CORE
Metadata, citation and similar papers at core.ac.uk
Provided by KnowledgeBank at OSU

OC TM

Applying Mathematics Through Floor Plan Design

Chris Bolognese, Upper Arlington High School

Architectural design allows a landscape (literally) for students to learn and apply mathematics. Through floor plan design, students showcase their understanding of an array of topics such as unit conversion, area and volume calculations, and transformations on various curves. This article describes two particular floor plan activities appropriate to two different levels (Geometry and Precalculus) that utilize two free technologies: Google Sketchup *and* Mac Grapher *software. Readers are invited to explore both of these technologies in their own classroom instruction.*

Introduction

The linkage of mathematics to everyday life is a ubiquitous theme in the new Common Core State Standards (2011). Through mathematical modeling, students can apply their mathematical understanding to authentic, real-life contexts. Architectural design offers an avenue that not only encourages student creativity and problem solving, but also is appropriate for a host of grade levels and mathematical content. Presented here are two activities that foster student interest and mathematical understanding that you are encouraged to adapt for your own students.

> *While students were given much freedom in this design process, they were encouraged to include a variety of shapes…*

Geometry: Dream House

My geometry students had just completed a comprehensive unit of the classification of polygons and proceeded to investigate how to determine the area of specific shapes, namely triangles, quadrilaterals, and circles. Their summative task was to design the floor plan of their dream house. Students started by sketching rough drafts of their two-dimensional floor plan. While students were given much freedom in this design process, they were encouraged to include a variety of shapes (triangles, rectangles, kites, trapezoids, parallelograms, sectors) to showcase the extent of their geometric understanding. For their drafts, students were also asked to determine a scale that would be appropriate for their house. This stipulation caused students to unite their design to realistic proportions, either by researching measurements of pre-existing floor plans online or by taking suitable measurements at their own home.

After receiving feedback on their initial designs, students went to the computer lab to recreate their designs in *Google Sketchup*. At no cost, this program is an excellent platform for students to visualize 2-D and 3-D geometric shapes while avoiding tedious geometric constructions. The software allows students to view their constructions at a host of vantage points. To illustrate the versatility of *Sketchup*, three different views of the same cylinder are shown in Figure 1: front, isometric, and top views. Toggling through the options "Camera," "Standard Views," changes the view for the user. Such a range of perspectives facilitates students' ability to reason spatially.

Sketchup has a large number of tools that are very straightforward to learn. In particular, the segment, rectangular, circular, and arc tools allow students to construct many types of regions with ease. The program can detect when parallel lines or perpendicular lines are

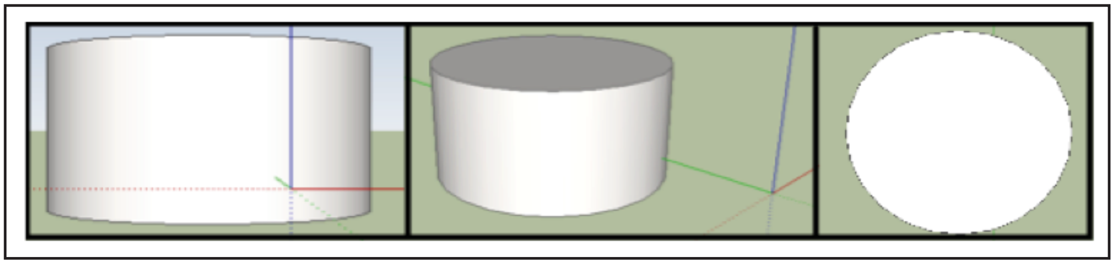


Fig 1 Three views of a cylinder: front view, isometric view, and top view.

being constructed and color codes when the user is in close proximity to one of these special types of lines. These tools also reinforce the definition of a polygon since any inner region turns blue once it is closed. Below is a screen capture of the toolbar with a brief description of some of the commonly used tools. If you are new to *Sketchup*, I invite you to try to construct a region (such as a square, trapezoid, or semicircle) and explore the function of each tool described in Figure 2.

A. **Selection Tool** - Used to select or highlight objects

B. **Line Tool** - Used to draw a segment (or series of segments)

C. **Rectangle Tool** - Used to draw a rectangle. Click once to establish a vertex then click again to establish the opposing vertex

D. **Circle Tool** - Click once to establish the center and click again to establish radius

E. **Arc Tool** - Click twice to establish diameter then drag out until a semi-circle is locked in

F. **Eraser Tool** - Click on a perviously constructed object to erase

G. **Tape Measure Tool** - Click the endpoints of a segment and the measure appears in the measurement box in the lower right corner

H. **Push/Pull Tool** - Click on a 2-D region and pull perpendicular to the plane in which the object lies to form a 3-D solid

I. **Move Tool** - Click and drag to select and move an object

J. **Offset Tool** - Click a region and drag to perform a dilation

K. **Orbit Tool** - Click and drag to rotate the perspective view

L. **Pan Tool** - Click and drag to shift the perspective view

M. **Zoom Tool** - Click and drag to zoom in or out

Fig 2 Commonly used *Sketchup* tools and their functions.

Once students' rooms were constructed, linear dimensions of each room's edges were determined by selecting "Tools," "Dimensions." With this option selected, students simply clicked on a segment and the linear measurement was displayed. An important activity for students was to determine exactly what measurements were necessary in order to calculate the area and perimeter. Some students tended to take redundant measurements while others left out important measures such as the height of a parallelogram or the diagonals of a kite. Finding such requisite measurements reinforced the parameters needed to apply the standard area formulas.

Students can vary some properties of their design such as color or units to personalize their design. While my students were required to use metric, under "Window," "Model Info," "Units," students can change the system and precision of measure. Using the "Text" option under the "Tools" menu, students can create labels for each room.

An important activity for students was to determine exactly what measurements were necessary in order to calculate the area and perimeter.

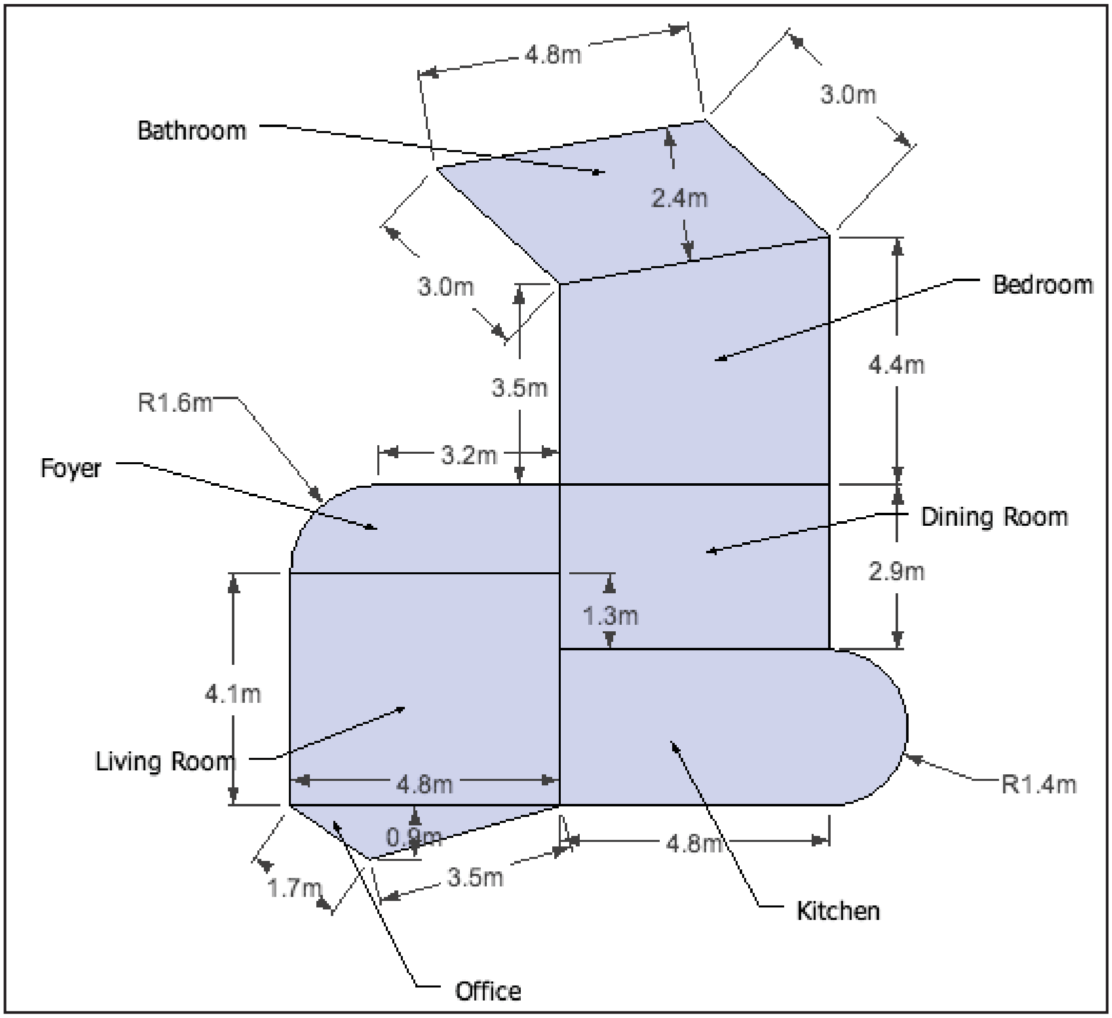


Fig 3 Sample floor plan design with required measurements.

Composite rooms such as the kitchen required students to adjust the traditional formulas to account for multiple shapes

Once their final sketches were complete, students exported the floor plan as a jpeg file to import into a word processing document. An exemplary design is illustrated in Figure 3 incorporating a variety of specific shapes.

Students then applied the perimeter and area formulas to calculate the area of each room and the total perimeter of their house. All work was presented electronically using *Microsoft Equation Editor.* Figure 4 illustrates example area calculations for the same floor plan displayed in Figure 3 created within *Microsoft Equation Editor.* Composite rooms, such as the kitchen, required students to adjust the traditional formulas to account for multiple shapes contributing to one total region. Students could verify their calculations using *Sketchup* as each region's area could be measured directly in the program by selecting "Tools," "Text" and clicking on the region desired. Students would then receive immediate feedback if their calculations agreed with the value displayed by *Sketchup*.

Once the total perimeter and area were computed with appropriate units, the final task was to research a bathroom tile online. Most manufactured tiles are square in shape with length measured in inches. Since linear dimensions in each room were taken in meters, this incongruence of units required students to use dimensional analysis to convert. Students concluded by calculating the number of tiles and their associated cost, including tax.

Although the activities described above kept students engaged, there is a host

Room	Shape	Calculations	Result
Office	Triangle	$A = \frac{1}{2}(4.8)(0.9)$	2.16 m^2
Living Room	Rectangle	$A = (4.8)(4.1)$	19.68 m^2
Foyer	¼ Circle & Rectangle	$A = \frac{1}{4}\pi(1.6)^2 + (3.2)(1.6)$	$0.64\pi + 5.12$ m^2
Kitchen	½ Circle & Rectangle	$A = \frac{1}{2}\pi(1.4)^2 + (4.8)(2.8)$	$0.98\pi + 13.44$ m^2
Dining Room	Rectangle	$A = (4.8)(2.9)$	13.92 m^2
Bedroom	Trapezoid	$A = \frac{1}{2}(3.5 + 4.4)(4.8)$	18.96 m^2
Bathroom	Parallelogram	$A = (4.8)(2.4)$	11.52 m^2
		Exact Total Area =	$84.82 + 11.62\pi$ m^2
		Approximate Total Area ≈	89.91 m^2

Fig 4 Sample area calculations for floor plan in Figure 3 using Microsoft Equation Editor

of other extension activities one could explore. Students could hold a competition based on design aesthetics or efficiency in use of perimeter or area. Also, all designs can be transferred to a 3-D perspective using the "orbit" tool. The "push/pull" tool can then be used to transform regions into solids. Students can then import pre-made furnishings to the house, such as windows or trees. Students can also imbed their 3-D house directly into the terrain on *Google Earth*. *Sketchup* also offers a "walk" option that allows the user to take a digital tour within the house.

In general, my Geometry students were completely engaged in this project as it was authentic and personal. It was helpful that many students had prior experience with the program and those that were inexperienced gained a lot of help from their peers. If you have never used the software before, it is very easy to get accustomed to. There are many tutorial videos on the web, such as "New to *Google Sketchup*" (2011) that you can use to become an expert at the program. I highly encourage you to try to have your students showcase their geometric understanding via this powerful software.

Precalculus: Dark Side Room Design

Motivated by the success of the Dream House project in Geometry, I wanted to create a similar floor plan activity for my Honors Precalculus students. During the unit on conic sections, students were asked to research different engineering applications of the familiar conic section curves: the parabola, the ellipse, and the hyperbola. There are many excellent resources that discuss the real-life applications of these curves, such as Britton (2010).

Uses in telescopes, GPS devices, and medical treatments are all possible due to the simple reflection properties of each curve. Rays emanating from the focus of a parabola reflect as parallel rays off the parabolic surface. In an ellipse, a light ray leaving one focus will always reflect back to the other focus. Lastly, in a hyperbola, light directed toward one focus will be reflected back to the other focus. Such properties can be fully investigated dynamically by using Baker and Wilkinson (2010).

To make use of these properties, students were given the task of creating floor plans of two separate rooms, the perimeter of which consisted solely of the conic curves.

In general, my Geometry students were completely engaged in this project as it was authentic and personal.

Each room was to be "simply connected" (meaning one could walk to any location in the room from any other location in the room) and could only possess one light source. The entire goal of the design was that each room was to have at least one dark spot due to light not being able to naturally access the region. This condition challenged students to completely understand how light rays would interact with each type of curve.

Once students had a basic sketch, they had to determine the equation of each curve in the room including any domain restrictions. This required students to be experts at transformations of the standard conic equations, especially to have curves share a focus or intersect at a desired point. While any form of graphing technology would be suitable, students used the Mac OS X program *Grapher*, available on any Mac under "Utilities" on the hard drive. The benefit of this program is it effortlessly graphs non-function relations, which is particularly useful for conic sections. After strategically placing their light source, students then drew a few sample light rays and their subsequent bounces to illustrate where the naturally dark regions would exist. An exemplary design with the required equations is illustrated in Figure 5. The design shows an elliptical room with two hyperbolic inner walls. The light source is placed at the rightmost focus.

Once students had a basic sketch, they had to determine the equation of each curve in the room including any domain restrictions.

Students concluded their project by writing both a precise description of their design process and an explanation of the functionality of each room. Students then shared their designs electronically for other classmates to see.

Conclusions

I hope these two floor plan activities encourage you to rethink the content you teach and find a way to possibly connect it to a real-life context while integrating the use of powerful software. Such activities allow for rich integration of technology while providing an engaging platform for students to exhibit their mathematical understanding.

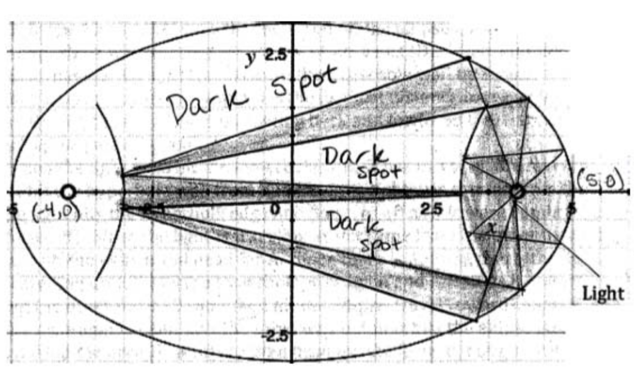
References

Baker, R. & Wilkinson, S. (2010). *Reflective Properties of Conic Sections.* Retrieved January 11, 2011, from http://demonstrations.wolfram.com/ReflectivePropertiesOfConicSections/

Britton, J. (2010). *Occurrence of the Conics.* Retrieved January 10, 2011, from http://britton.disted.camosun.bc.ca/jbconics.htm.

Common Core State Standards. (2011). Retrieved January 10, 2011, from http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf.

New to Google Sketchup. (2011). Retrieved January 10, 2011, from http://*Sketchup.Google*.com/training/videos/new_to_gsu.html



$$Ellipse: \frac{x^2}{25} + \frac{y^2}{9} = 1$$

$$Hyperbola: \frac{x^2}{9} - \frac{y^2}{7} = 1,\ x \in [-3.5, 3.5]$$

Fig 5 An exemplary conic room design with its associated equations

CHRIS BOLOGNESE, cbolognese@uaschools.org, is a mathematics educator at Upper Arlington High School and is the current OCTM newsletter editor. Three of his major areas of interest are mathematics contest problems, mathematics history, and dynamic geometry software. In his spare time, he plays bass for the Columbus-area indie rock band, *The Whiles*.

"Musical training is a more potent instrument than any other, because rhythm and harmony find their way into the inward places of soul, on which they mightily fasten, imparting grace, and making the soul of him who is rightly educated graceful..."
-- Plato