure. The derivation in figure 5 shows how we can do this, even when the subject is of a quantifier category. The resulting category is that of a sentence missing an object, to which Σ can apply.

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[9]Since the closure operator is defined to operate on sets of n-tuples, a curried relation must be decurried to combine with it, and then the result must be re-curried. We could of course eliminate these steps by defining closure directly over curried relations.

$$\cfrac{
\cfrac{x\colon np \vdash x\colon np \;\; \text{Ax}}{}
\quad
\cfrac{
\cfrac{
\cfrac{y\colon np \vdash y\colon np \;\; \text{Ax} \quad \cfrac{\phi(y)(x)\colon s \vdash \phi(y)(x)\colon \;\; \text{Ax}}{}}{\lambda w.\phi(w)(x)\colon s/np, y\colon np \vdash \phi(y)(x)\colon s} \;(/\text{L})
\quad
\cfrac{\delta\colon s \vdash \delta\colon s \;\; \text{Ax}}{} \;(\Uparrow \text{L})
}{\lambda w.\phi(w)(x)\colon s/np, Q\colon np \Uparrow s \vdash \delta\colon s}
}{}
}{\text{...}}$$

(Figure 6 derivation tree)

$$x\colon np, \lambda z\lambda w.\phi(w)(z)\colon np\backslash(s/np), Q\colon np \Uparrow s \vdash \delta\colon s \quad (\backslash \text{L})$$

Assoc

$$\cfrac{x\colon np, \phi\colon (np\backslash s)/np, Q\colon np \Uparrow s \vdash \delta\colon s}{\phi\colon (np\backslash s)/np, Q\colon np \Uparrow s \vdash \lambda x.\delta\colon np\backslash s} \;(\backslash \text{R}) \qquad \text{Cut}$$

Figure 6: Transitive verb + quantifier direct object; δ abbreviates $Q(\lambda y.\phi(y)(x))$

Figure 6 shows how an object can also be put together directly with a transitive verb. The step labeled "Assoc" abbreviates the Lambek derivable sequent given in (18).

(18) $\phi\colon (np\backslash s)/np \vdash \lambda z\lambda w.\phi(w)(z)\colon np\backslash(s/np)$.

These derived predicates give us three options for incorporating the summativity operator (in a way to be explained in a moment) to derive four readings for a sentence of the form "$Q_1\colon np \Uparrow s, \phi\colon (np\backslash s)/np, Q_2\colon np \Uparrow s$".

We have a cumulative one in (19a), and the same with reversed scopes in (19b). In these, both quantifiers lie outside the scope of summativity We also have an object dependent reading (19c) and a subject dependent one (19d). This last one is perhaps hardest to get for the relevant sentences, but I do not attempt to account for this asymmetry here.

(19) (a) $Q_1(\lambda x.Q_2(\lambda y.(\Sigma(\phi))(y)(x)))$

  (b) $Q_2(\lambda y.Q_1(\lambda x.(\Sigma(\phi))(y)(x)))$

  (c) $Q_1(\lambda w.(\Sigma(\lambda x.Q_2(\lambda y.\phi(y)(x))))(w))$

  (d) $Q_2(\lambda z.(\Sigma(\lambda y.Q_1(\lambda x.\phi(y)(x))))(z))$

Now we turn to the question of how Σ can get these scopes. Since type-logical grammar is radically lexicalized, the only way to introduce it into a sentence is by association with a lexical item. Since I argued above that summativity should be seen as a property of predication, I propose that it be incorporated lexically as part of the verbal predicate, but in a way that allows it to break free from the predicate. This can be accomplished by taking every lexical predicate we would normally assign some category $\phi\colon C$ and giving it the new category $\langle \phi, \Sigma \rangle\colon C \bullet \triangle(\forall X \leq \textit{Everb})(X/X)$. For instance, figure 7 shows a sample derivation of a (19c)-type object dependent reading for *three lawyers hired a secretary*, i.e. **three**(**law**)$(\lambda z.\Sigma(\lambda x.\mathbf{a}(\mathbf{sec})(\lambda y.\mathbf{hire}(y)(x)))(z))$ (abbreviated γ in the example). Here *hire*,

instead of category **hire**: *tv*, has category $\langle$**hire**, $\Sigma\rangle$: *tv* • ($\forall$X $\leq$ *Everb*)(X/X). The polymorphic part can be dissociated from the predicate as in the first step; since it is marked with $\triangle$, it need not combine directly with *hire*, but can permute to take a different argument as in the next step. When it finds one of some *Everb* category, here a yet-to-be-derived *np\s*, it can lose its $\triangle$, be instantiated to (*np\s*)/(*np\s*), and take the *np\s* as an argument.

7   Conclusion

Summativity has been shown to be a general characterization of the semantics of sentences with plural NPs, from which more specific construals can be derived as special cases. The way summativity is treated in section 3 ties together different traditions of approaching cumulative construals. As shown section 4, letting summativity apply to partially saturated predicates allows quantifier scope ambiguities to be captured in a representation that does not force further disambiguation between subconstruals of the summative one. The fragment in the final sections shows how this flexibility can be tied in a natural way to an explicit syntax.

Although our aim has been to maximize the generality of the analysis, the result may be too general in certain respects. First of all, as noted in footnote 7, the generality of the syntax-semantics interface actually undermines the generality of the semantics by allowing an undesirably specific construal as a combinatoric option. Secondly, as mentioned in section 6, it is not clear that subject dependent readings of the form in (19d) are actually possible. In that case, perhaps they should be ruled out in the syntax-semantics interface itself.

In a related vein, even if we take the most general construal as basic, natural language has mechanisms for forcing stronger readings. For instance, *each* appears to force distributivity, while *together* forces collectivity. It remains to be demonstrated exactly how this strengthening should be incorporated into the analysis given here.

Appendix

12 Proof (of proposition 11)

The definition of $^\star P$ w.r.t. L is equivalent to the definition of the complete sub-semilattice of L generated by P, which is equivalent, in fact for any arbitrary complete join semilattice L, to $\{x \in S \mid$ for some $C \subseteq P, C \neq \emptyset, x = \bigsqcup_L C\}$ (see (Link 1998) p. 364 for proof). Since $L^{(n)}$ is complete join semilattice by lemma 10(i), this holds for $^\star_n R$ as well, so 11(ii) and 11(iii) are clearly equivalent. We now show the equivalence of 11(i) and 11(ii). As a preliminary, notice that we can rewrite 11(i) as 12(i) by introducing explicit universal quantification over the k's (the projections of $\vec{a}$) and reducing the multiple existential quantifications over g_i's to a single existential quantification over the whole n-tuple $\langle g_1, \ldots, g_n\rangle$:

Figure 7: Derivation of the object dependent reading for *three lawyers hired a secretary*; γ abbreviates **three(law)**$(\lambda z.\Sigma(\lambda x.\mathbf{a}(\mathbf{sec})(\lambda y.\mathbf{hire}(y)(x)))(z))$

(i)

$$\mathbf{hire} : tv, \mathbf{a}(\mathbf{sec}) : np_{acc} \Uparrow s \vdash s \vdash \lambda x.\mathbf{a}(\mathbf{sec})(\lambda y.\mathbf{hire}(y)(x)) : np_{nom} \backslash s \quad \text{Prev}$$

(see (ii) below)

$$\mathbf{three}(\mathbf{law}) : np_{nom} \Uparrow s, \Sigma : (np_{nom} \backslash s)/(np_{nom} \backslash s), \lambda x.\mathbf{a}(\mathbf{sec})(\lambda y.\mathbf{hire}(y)(x)) : np_{nom} \backslash s \vdash \gamma : s \quad \text{/L}$$

Cut

(ii)

$$\mathbf{three}(\mathbf{law}) : np_{nom} \Uparrow s, \Sigma : (np_{nom} \backslash s)/(np_{nom} \backslash s), \mathbf{hire} : tv, \mathbf{a}(\mathbf{sec}) : np_{acc} \Uparrow s \vdash \gamma : s \quad \triangle\text{L}$$

$$\mathbf{three}(\mathbf{law}) : np_{nom} \Uparrow s, \Sigma : \triangle((np_{nom} \backslash s)/(np_{nom} \backslash s)), \mathbf{hire} : tv, \Sigma : \triangle((np_{nom} \backslash s)/(np_{nom} \backslash s)), \mathbf{hire} : tv, \mathbf{a}(\mathbf{sec}) : np_{acc} \Uparrow s \vdash \gamma : s \quad \triangle\text{P}$$

$$\mathbf{three}(\mathbf{law}) : np_{nom} \Uparrow s, \langle\mathbf{hire}, \Sigma\rangle : tv \bullet \triangle((np_{nom} \backslash s)/(np_{nom} \backslash s)), \mathbf{a}(\mathbf{sec}) : np_{acc} \Uparrow s \vdash \gamma : s \quad \bullet\text{L}$$

$$\mathbf{three}(\mathbf{law}) : np_{nom} \Uparrow s, \langle\mathbf{hire}, \Sigma\rangle : \forall X < Everb(tv \bullet \triangle(X/X)), \mathbf{a}(\mathbf{sec}) : np_{acc} \Uparrow s \vdash \gamma : s \quad \forall\text{L}$$

(see (iii) below)

$$\lambda x.\mathbf{a}(\mathbf{sec})(\lambda y.\mathbf{hire}(y)(x)) : np_{nom} \backslash s \vdash \lambda x.\mathbf{a}(\mathbf{sec})(\lambda y.\mathbf{hire}(y)(x)) : np_{nom} \backslash s \quad \text{Ax}$$

$$\mathbf{three}(\mathbf{law}) : np_{nom} \Uparrow s, \Sigma(\lambda x.\mathbf{a}(\mathbf{sec})(\lambda y.\mathbf{hire}(y)(x))) : np_{nom} \backslash s \vdash \gamma : s \quad \Uparrow\text{L}$$

$$\mathbf{three}(\mathbf{law}) : np_{nom} \Uparrow s, \Sigma : (np_{nom} \backslash s)/(np_{nom} \backslash s), \lambda x.\mathbf{a}(\mathbf{sec})(\lambda y.\mathbf{hire}(y)(x)) : np_{nom} \backslash s \vdash \gamma : s \quad \text{/L}$$

(iii)

$$z : np_{nom} \vdash z : np_{nom} \quad \text{Ax}$$

$$\Sigma(\lambda x.\mathbf{a}(\mathbf{sec})(\lambda y.\mathbf{hire}(y)(x))) : s \vdash \Sigma(\lambda x.\mathbf{a}(\mathbf{sec})(\lambda y.\mathbf{hire}(y)(x)))(z) : s \quad \text{Ax}$$

$$z : np_{nom}, \Sigma(\lambda x.\mathbf{a}(\mathbf{sec})(\lambda y.\mathbf{hire}(y)(x))) : np_{nom} \backslash s \vdash \Sigma(\lambda x.\mathbf{a}(\mathbf{sec})(\lambda y.\mathbf{hire}(y)(x)))(z) : s \quad \backslash\text{L}$$

$$\mathbf{three}(\mathbf{law})(\lambda z.\Sigma(\lambda x.\mathbf{a}(\mathbf{sec})(\lambda y.\mathbf{hire}(y)(x)))(z)) : s \vdash \gamma : s \quad \text{Ax}$$

$$\mathbf{three}(\mathbf{law}) : np_{nom} \Uparrow s, \Sigma(\lambda x.\mathbf{a}(\mathbf{sec})(\lambda y.\mathbf{hire}(y)(x))) : np_{nom} \backslash s \vdash \gamma : s \quad \Uparrow\text{L}$$

12(i) $(\forall k, 0 < k \leq n)(\forall x_k \sqsubseteq_L^\circ \pi_k(\vec{a}))(\exists \vec{g}_{x_k} \in S^{(n)})$:

$\qquad \pi_1(\vec{g}_{x_k}) \sqsubseteq_L \pi_1(\vec{a}) \wedge \cdots \wedge \pi_n(\vec{g}_{x_k}) \sqsubseteq_L \pi_n(\vec{a})$

$\qquad \wedge \; x_k \sqsubseteq_L \pi_k(\vec{g}_{x_k})$

$\qquad \wedge \; \vec{g}_{x_k} \in R$

Explicitly quantifying over the projections of $\vec{g}_{x_k}$ and $\vec{a}$, we get 12(ii):

12(ii) $(\forall k, 0 < k \leq n)(\forall x_k \sqsubseteq_L^\circ \pi_k(\vec{a}))(\exists \vec{g}_{x_k} \in S^{(n)})$:

$\qquad (\forall i, 0 < i \leq n)(\pi_i(\vec{g}_{x_k}) \sqsubseteq_L \pi_i(\vec{a}))$

$\qquad \wedge \; x_k \sqsubseteq_L \pi_k(\vec{g}_{x_k})$

$\qquad \wedge \; \vec{g}_{x_k} \in R$

We can pull the mention of R up to the quantification over $\vec{g}_{x_k}$. Finally, by the definition of $\sqsubseteq_n$, we can rewrite the second line of 12(ii) as $\vec{g}_{x_k} \sqsubseteq_n \vec{a}$. This yields 12(iii):

12(iii) $(\forall k, 0 < k \leq n)(\forall x_k \sqsubseteq_L^\circ \pi_k(\vec{a}))(\exists \vec{g}_{x_k} \in R)$:

$\qquad \vec{g}_{x_k} \sqsubseteq_n \vec{a} \; \wedge \; x_k \sqsubseteq_L \pi_k(\vec{g}_{x_k})$

11(i) implies 11(ii): Assume 11(i) holds; then 12(iii) above holds. For each $x_k \sqsubseteq_L^\circ \pi_k(\vec{a})$, let $G_{x_k} = \{\vec{g}_{x_k} \in R \mid \vec{g}_{x_k} \sqsubseteq_n \vec{a} \text{ and } x_k \sqsubseteq_L \pi_k(\vec{g}_{x_k})\}$. 12(iii) just tells us that each such G_{x_k} is non-empty. Furthermore, we know by atomicity that for any k ($0 < k \leq n$), there is always such an L-atom $x_k \sqsubseteq_L^\circ \pi_k(\vec{a})$, so there is indeed such a G_{x_k}.

For each k ($0 < k \leq n$), let $D_k = \bigcup_{x_k \sqsubseteq_L^\circ \pi_k(\vec{a})}(G_{x_k})$. Let $C = \bigcup_{0 < k \leq n}(D_k)$. Clearly $C \subseteq R$, since each $G_{x_k} \in R$. We claim C is an $L^{(n)}$-cover of $\vec{a}$.

The non-emptiness condition is straightforward to verify, since each D_k is non-empty. Now we must prove that $\vec{a} = \bigsqcup_n C$.

First of all, we show that $\vec{a}$ is an upper bound for C. By the definition of C, we know that for any $\vec{c} \in C$, there is a k ($0 < k \leq n$) s.t. $\vec{c} \in D_k$. Then, by the definition of D_k, there is an $x_k \sqsubseteq_L^\circ \pi_k(\vec{a})$ s.t. $\vec{c} \in G_{x_k}$; so by the defintion of G_{x_k}, $\vec{c} \sqsubseteq_n \vec{a}$.

Now, to see that $\vec{a}$ is the least upper bound of C, say there were some $\vec{b} \in S^{(n)}$ s.t. for all $\vec{c} \in C$, $\vec{c} \sqsubseteq_n \vec{b}$, but $\vec{a} \not\sqsubseteq_n \vec{b}$. Then by lemma 10(iii) above, there is some $L^{(n)}$-atom $\vec{u}$ s.t. $\vec{u} \sqsubseteq_n^\circ \vec{a}$ and some i ($0 < i \leq n$) s.t. $\pi_i(\vec{u}) \not\sqsubseteq_L \pi_i(\vec{b})$. We will refer to this $\pi_i(\vec{u})$ as u_i. Since $\vec{u} \sqsubseteq_n^\circ \vec{a}$, $u_i \sqsubseteq_L^\circ \pi_i(\vec{a})$ by lemma 10(ii) above. Pick an arbitrary $\vec{g}_{u_i}$ from G_{u_i}. Since $\vec{g}_{u_i} \in C$, $\vec{g}_{u_i} \sqsubseteq_n \vec{b}$ by assumption. This entails that $\pi_i(\vec{g}_{u_i}) \sqsubseteq_L \pi_i(\vec{b})$, by the definition of $\sqsubseteq_n$. However, by the definition of G_{u_i}, we know that $u_i \sqsubseteq_L \pi_i(\vec{g}_{u_i})$, so $u_i \sqsubseteq_L \pi_i(\vec{b})$. But recall that $u_i = \pi_i(\vec{u})$ and $\pi_i(\vec{u}) \not\sqsubseteq_L \pi_i(\vec{b})$. So there is no such $\vec{b}$, so $\vec{a} = \bigsqcup_n C$. Thus C is an $L^{(n)}$-cover of $\vec{a}$; since $C \subseteq R$, 11(ii) is proven.

11(ii) implies 11(i): Assume 11(ii) holds, i.e. that there is some non-empty $C \subseteq R$ s.t. $\bigsqcup_n C = \vec{a}$. Since $\bigsqcup_n C = \vec{a}$, we know by lemma 10(i) that for each i ($0 < i \leq n$), $\pi_i(\vec{a}) = \bigsqcup_L \pi_i[C]$. We now show 12(ii), thereby proving 11(i).

Pick an arbitrary k ($0 < k \leq n$) and assume that $x_k \sqsubseteq_L^\circ \pi_k(\vec{a})$. Then $x_k \sqsubseteq_L^\circ \bigsqcup_L \pi_k[C]$. So by sup-primes, there is a $\vec{g}_{x_k} \in C$ s.t. $x_k \sqsubseteq_L \pi_k(\vec{g}_{x_k})$. Now pick an arbitrary i ($0 < i \leq n$); since $\pi_i(\vec{g}_{x_k}) \in \pi_i[C]$, $\pi_i(\vec{g}_{x_k}) \sqsubseteq_L \bigsqcup_L \pi_i[C]$, i.e. $\pi_i(\vec{g}_{x_k}) \sqsubseteq_L \pi_i(\vec{a})$. Lastly, since $C \subseteq R$, $\vec{g}_{x_k} \in R$. □

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