WATER
WATER
WATER
WATER
WATER
WATER
WATER
WATER
WATER
WATER

PROJECTION COMPLETION
REPORT NO. 494X

HIERARCHICAL MODELING FOR THE PLANNING AND MANAGEMENT OF A TOTAL REGIONAL WATER RESOURCE SYSTEM

Joint Consideration of the Supply and Quality of Ground and Surface Water Resources

Yacov Y. Haimes
Professor of Systems Engineering
and Civil Engineering

Case Western Reserve University
Cleveland, Ohio

1976

United States Department
of the Interior

CONTRACT NO.
B-062-OHIO

**State of Ohio
Water Resources Center
Ohio State University**

HIERARCHICAL MODELING FOR THE PLANNING AND MANAGEMENT OF A TOTAL REGIONAL WATER RESOURCE SYSTEM: Joint Consideration of the Supply and Quality of Ground and Surface Water Resources

Submitted to

The Office of Water Research and Technology
U.S. Department of the Interior
Washington, D.C.

and

State of Ohio Water Resources Center
Ohio State University
Columbus, Ohio
Contract No. B-062-OHIO

by

Yacov Y. Haimes
Professor of Systems Engineering and Civil Engineering

Water Resources Program
Systems Engineering Department
Case Institute of Technology
Case Western Reserve University
Cleveland, Ohio

August 1976

PROJECT STAFF

Yacov Y. Haimes, Ph.D.

Principal Investigator

Leon S. Lasdon, Ph.D.

Co-Investigator

Prasanta Das, Ph.D. - *Research Assistant*

Yosef Dreizin, Ph.D. - *Research Assistant*

Hernan Lopez, M.S. - *Research Assistant*

Asok Sarkar, M.S. - *Research Assistant*

Cesar Bocanegra, M.S. - *Research Assistant*

Richard L. Perrine, Ph.D.

Project Consultant

University of California

Los Angeles

ACKNOWLEDGMENTS

We would like to express our gratitude to the following organizations and individuals who have been invaluable in our research on this project. We are grateful to the Office of Water Research and Technology, U.S. Department of the Interior for their support of this study, and to the Ohio State University, Water Resources Center, and in particular Dr. Robert C. Stiefel, Director; and Mr. James F. Tepple, Assistant to the Director, for their assistance and advice. We are grateful to the Miami Conservancy District, Dayton, Ohio and specifically to Messrs. Robert Schroer and Paul Plummer who provided data and significant advice and support; to the staff of the U.S. Geological Survey, Water Resources Division in Reston, Virginia, and particularly Drs. Nicholas C. Matalas and Thomas Maddock, III, for their comments, suggestions and contributions to this project. The collaboration and continuous exchange of information with the staff of the Water Resources Division, U.S.G.S. have significantly helped us on this project. Finally, we would like to thank the faculty members at Case Western Reserve University who served on the thesis committees of the graduate students involved in this project and the staff of the Water Resources Group. In particular to Marj Weber for her editorial assistance and Chris Lorenzo for her secretarial assistance.

EXECUTIVE SUMMARY

This research offers a new approach to the planning and management of complex, large-scale water resources systems. It utilizes the concepts and methodologies from systems engineering theory for the advanced structuring, formulating and solving of mathematical models. These models are aimed at the profound analysis of short- and long-term planning aspects of water resources.

A planning and management methodology for a regional water quality control is presented. The planning framework is developed based on a multiobjecitve analysis in order to take into consideration the conflicting objectives of surface water quality and the cost of expansion and operation of wastewater treatment plants (both secondary and tertiary). Multiobjective analysis in water resources systems has become particularly important in the context of the federal principles and standards for the planning of water and land resources. The objective of the guidelines is to place environmental concerns on a basis equal to economic development.

A regional water resources system may be a complex, large-scale system and may include many elements. In this study, the components included are ground and surface water and wastewater treatment plants.

The water quality objectives represent the levels of water quality parameters in different segments of the stream, over the entire planning horizon. The resulting levels of pollutants

depend on the net effluent discharges of various pollutants under consideration, as well as on the hydrologic characteristics of the stream.

Since the cost objective is in terms of dollars, while the water quality objectives are in terms of the pollutant levels (concentration), these objectives are noncommensurable, and a multiobjective optimization approach is desirable. The decision-maker is an individual or an agency who desires to simultaneously minimize the cost of wastewater treatment, along with the levels of water quality parameters.

A nonlinear programming is employed to determine the optimal schedule of construction and/or expansion of secondary and tertiary precesses at each plant location, meeting estimated effluent discharge levels at minimum present value cost. The cost function includes capital cost of secondary and tertiary units and variable operating cost of each process.

Water quality objectives represent the level of pollutant parameters (or other indicators) in the stream reaches over the planning period, and are developed by using a mass balance equation for conservative pollutants and the Streeter-Phelps equation for nonconservative pollutants. Two additional indices of assurance of satisfying the quality objectives and violation norm are also developed.

The cost and quality objectives are integrated to form a multiobjective planning problem. With cost as the primary objective and water quality as secondary objectives, the latter objectives are reformulated in the epsilon-constraints form. The epsilon-constraint problem is solved for different levels of pollutants in the stream, corresponding to different discharge policies. The non-inferior solutions, including the trade-offs along with optimal cost and corresponding levels of achievement of each objective may be submitted to the decision-maker for his evaluation of the Surrogate Worth function. Preferred solutions are obtained by satisfying the optimality criteria of the Surrogate Worth Trade-off method. The above developments are presented in Chapter 2 and Chapter 3.

Chapters 4 to 8 are devoted to a comprehensive modeling of a groundwater system, and to developing planning and management methodologies for efficient use of groundwater in general and conjunctive management of ground and surface water in particular. Both short- and long-term planning models of ground and surface water use are presented. In particular, it suggests procedures and methodologies for a comprehensive mathematical analysis of hydraulically connected multi-cell aquifer and multi-stream systems. The models consist of hierarchies of response functions relating the system's response to various activities affecting it.

Appropriate response functions are developed which exclusively allow for coupling a complex, large-scale water resources system with a management model. This is an appreciable step ahead in the state-of-the-art of analyzing conjunctive use of ground and surface water resources and is a major contribution of this study.

In Phases I and II of a previous related study, groundwater parameter identification models are developed and their usefulness is demonstrated. However, in those studies unknown parameters were assumed to be a continuous function of space, without taking into account the heterogeneous property of most aquifers. In this study an approach is adopted which takes into consideration the distributed nature of aquifer properties, by decomposing them into various cells whose geometric configurations are selected according to the geological characteristics of the aquifer. A sensitivity analysis of model output for errors introduced by input data and parameters is also carried out.

The multi-cell particular cell simulation procedure is discussed in Chapter 4 of this report. It provides the construction of mathematical models for numerically solving complex groundwater systems. The basic idea used is to decompose the system into a number of cells according to certain considerations. These considerations may involve geographical, geological and hydrological characteristics; administrative and operational judgments; or any other requirements associated with the particular need for the groundwater simulation model. The multicell mathematical model is used to approximate cells' boundary conditions associated with a given stress. These boundary conditions are used to isolate each particular cell's mathematical model. The following advantages are realized:

(1) The proposed procedure allows for applying mathematical simulation models to a large-scale and complex system, where the application of a regular compact simulation model on a digital computer is evidently inadequate.

(2) The restriction of computer capacity often needed in simulating a large aquifer system is best overcome by decomposing the model.

(3) The proposed procedure is evidently advantageous in cases where the interest is directed toward an isolated subsystem for a particular response. The modeling efforts can concentrate on the particular subsystem cell, while the rest of the system is accounted for through the aggregated multicell model.

(4) Data acquisition efforts are directed by the model's needs. This is an important factor in evaluating the model.

(5) The flexibility of the model's structure is an appreciable advantage in particular if an administrative scheme is considered. This characteristic is well illustrated by applying the management model to the tax-quota system in Chapter 8.

(6) Most developments later discussed are essentially based on the availability of the decomposed aquifer simulation model. It allows for production of response functions under any desired hierarchy.

The importance of the algebraic technological functions (A.T.F.) in a linear system is realized when the coupling of the physical system with a management framework is desired. Some real and meaningful advantages are associated with the hierarchy of the response functions as described below:

(1) It provides the systems analyst with a methodology by which to handle a large-scale and complex groundwater system within a management framework. The response functions superposition may be easily constructed in agreement with administrative or other considerations, not restricting the management model formulation.

(2) The amount of preparation work associated with the production of response functions for later use in management model formualtion is considerably reduced.

(3) If a large number of wells is considered in a management model, then the associated response functions matrices require an extensive computer capacity unless a certain weighting of the response is applied. This is possible via the proposed technique.

The stream-aquifer interactions add a most important aspect to this research. An important contribution is the analysis which considers a multi-stream system interacting with a complex groundwater system. Of particular interest is the superposition of functions relating infiltration from different streams to different aquifer cells. It provides a new analytical tool for coupling infiltration from a stream with management framework. The A.T.F. and the stream-aquifer response functions combined in the form developed in this study are the basis for analyzing a complex water resources system within a management framework.

The management model development and analyis presented in Chapter 6 constitutes a major contribution of this study. The quantitative analysis is made possible by utilizing the mathematical models previously developed. The following aspects are actually appreciated:

(1) The analysis provides a full demonstration of the advantages associated with previous developments in application to water resources management model formulation and solution perspective.

(2) An important contribution is made to the analysis of conjunctive use of ground and surface water systems. The proposed model is a first step in taking into account the distributed parameter characteristics of the systems involved in a water resources management model formulation.

TABLE OF CONTENTS

TABLE OF CONTENTS (continued)

TABLE OF CONTENTS (continued)

LIST OF FIGURES

LIST OF FIGURES (continued)

LIST OF TABLES

LIST OF TABLES (continued)

LIST OF TABLES (continued)

CHAPTER 1

INTRODUCTION

1.1 PREFACE

The population growth around the world, and the increased industrial activities and dependence on food and fibre production, have caused a critical demand for water and land resources. The increase in population places an increasing demand on municipal water consumption, requires greater facilities for water and land-based recreation facilities. At the same time, growing industrial and agricultural activities demand more water for industrial uses and for irrigation.

Effluent discharges from industrial wastewater and municipal sewage treatment plants into the streams and lakes often degrade the quality of water. Poor water quality may be unsuitable for recreation, fishing, and other nonwithdrawal uses; it may be harmful to fish and other aquatic life. Natural runoff from urban and agricultural lands also carries several polluting substances, including sediment, phosphorus, nitrogen, and other nutrients into the stream. Consequently, the problems of water quantity and water quality and interrelated.

In order to meet the growing demand for water in industrial, municipal, agricultural and other uses, expenditures must be made in construction, operation and maintenance of supply projects, such as reservoirs, dams, groundwater pumpage, desalination

plants, and distribution systems, including aquaducts, pipelines, and canals.

In order to maintain the given levels of water quality, investment in the construction of new wastewater treatment plants, expansion of existing plants , and operation of those plants must be made.

The economic development and environmental quality are thus in conflict, and often in competition with each other. An improvement in environmental quality may only be achieved at the expense of investing more in building wastewater treatment plants and applying appropriate waste treatment. A recent upsurge of public concern has resulted in redefining the federal guidlines for the future development of water and land resources [Federal Register, 1973].

The "principle and standards" for the planning of water and related land resources prepared pursuant to the Water Resources Planning Act of 1965 (Public Law 89-80) shows a considerable departure from past planning standards [Federal Register, 1973]. The objective is to place environmental concerns on a basis equal to economic development. The national economic development objective is reflected in an increase in the value of the nation's output of goods and services and an improvement in national economic efficiency. The environmental quality objective is reflected in the management, conservation, preservation, restoration, or improvement of the quality of natural and ecological resources.

Extensive research has been done on the problem of the conjunctive use of ground and surface water. A substantial portion of the United States' water supply comes from groundwater sources. Many other countries have found that using aquifers in conjunction with the available surface water has been an important factor in their development. This observation is even more important throughout most arid and semiarid regions. When water resources are a limiting factor in the development of a region, then their optimum utilization is society's main concern. Very sophisticated methods have been developed and successfully applied for the optimal planning, constructing, operating and controlling of surface water systems. This is due to the obvious desire to use extensively these most available, at-hand sources. The physics of runoffs, rainfall and streamflow, the mass-balance equations considering reservoirs, and the multipurpose surface water projects all are relatively well-developed and known. Full utilization of this knowledge paved the way to many excellent mathematical models aimed at the optimal solution to surface water problems. On the other hand, models on groundwater, to the extent that they have been developed to date, do not yet fully control this very important resource. This is due to the complicated physics associated with the law of flow in porous media. Scarce data raise the problem of error in identifying mathematical model parameters when such a model is assumed to approximate an actual system. As opposed to surface water systems, some elements essential to groundwater structure may hardly be measurable or even known, hence the problem of validating the model. Following the present line of evaluation of the world's resources scarcity, groundwater systems are limited absolutely, but unfortunately in too many cases are only partially and inefficiently utilized. The main reason for such neglect is the insufficient grounds for accurate planning and efficient operation. This is why so many recent studies analyzing water resources are devoted to the management of groundwater systems. These works are aimed at better using available water through optimal planning and operation. However, mathematical models resulting from to date studies are found to be limited in their applicability. A main reason for this is the complexity and the dimensionality associated with problems involving a distributed parameter groundwater system control. In many cases models are impractical because of certain simplifications assumed actually making the model unrealistic; or, being close to approximate reality, the mathematical formulation indices a substantial dimensionality limitation preventing the model from being applied to a real complex and large-scale system.

In the following discussion we frequently refer to the terms "complex" and "large-scale" systems. By "complex" we mean to include in the analysis non-homogeneous distributed parameter systems. The distribution is over time and space with irregular in shape boundaries. This is particularly true in groundwater systems. The "complexity" is even more severe if the system interacts with other physical systems such as surface water streams and reservoirs. Also coupling the physical considerations with administrative framework introduces more aspects making the system "complex".

The term "large-scale" is used to emphasize the involvement of large number of decisions, state variables, constraints and input-output relations in the model. It also means that various kinds of functional relations are associated with the modeled system.

Large-scale and complex groundwater system is therefore an aquifer system underlying a large area. Many different activities affect the system and are affected by it. To analyze such a system one must consider more than one functional associated with it, (hence the need for coordination between the various functions, or possibly multiobjective framework). Both space (number of wells) and time (planning horizon) play an essential role in the system's dimensionality.

The goal of this research is to develop an overall mathematical model made up of a hierarchy of submodels. A hierarchy of water quality submodels along with pollution control cost model are integrated in order to analyze and long-range planning for the Basin's surface water quality. Also developed are submodels that can be used as tools to analyse and plan the conjunctive use of ground and surface water resources.

1.2 MOTIVATION AND OBJECTIVE

The goal of this research is to develop long-term planning and management framework for a complex and large-scale water resources system. The development of planning objectives is carried out on a regional basis. The tendency towards regionalization of water resources development and control is due to a number of factors, such as economies of scale, access to advanced technology, hydrologic boundaries, jurisdictional power, comples network of water and land resources, etc., [Haimes and Macko, 1973]. The hydrologic boundaries of a watershet often extend beyond an area of local jurisdiction. Thus, a realistic planning framework may be developed and implemented when it is carried out on a regional basis.

Major efforts of this research may be attributed to the: (i) surface water quality control and management; (ii) groundwater response analysis; and (iii) conjunctive management of ground and surface water resources.

The groundwater system is represented in an analytical form. This enables one to model the response to both an imposed input and surface and groundwater interactions. By modifying recent developments in the field of groundwater management and using large-scale systems methods we have appreciably improved the state-of-the-art of using ground and surface water conjunctively. The final product comprises a step-by-step procedure, through which the optimal operation control of a large-scale and complex groundwater system, with or without a conjunctive surface water system, may be successfully achieved. The drawback associated with previous studies dealing with this same problem is considerably reduced. The well-established procedure should provide the implementation of a profound analysis for the benefit of water resources planning and operation.

In this research considerable effort is devoted to the integration of the mathematical models related to each planning objective. Some of the mathematical models available from earlier work in the field are modified and extended for that purpose.

The expansion and/or construction schedules of wastewater treatment plants and their operating policies must be determined for the entire planning period so that the water quality standards in the stream can be satisfied. The expansion and construction of secondary as well as tertiary treatment plants are considered in recognition of the Clean Water Act as amended in 1972. The future wastewater load at each plant is expected to increase due to population growth and increased industrial activities in the region. It is assumed that information on the wastewater load for the planning period is available and thus treated as a parameter in the wastewater treatment problem. The point source pollutants considered for the study include BOD and DO deficit levels in the stream.

Since the surface water quality objectives are noncommensurable to one another and to the economic objective, a multiobjective planning framework is applied. The Surrogate Worth Trade-off (SWT) method is utilized for this purpose [Haimes, Hall and Freedman, 1975].

A major part of this study's work was done under the project titled "Integrated System Identification and Optimization for Conjunctive Use of Ground and Surface Water," Phases I, II, and III, supported by the Office of Water Research and Technology, U.S. Department of the Interior, Washington, D.C. Full cooperation from the engineers of the Miami Conservancy District (MCD), Dayton, Ohio, provided us with a full-sized case study, to which most of the research results could be successfully applied and verified. Some of this study's contributions were used directly by the district.

The planning for groundwater use or the conjunctive use of ground and surface water can be efficiently achieved only when the state of groundwater levels in the basin is accurately known, and are explicitly coupled to the management and planning optimization model. The groundwater response model is coupled with the management model by developing algebraic technological functions. These functions should approximate the groundwater system to be coupled with a desired control scheme, taking explicitly into account most elements affecting the system. In dealing with a large-scale and complex aquifer system, the first step is to construct a mathematical model which is assumed to approximate the real system. A new procedure for that purpose is developed by decomposing the mathematical model into so-called multicell-particular cell models. This proves to be of great advantage, especially for large-scale, complex groundwater aquifer systems [Haimes, 1976].

CHAPTER 2

MODELING OF A COMPLEX SYSTEM OF SURFACE WATER POLLUTION CONTROL AND MANAGEMENT

2.1 WASTEWATER TREATMENT PLANT COST MODEL

2.1.1 Introduction

In this chapter the planning, operation, and expansion of a regional wastewater treatment plant management system is considered. The management model is designed for a region consisting of industries and cities near the stream where the river system is the main receiver of all treatment plant effluents. A dynamic planning model is considered in response to continued growth of waste production due to population and industrial growth in the region. It is assumed that there exist a number of wastewater treatment plants along the river. The objective of this dynamic planning model is to determine the most economical expansion schedule for these plants so that the increasing demand for wastewater treatment may be satisfied. The economic expansion schedule includes such factors as expansion capacity of each plant and the time of its expansion.

Many existing systems for management of water quality have grown more or less haphazardly, with extensions added to meet current exigencies, but without integrated plans for long-term development. Some planned development for water quality management has been attempted, but probably in only a few cases has an attempt

been made to fully deploy the techniques of systems analysis. It is the intention here to present a systematic approach to the water quality management of a river basin by applying an optimization model which integrates the cost of expansion and operation of a series of wastewater treatment plants with different water quality standards for a set of pollution constituents. The maximum tolerable level of pollutants in the stream not only depends on the specific utilization of stream water, but on various other factors. With increased affluence, there may be a public demand for a cleaner stream with higher quality standards, that is, reduced permissible waste loading. But also with the realization of high costs, there may be a demand for less stringent standards and higher permissible loading.

The cost of meeting the Clean Water Act of 1972 requiring communities to apply "best available technology" in wastewater treatment by 1983 is estimated to be $467 billion, [National Water Commision, 1973]. This is more than double the costs required to meet the water quality standards established under the Water Quality Act of 1965. The implementation of a true "no-discharge" policy by 1985, provided by the Clean Water Act, may even cost much more, if it is at all attainable. The basin wide wastewater treatment plants model developed in this chapter is able to examine the net savings in cost by gradually improving the water quality standards by imposing stricter effluent discharge standards, instead of meeting a "zero discharge" policy by 1985. The model is capable of analyzing the needed treatment efficiencies, operating levels and expansions of both secondary and tertiary treatment units, based on the net wastewater load and a net effluent discharge policy. By considering the secondary and tertiary treatment facilities as independent units, connected only by transport links, it is possible to apply appropriate cost functions for expansion as well as operation of these units, depending on the kind of treatment process used. Also, since any waste load entering the tertiary plant must have undergone secondary treatment, an incremental expansion and operational cost function (i.e., excess cost after secondary treatment) can be chosen appropriately.

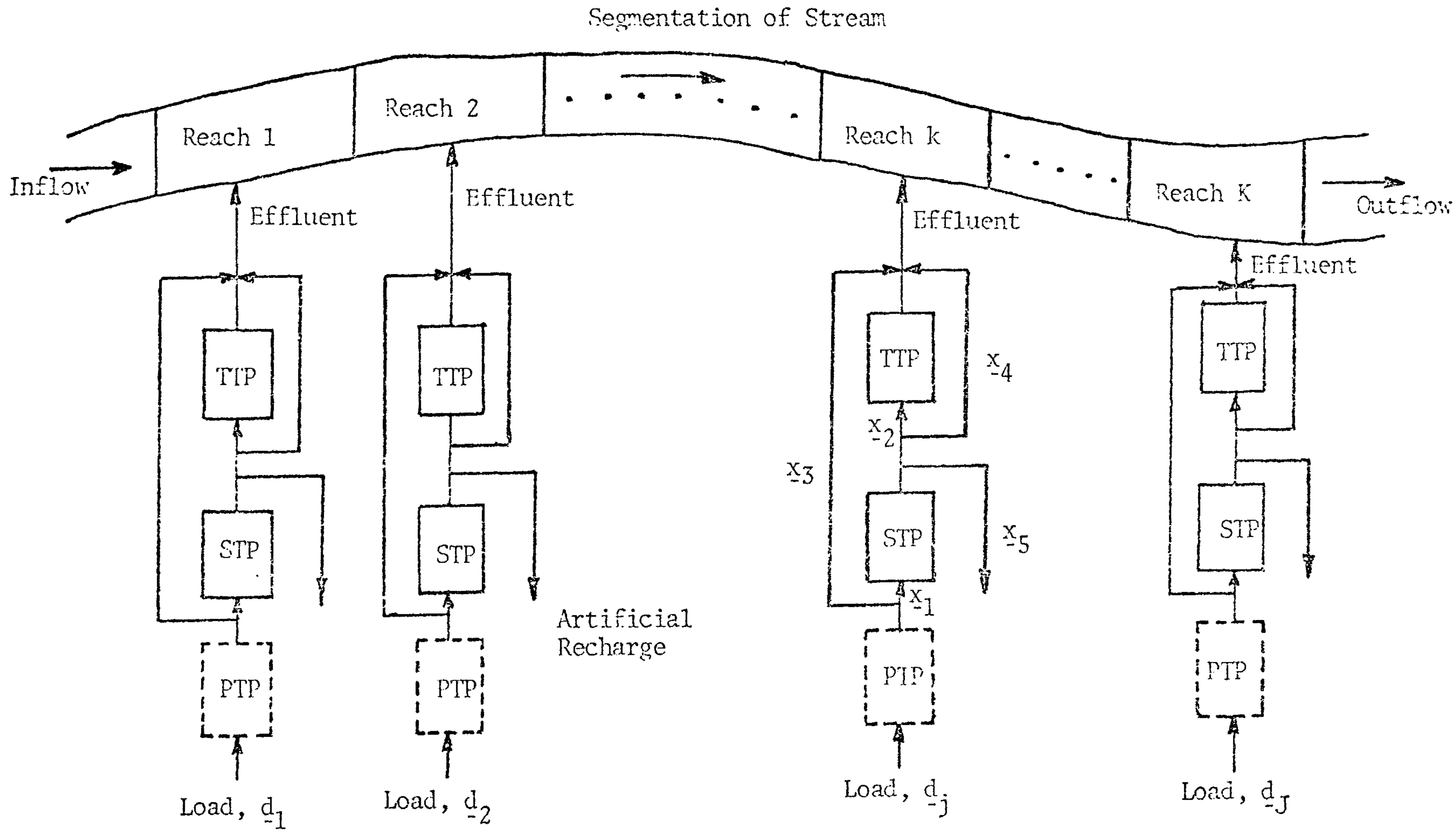
The reuse of wastewater as a supplemental source of water has long been recognized by many, such as Parizek et al [1968], and Sopper [1968]. The decision however, as to the proper use of waste effluent must be based on the relationship between water management and the available water supplies of the region. Artificial recharge of groundwater is primarily practiced as a way of conserving groundwater resources. A natural extension of this practice is to reuse treated wastewater for artificial groundwater recharge. Owen [1968] and Sopper [1968] indicated that a feasible method of wastewater renovation for reuse would be to apply partially treated wastewater to the land whereby it undergoes natural filtration through soil and finally recharges the groundwater system. The use of treated wastewater effluent is a relatively recent development in the United States. In

1970, Todd [1970] reported that 400 cities in the U. S. were using treated effluent for deliberate recharge of groundwater resources. It is evident that a coordinated use of waste effluent for artificial groundwater recharge will have a beneficial effect on the water table level. The extent of its use is determined by considering such factors as magnitude of water demand in a planning area, availability of other sources of water, and the economic trade-off between cost incurred in advanced treatment of additional wastewater and the cost of groundwater recharge. Wastewater reclamation and reuse through groundwater recharge is considered as a supply source in this model.

2.1.2 Mathematical Model Formulation

The stream we are concerned with is segmented into K number of reaches. A typical reach is denoted by a subscript k, where $k = 1,2,\ldots,K$. Let the number of locations along the river system where the treatment plants are located be J, where a typical wastewater treatment plant location is denoted by the subscript j, $j = 1,2,\ldots,J$. A particular reach may include multiple wastewater plants, depending on the number of reaches chosen and their length. However, there may be reaches without any treatment plants and subsequently there is no effluent discharge into those reaches. The hypothetical boundaries of stream reaches are drawn, taking into account such factors as location of plants, wasteload generation, hydrologic characteristics, existing water quality, as well as tributary flow rates. When effluents from more than one plant discharge into any reach k, the number of plants discharging into the k^{th} reach is represented by the subscript j_k. The configuration of the treatment plants is shown in Figure 2.1. The plants at each location j consist of secondary and tertiary treatment processes. These two processes are independent, except that they are interconnected through flow variables. It is assumed that raw wastewater load at each plant must undergo at least primary treatment. The configuration of plant facilities considered allows us several alternatives in connection with wastewater flow. The effluent volume from a primary treatment facility may be subjected to secondary treatment, it may be discharged directly into the stream, or a portion of it may be further treated in a secondary treatment facility and the rest discharged into the stream. The net effluent volume from the secondary treatment plant may be further subjected to three different alternatives; it can be transported to a tertiary treatment plant for further pollutant removal; discharged into the stream, or utilized for groundwater recharge. The decision variables, constraints and objective functions are introduced next.

The decisions at both the secondary and tertiary treatment plants include whether and how to schedule construction and expansion of the individual unit and what are their operational policies. The operational policies include how much wastewater can be treated in secondary and tertiary units, how much secondary effluent to utilize for artificial groundwater recharge, and what



PTP = Primary Treatment Plant
STP = Secondary Treatment Plant
TTP = Tertiary Treatment Plant

Figure 2.1. Wastewater Treatment Plants Configuration

will be the effluent load discharged into the stream reach. The net effluent discharge into the stream is dependent on the water quality requirements of the stream. Let q_{1jn} represent the capacity expansion of the j^{th} secondary treatment plant in the planning period n, where j = 1,2,...,J, and n = 1,2,...,N. Similarly, let q_{2jn} be the capacity expansion of the j^{th} tertiary treatment plant in the planning period n. Other decision variables are related to the operational level of the plants. Let x_{1jn} be the quantity (million gallons per day) wastewater load treated in the secondary plant j during a period n, where j = 1,2,...,J and n = 1,2,...,N. Similarly, the operating level, or the quantity of secondary effluent subjected to further treatment in the tertiary treatment plant at location j during a period n is denoted by x_{2jn}. It is assumed that the wastewater load curve at each plant location for the entire planning period is known and hence treated as exogeneous variables. The increased population in urban areas over the planning period results in an increased wastewater load in the municipal plants. For the analysis of the case study, population projection of OBER's Series E is used to determine the increase in wastewater demand over the planning period. Similarly, for the industrial plants, an increase in industrial activities places an increasing waste load at each industrial plant.

Let the demand function at plant location j over the planning period be denoted by a vector $\underline{d}_j$, where $\underline{d}_j$ is a (Nx1) dimensional column vector,

$$\underline{d}_j = \begin{bmatrix} d_{j1} \\ d_{j2} \\ \vdots \\ d_{jn} \\ \vdots \\ d_{jN} \end{bmatrix}$$

where d_{jn} represents the wastewater load at j^{th} plant during a time interval n. Due to the general configuration of the plants, several other decision variables must be defined. The wastewater load with primary treatment either enters the secondary treatment plant or a fraction of the load can be transported to the point at which it is discharged into the stream reach. Let x_{3jn} be the volume of wastewater (in millions of gallons per day) after primary treatment discharging directly into the stream reach from j^{th} plant during a planning period of n, $j = 1,2,\ldots,J$ and $n = 1,2,\ldots,N$. The effluent from the secondary treatment plant is again subjected to several alternatives. The secondary effluent may be treated further in a tertiary plant for advanced removal, or it may be discharged into the stream. Also the secondary effluent can be reused as a supply source for groundwater recharge.

Let x_{4jn} represent the quantity of secondary effluent (million gallons per day) discharged directly into the receiving water body from j^{th} plant during n^{th} time interval without further treatment, whereas x_{5jn} is the amount of secondary effluent reclaimed for groundwater recharge from the plant j, in the period n, where $j = 1,2,\ldots J$; $n = 1,2,\ldots,N$. The decisions relating to treatment levels in the secondary and the tertiary plants are defined next.

Let z_{1jn} be the percentage removal of biological oxygen demanding (BOD) load in the secondary treatment plant at location j during period n. Similarly, let z_{2jn} represent the BOD removal efficiency in the tertiary treatment plant at location j during period n, whereas z_{3jn} is the phosphorus removal efficiency in tertiary plant j during the n^{th} period, for all $j = 1,2,\ldots,J$, and $n = 1,2,\ldots,N$.

The purpose of the model as stated already is to determine the minimum cost of expanding, operating and maintaining wastewater treatment plants consisting of secondary process, tertiary process, and a provision for reusing treated effluent as an indirect supply source through groundwater recharge. By considering the secondary and tertiary plants separately, it is possible to apply appropriate cost functions for expansion as well as operation and maintenance of the individual processes in the plants. The allocation of the waste load at secondary and tertiary treatment plants is determined in the optimization process by four important determinants:

(i) Cost of activities in the secondary units,

(ii) Cost of activities in the tertiary units,

(iii) Water quality requirements in the surface stream,

(iv) Cost of groundwater recharge by secondary effluent.

The costs of wastewater treatment plants are quantified, based on the capital cost functions associated to various treatment processes developed by Smith [1968]. Frankel [1965] presented a series of treatment processes and the biological oxygen demand (BOD) load removal efficiency of each of these processes. It is shown that with a high-rate trickling filters process in the secondary plant for different loading parameters, a BOD removal efficiency of as high as 85% can be achieved. In the tertiary treatment plant, clorination and chemical precipitation can be used which is capable of removing up to 99% of the BOD load. Clorination and chemical precipitation is applicable only after the wastewater has been treated for secondary removal. Thus an incremental cost for tertiary treatment, over and above the secondary treatment is considered. The specific tertiary costs of interest are those of phosphorus and BOD removal, representing an additional requirement for a given basic facility. The capital cost functions for both secondary and tertiary treatment plants' expansions are represented as functions of installed capacity. Smith [1968] presented an exponential type capital cost function for both secondary and tertiary treatment plants. The expansion cost shows economics of scale, which means that each additional unit of capacity is less costly than the previous one. In other words, average per unit cost for a bigger plant is less than that for a smaller sized plant. Deredec [1972] and Michel [1970] also showed that the capital cost of a wastewater treatment plant shows economies of scale with increasing capacity, hence, an exponential cost function is a good representation of plant expansion. The cost functions presented by Smith [1968] are in 1968 dollars. Kaplan [1975] converted these costs to 1975 dollars using the EPA sewage treatment Plant Construction Cost Index [Engineering News-Record, 1975].

The functional representation of capital costs for secondary and tertiary treatment plants are:

$$\phi_1^s(q_{1jn}) = A_1[q_{1jn}]^{\alpha_1} \qquad 0 < \alpha_1 < 1 \tag{2.1}$$

$$\phi_1^t(q_{2jn}) = A_2[q_{2jn}]^{\alpha_2} \qquad 0 < \alpha_2 < 1 \tag{2.2}$$

$$j = 1,2,\ldots,J; \quad n = 1,2,\ldots,N$$

where, q_{1jn} and q_{2jn} represent the expansion of secondary and tertiary treatment facilities respectively in the j^{th} plant location, during the n^{th} period; $\phi_1^s(q_{1jn})$ represents the fixed cost function for secondary treatment plant expansion, whereas $\phi_1^t(q_{2jn})$ is the incremental cost of tertiary treatment plant expansion. The exponents α_1 and α_2 are greater than zero but less than one, indicating the existence of economies of scale.

The operational variable cost depends primarily on the operating level or the amount of wastewater (millions of gallons per day) treated, and the percentage removal or the treatment efficiency. A great number of studies have examined the operating and maintenance (O&M) costs of wastewater treatment plants of various capacities and treatment levels. Michel [1970] estimated the operation, maintenance and replacement costs due to labor, chemical and electrical power costs, according to plant size and treatment process. Smith [1968], Shah and Reid [1970] examined the O&M costs of wastewater treatment plants for different plant sizes and treatment levels. Shah and Reid [1970] developed a cost function by multiple regression analysis where key variables were population, flow rates, and plant efficiencies.

The annual operating and maintenance cost functions are developed by using the data presented by Frankel [1965] for various levels of flow treated and the treatment levels. Data compiled by Frankel [1965] indicates that if the treatment level is greater than 45% (equivalent to primary removal), then the operations and maintenance cost in a secondary plant is independent of the treatment level but vary linearly with the volume of wastewater treated, whereas the O&M cost for tertiary treatment depends both on the quantity of wastewater flow and the level of treatment. Assuming a treatment level greater than 85%, the operation and maintenance cost for tertiary treatment can be presented by a quadratic function of treatment level. The cost functions can be quantified in the functional form as follows:

$$\phi_2^s(x_{1jn}, z_{1jn}) = a_0 + a_1 x_{1jn} \tag{2.3}$$

$$0.45 \leq z_{1jn} \leq .85$$

$$\phi_2^t(x_{2jn}, z_{2jn}) = b_0 + b_1 x_{2jn} + b_2(z_{2jn} - 0.85)^2$$

$$+ b_3 x_{2jn}(z_{2jn} - 0.85)^2 \tag{2.4}$$

$$0.85 \leq z_{2jn} \leq 1.0$$

where $\phi_2^s(x_{1jn}, z_{2jn})$ is the annual O&M cost for secondary treatment, for a flow treated of x_{1jn} MGD, and a treatment level of z_{1jn}. The subscript j and n are used to identify the plant and the period of analysis. The O&M cost function for tertiary treatment is $\phi_2^t(x_{2jn}, z_{2jn})$, where x_{2jn} is the flow treated and z_{2jn} represents the treatment level in the tertiary plant. Again the subscript j indicates the plant, and n is used for the period of analysis, $j = 1,2,\ldots,J$ and $n = 1,2,\ldots,N$. The values of the coefficients a_0, a_1, b_0, b_1, b_2 and b_3 are determined by regression analysis.

For the secondary plant, O&M cost function is obtained by performing a regression analysis with flow rate (in MGD) as independent variable and cost as dependent variable. Similarly, for tertiary treatment plant, O&M cost function developed is quadratic in the treatment level and linear in the plant size. Again the operating costs presented in (4.3)-(4.4) using Frankel's data is in 1965 dollars. The Wholesale Price Indices for Electric, Power, Chemicals and Allied Products, and Industrial Commodities [U.S. Department of Labor, Bureau of Labor Statistics, 1975] are employed to represent the cost functions in 1975 dollars.

These costs enter into the cost objective only when the wastewater load at a particular plant location contains the pollutant under investigation. The annual operating cost at a given plant is then simply the sum of the annual operating costs of the various pollutant removal at that particular plant. The total cost in wastewater treatment plant model is then the sum of the plants' expansion, as well as operation and maintenance costs of all plants over the planning period.

Let $\hat{f}_2(q_1, q_2, \underline{x}, \underline{z})$ represent the total present value cost of wastewater treatment plant expansion, operation-and-maintenance, of all plants for the entire planning horizon. For simplicity, vector notation is used wherever convenient. The variables are defined next.

Let q_1 be a (NJx1) column vector of the secondary treatment plants' expansion capacities over the entire planning period. Thus,

$$q_1 = \begin{bmatrix} q_{11} \\ \hline q_{12} \\ \hline \vdots \\ \hline q_{1j} \\ \hline \vdots \\ \hline q_{1J} \end{bmatrix}$$

where $\underline{q}_{ij}$ is represented as a (Nx1) dimensional column vector as:

$$\underline{q}_{1j} = \begin{bmatrix} q_{ij1} \\ q_{1j2} \\ \vdots \\ q_{1jn} \\ \vdots \\ q_{1jN} \end{bmatrix}$$

For the tertiary treatment plants, $\underline{q}_2$ is a (NJx1) column vector of expansion capacities over the planning period. Hence,

$$\underline{q}_2 = \begin{bmatrix} \underline{q}_{21} \\ \hdashline \underline{q}_{22} \\ \hdashline \vdots \\ \hdashline \underline{q}_{2j} \\ \hdashline \vdots \\ \hdashline \underline{q}_{2J} \end{bmatrix}$$

where $\underline{q}_{2j}$ can be represented as a (Nx1) dimensional column vector as:

$$\underline{q}_{2j} = \begin{bmatrix} q_{2j1} \\ q_{2j2} \\ \vdots \\ q_{2jn} \\ \vdots \\ q_{2jN} \end{bmatrix}$$

Let q_{1j0} be the existing initial capacity of j^{th} secondary treatment plant, and q_{2j0} be the initial capacity of j^{th} tertiary treatment plant.

In order to simplify the expression, the operating flow variables $\underset{\sim}{x}_1$, $\underset{\sim}{x}_2$, $\underset{\sim}{x}_3$, $\underset{\sim}{x}_4$, and $\underset{\sim}{x}_5$ are grouped together and represented as $\underset{\sim}{x}$, where

$$\underset{\sim}{x} = \begin{bmatrix} \underset{\sim}{x}_1 \\ \hdashline \underset{\sim}{x}_2 \\ \hdashline \underset{\sim}{x}_3 \\ \hdashline \underset{\sim}{x}_4 \\ \hdashline \underset{\sim}{x}_5 \end{bmatrix}$$

where each of the variables $\underset{\sim}{x}_\ell$, $\ell = 1,2,3,4,5$ represents the wastewater flow in MGD through the plants at different segment ℓ, as depicted in Figure 2.1. Each of the flow variables $\underset{\sim}{x}_\ell$ for $\ell = 1,2,3,4,5$ is a (NJx1) column vector represented as follows:

$$\underset{\sim}{x}_\ell = \begin{bmatrix} \underset{\sim}{x}_{\ell 1} \\ \hdashline \underset{\sim}{x}_{\ell 2} \\ \hdashline \vdots \\ \hdashline \underset{\sim}{x}_{\ell j} \\ \hdashline \vdots \\ \hdashline \underset{\sim}{x}_{\ell J} \end{bmatrix}$$

where $\underset{\sim}{x}_{\ell j}$ is a (Nx1) dimensional column vector of ℓ^{th} operating variable in j^{th} location treatment plant for the entire planning horizon. Thus,

$$\underline{x}_{\ell j} = \begin{bmatrix} x_{\ell j1} \\ x_{\ell j2} \\ \vdots \\ x_{\ell jn} \\ \vdots \\ x_{\ell jN} \end{bmatrix}.$$

where $x_{\ell jn}$ is the flow through ℓ^{th} segment in j^{th} plant in the period n. Again, x_{1jn} is the amount of wastewater load treated in a secondary plant whereas x_{2jn} is the amount treated in tertiary plant j during period n.

The volume of wastewater discharged into the stream reach after primary treatment is x_{3jn}, and x_{4jn} is the volume of secondary effluent discharged directly into the receiving stream from the j^{th} plant during the n^{th} time interval, whereas x_{5jn} is the amount of secondary effluent reclaimed for groundwater recharge, $j = 1,2,\ldots,J$; $n = 1,2,\ldots,N$. Finally, decisions relating to the treatment efficiencies of the plants over the entire planning period are expressed by a vector $\underline{z}$ as,

$$\underline{z} = \{\underline{z}_\ell\} \ , \ \ell = 1,2,$$

where each of the variables $\underline{z}_\ell$, $\ell = 1,2,$ represents the percentage removal efficiency of ℓ^{th} pollutant element. Each of the treatment efficiency variables $\underline{z}_\ell$ is a (NJx1) column vector which can be represented as follows:

$$\underline{z}_{\ell} = \begin{bmatrix} \underline{z}_{\ell 1} \\ \hdashline \underline{z}_{\ell 2} \\ \hdashline \vdots \\ \hdashline \underline{z}_{\ell j} \\ \hdashline \vdots \\ \hdashline \underline{z}_{\ell J} \end{bmatrix}$$

where $\underline{z}_{\ell j}$ is a (Nx1) dimensional column vector of ℓ^{th} treatment element in the j^{th} location over the planning period. Hence,

$$\underline{z}_{\ell j} = \begin{bmatrix} z_{\ell j1} \\ z_{\ell j2} \\ \vdots \\ z_{\ell jn} \\ \vdots \\ z_{\ell jN} \end{bmatrix}$$

where $z_{\ell jn}$ represents the treatment efficiency of pollutant element ℓ, in the j^{th} plant over a period n. Again, z_{1jn} is the percentage removal efficiency of BOD load in the secondary plant and z_{2jn} represents the removal efficiency of BOD load in the tertiary treatment plant, where the subscripts j and n indicate the plant's location and the period of analysis.

The level of pollutants in the surface water depends on the net discharge of each pollutant from treatment plants, the initial pollutant load in the stream, the tributary inflow, and streamflow condition. Each of the pollutnats can be viewed as an objective, the levels of which can be imposed on the model based on the decision maker's evaluation of the water quality standard. In Section 2.3, stream environment quality objectives are presented for BOD load and DO deficit levels. A multiobjective formulation of all noncommensurable objectives are then presented in Chapter 3.

The overall optimization problem for point source pollution control can be formally stated as follows:

$$\min \{ \hat{f}\ (q_1, q_2, \underline{x}, \underline{z}) = \sum_{n=1}^{N} \sum_{j=1}^{J} [\phi_1^s(q_{1jn}) + \phi_1^t(q_{2jn}) + \phi_2^s(x_{1jn}, z_{1jn})$$

$$+ \phi_2^t(x_{2jn}, z_{2jn})$$

$$+ \phi_3(x_{5jn})\ ] (1 + \rho)^{-(t_n - t_1)} \} \tag{2.5}$$

subject to:

(i) Resource Demand Constraints:

$$x_{1jn} + x_{3jn} = d_{jn} \tag{2.6}$$

$$j = 1,2,\ldots,J; \quad n = 1,2,\ldots,N$$

(ii) Project Utilization/Capacity Constraints:

$$x_{1jn} \leq \sum_{\eta=1}^{n} q_{1j\eta} + q_{1j0} \quad \text{(Secondary unit)} \tag{2.7}$$

$$x_{2jn} \leq \sum_{n=1}^{n} q_{2jn} + q_{2j0} \text{ (Tertiary Unit)} \qquad (2.8)$$

$j = 1,2,\ldots,J; \quad n = 1,2,\ldots,N$

(iii) Secondary to Tertiary Plant Transport Constraints:

$$x_{1jn} - x_{2jn} - x_{4jn} - x_{5jn} = 0 \qquad (2.9)$$

$j = 1,2,\ldots,J; \quad n = 1,2,\ldots,N$

(iv) BOD Removal Efficiency Constraints (secondary and tertiary).

$$0.45 \leq z_{1jn} \leq 0.85 \qquad (2.10)$$

$$0.85 \leq z_{2jn} \leq 1.0 \qquad (2.11)$$

$j = 1,2,\ldots,J; \quad n = 1,2,\ldots,N.$

(v) Phosphorus Removal Efficiency Constraints:

$$0.8 \leq z_{3jn} \leq 1.0 \qquad (2.12)$$

$j = 1,2,\ldots,J; \quad n = 1,2,\ldots,N.$

(v) Groundwater Recharge Capacity Constraints:

$$x_{5jn} \leq g_n \qquad (2.12)$$

$j = 1,2,\ldots,J; \quad n = 1,2,\ldots,N.$

(vi) Nonnegativity Constraints: .

$$\underline{q}_1, \underline{q}_2, \underline{x}, \underline{z} \geq \underline{0} \qquad (2.13)$$

The cost function (2.5) presents the total cost of expansion and operation-and-maintenance of Basin-wide wastewater treatment plants. The costs of expansions of secondary and tertiary treatment plants are represented by $\phi_1^s(q_{1jn})$ and $\phi_1^t(q_{2jn})$ respectively. The functional relationships of $\phi_1^s(q_{1jn})$ and $\phi_1^t(q_{2jn})$ are presented in (2.1)-(2.2). The operating costs of secondary and tertiary treatments are given by $\phi_2^s(x_{1jn}, z_{1jn})$ and $\phi_2^t(x_{2jn}, z_{2jn})$. The functional relationships for $\phi_2^s(x_{1jn}, z_{1jn})$, and $\phi_2^t(x_{2jn}, z_{2jn})$ are presented in (2.3)-(2.4). The cost of groundwater recharge is presented by $\phi_4(x_{5jn})$. Finally, $\hat{f}$ $(g_1, g_2, \underline{x}, \underline{z})$ is expressed in present value cost, by applying an appropriate discounting factor. For the case study problem a discount rate, $\rho = 6.125\%$ is used. The constraint (2.6) indicates that the wastewater load generated at each plant location is subjected to the treatment alternatives as depicted in Figure 2.1 Constraints (2.7)-(2.8) imply that the total capacity of secondary and tertiary facilities at each location at any time should be at least equal to the wastewater flow volume. The allocation of secondary effluents in different alternatives such as tertiary treatment and groundwater recharge, is satisfied by constraint (2.9). Constraints (2.10)-(2.11) indicate the lower and upper bounds on the BOD and phosphorus removal efficiency in secondary and tertiary treatment plants. The groundwater recharge capacity constraint is presented in (2.12), where g_n represents the capacity of recharge facility in the time period n.

In the following section, a stream water quality model is presented. Net pollutant discharges from wastewater treatment plants into the stream over the planning period constitute input for the stream quality model. In particular, pollutants considered are the BOD load, DO deficit, and phosphorus levels.

2.2 STREAM WATER QUALITY MODEL

2.2.1 Introduction

Until recent years, analysts in the field of water and related land resources emphasized economic objectives in planning while at the expense of environmental qualities, recreational opportunities and other related objectives. The federal principles and standards for the planning of water and related land resources systems [Federal Register, 1973] prepared pursuant to the Water Resources Act of 1965 (Public Law 89-90), show a considerable departure from past planning standards. The "Principles and Standards" specify that the overall purpose of water and related land resources planning will be directed toward improvement in the overall quality of life through contributions to the objectives of national economic development and environmental quality. These two broad-based planning objectives have been established to place environmental concerns on a basis equal to economic development. The Clean Water Act as amended in 1972 (Public Law 92-500) [Federal Water Pollution Control Act, 1972] focuses attention of the elimination of all point source pollutants from the nation's water by 1985. The attainment prohibitive, since in most wastewater treatment processes, cost increases exponentially with treatment efficiency. For example, the cost of cleaning up the last one percent of pollution may be double that of eliminating the first 99 percent [National Water Commission, 1973]. The point sources of pollutants are characterized by those waste constituents such as outfall of domestic sewage and industrial waste from municipal and industrial treatment plants respectively, whose points of entry into water courses are known.

2.2.2 Mathematical Modeling of Quality Objectives

The objective of the stream quality is to minimize the critical quality components over the entire planning period in all major streams in the Basin. The minimization of pollutant levels in streams is carried out by observing the level of each pollutant at all reaches of the streams. It should be noted that a minimum acceptable level of quality for each component is a subjective factor. For example, municipal, industrial, agricultural, and recreational users may demand water of varying quality.

The stream is decomposed into a number of hypothetical reaches. The length of each reach and location of its boundaries are fixed by considering such factors as the locations of treatment plants, their effluent discharge rates, hydrologic characteristics of the stream and the existing level of quality parameters.

The environmental quality objectives should be responsive to the publicly expressed concern over the environmental effects of specific resource management measures within the planning area. In addition, the trade-off analysis by the decision-maker in multiple objective planning should properly represent the point of view of affected groups where alternative planning objectives are compared so that impacts measured in noncommensurable units may be traded-off against one another. Hence the planner is responsible for formulating his objectives in such a way that the decision-maker has a basis for an effective choice which in fact represents society's choice. A decision-maker's interest in environmental quality objective may be many-fold. A decision-maker may not be satisfied simply by looking at the worst case level of each of the quality constituents in the stream over the planning period. Also, he may not be satisfied to look at the overall basinal water quality problems formulated to represent the total load of each pollutant imposed on the water body. The decision-maker must be presented with quality measurement criteria which can adequately form a basis for trade-off analysis.

When municipal sewage and industrial wastes are discharged into surface water, its dissolved oxygen level becomes depleted. This is due to the oxygen demanded by biodegradable materials in the process of their decomposition. This can be remedied by removing most of the BOD load from municipal and industrial wastes through treatment before discharging them into the streams. In other words, treatment can be loosely interpreted as reducing dissolved oxygen deficit. Obviously, a trade-off exists between an acceptable environmental quality in surface water and cost (capital and operational) associated with the removal of pollutants such as BOD load, etc. The basis for a trade-off analysis as presented in this study depends not only on the level of pollutants in the stream, but also on two other indices of measurement. These indices are the assurance that the pollutant level taken as the stream standard is satisfied and a norm to measure the extent of violation of that stream standard. These indices certainly give the decision-maker a better perspective in determining the relative worth of costs incurred in improving the level of water quality in the stream.

The decision-maker while looking at the number of violations of the stream standard, can at the same time be presented with the extent of deviation in the standard at the points of violation. The first objective is termed the assurance level and the second objective is termed the violation norm. The four quality objectives considered in this study are then:

(i) Biological Oxygen Demand (BOD),

(ii) Dissolved Oxygen Deficit (DO deficit),

(iii) Level of Assurance,

(iv) Violation Norm.

The resulting level of BOD load and DO deficit in the stream depends on the net BOD load discharged from treatment plants, and the tributary contribution of dissolved oxygen into the stream. The net loads, however depend on the operational variables of the treatment plants. These variables include the amount of wastewater treated in the secondary and tertiary plants, the pollutant removal efficiencies of each plant, etc.

The stream hydrology can be defined adequately by simulating the stream flow over time and space. In order to simulate the stream flow, a large number of Basin parameters such as rainfall, runoff, Basin topology, soil moisture conditions, vegetation cover, and many other parameters are required to be known or estimated [Crawford and Linsley, 1966; Ricca, 1972; Haimes et al, 1973]. Although the hydrologic cycle is fairly easy to describe in qualitative terms, the extension of this qualitative knowledge to a more quantitative ground is quite difficult. The accuracy of the model depends not only on the availability of algorithms and simulation strategies, but alo on the availability of extensive and reliable data. However, a critical low flow condition can be considered, in which case the pollutant load will describe the worst situation in the stream. A critical low flow period is defined as the low seven-day or one-month flow occurring once in ten years [Hall and Dracup, 1970]. A low flow condition occurs in summer months when the water temperature is high. At high temperature, the saturation level of dissolved oxygen is reduced. At the same time, wastewater contributes a substantial volume to river flow due to less water in the stream. Thus, if a critical lowflow condition is adopted for water quality analysis, the quality levels will be satisfied under improved stream flow condition for a chosen treatment and land management policy. Analysis based on critical low flow condition is justified for the following tractibility in modeling:

(i) Stream flow dynamics can be presented by a one-dimensional differential equation.

(ii) The modeling effort is simplified.

(iii) Data needs are greatly reduced.

(iv) Computational complexity is considerably reduced.

(v) Model output can only describe the worst pollution distribution in the stream.

In this study only nonconservative types of pollutants are considered. The BOD load and DO deficit level are nonconservative pollutants. Nonconservative pollutants are subjected to decomposition and dilution and their concentration in a stream may depend on the other interacting pollutants [Eckenfelder, 1970].

2.2.3 Quality Objectives

The equations describing the distribution and concentrations of sediment and phosphorus are developed. The stream is segmented into a total of K number of hypothetical reaches, where the subscript k indicates the k^{th} reach. Associated with each reach k, is BOD load from point source combinations being discharges into the stream from industrial and municipal treatment plants in the Basin. Under critical low flow conditions, the time of travel for a unit volume of water from one position to another is directly proportional to the travel distance. In other words, for constant flow, time and distance are equivalent measures. However, when the effluent discharge is superimposed on the critical low flow condition in the stream, the time of travel can no longer be assumed to be constant, but is dependent on the velocity of flow which in turn depends on the total flow volume at each reach and the reaches upstream. The equations developed are quite general. All component inputs are introduced. If an input component is not applicable for the Basin, it can be deleted in the computer model. In order to identify a treatment plant j which discharges its effluent in a reach k, the notation is now slightly modified. Let j_k represent the wastewater plant which discharges into a reach k, and let k_j indicate the reach into which plant j discharges its effluent.

A mathematical relationship exists between biological oxygen demand and dissolved oxygen in the stream. When biological oxygen demanding material is discharged into the stream, its dissolved oxygen level tends to be depleted due to the oxygen demanded by the biodegradable materials in the process of decomposition. Streeter and Phelps

[1925] have shown that the decomposition rate of BOD materials and the rate at which it uses the dissolved oxygen depend exponentially on deoxygenation and reaeration coefficients. The values of these coefficients are in general temperature dependent. Since the analysis is based on the critical low flow condition, they are assumed to be constant within each reach.

By slightly modifying the notation as described earlier in Section 2.2, let $\hat{w}_{1j_k n}(\underline{x},\underline{z})$ be the net discharge of BOD load at plant j discharging into k^{th} reach in the time period n, $j = 1,2,\ldots,J$; $n = 1,2,\ldots,N$. The treatment plant's configuration at any location yields the following equation:

$$\hat{w}_{3j_k n}(\underline{x},\underline{z}) = w_{jn}(x_{4jn}(1-z_{1jn}) + x_{2jn}(1-z_{2jn}) + x_{3jn}) \qquad (2.14)$$

where $\underline{x}$ and $\underline{z}$ are respectively the operating and plant efficiency variables related to j^{th} plant, and w_{jn} represents the gross BOD load per unit volume of wastewater (lbs./MGD) generated at plant location j during period n. The level of BOD load at the sampling point in the 1st reach at any period n can be described by the following equation [Streeter and Phelps, 1925].

$$\hat{f}_{1ln}(\underline{x},\underline{z}) = [c_{10} + \hat{w}_{1j_1 n}(\underline{x},\underline{z}) + q^{t}_{1ln} + q^{\sigma}_{1ln}]\exp(-d_1\hat{t}_{1n}(x)) \qquad (2.15)$$

where,

c_{10} = BOD load at the beginning of 1st reach in pounds/day.

$\hat{w}_{1j_1n}(x,z)$ = Total BOD load discharged into reach 1 during the time period n in pounds/day.

σ^t_{11n} = Tributary flow contribution of BOD load into 1st reach; if no tributary flow exists, $\sigma^t_{31n} = 0$.

σ^g_{11n} = Groundwater flow contribution of BOD load into reach 1; if no groundwater interaction exist, $\sigma^g_{31n} = 0$.

d_1 = Deoxygenation coefficient in reach 1.

$\hat{t}_{1n}(\underline{x})$ = Time of travel in reach 1 in time period n.

For a two-reach stream segment, the BOD load at the sampling point in the 2nd reach is given by the sum of residual BOD load after decomposition from 1st reach and the net load directly into reach 2. Thus,

$$\hat{f}_{12n}(\underline{x},\underline{z}) = [\hat{f}_{11n}(\underline{x},\underline{z}) + \hat{w}_{1j_2n}(\underline{x},\underline{z}) + \sigma^t_{12n} + \sigma^g_{12n}]\exp(-d_2\hat{t}_{2n}(\underline{x})) \qquad (2.16)$$

Substituting for $\hat{f}_{31n}(\underline{x},\underline{z})$ from (2.15) in (2.16), we get:

$$\hat{f}_{12n}(\underline{x},\underline{z}) = c_{10}\exp(-\sum_{i=1}^{2}(d_i\hat{t}_{in}(\underline{x})) + \sum_{j=1}^{j_2}\{\hat{w}_{1jn}(\underline{x},\underline{z})\}\exp(-\sum_{\ell=2_j}^{2} d_\ell\hat{t}_{\ell n}(\underline{x}))$$

$$+ \sum_{i=1}^{2}(\sigma^t_{1in} + \sigma^g_{1in})\exp(-\sum_{\ell=i}^{2} d_\ell\hat{t}_{\ell n}(\underline{x})) \qquad (2.17)$$

$$n = 1,2,\ldots,N$$

A similar development can be extended to a k reach stream by recursive formulation to yield:

$$\hat{f}_{1kn}(\underline{x},\underline{z}) = c_{10} \exp(-\sum_{i=1}^{k} d_i \hat{t}_{in}(\underline{x})) + \sum_{j=1}^{j_k} \{\hat{w}_{1jn}(\underline{x},\underline{z})\} \exp(-\sum_{\ell=k_j}^{k} d_\ell \hat{t}_{\ell n}(\underline{x}))$$

$$+ \sum_{i=1}^{k} (\sigma_{1in}^{t} + \sigma_{1in}^{g}) \exp(-\sum_{\ell=i}^{k} d_\ell \hat{t}_{\ell n}(\underline{x})) \qquad (2.18)$$

$$k = 1,2,\ldots,K \; ; \quad n = 1,2,\ldots,N$$

where $\hat{f}_{1kn}(\underline{x},\underline{z})$ is the total BOD load (pounds/day) in reach k contributed by residual upstream loads, input loads from treatment plants discharging directly into that reach and groundwater and tributary inflow, if any. Again, the net BOD load from treatment plants depend on the operational policy $\underline{x}$ and treatment efficiencies $\underline{z}$.

The Streeter-Phelps equation [Streeter and Phelps, 1925] is universally accepted for describing dissolved oxygen deficit level in streams. The resulting level of dissolved oxygen deficit can be expressed by a linear first order differential equation which relates dissolved oxygen deficit to the BOD load, the initial oxygen saturation deficit, and initial oxygen demand. The parameters in the equation (2.18) are deoxygenation and reoxygenation coefficients. Since in general there may be tributary flow and groundwater-surface water interaction, the contribution of dissolved oxygen to the mainstream through tributary and groundwater inflow are also taken into consideration. The reoxygenation coefficient in general depends on the hydrologic characteristics of the stream, and the temperature of water [Hass, 1970]. The hydrologic characteristics include the velocity of the stream, the depth of stream water, etc. Since the net stream flow at any reach depends on the net discharge of wastewater in that reach and reaches upstream, and the tributary inflow, the reoxygenation coefficient is assumed to be a function of operational policies of wastewater plants during each time period. Let the dissolved oxygen deficit in reach k during time interval n be given by $\hat{f}_{2kn}(\underline{x},\underline{z})$, where the vectors $\underline{x}$ and $\underline{z}$ are the operational and pollutant removal variables respectively. The dissolved oxygen deficit at the end of the first reach in the stream at time period n, denoted by $\hat{f}_{21n}(\underline{x},\underline{z})$ is given by the following equation [Streeter and Phelps, 1925].

$$\hat{f}_{21n}(\underline{x},\underline{z}) = (c_{20} - \hat{w}_{2j_1n}(\underline{x},\underline{z}) - \sigma^{t}_{21n} - \sigma^{g}_{21n}) \exp(-r_{1n}(\underline{x})\hat{t}_{1n}(\underline{x}))$$
$$+ \hat{f}_{11n}(\underline{x},\underline{z}) \frac{d_1}{r_{1n}(\underline{x})-d_1} (\exp(-d_1\hat{t}_{1n}(\underline{x})-\exp(-r_{1n}(\underline{x})\hat{t}_{1n}(\underline{x})) \quad (2.19)$$
$$n = 1,2,\ldots,N$$

where,

c_{20} = Initial dissolved oxygen deficit at the head of reach 1 in pounds/day.

$\hat{w}_{2j_1n}(\underline{x},\underline{z})$ = Net added dissolved oxygen due to effluent discharge into reach 1 during period n.

σ^{t}_{21n} = Contribution of dissolved oxygen due to tributary inflow in reach 1 during planning interval n.

σ^{g}_{21n} = Contribution of dissolved oxygen due to groundwater flow into reach 1 during the time period n.

$r_{1n}(\underline{x})$ = Reoxygenation coefficient in reach 1 during time period n.

The initial DO deficit level c_{20} depends on the saturation level at the head of reach 1, g_o, in pounds/cu.ft., the initial flow at the head of reach 1, s_o, in cu.ft/day, and h_o, the initial dissolved oxygen level at the head of reach 1 in pounds/day. Thus, $c_{20} = g_o s_o - h_o$. By considering a two-reach stream segment, the DO deficit level at the end of the second reach during a period n can be written as:

$$\hat{f}_{22n}(\underline{x},\underline{z}) = (\hat{f}_{21n}(\underline{x},\underline{z}) - \hat{w}_{2j_2n}(\underline{x},\underline{z}) - \sigma^t_{22n} - \sigma^g_{22n}) \exp(-r_{2n}(\underline{x})\hat{t}_{2n}(\underline{x}))$$

$$+ \hat{f}_{22n}(\underline{x},\underline{z}) \frac{d_2}{r_{2n}(\underline{x}) - d_2} (\exp(-d_2\hat{t}_{2n}(\underline{x}) - \exp(-r_{2n}(\underline{x})\hat{t}_{2n}(\underline{x})) \quad (2.20)$$

$$n = 1,2,\ldots,N.$$

In (6.9), $\hat{f}_{22n}(\underline{x},\underline{z})$ is expressed as a function of residual DO deficit $\hat{f}_{21n}(\underline{x},\underline{z})$, from previous reaches, the net added DO from treatment plants directly into the reach under consideration, $\hat{w}_{2j_2n}(\underline{x},\underline{z})$, as well as the net BOD load $\hat{f}_{22n}(\underline{x},\underline{z})$ in reach 2, and of other parameters.

The above formulation can be extended to a k reach stream segment, and the DO deficit $\hat{f}_{2kn}(\underline{x},\underline{z})$ in reach k is expressed as:

$$\hat{f}_{2kn}(\underline{x},\underline{z}) = (\hat{f}_{2k-1,n}(\underline{x},\underline{z}) - \hat{w}_{2j_{k-1}n}(\underline{x},\underline{z}) - \sigma^t_{2k-1,n} - \sigma^g_{2k-1,n})$$

$$\exp(-r_{k-1,n}(\underline{x})\hat{t}_{k-1,n}(\underline{x}) + \hat{f}_{3k-1,n}(\underline{x},\underline{z}) \frac{d_{k-1}}{r_{k-1,n}(\underline{x}) - d_{k-1,n}}$$

$$(\exp(-d_{k-1}\hat{t}_{k-1,n}(\underline{x})) - \exp(-r_{k-1,n}(\underline{x})\hat{t}_{k-1,n}(\underline{x}))) \quad (2.21)$$

$$k = 1,2,\ldots,K \; ; \quad n \quad 1,2,\ldots,N.$$

Equation (2.21) is further modified to express in terms the decisions related to plants' operations and pollutants' exponential decay factors to obtain in the following form.

$$\hat{f}_{2kn}(\underset{\sim}{x},\underset{\sim}{z}) = (g_o s_o \cdot h_o) \exp\left\{-\sum_{\ell=1}^{k} r_{\ell n}(\underset{\sim}{x})\hat{t}_{\ell n}(\underset{\sim}{x})\right\}$$

$$- \sum_{\ell=1}^{k} (\sigma^{t}_{2\ell n} + \sigma^{g}_{21n}) \exp\left\{-\sum_{i=\ell}^{p} r_{in}(\underset{\sim}{x})\hat{t}_{in}(\underset{\sim}{x})\right\}$$

$$- \sum_{j=1}^{j_k} \hat{w}_{2jn}(\underset{\sim}{x},\underset{\sim}{z}) \exp\left\{-\sum_{\ell=k_j}^{k} r_{\ell n}(\underset{\sim}{x})\hat{t}_{\ell n}(\underset{\sim}{x})\right\}$$

$$+ c_{i0} \sum_{\ell=1}^{k} \left[\frac{d_\ell}{r_{\ell n}(\underset{\sim}{x})-d_\ell} (\exp(-d_\ell \hat{t}_{\ell n}(\underset{\sim}{x}) - \exp(-r_{\ell n}(\underset{\sim}{x})\hat{t}_{\ell n}(x)))\right.$$

$$\left. \exp\left\{-\sum_{\nu=1}^{\ell-1} d_\nu \hat{t}_{\nu n}(\underset{\sim}{x})\right\} \exp\left\{-\sum_{\nu=\ell+1}^{k} r_{\nu n}(\underset{\sim}{x})\hat{t}_{\nu n}(\underset{\sim}{x})\right\}\right]$$

$$+ \sum_{j=1}^{j_k} \left\{\hat{w}_{1jn}(\underset{\sim}{x},\underset{\sim}{z})\left[\sum_{\ell=k_j}^{k} \left(\frac{d_\ell}{r_{\ell n}(\underset{\sim}{x})-d_\ell} (\exp\{-d_\ell \hat{t}_{\ell n}(\underset{\sim}{x})\} - \exp\{-r_{\ell n}\hat{t}_{\ell n}(\underset{\sim}{x})\})\right.\right.\right.$$

$$\left.\left.\left. \exp\left\{-\sum_{\nu=k_j}^{\ell-1} d_\nu \hat{t}_{\nu n}(\underset{\sim}{x})\right\} \exp\left\{-\sum_{\nu=\ell+1}^{k} r_{\nu n}(\underset{\sim}{x})\hat{t}_{\nu n}(\underset{\sim}{x})\right\}\right]\right]\right\} \tag{2.22}$$

$$k = 1,2,\ldots,K; \quad n = 1,2,\ldots,N.$$

The water quality standards for BOD and DO deficit levels are expressed as concentration of pollutants in the stream. In order to express pollutant level in concentration, net stream flow must be calculated. Net streamflow, $s_{kn}(\underset{\sim}{x})$, at the end of reach k during period n is the sum of critical flow S_o in cu.ft/day, added flow due to tributary inflows, v_k in reach k and reaches upstream, and the net effluent volume $u_{kn}(\underset{\sim}{x})$ directly discharged to reach k during time period n and all reaches upstream. The net effluent discharge $u_{kn}(\underset{\sim}{x})$ in turn, depends on the operational decision $\underset{\sim}{x}$ of secondary and tertiary plants. Therefore, the stream flow in reach k over a period n is:

$$s_{kn}(\underset{-}{x}) = S_o + \sum_{\ell=1}^{k} \nu_\ell + \sum_{j=1}^{j_k} u_{jn}(\underset{-}{x}) \tag{2.23}$$

$$k = 1,2,\ldots,K \; ; \quad n = 1,2,\ldots,N.$$

The concentration of pollutant p in any reach k can be obtained simply by dividing net load of pollutant p in reach k by the net stream flow $s_{kn}(\underset{-}{x})$ in that reach at any period n. Since net load of pollutants are in pounds/day and stream flow is expressed in ft^3/day, the concentration is expressed in pounds/ft^3. However, an appropriate conversion factor may be used to convert pounds/ft^3 to other unit such as, mg/litre.

Let f_{pkn} be the concentration of pollutant p in reach k over a period n.

Thus, for BOD load and DO deficit level which depend only on point source pollutant discharge, are given by:

$$f_{pkn}(\underset{-}{x},\underset{-}{z}) = \hat{f}_{pkn}(\underset{-}{x},\underset{-}{z})/s_{kn}(\underset{-}{x}) \tag{2.24}$$

$$p = 1,2; \; k = 1,2,\ldots,K: \; n = 1,2,\ldots,N.$$

Thus the resultant level of pollutants, namely BOD and DO deficit in the stream over the planning period are now obtained for a particular policy of plants' operation.

It has been mentioned earlier in the chapter that presenting the level of pollutants in all stream reaches over the planning period may not be adequate or practical. At the same time, presenting the decision maker with only the worst case condition of quality with respect to each pollutant over the planning period is not adequate, since the decision maker does not have knowledge on the frequency of violation of quality standard, nor does he know the number of observation points where quality level exceeds the prescribed standard. Hence two additional indices of quality measurement are considered: (i) level of assurance, and (ii) violation norm.

Kaplan [1975] introduced an assurance level of satisfying a water quality standard by defining a water quality standard as that level of quality which will be violated by some specified fraction of the total number of observations. Considering a total of P pollutants, measured at K reaches over a period of N intervals, a total of PKN observations can be generated. The number of observations (or data points) included in actual analysis can be considerably less than PKN, since it may not be necessary to observe pollutant level at all reaches. Only those critical reaches where stream standards are most likely to be violated may be sufficient. For each pollutant a total of KN observations can be generated. By using notations, similar to those used by Kaplan [1975], let $G_p(\underline{x},\underline{z},\hat{\underline{z}})$ be a random variable with discrete probability function so that $P_p(G_p(\underline{x},\underline{z},\hat{\underline{z}}) = f_{pkn}) = \frac{1}{KN}$, where $P_p(G_p(\cdot) = f_{pkn})$ represents the probability that the value of a random variable G_p is equal to f_{pkn}. Thus,

$$\sum_{k=1}^{K} \sum_{n=1}^{N} P_p(G_p(\underline{x},\underline{z},\hat{\underline{z}}) = f_{pkn}) = 1$$

Let $D_p(f_{pkn})$ be a discrete distribution function representing the probability of having the value of random variable G_p less than or equal to some specified standard f_{pkn}, i.e., $D_p(f_{pkn}) = P_p(G_p(\underline{x},\underline{z},\hat{\underline{z}}) \le f_{pkn})$. The distribution function $D_p(f_{pkn})$ is thus described by choosing a set of assurance levels and determining the corresponding levels of water quality. A water quality standard may be defined as that level of quality which will be violated by a specified fraction of the total number of observations. The lesser the number of violations of water quality standards, the greater the level of assurance of satisfaction. The assurance level can thus be represented by a scalar quantity, say α_p, for pollutant p, p = 1,2. The range of values of α_p varies from zero to one, i.e., $0 \leq \alpha_p \leq 1$. For a pollutant p, $\alpha_p=1$ indicates an assurance level of 100% for some specified standard for pollutant p. Similarly, $\alpha_p=0$, indicates an assurance level of 0%. In other words, the quality standard for pollutant p is violated at all observation points KN at all reaches over the planning period. Thus, for each pollutant p, the expected number of standard violations is presented by a scalar quantity $(1-\alpha_p)KN$, having an assurance of α_p, p = 1,2. There may be several criteria in establishing a water quality standard. An average value of pollutant level over all reaches over the planning period may be selected as a criterion. A worst case level of pollutant at all reaches over the entire planning period may also be chosen as a standard. If a worst case level of a given pollutant for all reaches, over the entire planning period is selected as standard, then this standard will have a model expected frequency of violation of 0% or equivalently an assurance of 100%. Hence, for worst case level of a given pollutant p, α_p is always equal to one. However, if an average value of water quality level for each pollutant p is selected, then the model expected frequency of violation of water quality standard is not necessarily 0%, unless the water quality level in all reaches over the entire planning period is uniform. The later criterion is selected in this study. Referring to (6.15)-(6.16), the stream quality objectives are modified to include the above criteria. Let f_p represent either the worst case or the average quality level of objective for a given pollutant p. Hence, if the first criteria is chosen, then we may write:

$$f_p(\underline{x},\underline{z}) = \max_{n\epsilon N} \max_{k\epsilon K} f_{pkn}(\underline{x},\underline{z}) \qquad \text{for } p = 1,2 \qquad (2.25)$$

However, if an average level of pollutants in the stream over the planning period is chosen, then the quality objective f_p for a given pollutant p can be written as:

$$f_p(\underline{x},\underline{z}) = \frac{1}{KN} \left\{ \sum_{k=1}^{K} \sum_{n=1}^{N} f_{pkn}(\underline{x},\underline{z}) \right\} \qquad \text{for } p = 1,2 \qquad (2.25a)$$

An additional index of quality measurement is introduced along with the assurance objective that indicate the extent of violation of quality standard at the observation points for each pollutant p. The decision-maker while considering the number of violations of stream standard, is also presented with the amount of violation at the observation points. In other words, the amount of deviation in the stream quality levels from a specified standard is also considered as a criterion of the decision-maker's assessment for trade-off analysis. The extent of violation is presented to the decision-maker by introducing a violation or error norm. In this case an absolute norm is adopted.

Let the level of quality for a given pollutant p at the observation points where the standard is violated with a specified assurance of α_p and for a net pollutant discharge of $\hat{w}_{pkn}(\underline{x},\underline{z})$ into the stream for all k=1,2,...,K and n=1,2,...,N, be donoted by $\bar{f}_{pv}(\underline{x},\underline{z})$. The subscript v denotes the data points where the standard is violated, $v = 1,2,\ldots V_p$. The total number of observation points where a given pollutant p has violated the specified quality standard is denoted by V_p. In practice, for a pollutant p, $V_p = (1-\alpha_p)KN$ for an assurance of α_p. In general, using a vector notation, let $\underline{\alpha}$ represent the assurance of satisfying quality standard for all the pollutants under analysis, where, $\underline{\alpha} = [\alpha_1,\alpha_2]^T$. The water quality standard for a pollutant p which can be achieved by an assurance of $\underline{\alpha}$ may be defined as $f_{pv}(\underline{x},\underline{z},\underline{\alpha})$ at the observation point v, $v = 1,2,\ldots,V_p$. Let the violation norm for a given pollutant p be denoted by β_p which is also a scalar quantity. Thus, β_p can be represented as an absolute norm:

$$\beta_p = \underset{v \varepsilon V_p}{\text{Max}} \, ||f_{pv}(\underline{x},\underline{z},\underline{\alpha}) - \bar{f}_{pv}(x,z)|| \tag{2.26}$$

where β_p thus represents the maximum level of violation of water quality standard for a pollutant p, p = 1,2.

By including a deviation norm as a measure of environmental quality, the decision-maker is likely to arrive at a better judgment. In a sense, the assurance level can be viewed as risk and uncertainty, whereas the violation norm is a measure of the sensitivity in the attainment of other performance objectives. Hall and Haimes [1975], Haimes, Hall and Freedman [1975] justified the need for including these soft objectives in water resources planning. The water quality objectives $f_p(\underline{x},\underline{z},\underline{\alpha},\underline{\beta})$ for each pollutant can now be formulated as that level of pollutant p in the stream for a specified attainment level of assurance $\underline{\alpha}$, and a violation norm of $\underline{\beta}$. The vector $\underline{\beta}$ represents the violation norm for each of the pollutants p, p = 1,2, where $\underline{\beta} = [\beta_1, \beta_2]^T$. Again, p = 1 indicates BOD level, and p = 2 represents the DO deficit level.

In summary, water quality is analyzed by jointly considering the effect of pollutants from point sources (wastewater plants) in the Basin. The mathematical models are developed to analyze the stream quality responsive to the pollutants under consideration. The wastewater treatment planning model considers capital expenditures and operational costs for expansion as well as operation and maintenance of pollution control facilities. The formulation of planning model, both secondard and testiary treatment processes are taken into account. In the following chapter, the cost objectives and stream quality objectives are utilized in a multiobjective analysis framework.

CHAPTER 3

MULTIOBJECTIVE INTEGRATED PLANNING MODEL FOR SURFACE WATER POLLUTION CONTROL

3.1 INTRODUCTION

This chapter presents a method of integrating the planning models developed in Chapter 2. The cost objective of the planning models is the operation and expansion of wastewater treatment plants in the Basin. The optimization problem for the Basin-wide planning involves a large number of decision variables and many constraints. Thus its direct solution is computationally difficult.

Also a single model approach for the planning of water resources is deficient and incapable of representing all the couplings among the various systems components and activities. The hierarchical-multiobjective modeling is a natural approach which is responsive to the large scale and complexity of these systems. This approach is essential for handling the planning of large-scale water resources and environmental systems, while taking into consideration the multiple objectives and goals as well as all systems' interactions. Since the hierarchical-multiobjective analyses are complementary to each other and are part of an overall approach in the decisionmaking process, a brief discussion on these approach is presented in this chapter [Haimes, 1976].

The concept of the multilevel approach is based on the decomposition of large-scale, complex systems and the subsequent modeling of them into "independent" subsystems. This decentralized approach, by utilizing the concepts of strata, layers and echelons, enables the system analyst to analyze and comprehend the behavior of the subsystems at a lower level and to transmit the information obtained to fewer subsystems at a higher level. Each subsystem is separately and independently optimized, with perhaps different optimization techniques being applied, based on the nature of the subsystem models as well as on the objectives and constraints of the subsystems. This is termed a first-level solution. The subsystems are joined by coupling variables which are manipulated at a second or higher level in order to arrive at the optimal solution of the whole system. This is termed the second- or higher-level solution. One way to achieve subsystem "independence" is by first relaxing one or more of the necessary conditions for optimality, and then satisfying these conditions at the second level.

Decomposition and multilevel optimization approaches have several significant advantages in solving large-scale, complex optimization problems over conventional optimization methods. For example, by decomposing the problem into several subproblems (subsystems), a conceptual simplification of a complex system is achieved. This is especially important for highly coupled systems, where the outputs of one subsystem are the inputs to others. The decomposition yields a reduction in the dimensionality of the problem at hand at the expense of having to solve several subproblems of smaller dimensions. This in turn reduces the computational effort involved, such as problem formulation time, programming effort, debugging effort, and the number of cards to be punched, etc. A significant advantage of the multilevel approach is that none of the system model functions needs to be linear, and thus more flexible mathematical models can be constructed to represent the real system. Note that a major shortcoming and deficiency of classical systems engineering practices is that they often result in an imbalance between system modeling and system optimization. This is reflected by the vast number of linearized models in the literature that take advantage of the Simplex method and its extensions. By applying the decomposition and multilevel optimization techniques, no such costly sacrifice of realism in modeling is needed, as more representative and sophisticated nonlinear multivariable dynamic mathematical models can be constructed. Furthermore, interactions among subsystems can be handled, since at the lower levels the subsystems' "independences" are achieved via pseudo variables. The above trade-off between system modeling and system optimization is minimized by the applicability of the approach to both static and dynamic systems. Thus the time domain which plays an important role in a water resources system need not be imbedded or ignored in the analyses (as is the case in static models; e.g., linear programming). Therefore the water resource system can be modeled by both static algebraic equations and dynamic differential equations. Both centralized and decentralized decisionmaking processes can be considered via the hierarchical-multilevel approach. This is especially important for regional water resources management, including regional water quality control and pollution abatement.

Four major sources of complexity arise in attempting the modeling task for environmental and water resources planning. The sources of complexity, which are due to the coupling in natural systems, are listed below [Haimes and Macko, 1973].

(i) *Temporal Coupling:* The planning horizon in such studies spans periods which vary from 15 to 50 years. The dynamic changes in the demographic, economic, hydrologic, and other elements should be accounted for.

(ii) *Political-Geographical Coupling:* The basin or the region is often divided into several major planning subareas based on political-geographical boundaries, which cross hydrologic boundaries.

(iii) *Hydrologic Coupling:* An alternative subdivision of a river basin or region is on a hydrologic basis. In particular, the analysis of flood plains and water quality is made on a hydrologic basis.

(iv) *Functional Coupling:* The various planning objectives and goals (e.g., to control flooding, enhance recreation opportunities, enhance water quality, etc.) are coupled with each other so that improving one objective may affect all others.

Clearly, each of the above classes of coupling provides a basis for a different system decomposition with a corresponding hierarchy of models. Figure 3.1 depicts such a hierarchy of two layers, where the first is the decomposition layer and the second is the coordination layer. The first layer is composed of two levels. The second level constitutes m planning subareas based on the geographical-political decomposition. The first level constitutes n objective functions in the planning study based on the functional decomposition. The second layer is the overall hierarchical coordination layer where a multiobjective optimization method may be applied. The temporal and hydrological coupling are analyzed implicitly. Other hierarchical structures are possible and their choice depends on the specific needs and goals of the systems analyst as well as on the type and availability of data.

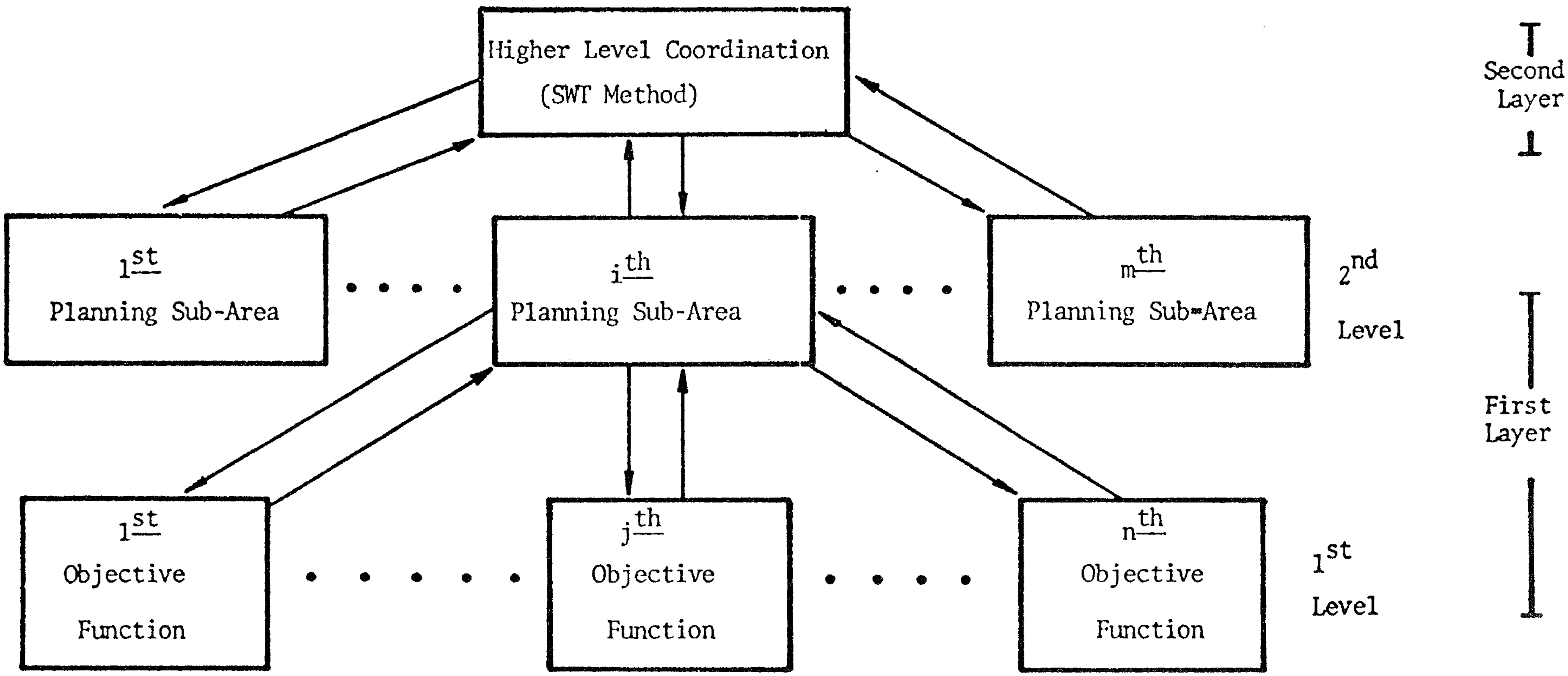


Figure 3.1 Hierarchical Modeling for Environmental Water Resources Planning

A hierarchical structure based on hydrologic decomposition in the Basin may also be suitably applied to the solution of the integrated multiobjective planning problem. The overall Basin-wide cost function is decomposed functionally into smaller subproblems, each of which represent facilities within a hydrological sub-basin. A multiobjective optimization problem is formulated recognizing that the stream quality objectives are noncommensurable with the cost objective. At the first level, the primal Lagrangian problem for each subsystem is solved for a minimum cost strategy with a fixed Lagrange multiplier chosen at the second level. The optimum values of the decision variables related to the wastewater treatment problem are utilized to determine the resulting water quality of the major streams in the Basin. Based on a specified water quality standard for individual pollutants, the resulting number of violation of stream standards and the violation norm are calculated.

The wastewater treatment problem considers the expansion and operation of plants at each location. Since the projected load of raw wastewater increases over time at each plant location, the expansion and operation of plants are considered jointly. The expansion schedule at each plant includes both secondary and tertiary treatment facilities. This is particularly important in the light of U.S. Public Law 92-500. Since the benefits from most wastewater treatment processes are subject to severely diminishing return with the increase in treatment efficiency, the model will enable one to examine the cost savings of gradually improving the quality of discharged effluent as opposed to implementing a 'zero discharge' policy by 1985 and the requirement of "best available technology" by 1983 for wastewater treatment plants.

The stream quality objectives are developed in Chapter 2, taking into account the hydrologic characteristics of stream, the effluent discharge volume at stream reaches, the net pollutant discharge, tributary inflows, groundwater contribution of pollutants, travel time and other parameters. The operational policies of the wastewater treatment plants are used to determine the net discharge of a pollutant into stream reaches. For different values of net discharges allowed for each pollutant, a different set of water quality level in the stream is obtained. In the next section, the submodels presented in the previous chapter are integrated to form a multiobjective optimization problem.

3.2 MULTIOBJECTIVE INTEGRATED MODEL

The overall multiobjective problem is presented in this section. The cost function is the present-value cost of capacity expansion and operations of the secondard and tertiary treatment facilities, $\hat{f}_2(q_1, q_2, \underline{x}, \underline{z})$. The effect of water withdrawal from streams and groundwater in the Maumee River Basin though affect the prevailing stream quality in some instances, is negligible. Hence the ground and surface water supply management and optimization problem is treated separately.

An improvement in water quality standard can be achieved by decreasing the pollutants discharged into the stream. Any decrease in pollutants however results in an incremental cost in wastewater treatment and land management. In other words, there exists a trade-off between the cost of pollution control and water quality. A multiobjective optimization of management cost and water quality objectives is thus necessary. The Surrogate Worth

Trade-off method [Haimes and Hall, 1974a; Haimes, Hall, and Freedman, 1975] is utilized for multiobjective analysis. It involves a vector optimization of noncommensurable objectives with an appropriate set of constraints. The vector optimization problem

$$\min\ [f_o(q_1, q_2, \underset{\sim}{x}, \underset{\sim}{z}),\ f_1(\underset{\sim}{x}, \underset{\sim}{z}, \underset{\sim}{\alpha}, \underset{\sim}{\beta}),\ f_2(\underset{\sim}{x}, \underset{\sim}{z}, \underset{\sim}{\alpha}, \underset{\sim}{\beta})]$$

where $f_o(q_1, q_2, \underset{\sim}{x}, \underset{\sim}{z})$ represents the combined present-value cost of wastewater treatment; $f_1(\underset{\sim}{x}, \underset{\sim}{z}, \underset{\sim}{\alpha}, \underset{\sim}{\beta})$ and $f_2(\underset{\sim}{x}, \underset{\sim}{z}, \underset{\sim}{\alpha}, \underset{\sim}{\beta})$ are water quality objectives with respect to BOD load and DO deficit level respectively in the stream, with level of assurance $\underset{\sim}{\alpha}$ and violation norm of $\underset{\sim}{\beta}$.

The wastewater treatment plant model has a number of physical and environmental constraints which must be satisfied. Constraints (2.6) - (2.13) associated with the wastewater treatment problem are presented in Chapter 2.

3.2.1 Multiobjective Formulation

A multiobjective optimization problem is then formulated as follows:

$$\min_{\underset{\sim}{q_1}, \underset{\sim}{q_2}, \underset{\sim}{x}, \underset{\sim}{z}} \begin{bmatrix} f_o(\underset{\sim}{q_1}, \underset{\sim}{q_2}, \underset{\sim}{x}, \underset{\sim}{z},) \\ f_1(\underset{\sim}{x}, \underset{\sim}{z}, \underset{\sim}{\alpha}, \underset{\sim}{\beta}) \\ f_2(\underset{\sim}{x}, \underset{\sim}{z}, \underset{\sim}{\alpha}, \underset{\sim}{\beta}) \end{bmatrix} \tag{3.1}$$

$$x_{ijn} + x_{3jn} = d_{jn} \tag{3.2}$$

$$j = 1,2,\ldots,J \; ; \quad n = 1,2,\ldots,N$$

$$x_{1jn} \leq \sum_{n=1}^{n} q_{1jn} + q_{1jo} \tag{3.3}$$

$$x_{2jn} \leq \sum_{n=1}^{n} q_{2jn} + q_{2jo} \tag{3.4}$$

$$j = 1,2,\ldots,J \; ; \quad n = 1,2,\ldots,N$$

$$x_{1jn} - x_{2jn} - x_{4jn} - x_{5jn} = 0 \tag{3.5}$$

$$j = 1,2,\ldots,J \; ; \quad n = 1,2,\ldots,N$$

$$0.45 \leq z_{1jn} \leq 0.85 \tag{3.6}$$

$$0.85 \leq z_{2jn} \leq 1.0 \tag{3.7}$$

$$j = 1,2,\ldots J \; ; \quad n = 1,2,\ldots,N$$

$$x_{5jn} \leq g_n \tag{3.8}$$

$$j = 1,2,\ldots J \; ; \quad n = 1,2,\ldots,N$$

$$\underline{q}_1, \; \underline{q}_2, \; \underline{x}, \; \underline{z}, \quad \geq 0 \tag{3.9}$$

Haimes [1970] used an ε-constraint approach to solve a vector optimization problem. Here the ε-constraint approach is further utilized to derive the surrogate trade-off ratios. The water quality objectives $f_p(\underline{x},\underline{z},\underline{\alpha},\underline{\beta})$ for p = 1,2 represent that level of pollutant p in the stream which will be violated at $(1-\alpha_p)KN$ number of observation points, and will have a violation norm of β_p. The multiobjective problem can be solved for various levels of net pollutant discharge from treatment plants. The resulting level of pollutant concentrations in the stream is obtained by solving the appropriate water quality equations presented in Chapter 2. The maximum concentration or the worst quality case of each pollutant at one or more KN mumbers of data points can then be regarded as having an assurance level of 100% i.e., zero number of violations, thus the corresponding value of the violation norm is also zero. Hence, an ε-constraint problem can be solved for different levels of net pollutant discharge policies resulting in a different set of planning for operational policies in the wastewater treatment plants. Note that the concentration in the stream reaches k during a period n for pollutant p is given by $f_{pkn}(\underline{x},\underline{z})$. Let ε_{pkn} be the maximum allowable level of pollutant p in stream reach k during time period n, $p = 1,2$; $k = 1,2,\ldots,K$; $n = 1,2,\ldots,N$. Since a uniform pollutant level is considered over the planning period, the subscripts k and n may be dropped from ε_{pkn}. Considering the cost objective $f_o(\underline{q}_1,\underline{q}_2,\underline{x},\underline{z},)$ as primary, and the quality objectives as secondary, the ε-constraint problem may be written as follows:

$$\min_{\underline{q}_1,\underline{q}_2,\underline{x},\underline{z},} \{f_o(\underline{q}_1,\underline{q}_2,\underline{x},\underline{z})\} \tag{3.10}$$

$$f_{pkn}(\underline{x},\underline{z}) \leq \varepsilon_p, \quad p = 1,2 \tag{3.11}$$

$$k = 1,2,\ldots,K; \quad n = 1,2,\ldots,N$$

$$x_{1jn} + x_{3jn} = d_{jn} \tag{3.12}$$

$$j = 1,2,\ldots,J; \quad n = 1,2,\ldots,N$$

$$x_{1jn} \leq \sum_{\eta=1}^{n} q_{1j\eta} + q_{1jo} \tag{3.13}$$

$$x_{2jn} \leq \sum_{\eta=1}^{n} q_{2j\eta} + q_{2jo} \tag{3.14}$$

$$j = 1,2,\ldots,J; \quad n = 1,2,\ldots,N$$

$$x_{1jn} - x_{2jn} - x_{4jn} - x_{5jn} = 0 \qquad (3.15)$$

$j = 1,2,\ldots,J; \quad n = 1,2,\ldots,N$

$$0.45 \leq z_{1jn} \leq 0.85 \qquad (3.16)$$

$$0.85 \leq z_{2jn} \leq 1.0 \qquad (3.17)$$

$j = 1,2,\ldots,J; \quad n = 1,2,\ldots,N$

$$x_{5jn} \leq g_n \qquad (3.18)$$

$j = 1,2,\ldots,J; \quad n = 1,2,\ldots,N$

$$\underline{q}_1, \underline{q}_2, \underline{x}, \underline{z}, \hat{\underline{z}} \geq \underline{0} \qquad (3.19)$$

where (3.11) are quality objectives formulated as ε-constraints. Constraints (7.16)-(7.17) are related to nonpoint source pollutants indicating its upper and lower bounds. Constraints related to wastewater treatment problems are presented by (7.18)-(7.25). In (3.11) the right-hand side ε_p may be varied parametrically. The ε-constraint problems (3.10) - (3.19) are solved for each parametric value of ε_p and corresponding total optimal cost and optimal decision variables are obtained. Again, instead of considering different values of pollutant level in the stream reaches over the planning period, a uniform standard with respect to each pollutant p, is assumed for all reaches and over the entire planning horizon.

The ε-constraint approach [Haimes and Wismer, 1972; Haimes, 1970; and Haimes, Hall, and Freedman, 1975] is utilized providing the information needed to generate the trade-offs. The solution of the optimization problem described by (3.10)-(3.19) for binding ε-constraints (3.12) would generate noninferior solution and corresponding trade-offs. By varying the right-hand sides of (7.14)-(7.15), noninferior region may be generated. Forming Lagrangian $L(\underline{q}_1,\underline{q}_2,\underline{x},\underline{z})$ for the above problem, yields:

$$\min\left\{L(\underline{q}_1,\underline{q}_2,\underline{x},\underline{z}) = f_o(\underline{q}_1,\underline{q}_2,\underline{x},\underline{z}) + \sum_{k=1}^{K}\sum_{n=1}^{N}\left[\lambda_{1kn}(f_{1kn}(\underline{x},\underline{z}) - \varepsilon_1) + \lambda_{2kn}(f_{2kn}(\underline{x},\underline{z}) - \varepsilon_2)\right]\right\} \tag{3.20}$$

s.t.

$$(\underline{q}_1,\underline{q}_2,\underline{x},\underline{z})\in S$$

where S is a set of decision variables satisfying constraints (3.13)-(3.19) of the original problem, and λ_{pkn} is the Lagrange multipliers associated to the ε-constraints for $p = 1,2;$ $k = 1,2,\ldots,K;$ $n = 1,2,\ldots,N$. Only the Kuhn-Tucker conditions for optimality [Kuhn and Tucker, 1950], for which the following conditions hold are of interest here:

$$\lambda_{1kn}(f_{1kn}(\underline{x},\underline{z}) - \varepsilon_1) = 0 \tag{3.21}$$

$$\lambda_{2kn}(f_{2kn}(\underline{x},\underline{z}) - \varepsilon_2) = 0 \tag{3.22}$$

$$\lambda_{1kn},\ \lambda_{2kn} \geq 0 \tag{3.23}$$

$$k = 1,2,\ldots,K; \quad n = 1,2,\ldots,N.$$

Clearly, if $(f_{pkn}(\cdot) - \varepsilon_p) < 0$ for any $p = 1,2;$ $k = 1,2,\ldots,K$ and $n = 1,2,\ldots,N$, the corresponding multiplier $\lambda_{pkn} = 0$, and the solution is not guaranteed to be a noninferior point. However, for $f_{pkn}(\cdot)-\varepsilon_p=0$, corresponding λ_{pkn} is either zero or nonzero positive. When each of the multipliers λ_{pkn} is positive, the corresponding solution is a noninferior one.

Let λ_{pkn} be a function of ε_1, ε_2. where $\lambda_{pkn}(\varepsilon_1, \varepsilon_2)$ indicates marginal increase in cost objective $f_o(\underline{q}_1,\underline{q}_2,\underline{x},\underline{z})$ incurred in reducing the level of p^{th} pollutant, ε_p, by one unit in reach k during the time period n, given satisfactory levels of other objectives ε_i, $i \neq p$. From (3.20) one may obtain:

$$\lambda_{pkn}(\varepsilon_1, \varepsilon_2) = -\frac{\partial L}{\partial \varepsilon_p}, \qquad (3.24)$$

$$p = 1,2; \quad k = 1,2,\ldots K; \quad n = 1,2,\ldots,N.$$

However, at the minimum of Lagrangian, the constraints (3.21) - (3.22) are all satisfied, so that,

$$L(\underline{q}_1,\underline{q}_2,\underline{x},\underline{z}) = f_o(\underline{q}_1,\underline{q}_2,\underline{x},\underline{z})$$

Therefore,

$$\lambda_{pkn}(\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4) = -\frac{\partial f_o}{\partial \varepsilon_p}, \quad p = 1,2, \qquad (3.25)$$

$$k = 1,2,\ldots,K; \quad n = 1,2,\ldots,N.$$

Since we are interested in noninferior solution, for which the ε-constraints are binding for all nonzero multipliers, (3.24) can be finally written as:

$$\lambda_{pkn}(\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4) = -\frac{\partial f_o}{\partial f_{pkn}} \qquad (3.26)$$

$$p = 1,2; \quad k = 1,2,\ldots,K; \quad n = 1,2,\ldots,N.$$

As discussed earlier in this chapter, the above optimization problem involves a large number of decision variables and constraints, thus a hierarchical decomposition may be of advantage in reducing computational complexity. Each subsystem problem is handled by a conventional optimization technique. The above problem may also be solved by using an efficient nonlinear programming algorithm. The Generalized Reduced Gradient (GRG) algorithm [Lasdon et al, 1973] for nonlinear optimization is found to be quite efficient in solving the ε-constraint problem presented above. A two-level optimization scheme based on a basin's hydrologic boundaries may also be suitably used. Such a hydrologic decomposition structure is shown in Figure 3.2.

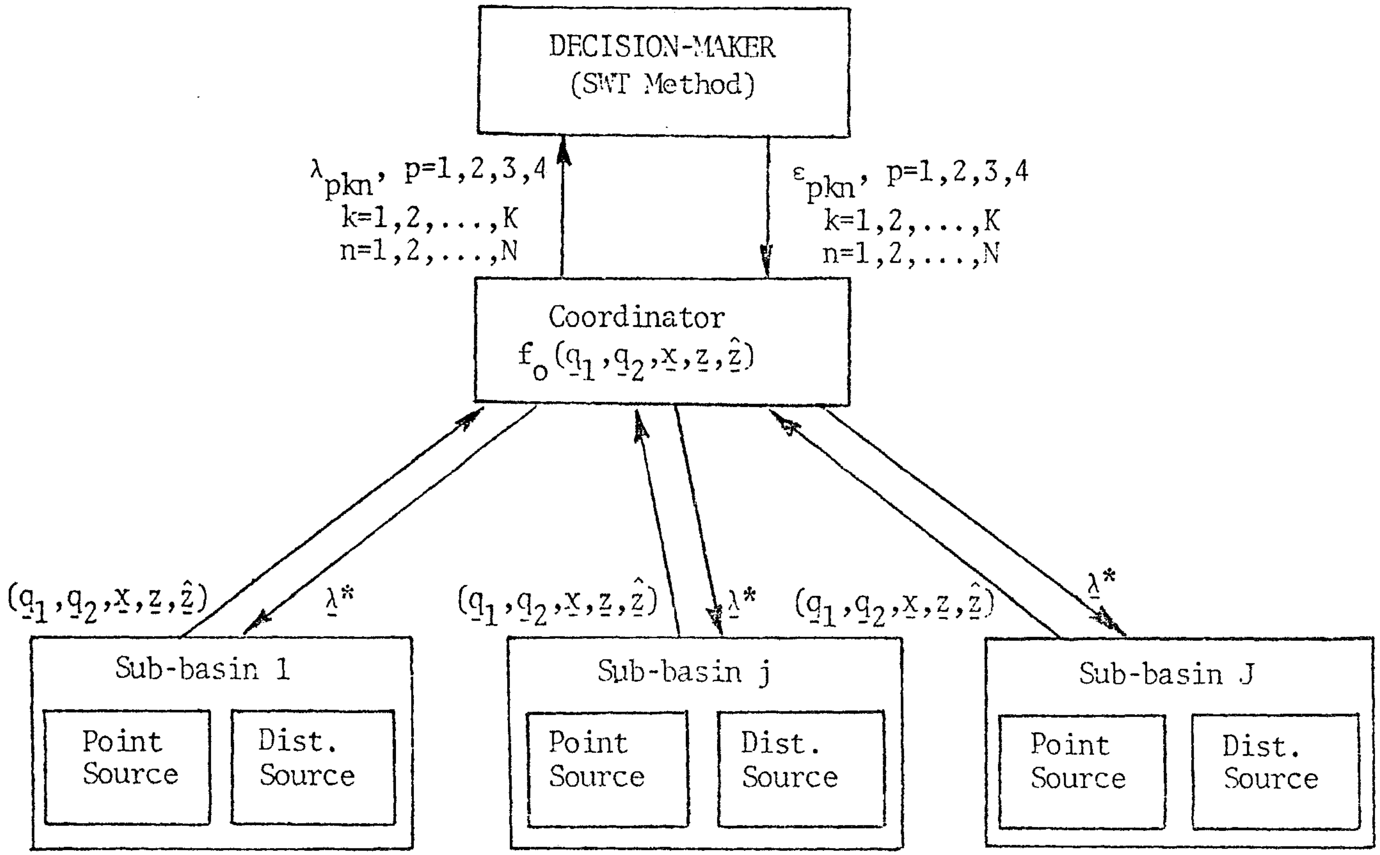


Figure 3.2. Hydrological Decomposition Structure

3.2.2 Decision-Maker and the Surrogate Worth Function

Before applying the Surrogate Worth Trade-off Method to the integrated problem, it is necessary to outline the following definitions [Haimes and Hall, 1974a].

Noninferior Solution: A noninferior solution is one in which no decrease can be obtained in any of the objectives without causing a simultaneous increase in at least one of the other objectives.

Indifference Band: The indifference band is defined to be the subset of the noninferior set where the improvement of one objective function is equivalent in the mind of the decision-maker to the necessary degradation of others.

Preferred Solution: A preferred solution is defined to be any noninferior feasible solution which belongs to the indifference band.

By varying the right-hand side of epsilon-constraints (3.12), all noninferior solutions may be generated. However, in our study, we have adopted a uniform basin wide effluent discharge policy. A particular combination of operating policies for wastewater plants results in a set of pollutant levels in stream reaches over the planning period. The average value of quality level with respect to each pollutant for the entire planning period, ε_p, the level of assurance, α_p, and violation norm, β_p, are calculated for each pollutant p. Thus, a set of uniform effluent discharge policies for all treatment facilities can be adopted and for each policy a corresponding water quality distribution over the stream reaches can be obtained. At this point, the Lagrangian problem (3.20) is solved, utilizing the stream quality values distributed over stream reaches, as obtained from water quality models in Chapter 2. The Lagrangian problem would generate a set of trade-off values corresponding to each water quality models of Chapter 2. The Lagrangian problem would generate a set of trade-off values corresponding to each water quality objective. Once the set of trade-off values corresponding to solutions within the noninferior region are generated, the decision-maker must select a preferred solution from those candidate solutions, based on his subjective preferences. At this point, there are several alternatives open as to the interaction with the decision-maker. The decision-maker may be presented with an average value of trade-off with respect to each pollutant, i.e.,

$$\lambda_{op}(\varepsilon_1, \varepsilon_2) = \frac{1}{KN} \sum_{k=1}^{K} \sum_{n=1}^{N} \lambda_{pkn}, \quad p = 1,2.$$

Alternatively, the decision-maker may be interested in maximum value of trade-off over all reaches and for all time period. Thus,

$$\lambda_{op}(\varepsilon_1, \varepsilon_2) = \max_{k \varepsilon K} \max_{n \varepsilon N} \{\lambda_{pkn}\} \quad p = 1,2.$$

The systems analyst (coordinator) interacts with the decision-maker by presenting him the total cost involved in attaining given levels of environmental quality objectives (represented by average value, ε_p, $p = 1,2$ and the trade-offs $\lambda_{op}(\varepsilon_1,\varepsilon_2)$ along with levels of assurance $\underline{\alpha}$ and violation norm $\underline{\beta}$. The decision-maker is asked to give his evaluation of the worth of $\lambda_{op}(\varepsilon_1,\varepsilon_2)$ marginal units of total cost incurred in improving an additional unit of p^{th} quality objective, given the attainment levels of ε_p for all p, $p = 1,2$ and given the levels of assurance $\underline{\alpha}$, and violation norm $\underline{\beta}$.

By asking the decision-maker sufficient questions at various points within noninferior region, the Surrogate Worth function $W_{op}(\varepsilon_1,\varepsilon_2)$ can be constructed for each two objective functions The optimal solution (often known as preferred solution) is those values of water quality objectives ε_p^*, $p = 1,2$ and total cost objective $f_o^*(\varepsilon_1^*,\varepsilon_2^*)$ where the decision-maker is simultaneously indifferent to all trade-offs.

In summary, this chapter presents a multiobjective model formulation by integrating the submodels presented in Chapter 2. Since the cost objective is noncommensurable and in direct conflict with water quality objectives, an optimal solution involves a vector optimization problem. The Surrogate Worth Trade-off method is utilized in solving this multiobjective problem.

Two different water quality components are included in the study. These are BOD load and DO deficit. In addition, the level of assurance of satisfying the water quality standard and violation norm indicating the extent of violation of standards are included as a measure of performance. A uniform effluent discharge policy is adopted for both point and nonpoint source pollutants. Different effluent discharge standards result in different pollution levels in the stream. The distribution of pollution levels in the stream over the planning period is then obtained by solving the water quality model equations presented in Section 2.2. Either a worst level or the mean value of quality distributed over the stream, with respect to each pollutant for a specific effluent discharge policy can be taken as standard.

Once the distribution of pollutant levels over the stream segments are obtained for any specified effluent discharge policy, the Lagrangian problem is solved by substituting those values of quality levels at the right-hand sides of ε-constraints. The optimal solution of Lagrangian problem yields the trade-offs between the cost and water quality objectives (when the trade-offs are positive). The Surrogate Worth function is constructed by asking the decision-maker to give his evaluation of preferences among trade-offs, given the levels of attainment of all objectives.

The optimal solution of multiobjective problem is the point where all the Surrogate Worth functions are simultaneously zero. However, it may be possible that no solution is obtained where the Surrogate Worth functions are simultaneously zero. In that case, additional noninferior points may be presented to the decision-maker or, alternatively, the optimal solution is chosen for which a majority of the optimality conditions are satisfied.

In the following chapter, modeling of a complex groundwater system by decomposition and superposition approach is presented, which is a further extension of our work of Phase I and II of this project.

CHAPTER 4

MODELING OF A COMPLEX, LARGE-SCALE GROUNDWATER SYSTEM -- THE DECOMPOSITION AND SUPERPOSITION APPROACH

4.1 INTRODUCTION

In Phase I the application of the decomposition and superposition approach as a modeling procedure for a multicell aquifer groundwater system was introduced [Haimes, 1973]. A hierarchy of response functions was developed in Phase II [Haimes, 1974], relating the complex system response to imposed input. The above developments laid the groundwork to practically establish mathematical models for coupling physical water systems with administrative, economical and other considerations. This study is therefore devoted to two aspects of the desired analysis:

1) To establish the multicell-particular cell simulation procedure as a major tool for large-scale groundwater system analysis. Two chapters summarize this goal. The first contains the model itself, repeated from Phase I, but with well-established procedures and mathematical justifications. The second contains the identification schemes as developed in Phases I and II but modified to use the decomposition of groundwater approach. Sensitivity analysis is applied to point out the advantages associated with the modified approach.

2) The second aspect of the desired analysis is to analyze, develop and apply a management model for the conjunctive use of a large-scale, complex groundwater system with other water resources. Three chapters

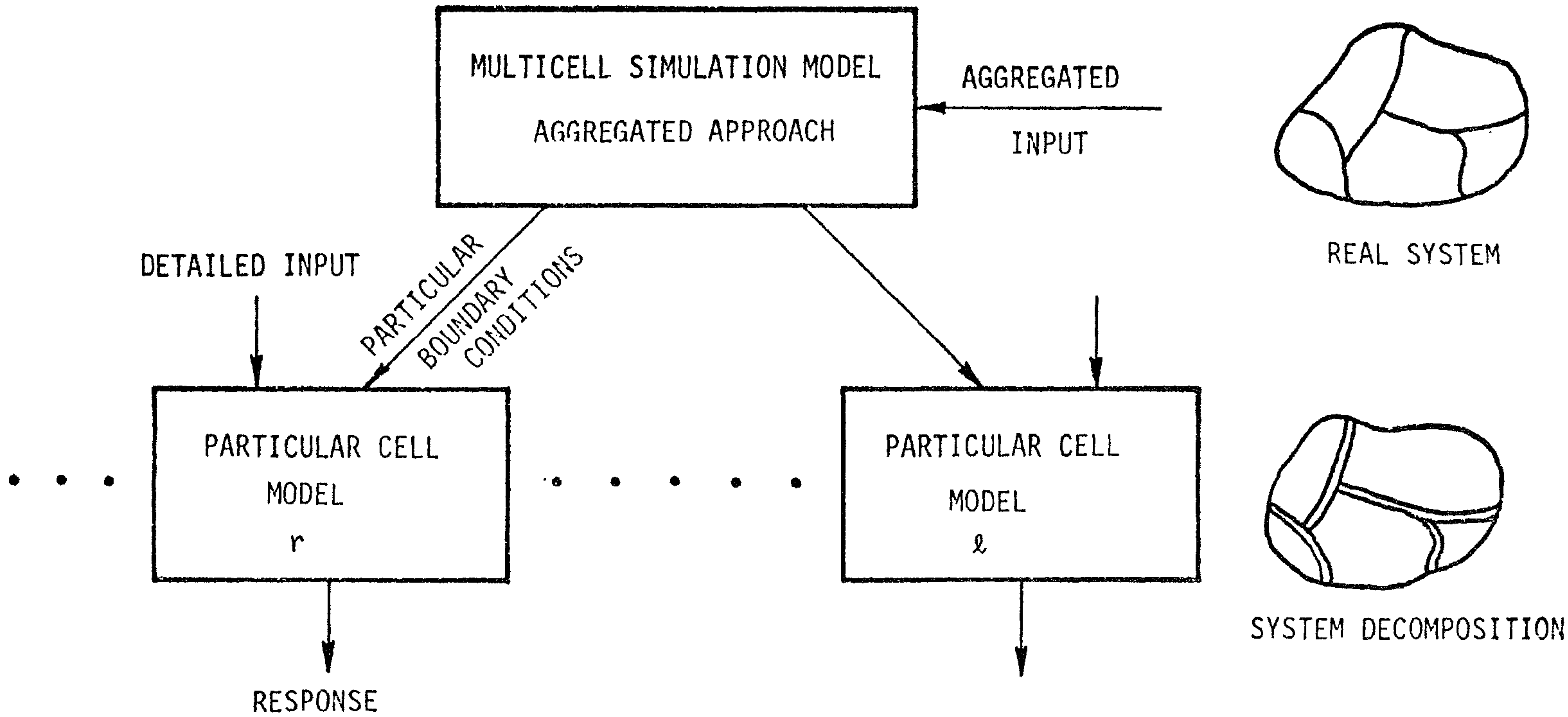
are devoted to this purpose. In one we formulate a general model where the distributed parameter system is explicitly considered. Next the model is applied to the case study described in Phases I and II, namely the Fairfield-New Baltimore area, Dayton, Ohio. Finally, based on the general model, an example problem is solved where the conjunctive use of groundwater, streams and a surface reservoir is considered. The discussion is completed by introducing a multiobjective analysis to that same area.

4.2 THE NEEDS FOR MODEL DECOMPOSITION

The groundwater simulation model plays an important role in all studies on groundwater systems: Prickett and Lonnquist [1971], Tyson and Weber [1964], Pinder and Bredehoeft [1968], Bear et al [1972], and Haimes [1973]. A simulation model will also be used in this study as the basis for developing ways of coupling the physical system with management models. However, there are many disadvantages to digital simulation models developed and used in groundwater systems modeling problems. While the traditional approach, Prickett and Lonnquist, [1971], may be appropriate for systems governed by a single partial differential equation, applying it to systems whose portions are governed effectively by different equations may make the modeling difficult. Another disadvantage occurs when the system consists of several combined unit aquifers. Although each unit is affected by the others, an input from within a unit has a greater influence than an input from outside, Haimes et al [1968]. Thus, points within and outside a given unit deserve different weightings in the model. Finally, for any real water resources system, it is likely that detailed analysis will require extensive computer capacity followed by a considerable amount of input data which may prove to be an important restriction, Maddock [1973]. In particular, this difficulty prevails for a large-scale aquifer system where direct use of traditional techniques may prove inadequate.

In the following a new approach to the construction of a groundwater simulation model is proposed, (Figure 4.1). The basic principle is to apply system decomposition techniques in constructing a hierarchy of simulation models. These models are aimed at determining a particular response to overall distributed activities (pumpage, recharge, etc.) throughout the system. The idea of the multicell model, Bear et al [1972], is used farther up in the model's hierarchy for determining boundary conditions for a particular cell where the point(s) of interest has been located. The particular cell, while isolated from the rest of the system by means of the computed boundary conditions, is now modeled from an accurate analysis. This proposed modeling procedure may provide an improved solution to some of the difficulties of traditional groundwater simulation models:

1. For a large-scale, complex system, where a compact simulation model on a digital computer is evidently inadequate, the proposed technique may prove to be a real advantage. The principle of water balance equations used in formulating the multicell model provides a first approximation for the interactions between different parts of the system. Thus vertical flows as well as horizontal flows are computed along with other conditions along interfaces. These are then used as boundary conditions for decomposing the system into subsystems each of which, while isolated, is easily modeled and solved. There is no standard procedure,



Assumptions: 1) Error due to aggregation is small (function of distance).
2) Solution is unique.

FIGURE 4.1. SIMULATION MODELS HIERARCHY

however, for decomposing the system, and it is the ingenuity and experience of the system analyst that are required for an improved model structure.

2. The extensive computer capacity that is often needed introduces an important restriction to applying groundwater models. This restriction is best overcome by decomposing the model. In many cases, a groundwater simulation model is viewed as an operational tool which is used periodically. This view requires frequent running of the simulation program using mini- or middle-sized digital computers "on-line." A step-by-step procedure may permit a large-scale groundwater system to be simulated on a computer with a limited capacity.

3. The unavailability of input data with which to identify a groundwater system to be modeled by digital simulation is in most cases the main source of errors in the model's results, Bear et al [1972]. Under a given budget for data collection, it is essentially the vicinity of the interesting area that is expected to affect the model results the most, Haimes et al [1968]. Hence, data collection efforts should be concentrated mainly on identifying that part of the system. The proposed technique offers the advantage of considering in detail a particular cell while the rest of the system is aggregated by means of the multicell. Obviously, this advantage is greatly appreciated where the interest is on an isolated subsystem. It may not be so where interest in the system response is equally distributed over all or most of the system.

4. The hierarchy of models structure in the proposed modeling technique (Figure 4.1), is actually not restricted by the geological or hydrological conditions of the modeled area. Hence, the lower level subsystems may be defined subject to administrative considerations. This may be desirable in cases where the groundwater model essentially couples the system with some management model where an administrative scheme controls well pumpages and artificial recharges. The advantage of having the structure of the simulation model follow that of the management model is evident.

The remainder of this chapter is devoted to a general discussion of groundwater simulation models, including the multicell model and the particular cell model comprising the model decomposition context. Some of the essential conditions and assumptions underlying the proposed technique are discussed and analyzed. Applications to a case study illustrate the procedures, pointing out the advantages of the proposed technique as opposed to other methods.

4.3 GROUNDWATER SIMULATION MODELS

A brief discussion aimed at introducing groundwater mathematical models can be found in Bear et al [1972]. Prickett and Lonnquist [1971] analyze digital computer aquifer simulation models more profoundly. A detailed formulation for developing groundwater simulation models is found in Pinder and Bredehoeft [1968], regarding unsteady-state flow of a fluid in a confined aquifer. A three-dimensional flow equation system is discussed by Bredehoeft and

Pinder [1970]. A brief list of possible mathematical models to approximate groundwater flow under different conditions is given by Haimes [1973] , based on Bear [1972] and others.

The common feature of most digital simulation models developed to date is that they are constructed to solve sets of equations with associated boundary conditions. These equations are assumed to describe mathematically the flow in the aquifer system. Because of the complexity of boundary conditions in the real world, no explicit solution is yet available, and hence the digital computer program is essentially for solving the mathematical model's response to a specified stress imposed on the system. The technique basically used is to solve numerically the set of equations while satisfying the boundary conditions. The procedure is to simultaneously solve the system equations, while taking into account initial and boundary conditions and the particular set of forcing functions for which the system response is desired.

The discussion in Section 4.2 on the disadvantages of commonly used simulation models relates directly to the above approach.

The decomposition approach however, suggests a different way of solving the same mathematical model, arriving at the solution in a step-by-step procedure. In that procedure, the final step corresponds to the solution of the so-called particular cell model. The solution to this model is possibly subject to boundary conditions determined by previous steps via the multicell model. The mathematical model which is used in our study is now represented.

Darcy's law and Jacob [1950], provided Pinder and Bredehoeft [1968] the basis for showing that for two-dimensional laminar flow in an anisotropic, non-homogeneous porous medium, the hydraulic head $h(x,y,t)$ is given by the partial differential equation

$$\frac{\partial}{\partial x}\left(T\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(T\frac{\partial h}{\partial y}\right) = S\frac{\partial h}{\partial t} + q(x,y,t) \tag{4.1}$$

where $T(x,y)$ is the transmissivity coefficient, $S(x,y)$ is the storage coefficient, and $q(x,y,t)$ is the flow per unit of aquifer depth leaving the aquifer. For a particular cell model the term $q(x,y,t)$ represents the net effect of recharge and discharge from the aquifer cell. In the following discussion we assume that induced in this term are pumpages from wells and flows into and out of the cell due to interactions with its neighbors.

define

$$q(x,y,t) = \sum_{k=1}^{M} Q(k,t)\,\delta(x-x_k)\,\delta(y-y_k) + \sum_{j=1}^{J} W(j,t)\,\delta(x-x_j)\,\delta(y-y_j) \tag{4.2}$$

where $Q(k,t)$ is the pumpage at well k and $W(j,t)$ is the flow leaving the cell through the j^{th} section of the boundary line defined between the cell and its neighbors, at time t. δ is the Dirac delta function. $W(j,t)$ is determined by the multicell model, and its derivation is shown in the following section, for all boundary line sections j, $j = 1, \ldots, J$.

In addition to the boundary lines j, $j = 1,\ldots,J$, the aquifer cell may contain no-flow boundaries which we denote by λ, so that

$$\frac{\partial h(\lambda,t)}{\partial n} = 0 \qquad (4.3)$$

where n is the normal direction to the boundary and $\frac{\partial h}{\partial n}$ is evaluated on the boundary.

We also denote by μ the boundary line where constant head boundaries are induced on the aquifer cell so that

$$h(\mu,t) = h(\mu) \quad , \qquad t \in [0,T] \qquad (4.4)$$

the initial conditions are

$$h(x,y,0) = g(x,y) \qquad (4.5)$$

corresponding to conditions before any external activity is imposed on the system.

The finite difference approach is discussed by Pinder and Bredehoeft, [1968], and others, using the alternating direction implicit iterative procedure (Peaceman and Rachford [1955]), for solving the model equations. In our study, the simulation program developed by Maddock [1969], was used for the case study verification, and applied to the particular cell for its solution.

We are now in a position to assume that the set of equations defined by (4.1) - (4.5) is specified and that the only information necessary to completely solve the model is the flow function $W(j,t)$, for some j and all $t \varepsilon [0,T]$. We next show that the multicell model may assist in deriving this function.

4.4 MULTICELL MODEL FORMULATION

The multicell approach to modeling groundwater makes use of a set of water balance equations, of which each represents a mass balance applied to a particular cell. For a single cell representing an area within an aquifer and surrounded by impervious boundaries, the balance equation takes the form, Bear et al [1972]:

$$Q \cdot \Delta t = [h(t + \Delta t) - h(t)] \cdot A \cdot S$$

where

Δt = period for which the balance is written

Q = net inflow into the cell

A = area of cell

$h(t)$ = average water level elevation in the cell at time t

S = aquifer storativity at the cell (averaged)

Applying the same principle of water balance to a multicell system, taking into account the interflow between adjacent cells, leads to a set of difference equations [Bear and others]. The form of these equations is identical to the form of those which result from the discretization of a partial differential equation used to approximate the aquifer system.

The thickness of an aquifer usually is small compared with its lateral dimensions. For an unconfined flow in non-homogeneous medium, in which the storage coefficient is assumed to be independent of water table elevation while transmissivity is not, the following difference equation for the r^{th} cell and the $m+1$ period may be used, Yu and Haimes [1974]:

$$\sum_{\ell} \{R_{\ell,r}[h(\ell,i) - h(r,i)] + U_{\ell,r}[(h(\ell,i))^2 - (h(r,i))^2]\}$$

$$= V_r[h(r,i+1) - h(r,i)] + Q(r,i) \qquad (4.7)$$

where

$$R_{\ell,r} \triangleq \frac{W_{\ell,r}\, C_{\ell,r}}{L_{\ell,r}} \qquad U_{\ell,r} \triangleq \frac{W_{\ell,r}\, K_{\ell,r}}{2 \cdot L_{\ell,r}}$$

$$V_r \triangleq \frac{A_r\, S_r}{\Delta t} \qquad C_{\ell,r} \triangleq K_{\ell,r}(E_{\ell,r} - F_{\ell,r})$$

$h(\ell,i)$ = water table elevation at the ℓ^{th} cell during the i^{th} time step

$Q(r,i)$ = net outflow from the r^{th} cell during the i^{th} time step

$W_{\ell,r}$ = length of the perpendicular sector associated with the segment between cells ℓ and r

$L_{\ell,r}$ = distance between the centers of nodes ℓ and r.

$K_{\ell,r}$ = hydraulic conductivity averaged between cells ℓ and r.

$E_{\ell,r}$ = effective aquifer depth averaged between cells ℓ and r

$F_{\ell,r}$ = elevation at the top of the aquifer averaged between cells ℓ and r

A_r = area of r^{th} cell

S_r = storage coefficient averaged over the r^{th} cell

The non-linear term on the left in Equation (4.7) stands for the flow from the neighboring ℓ^{th} cell into the r^{th} cell during the i^{th} period.

The first term on the right side is the quantity of water stored in the r^{th} cell during one period while the second term is the pumping flow rate from the r^{th} cell during the i^{th} period. Hence, equation (4.7) states a balance condition for the sum of all flows entering a cell from its surroundings as balanced by storage and pumpage.

One should note that the multicell approach is an over-simplification of the real system. Boundary conditions must be simplified as well. Constant flow may be handled through inflow to a particular cell. Constant head requires a fixed head for the cell at all times. No-flow requires that the hydraulic conductivity be set at zero between cells and the construction of an imaginary neighboring cell.

The multicell model provides approximate inflows and outflows for each cell in the modeling procedure. These values may be computed for each time step together with averaged water heads.

The flow between the ℓ^{th} cell and the r^{th} cell during time period i is:

$$R_{\ell,r}[h(\ell,i) - h(r,i)] + U_{\ell,r}[(h(\ell,i))^2 - (h(r,i))^2] \qquad (4.8)$$

Equation (4.8) is essentially the required flow function $W(j,t)$ (Equation (4.2)) where j corresponds to a particular neighboring cell, ℓ.

For the particular cell, a more detailed formulation may be used, and the above computed flow is then distributed along the boundary line according to spatial and hydrological considerations.

In the following section we shall state and prove the mathematical ground for the proposed procedure.

4.5 ANALYTICAL JUSTIFICATION FOR MODEL SUPERPOSITION

A new groundwater simulation procedure was developed and stated in the previous sections. System decomposition and response superposition are featured in that approach, together with input aggregation and crude approximations of some of the functions such as W(j,t) (Equation (4.2)). In the following we state and prove some of the arguments essentially underlying the basics of the proposed technique.

4.5.1 An Error Function and the Aggregation via the Multicell Model

The time-dependent effect of activities such as pumping or recharge imposed on an aquifer is distributed unequally throughout the system. In particular, at time $t > 0$, the response distribution depends on the aquifer physical characteristics, namely transmissivity and storativity (T,S) coefficients, the boundary conditions and the distance between the activated point and the interesting point, Bear [1972]. In developing the modeling superposition procedure, a basic assumption is that the response is strongly influenced by near-well properties rather than by those further away, Haimes et al [1968]. Consequently the groundwater simulation model structure provides aggregation of pumpages in all other cells. Pumping from wells inside the particular cell is considered to minimize the induced error more accurately. This basic assumption is intuitively obvious, and may be analytically proved for the following classical case.

Consider transient radial flow through a homogeneous, unconfined aquifer. We get the equation, Jacob, [1950]:

$$\frac{1}{r}\frac{\partial}{\partial r}\left[rh\frac{\partial h}{\partial r}\right] = S/k\,\frac{\partial h}{\partial t} \tag{4.9}$$

where h is hydraulic head, r the radial coordinate, S the storage coefficient, and k the hydraulic conductivity. Let Q be a constant (positive) well production at the origin. Initial and boundary conditions are:

$$\begin{aligned} \lim_{t\to 0} \quad h(r,t) &= h_o \\ \lim_{r\to\infty} \quad h(r,t) &= h_o \\ \lim_{r\to 0^+} \quad r\,\frac{\partial h^2}{\partial r} &= \frac{Q}{\pi k} \end{aligned} \tag{1.10}$$

where h_o is the initial hydraulic head in the aquifer.

Haimes et al [1968], show that if drawdowns are small compared with the aquifer thickness, transmissivity coefficient is defined $T = k\bar{h}$ where $\bar{h}$ is the mean value of h, and the solution to (4.9) subject to (4.10) is:

$$h = h_o + \frac{Q}{4\pi T}\,E_i\left(-\frac{sr^2}{4Tt}\right), \qquad E_i(x) \triangleq \int_x^\infty -\frac{e^{-u}}{u}\,du \tag{4.11}$$

A sensitivity analysis for that case may be done to determine the sensitivity of the solution to certain parameters. Rewrite Equation (4.11):

$$h = h_o + \frac{Q}{4\pi T}\left(- \lim_{A\to\infty} \int_{\frac{sr^2}{4Tt}}^{A} \frac{e^{-u}}{u} \, d u\right) \qquad (4.12)$$

Through aggregating pumpage from different wells at a single point (the multicell model principle) we in fact are changing the variable r , which is the distance from the origin. The sensitivity of the solution h to perturbations in r is approximated by the following equation:

$$\frac{\partial h}{\partial r} = \frac{Q}{4\pi T} \cdot \left[\frac{Sr}{2Tt} \; e^{-Sr^2/4Tt} \Big/ \frac{Sr^2}{4Tt}\right]$$

$$= \frac{Q}{2\pi Tr} e^{-\frac{Sr^2}{4Tt}} = \frac{C_1}{re^{C_2 r^2}} \qquad (4.13)$$

where

$$C_1 = \frac{Q}{2\pi T} \qquad\qquad C_2 = \frac{S}{4Tt}$$

The effect of perturbating r by δr on the computed head h at a point located at a distance r may be approximated as:

$$\delta h \cong \frac{C_1}{re^{C_2 r^2}} \delta r \qquad (4.14)$$

It is evident, that as the distance r between the pumping well and the measuring point is larger, the error caused in computing the drawdown at $r \pm \delta r$ is reduced, and is approximated by the expression (4.14).

Such a sensitivity analysis, if performed for more complex systems which are nonhomogeneous with irregular boundaries, is expected to be more tedious if possible at all. Later in this study, application of the proposed procedure to the real system case study shows induction of negligible error due to the superposition technique as compared with a much more detailed one. Furthermore the modeling efforts are considerably easier.

4.5.2 The Uniqueness of the Decomposition Approach Solution

Given a system which may be described by a set of partial differential equations and the associated boundary and initial conditions, the solution strategy basically suggested in this study is as follows:

1. Solve the system equations (via the multicell model).
2. Use the solution to compute boundary conditions for a particular subsystem (particular cell).
3. Solve the particular cell model. This solution is subject to the boundary conditions derived from the multicell model. This solution is applied to solve for the system response inside the cell.

Dealing with the problem of flow in a porous media, the mathematical model used for describing the system is comprised of the diffusion equation, namely partial differential equation of the parabolic type, Bear [1972].

Consider the one-dimensional operator L: $Ly = 0$ (4.15)

where

$$L = \frac{\partial}{\partial t}(\cdot) - D\frac{\partial^2}{\partial x^2}(\cdot) \qquad x \in [0,1] \quad t \in [0,T] \tag{4.16}$$

and boundary conditions:

$$y(x,0) = g(x) \tag{4.17}$$

$$y(0,t) = y(1,t) = 0 \tag{4.18}$$

The solution for this case is explicitly known to be (Roach, [1970]

$$y(x,t) = \sum_{i=1}^{\infty} \exp\left[-D(i\pi)^2 t\right]\cdot\left[\int_0^1 g(x)\sin i\pi x\, dx\right]\cdot \sin i\pi x \tag{4.19}$$

Assume now that the solution (4.19) is used to specify the value of y corresponding to the values of the spatial variable $x = a$, $x = b$ such that $0 < a < b < 1$.

$$\begin{aligned} y(a,t) &= y_1(a,t) = h_1(t) \\ y(b,t) &= y_2(b,t) = h_2(t) \end{aligned} \tag{4.20}$$

A particular problem for $x \in [a,b]$ is now performed. We now consider the operator L': $L' y_p = 0$ (4.21)

where

$$L' = \frac{\partial}{\partial t}(\cdot) - D\frac{\partial^2}{x^2}(\cdot) \qquad x \in [a,b]\,, \quad t \in [0,\infty] \tag{4.22}$$

and boundary conditions:

$$y_p(x,0) = g(x) \tag{4.23}$$

$$y_p(a,t) = h_1(t) \tag{4.24}$$

$$y_p(b,t) = h_2(t) \tag{4.25}$$

The solution for the problem stated in (4.21) - (4.25) is

$$y_p(x,t) = f_p(x,t) \qquad x \in [a,b] \quad t \in [0,\infty] \tag{4.26}$$

(4.26) is assumed to pertain to a unique solution for operator L' and the associated boundary conditions.

The procedure stated at the beginning of this discussion (1) - (3), is essentially illustrated through the derivations in (4.15) - (4.26).

THEOREM: The solution $y(x,t)$ in (4.19) is identical to the solution $y_p(x,t)$ in (1.26) for all $x \in [a,b]$, $t \in [0,\infty]$ if and only if $y(x,t)$ is a unique solution of operator L and $y_p(x,t)$ is a unique solution of operator L'.

PROOF: Let Z_1, Z_p be two distinct solutions for (4.19) and (4.26), respectively, $x \in [a,b]$.

define $\quad Z = Z_1 - Z_p \qquad (4.27)$

$$L^2 = \frac{\partial}{\partial t}(\cdot) - \frac{\partial^2}{\partial x^2}(\cdot) \qquad x \in [a,b] \qquad t \in [0,\infty] \tag{4.28}$$

$$L^2 Z = L^2(Z_1 - Z_p) = L^2 Z_1 - L^2 Z_p$$

$$= \left(\frac{\partial}{\partial t} Z_1 - \frac{\partial^2}{\partial x^2} Z_1\right) - \left(\frac{\partial}{\partial t} Z_p - \frac{\partial^2}{\partial x^2} Z_p\right)$$

$$= 0 - 0 = 0 \tag{4.29}$$

$$Z(x,0) = Z_1(x,0) - Z_p(x,0) = g(x) - g(x) = 0 \tag{4.30}$$

$$Z(a,t) = Z_1(a,t) - Z_p(a,t) = h_1(t) - h_1(t) = 0 \tag{4.31}$$

$$Z(b,t) = Z_1(b,t) - Z_p(b,t) = h_2(t) - h_2(t) = 0 \tag{4.32}$$

(4.29) - (4.32) hold true provided both Z_1 and Z_p each constitute a unique solution for L and L', respectively.

Equations (4.27) - (4.32) constitute a problem whose solution is $Z(x,t) = 0 \quad \forall\, x,t$, Mikhlin [1970], and consequently

$$Z_1(x,t) = Z_p(x,t) \qquad x \in [a,b] \qquad t \in [0,\infty] \tag{4.33}$$

To conclude this part of our discussion, the multicell-particular cell modeling technique approximates the unique solution for the drawdown distribution provided both mathematical models each constitute a unique solution.

The hierarchy of groundwater simulation models (Figure 4.1) is based on the analytical groundwork which the previous discussion provides. Thus, we first solve the multicell simulation model. This model will serve as the higher level in the simulation hierarchy.

Consequently, we have the particular cell model solution lower in the hierarchy. The higher level provides the lower level with boundary flow equations which in turn are used in the particular cell model formulation to specify the "rest of the world" effect on the modeled subarea. The procedure described here was applied to the case study as discussed and summarized in Phase I.

A most appreciable advantage of the proposed procedure is that the digital computer time consumed is small. In order to determine 10 years' drawdown at wells located in a particular area (Cell 4), Maddock's groundwater simulation program, Maddock [1969], on the UNIVAC 1108 consumed 59 seconds to simulate the overall aquifer system in one single stage. The two-stage simulation, however, consumed less than 14 seconds, of which the particular cell simulation (with Maddock's program) consumed 10 seconds.

CHAPTER 5

IDENTIFICATION OF GROUNDWATER PARAMETERS IN A MULTICELL SYSTEM

5.1 INTRODUCTION

Groundwater is a vital source of water supply. Its wise management presents numerous problems of varying degrees of complexity. Thus a broad approach is required to analyze and solve these problems. One of the problems is that there are not enough data available on the system being modeled. Thus water resources systems analysts develop a nonrepresentative model of the system, which often results in an erroneous output from the model. This chapter is concerned with developing a reasonably representative model of a groundwater system, using additional information so that a model output with a high degree of accuracy can be obtained. Hence, in the process of evaluating groundwater as a continuous source of water supply, the analyst may consider the following questions:

(1) What system model has to be built in order to closely represent the real system?

(2) What are the errors involved in modeling?

(3) What are the effects of model errors on the output of predicted water levels?

The purpose of this chapter is to answer the above fundamental and important questions faced in modeling a groundwater system. Attention is primarily directed toward a sensitivity analysis of identifying parameters of confined aquifer models.

5.1.1 Motivation

Identification of unknown aquifer parameters is essential for making optimal decisions in the planning of a water resources system where groundwater or the conjunctive effect of ground and surface hydrology is considered. Obtaining the required aquifer system parameter values directly by an extensive observation system would be very difficult. For this reason most of the parameter values used are deduced from the behavior of the real system rather than from direct observation. Mathematical models which approximate a real system play an important part in this regard. The basic motivation of this chapter is to identify the unknown parameters so that the mathematical model closely represents the real system response.

Applying this motivation to this phase of the project accomplishes the following:

(1) it develops a drawdown forecast model.

(2) it analyzes sensitivity of computed head values to systematic changes in different model parameters.

(3) it uses the Fairfield-New Baltimore area in Southern Ohio as a case study.

5.1.2 Objective

The main objective can be described as follows:

(1) To develop an efficient means of identifying the parameter of an aquifer system that is confined, unconfined (when drawdown is small compared to the saturated thickness) or both, using additional information so that the model becomes less sensitive to error in parameter identification. To do this, the aquifer is decomposed into blocks known as cells according to available hydrological and other information. A set of difference equations is established for particular cells based on the interflow between adjacent cells. To obtain an accurate estimate of drawdown at a given point of interest, one can isolate the cell in which the point of interest is located. This cell may then be modeled in greater detail, using a mathematical model which considers the particular boundary conditions related to the adjacent cells as a function of time. This decomposition approach uses much more available information than any other approach developed for identifying aquifer parameters in groundwater systems.

(2) To show that the above decomposition approach to parameter identification for predicting drawdown of groundwater systems yields better results than earlier work in this area. Note that earlier parameter identification (presented in Phase I and II) considers (i) to be the whole aquifer as a single cell and (ii), the transmissivity, to be spatially distributed in two-dimensional coordinates.

The scope of the following is limited to these assumptions:

(1) The aquifer model can be described by a linear parabolic partial differential equation.

(2) Transmissivity is decomposed on a two-dimensional space.

(3) Storage coefficient as well as the initial and boundary conditions of the aquifer, together with the recharge and withdrawal, are known.

5.1.3 Literature Survey

Practical water resources problems are governed by partial differential equations containing a number of physical parameters. These unknown parameters are usually determined empirically. However, over the past several years, investigators have presented theoretical ways of identifying them from data observed in the field. Thus the theoretical ways of identifying these parameters are equivalent to the problem of parameter identification of a partial differential equation. This area is not well developed and many problems remain unsolved as yet. The problem stems from the fact that the theory of partial differential equations is complex and difficult to apply. Most partial differential equations of interest in engineering have no analytical solutions, and the existing numerical techniques to solve them are not completely satisfactory.

For identification of partial differential equations, most techniques focus on identifying a constant parameter in a one-dimensional system, whereas this chapter focuses on identifying varying parameters in a multidimensional system. The literature dealing with parameter identification in unsteady groundwater flow governed by a partial differential equation is widespread.

To the problem of water resources analysis, Yeh and Tauxe [Yeh and Tauxe, 1971] applied quasi-linearization in identifying the parameters of a homogeneous and isotropic confined aquifer system. A further extension of this model to a finite leaky aquifer system was studied by Marino and Yeh [Marino and Yeh, 1973]. The major criticism of quasilinearization is its small region of convergence. Also, for systems of more than one dimension, it produces large sets of ordinary differential equations which are obtained by transforming partial differential equations, thus increasing considerably the problem's dimensionality.

For a particular identification of aquifer parameters, Haimes, et al [1968], applied decomposition and multilevel optimization techniques where the aquifer system model is decomposed into a set of independent subsystems each of which is described by a one-dimensional, constant-parameter partial differential equation. This approach is appealing for its relative simplicity. However, it cannot handle complex boundary characteristics which cause interference with well response, since the image equations (which describe interactions among subsystems) become rather complicated. Also, variable recharge produced by lakes and/or rivers cannot be handled, since the input-output water balance of the aquifer is assumed constant (indeed, the computational simplicity of the method would be spoiled since no analytical solution for the subsystems' equations exists). Other comments on this approach can be found in Birkhoff and Varga [Birkhoff and Varga, 1959]. In this chapter, both complex boundaries and recharge patterns can be handled with the scheme developed in section 5.2.

Falkenbarg [Falkenbarg, 1971] identified variable parameter one-dimensional equations by transforming the partial differential equation into an integral equation representation. Using a functional approach, he generates an approximate solution for the distributed system, using the integral equation. This approximate solution is then used to identify the equation parameters on a least-square basis. Extensions of this methodology to handle two-dimensional partial differential equations has not been done up to now and therefore cannot be applied here.

Kleinecke [Kleinecke, 1971] transforms the partial differential equation into a set of difference equations, and using an equation balance error criterion, formulates the aquifer model calibration problem as a linear programming problem. The validity of this approach has been questioned because of the difficulty of accurately estimating time and spatial derivatives using discrete data on the function being identified. The approach in general seems to be very sensitive to the level of measurement error and therefore of little use.

Karplus and Kawamoto [Karplus and Kawamato, 1966] apply sensitivity analysis to identify constant parameters in a multidimensional partial differential equation. Senfield [Senfield, 1971] follows the same approach. The identification problem is posed as a minimization problem. Solution of the partial differential equation is required to match the measured response of the physical system. The parameters are identified on a least-squares basis using a steepest-descent method. The main drawback of this approach is the slow convergence rate of the steepest-descent method. This, combined with the number of sensitivity equations (equal to the parameters being identified) that have to be resolved at each iteration, may be an overburden from a computational viewpoint.

Phillipson [Phillipson, 1971] solves the problem of identifying initial and boundary conditions for systems described by linear parabolic and second-order hyperbolic partial differential equations. He casts the problem within a variational framework and characterizes extremals of quadratic functionals constrained by a partial differential equation by applying known results from the theory of optimal control of distributed parameter systems developed by Lions [Lions, 1971].

In Phase I we formulate the identification problem using steps similar to those of Phillipson [Phillipson, 1972]. On the other hand, we use Lions [Lions, 1971] for solving the quadratic approximation of the parameter identification as a variational problem.

The different methodologies of identifying parameters mentioned above have some features in common -- they all primarily assume an aquifer either as a single cell or as a one-dimensional flow system or both. These assumptions have the following problems:

(1) Considering an aquifer as a single cell leads to assuming a homogeneous property of the aquifer. In the real world, the discontinuity of soil characteristics in an aquifer causes the aquifer to have non-homogeneous properties. Hence the assumption of homogeneity is erroneous.

(2) Groundwater flow is multidimensional. Hence the assumption of one-dimensional flow becomes nonrepresentative of the actual groundwater flow.

In general, errors associated with mathematical assumptions results from using a relatively simple mathematical expression to represent a complex natural physical system. To cope with this problem reasonably, this chapter implements a better procedure for groundwater system modeling. In this procedure the whole aquifer is decomposed into different cells, taking into account the fact that interflow between adjacent cells results in a set of difference equations. In chapter 1 this procedure is discussed.

To identify the parameter (transmissivity) of a particular cell, the cell is modeled in greater detail and calibrated via Marquardt's Non-linear Algorithm [Marquardt, 1963].

Consequently in this approach, by decomposing the system and considering multidimensional flow, we assign more importance to the nonhomogeneous soil characteristics and the two-dimensional flow pattern of an aquifer. Finally, additional information generated due to disintegration of the aquifer system leads to a parameter identification procedure which results in a less sensitive output, even if some error exists in basic input information.

5.1.4 Aquifer Identification Problem

Using the models described in Chapter 1, Equations (4.1) and (4.7), to forcast aquifer water levels, the following information for each cell should be obtained:

1. Length of the perpendicular sector associated with the segment between cells, W
2. Distance between centers of cells, L
3. Hydraulic conductivity averaged between cells, K
4. Effective aquifer depth averaged between cells, E
5. Elevation at the top of the aquifer averaged between cells, F
6. Water elevation, h
7. Forcing function or pumpage, Q
8. Storage function, S
9. Transmissivity function, T
10. Initial conditions
11. Boundary conditions

Determining the above eleven types of input data or parameters comprises the aquifer system identification problem, and identifying each of these parameters is difficult. For example, identifying Q requires determining the pumpage and recharge pattern, rain infiltration, river and lake percolation, and leakage and losses to make a water balance of the total water input to the aquifer. A similar puzzle is determining the aquifer's initial and boundary conditions (I.C. & B.C.). This is known as a state identification problem. Transmissivity and storativity are highly variable discretely distributed parameters. This is due to the wide variety of geological materials and structures an aquifer can be composed of. Such characteristics pose serious problems in identifying aquifer model transmissivity and storativity. In general the eleven points mentioned above are related to each other and can be considered a single problem composed of many subproblems. This chapter addresses itself to a single subproblem: Identifying the particular cell transmissivity function using more hydrological and geological information. It is assumed that pumpage, elevation, storage function, conductivity, I.C. and B.C. are already known. The problem can be stated as follows:

Given the following information on each cell

(1) initial and boundary conditions

(2) storage coefficient

(3) conductivity

(4) well pumpage records

(5) water elevation records

(6) topology

estimate the value of T (model transmissivity function) on the basis of the above information, using some curve-fitting criterion. Some factors which complicate the solution to this problem, are:

1. Since the aquifer water sources are random variables, it is difficult to estimate accurately the input function (Q) of each cell.
2. As it is not feasible to collect data for an entire particular-cell, crude discretely distributed data are used to estimate the overall distributed parameter function of a cell.
3. It is difficult to determine initial conditions, boundary conditions and topology of each cell.

5.1.5 Aquifer System Identification

Due to the heterogeneous property of most aquifers, the assumption that the groundwater system has distributed rather than lumped parameters is inherently more realistic. In this regard, two basically different approaches may be used to get useful representations for the heterogeneous properties of the present system. One approach is to subdivide the aquifer into a finite number of areas of specified geometry, each of which is assumed to be homogeneous with respect to transmissivity and storage. The simplest such case is the analysis of a lumped system for which the entire aquifer is considered to have homogeneous transmissivity and storage. The second approach is to define aquifer properties through a functional relationship which provides spatial variation. In this chapter a mixed approach of the above two methods will be considered. The whole aquifer is subdivided into a finite number of blocks known as particular cells, each of which has

(1) Constant storativity and

(2) Spatially distributed transmissivity.

Thus the identification problem in groundwater hydrology involves determining the distribution of parameters which characterize a particular cell from observations of pumping and recharge rates, flow at boundaries, water levels, and topology.

In order to predict future system response of a particular cell using equation (5.3) one should know the following about each cell:

(1) Boundary conditions including additional interflow information between cells obtained from multicell model equation (5.2).

(2) Production rates (i.e., rate of pumping, Q).

(3) Values of T and S.

It is easy to obtain the first two pices of information from observed data at specified locations, whereas collecting data for (3) creates a problem since no detailed knowledge of the variation of T(x,y) and S(x,y) is available. One way to handle this is to formulate an inverse problem. Thus, utilizing the observed information as input, an inverse problem in the aquifer system can be formed:

Given some function

$$F(h - \hat{h})$$

where $\hat{h}$ = observed head &

$h = h(T,S)$ = calculated head

How must T and S be chosen so that F is minimized? An answer to this question enables one to predict accurately the system response to future modes of operation. So it can be assumed a useful description of the system is given by specifying T and S which will minimize an appropriate criterion function.

5.2 IDENTIFICATION PROBLEM

5.2.1 Introduction

The important step in the identification of a parameter problem is to choose the model topology for the system being considered. In addition, one will need to determine the existence and iniqueness of a solution to the model and to have the capability of solving the equations governing it. Selecting the model for the aquifer has already been discussed in Chapter 4. The next step, developing an identification algorithm for model identification, is the main topic of this Chapter.

5.2.2 Composition of the Identification Problem

As mentioned in the last chapter, the mathematical model of the present system consists of two parts

(1) multicell model

(2) particular cell model

5.2.2.1 *Multicell model contribution for parameter identification problem*

The multicell model described by equation (4.7) is repeated for convenience below

$$\sum_j \frac{W_{ji}C_{ji}}{L_{ji}} (h_{jm} - h_{im}) - \frac{W_{ji}K_{ji}}{2L_{ji}}[(h_{jm})^2 - (h_{im})^2]$$
$$= \frac{A_i S_i}{\Delta t} (h_{i,m+1} - h_{i,m}) - Q_{i,m} \qquad (5.1)$$

The flow between the j-th cell and the ith cell during time period m is:

$$\sum_j \frac{W_{ji}C_{ji}}{L_{ji}} (h_{jm} - h_{im}) - \frac{W_{ji}K_{ji}}{2L_{ji}} [(h_{jm})^2 - (h_{im})^2] \qquad (5.2)$$

5.2.2.2 *Particularcell Parameter Identification*

For the particularcell, a more detailed formulation is used, and the above computed flow (5.2) is then distributed along the boundary line according to spatial and hydrological considerations.

The particularcell model under consideration as described by equation (5.1) can be written more specifically for cell j as

$$\frac{\partial}{\partial x}(T_j \frac{\partial h}{\partial x}) + \frac{\partial}{\partial y}(T_j \frac{\partial h}{\partial y}) = S_j \frac{\partial h}{\partial t} + Q_j \qquad (5.3)$$

$$h_j(x,y,o) = h_{oj} \qquad (5.4)$$

$$\frac{\partial h}{\partial \vec{n}} \mid r_i = o_j \quad h_j(x,y,t) \mid r_2 = h_{1j} \qquad (5.5)$$

$$Q_j(x,y,t) \in R_j \qquad (5.6)$$

where $h_j(x,y,t)$ = drawdown at location (x,y) of cell j and time t. $Q_j(x,y,t)$ = net discharge rate per unit area, including recharge, leakage etc. at location (x,y) of cell j and time t. The initial and boundary conditions of the system are respectively given by (5.4) and (5.5) r_1 and r_2 denote the boundary geometry, R_j in equation (5.6) is the domain of (5.4) - (5.5)

The model described in (5.3) - (5.6) is not completely determined because the function $T_j(x,y)$ is unknown; therefore, the question arises as how to determine $T_j(x,y)$. The identification of the function $T_j(x,y)$ for a particular cell is known as a parameter identification, system identification, parameter estimation or model calibration.

Since the transmissivity value, $T_j(x,y)$ is not known, the response $h_j(x,y,t)$ cannot be computed from (5.3) - (5.6) The identification problem is to estimate the value of the transmissivity function $T_j(x,y)$, so that a specified performance criterion is satisfied. Choosing a performance criterion however, depends on many factors, including, for example, the model representing the physical system, the number of data points, the sensitivity of parameters as related to performance function, etc. A least-square norm of the output error, i.e., between observed and calculated values for the water head, is selected as the performance function.

This function $J_j(T(x,y))$ is expressed as

$$J_j(T(x,y)) = \int_0^t \int_{\Omega j} [h_j(x,y,t;T) - \hat{h}_j(x,y,t)]^2 \, dt \, d\Omega_j \qquad (5.7)$$

where

Ω_j = the area of cell j

$h_j(x,y,t,T)$ = the model output for a given function $T_j(x,y)$

$H_j(x,y,t)$ = the observed value of the waterhead of various points in space and time over the area of cell j

Complete knowledge of a specific cell's geology is required to determine the mathematical structure of $T_j(x,y)$. The difficulties involved in determining transmissivity from physical measurements force hydrologists to pursue indirect methods. Accordingly, a scond-order polynomial representation of transmissivity function is utilized. The representation of transmissivity as a linear function in spatial coordinates was originally developed in Phase I, then it was modified to a second order polynomial in Phase II. The second-order polynomial representation of $T_j(x,y)$ which belongs to the space of positive polynomials in x and y is

$$T_j(x,y) = b_1 x^2 + b_2 y^2 + b_3 x + b_4 y + b_5 \qquad (5.8)$$

where b_1, b_2, b_3, b_4 and b_5 are unknown coefficients to be estimated.

The identification problem can now be stated as follows:

$$\text{Minimize } J_j(T_j(x,y)) = \text{Min} \left\{ \int_0^t \int_{\Omega j} [h_j(x,y,t,T) - \hat{h}_j(x,y,t)]^2 \, dt \, d\Omega j \right\} \qquad (5.9)$$

Subject to the constraints set

$$\left.\begin{aligned} &\frac{\partial}{\partial x}(T_j(x,y) \frac{\partial h}{\partial x}) + \frac{\partial}{\partial y}(T_j(x,y) \frac{\partial h}{\partial y} = S_j \frac{\partial h}{\partial t} + Q_j(x,y,t) \\ &h_j(x,y,o) = h_o \\ &\frac{h}{\partial \vec{n}} \Big| r_i = o_j \; ; \; h_j(x,y,t) \Big| r_2 = h_1 \\ &Q_j(x,y,t) \;\; \varepsilon \; R_j \end{aligned}\right\} \qquad (5.10)$$

The search for a transmissivity function $T_j(x,y)$ which minimizes the objective function (5.9) constitutes the identification algorithm for a particular cell. The Marquardt Algorithm for least-squares estimation of nonlinear parameters [Lopez, 1973] as used for parameter identification is found to be an effective approach in this regard.

Once the parameters (b_1, b_2, b_3, b_4 & b_5) representing spatially distributed transmissivity function $T_j(x,y)$ of cell j is identified, the next task will be to find the average value of tansmissivity for cell j, $T_{j\,av}$ as follows:

$$T_{j\,av} = \frac{\int_x \int_y T_j(x,y)dxdy}{\int_x \int_y dx\,dy} \tag{5.11}$$

where $\int_x \int_y T_j(x,y)\,dx\,dy$

is the sum of transmissivities at different points over the entire particularcell j

and $\int_x \int_y dx\,dy$ is the total area of cell j

5.2.3 Iterative Procedure for Identification Problem

Consider a number of cells constituting an aquifer. It is assumed that within the times considered there is no change in the aquifer's boundary conditions. Thus based on geohydrological considerations, a two-dimensional system model comprised of cells can be formed. Water in adjacent cells can flow from one to another. Hence for an n-cell aquifer system, the following approach is proposed as a solution to the identification problem:

(1) Make an initial guess for the vector $\underline{T}_{av}$

$$\underline{T}^{o}_{av} = \begin{bmatrix} T^{o}_{1_{av}} \\ T^{o}_{2_{av}} \\ T^{o}_{i_{av}} \\ T^{o}_{j_{av}} \\ T^{o}_{n_{av}} \end{bmatrix} \qquad (5.12)$$

(2) Substitute T in the relation

$$K_{ji} = \frac{T_{ji}}{D_{ji}} \qquad (5.13)$$

where K_{ji} = conductivity averaged between cells j and i

$T_{ji} = \frac{T_{j\,av} + T_{i\,av}}{2}$ = transmissivity averaged between cells j and i

D_{ji} = flow depth averaged between cells j and i to get the conductivity K_{ji}

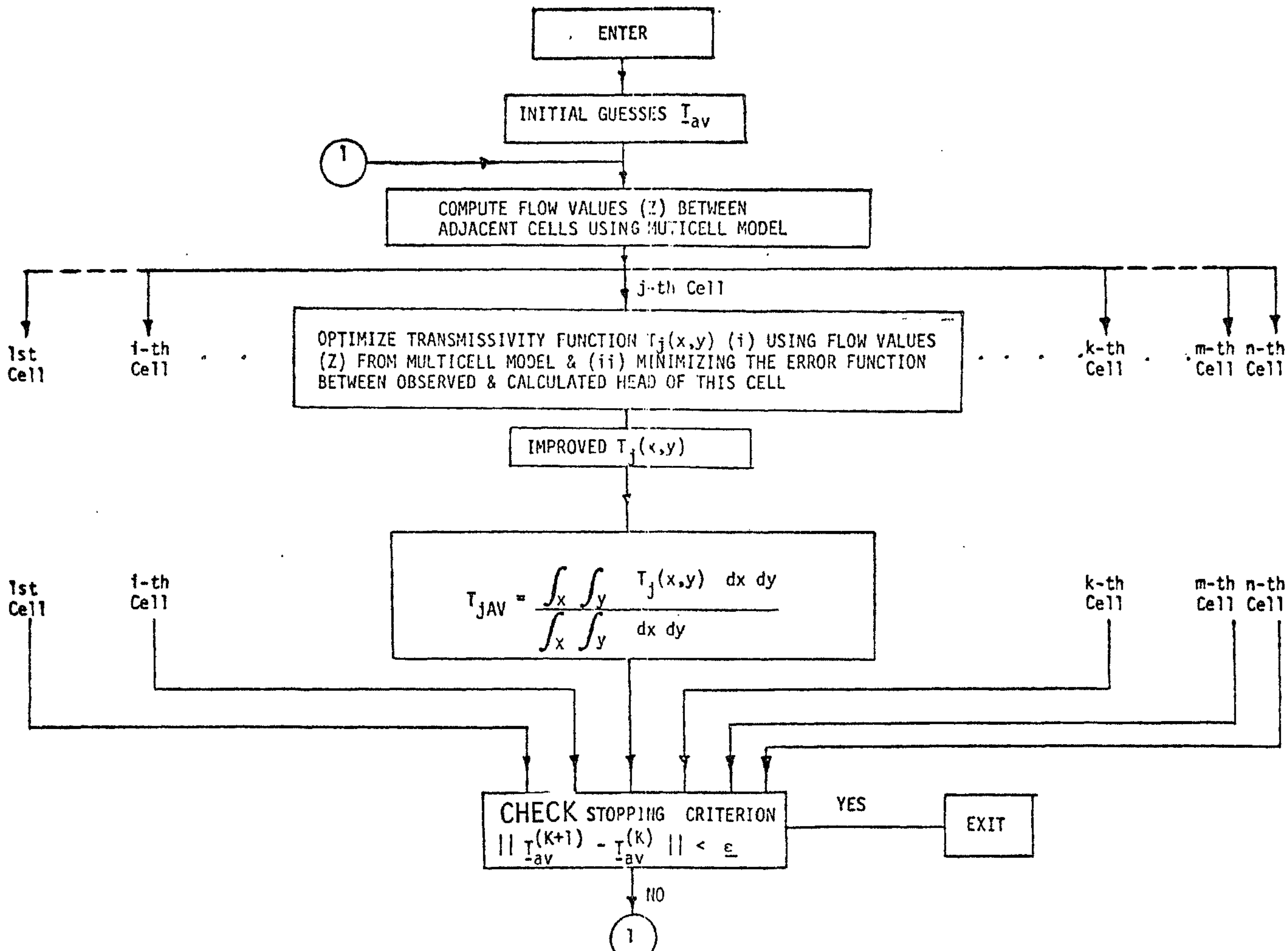
(3) Solve multicell model equation (5.1) to compute flow values between adjacent cells and water head at different times. To do so, use the information generated in step (2) above.

(4) Optimize transmissivity function T(x,y) for each particular cell by minimizing the error function between observed and calculated values of drawdown at specified points for each cell. Calculated values of drawdown are subject to flow values of multicell model equation (5.1).

(5) Transform improved $T(x,y)$ of step (4) into average transmissivity T_{av} using equation (5.11) - for each cell.

(6) Compare the average transmissivity vector $\underline{T}_{av}$ obtained in step (5) with the initial guess of $\underline{T}_{av}$ in step (1). If this difference is less than a vector of convergence factor $\underline{\varepsilon}$, then stop the procedure. Otherwise go to step (1) (use improved $\underline{T}_{av}^{(K+1)}$ obtained in step (5) rather than initial guess $\underline{T}_{av}^{(K)}$).

A flow diagram of the identification algorithm is depicted in Figure (5.1) The preceding theoretical concept was put on the Univac 1108 digital computer in fortran language to achieve our results.



BASIC SCHEME FOR THE ITERATIVE PROCESS

Figure 5.1

5.3 CASE STUDY

5.3.1 Introduction

The purpose of this section is to illustrate the feasibility of the modeling technique proposed in the last chapter by means of a case study. The Fairfield-New Baltimore aquifer in the lower great Miami River Valley of southern Ohio is a typical example of a large water resources system. This example is well suited to testing the methodologies developed in this chapter. Even though the system is described in detail in Phases I and II, we represent it here for the completeness of the report.

5.3.2 Description of Real Aquifer System: Miami Conservency District

The area modeled for the validation of the identification algorithm is the Fairfield-New Baltimore area of the Miami Conservency District which consists of 32 square miles of the Great Miami River Valley southwest of Hamilton, Ohio. The area modeled possesses a sand and gravel aquifer that is bounded by the bedrock walls of the Great Miami River Valley. These walls form the boundary of the aquifer, with the exceptions of the west and the north, where the boundaries are arbitrary. For the west boundary the dry fork of the White Water River, located about two miles west of New Baltimore was selected. For the northern boundary a line through Fairfield near the southern city limit of Hamilton was chosen.

Geologically, the aquifer under study consists of glacial outwash, sands, and gravels of the Pleistocene Age. From the hydrogeological point of view, the aquifer area can be conveniently divided into three parts as follows:

In the central part of the area the aquifer material consists of stratified sand and gravel situated 150-200 feet below ground surface. Widely scattered lenses of clay and silt are also present but do not cover a sufficient area to cause any perceptible confining effects. In the southwest corner the sand and gravel is only about 80 feet thick.

Along the eastern edge of the area some three square miles consist of a sand and gravel aquifer which is about 100 to 150 feet thick and is overlain by about 100 feet of clay and silt.

In the western-most portion of the Fairfield-New Baltimore area, which covers about eight square miles, the aquifer is about 200 feet thick and is capped with a complex layer of till, silt and clay.

Groundwater is unconfined throughout most of the area. However, the mathematical condition that the drawdown be small as compared to the saturated thickness of the aquifer is satisfied. This condition permits use of the identification technique developed in this work.

The hydrologic and geologic characteristics of the Fairfield-New Baltimore aquifer have been extensively studied and a report [Spieker, 1968] provides an excellent source of information for the area.

5.3.2.1 *Estimation of the Input-Output Water Balance*

Concerning the hydrologic boundaries (i.e., boundary conditions), the aquifer is bounded by the vertical bedrock wall of the buried Miami Valley. The permeability of this rock is slight, yet it can contribute a significant amount of water to the system due to the very large contact area, therefore, a leakage boundary is introduced into the model. A second source of water is provided by the Great Miami River which traverses the aquifer as shown (Figure 5.2). The river strongly interfaces with the aquifer and is one of the most important components of the ground and surface water system.

The input-output water balance of the aquifer is made up of the following components:

(i) Recharging of Induced Stream Infiltration

This is a difficult system input to estimate. It is a highly variable quantity whose interaction with the aquifer depends on many factors, such as width and depth of the river, velocity of the streamflow, permeability of the streambed. The most critical of all these factors is the stream infiltration rate under conditions of low streamflow. Two estimates of this factor have been made for the area in question and, based on them, a range of 240,000 to 500,000 gpd per acre has been determined as the expected range of variation for the maximum infiltration rate all year round [Spieker, 1968]. Such a range indicates that the river is a large source of water for the aquifer; consequently, in the aquifer model the river has been modeled as a constant head boundary.

(ii) Recharge from Boundaries

The perimeter of the aquifer modeled is 220,000 feet, of which 180,000 feet are along the bedrock valley walls. The permeability has been estimated to be on the order of 1.5 gpd per sq. ft. These figures, when multiplied by the total area, yield 6.8 mgd coming from the bedrock formations into the aquifer. This last figure is used in this study.

(iii) Pumping

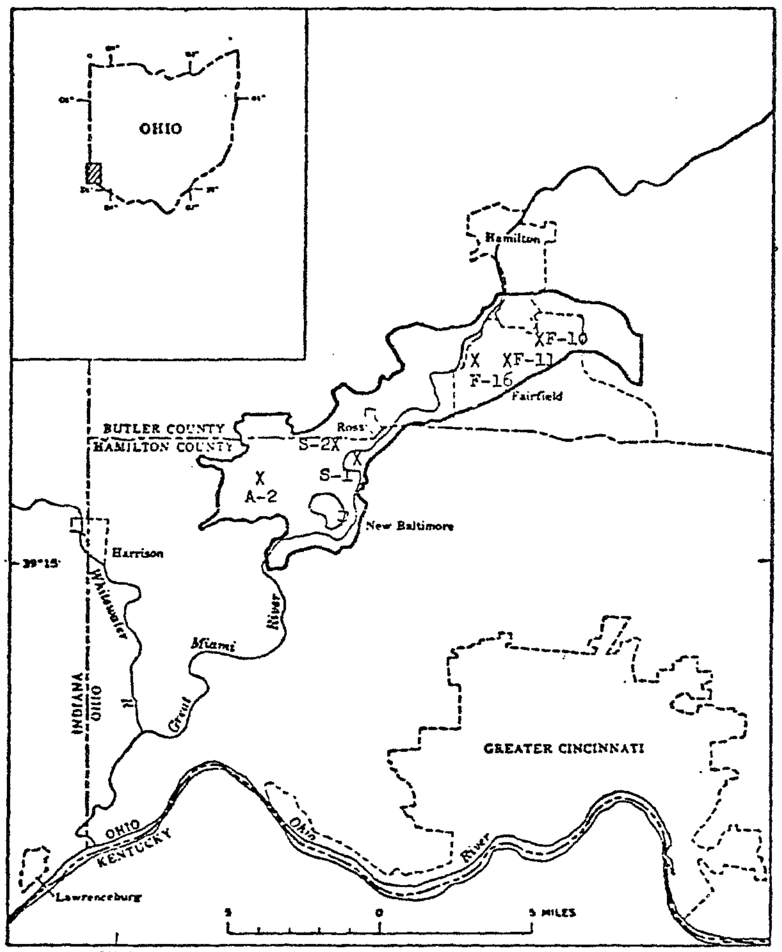
Pumping is concentrated in three well fields, namely, the well fields of Hamilton south (Fairfield), the Southwestern Ohio Water Co., and the U.S. Atomic Energy Commission. Pumping started in 1943 with eleven wells in Fairfield. These were operated from 1943 to 1945. Then, from 1945 to 1952 there was no significant pumping in the area. In 1952 Southwestern Co. installed a new well, S-1 (Figure 5.4). It was pumped from 1952 to 1955 at an average rate of 10 mgd. In 1955 a second well was installed, S-2 (Figure 5.4), The combined pumpage of S-1 and S-2 from 1955 to 1962 averaged 13.8 mgd. In 1956 the city of Hamilton installed a new well field (F-16, F-10, F-11) which was pumped from 1956 through 1962 at an average of 7.5 mgd. The U.S. Atomic Energy Commission well field A-2, has been pumped at an average of 1 mgd since 1952.

(iv) Initial Conditions

Records of water level in the area were not kept until pumping had started; therefore, it is difficult to determine the initial conditions of the system. Spieker [Spieker, 1968] estimated those conditions based on existing hydrographs of the area, present water level measurements, models' results, and river stages. In the present work, initial conditions for groundwater levels in the area were considered according to Spieker.

For the Fairfield-New Baltimore area only four reliable pumping tests have been performed to determine the aquifer transmissivity. Locations of test points are shown as T_1,T_2,T_3,T_4, (Figure 5.4). The storage coefficient has been considered based on Spieker.

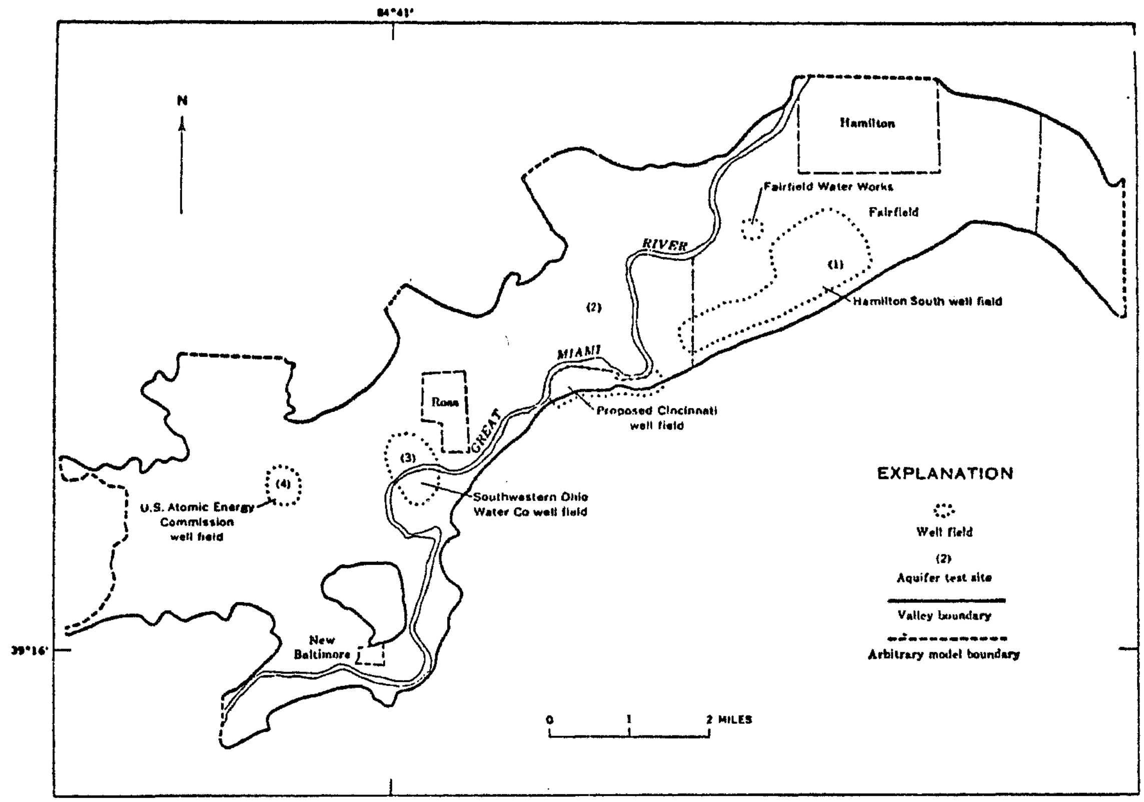
The construction and validation of an aquifer model for the Fairfield-New Baltimore area is an important step in this project since no prediction of the real system behavior can be made without such a component.



Ground Water In The Lower Great Miami River Valley, Ohio

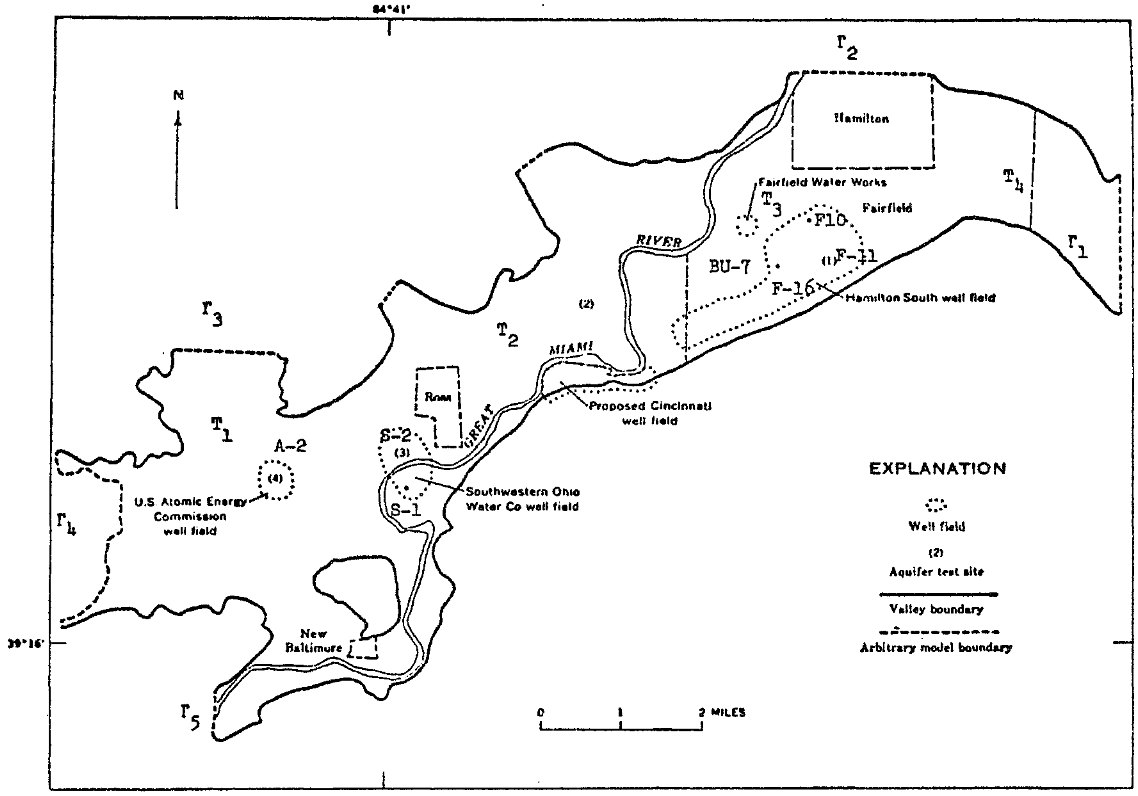
Fig. 5.2 - Location of the Fairfield-New Baltimore area, lower Great Miami River valley

Well Locations Marked (X)



DESCRIPTION OF THE LOWER GREAT MIAMI RIVER VALLEY, OHIO

Figure 5.3



—Location of existing well fields and of the proposed Cincinnati well field, Fairfield-New Baltimore area. Arbitrary limits of the modeled area, beyond which the aquifer extends, are indicated by dashed lines.

GROUND WATER IN THE LOWER GREAT MIAMI RIVER VALLEY, OHIO

Figure 5.4

5.3.3 The Aquifer Model

The modeling of the real system described in the previous sections is described in this section. A computer program was written to simulate the aquifer. The system was divided into cells with differing characteristics (See Fig. 5.5). The data utilized include pumpage water elevations and cell boundary conditions and were taken from Spieker [Spieker, 1968]. An explicit computation scheme can be used, if care is taken to avoid the stability problem by choosing an appropriately small time step. The semi-pervious bedrock which forms the natural boundaries for the groundwater system can be handled as part of the water balance of each cell (constant inflow). The river can be handled as constant head cells. Initial waterhead values in all cells are part of the input to the program. For each time period (one year) the forcing function (pumpage) at each cell is given.

The simulation model can produce two types of output:

(i) For each time period, the interflow between adjacent cells is provided.

(ii) For each time period the averaged water level is predicted in all cells.

Cells #4, #5 and #6 [See Fig. 5.5] were considered in this work due to the location of observation wells (F-10, F-11, F-16, S-1, S-2, and A-2) within these cells. Infiltration rates and the complete pumping history of these cells from 1952 to 1962, which were obtained from the Miami Conservancy District, are

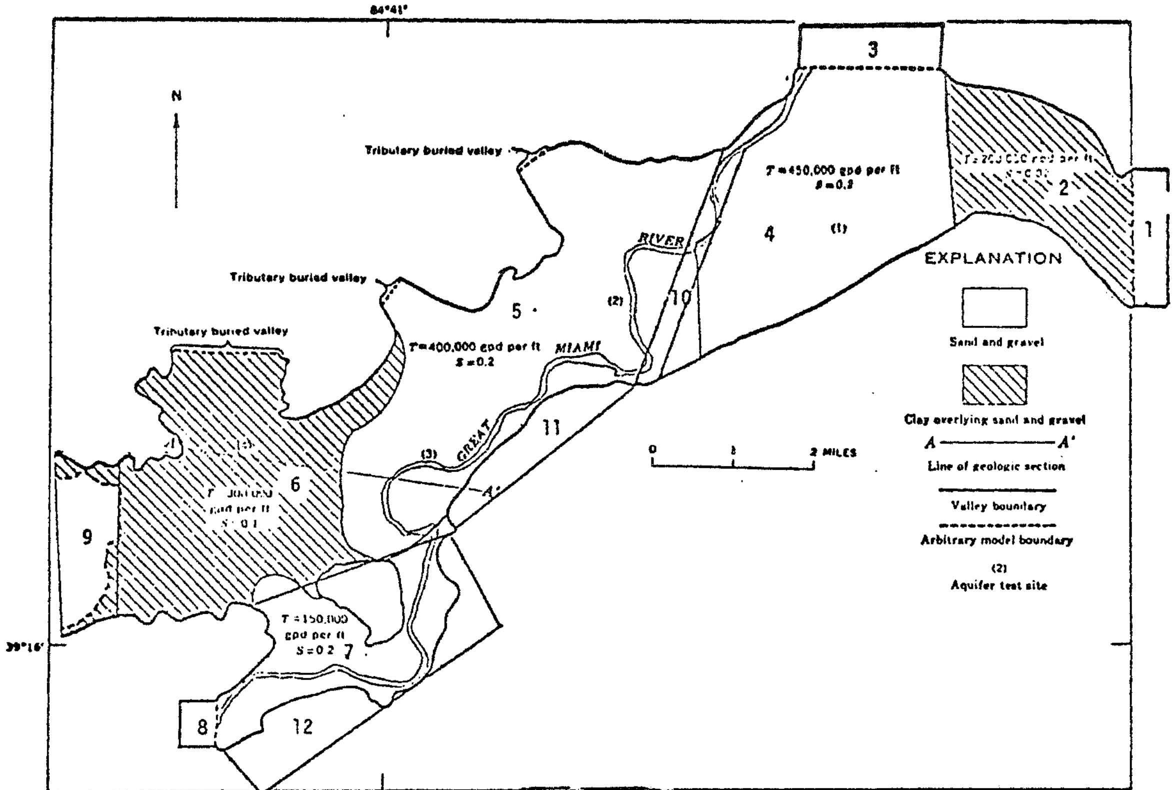


Figure 5.5

Analog Study of Increased Pumping Effects, Fairfield-New Baltimore Area

Generalized geology and coefficients of transmissibility (T) and storage (S) of the Fairfield-New Baltimore area. Cells assignment--Cells 10, 11, and 12 represent the river.

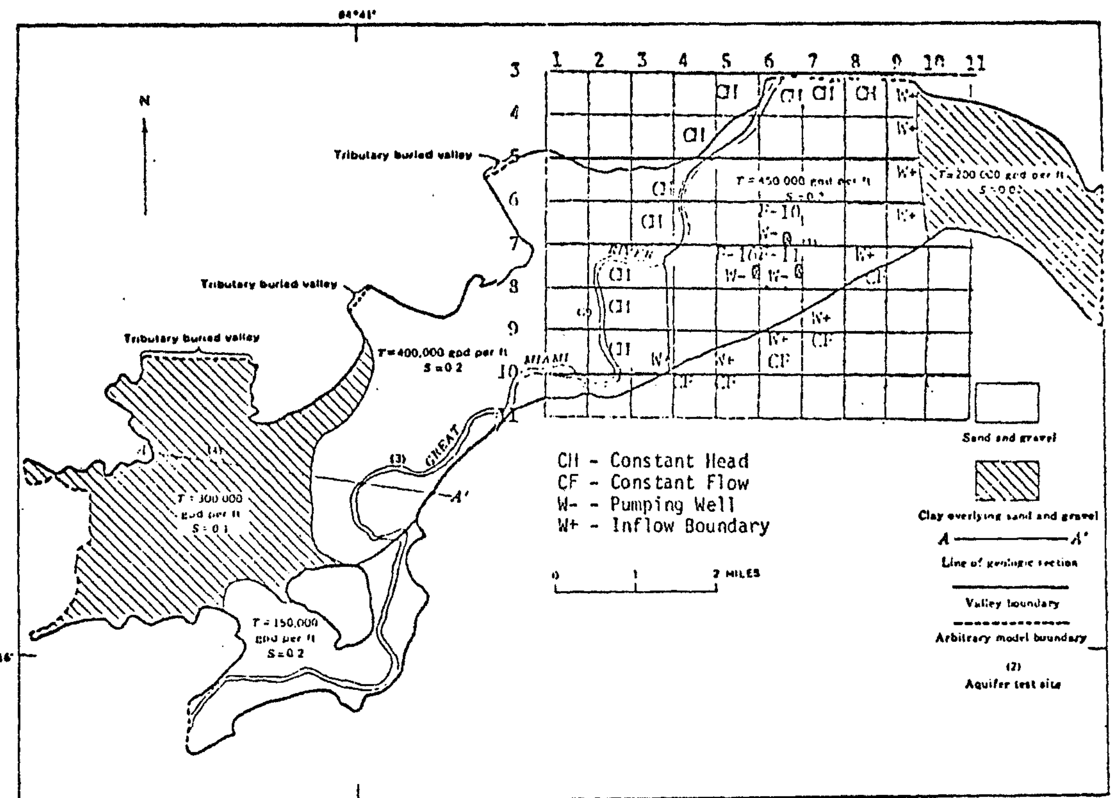
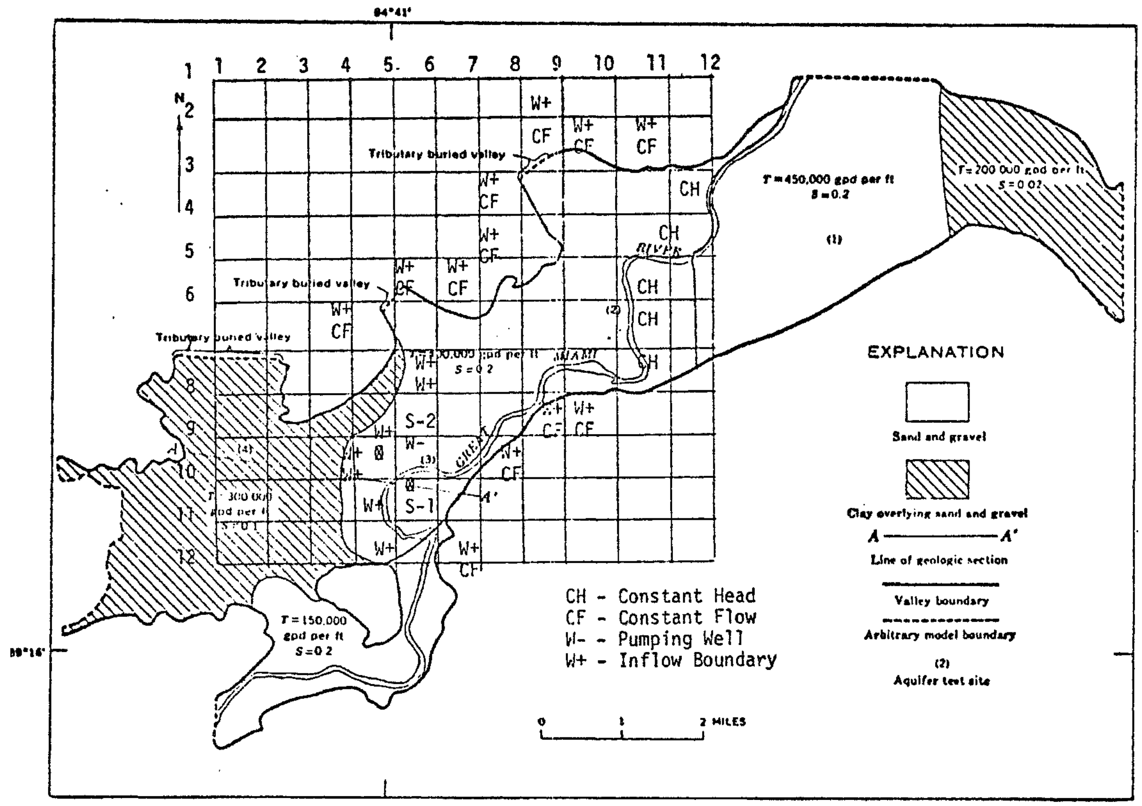


Figure 5.6 - Generalized geology and coefficients of transmissibility (T) and storage (S) of the Fairfield-New Baltimore area.

Cell #4 Discretization for the Detailed Modeling



Generalized geology and coefficients of transmissibility (T) and storage (S) of the Fairfield-New Baltimore area.

Cell #5 Discretization for the Detailed Modeling

Figure 5.7

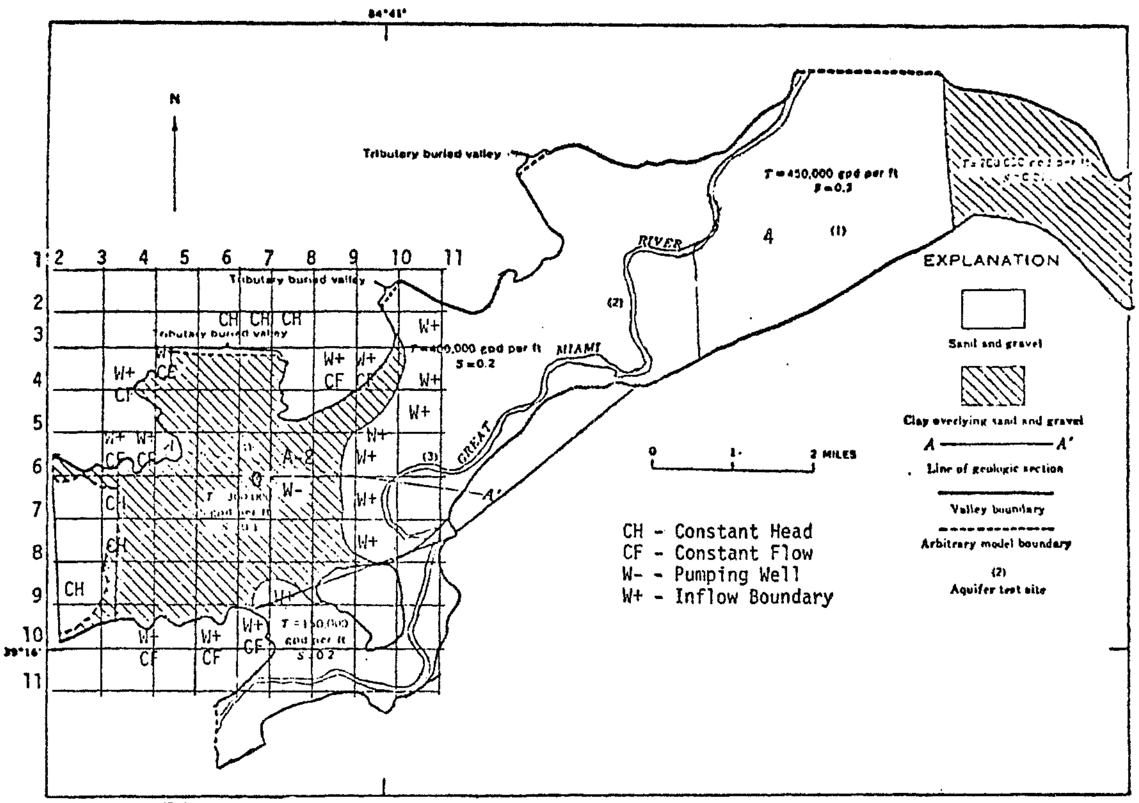


Figure 5.8 Generalized geology and coefficients of transmissibility (T) and storage (S) of the Fairfield-New Baltimore area.

Cell #6 Discretization for the Detailed Modeling presented in Tables 5.1 and 5.2. A breakdown per month can be obtained from Spieker [1968]. Location of the pumping wells is shown in Figure 5.4. Table 5.3 summarizes the characteristics of the cells under study. Figures 5.6, 5.7, and 5.8 indicate the constant head and recharging boundaries of the concerned cells.

To show the possible applications of the methodology developed in this chapter to the case under study, boundary conditions were taken for these three cells from the results of the multicell model. The method used for identifying the transmissivity function parameters of these cells is an iterative gradient algorithm developed by Lopez [Lopez, 1973] based on the maximum neighborhood method [Marquardt, 1963]. Once the parameters defining the transmissivity function have been estimated, the appropriate next test of the calibrated equipment model is how well it predicts the aquifer's response to any demand placed on it.

5.3.4 Needs for Additional Information in Aquifer Modeling

The decomposition approach of aquifer modeling in this chapter stems from the intuition of developing an accurate groundwater model for great Miami River Basin using additional available information on the groundwater system. To do this, it is worthwhile to answer the following questions:

(1) What kind of modeling errors can we come up with in developing an accurate model?

(2) How can those errors be minimized?

Well Name	Cell Location	Pumping Periods 1952	1953	1954	1955	1956	1957	1958	1959	1960	1961	1962
A-2	6	155	155	155	155	155	155	155	155	155	155	155
S-1 / S-2	5	1512	1835	1762	2155	2031	2260	2019	2298	2223	2004	1951
F-10	4	0	0	500	0	338	377	381	372	356	354	357
F-11	4	0	500	0	0	423	471	477	465	445	443	446
F-16	4	500	0	0	500	338	377	381	372	356	354	357

Table 5.1

PUMPING HISTORY FAIRFIELD-NEW BALTIMORE AQUIFER. FIGURES ARE GIVEN IN FT^3/SEC. *100. DATA FROM 1958-62 WERE NOT USED IN THE IDENTIFICATION OF T

CELL #4	Boundary Points (I,J)	(7,8)	(7,9)	(8,7)	(9,4)	(9,5)	(9,6)			
		(See Fig. 4.5 for location of this coordinates)								
	Infiltration Rate	5	5	5	5	5	5			
CELL #5	Boundary Points (I,J)	(3,11)	(3,10)	(3,9)	(3,8)	(4,8)	(5,8)	(6,7)	(6,6)	(6,4)
		(See Fig. 4.6 for location of this coordinates)								
	Infiltration Rate	18	12	12	12	12	12	12	12	6
	Boundary Points (I,J)	(6,5)	(7,4)	(8,10)	(8,9)	(9,7)	(9,8)	(10,6)	(11,6)	
	Infiltration Rate	6	6	6	6	6	6	12	6	
CELL #6	Boundary Points (I,J)	(4,5)	(5,5)	(6,4)	(6,5)	(3,8)	(4,8)	(4,9)		
		(See Fig. 4.7 for location of this coordinates)								
	Infiltration Rate	12	12	12	12	6	12	12		
	Boundary Points (I,J)	(3,10)	(9,4)	(9,5)	(9,6)	(9,7)	(9,8)			
	Infiltration Rate	6	12	12	12	12	2			

Table 5.2 Infiltration Rates Fairfield-New Baltimore Aquifer (Units: ft^3/sec *100)

CHARACTERISTIC	CELL NO.	DESCRIPTION
Aquifer Type	4	Unconfined small marginal areas are of semi-confined type.
	5	
	6	
Storage Coefficient, s (Dimensionless)	4	0.2
	5	0.2
	6	0.1
Transmissivity Coefficient, T (ft/sec)	4	Unknown
	5	
	6	
Initial Head (in ft.)	4	552
	5	532
	6	524
Boundary Conditions	4	East & West: Inflows from Cell #2 & Const. Head North & South: Const. head & Constant flow
	5	East & West: Inflow from Cell #6 North & South Constant flow
	6	East & West: Inflow from Cell #5 & Constant head North & South: Constant flow
Wells	4	F-10, F-11, F-16
	5	S1 & S2
	6	A2
Approximate Area (in sq. miles)	4	7
	5	9
	6	8

Table 5.3

AQUIFER DATA: FAIRFIELD-NEW BALTIMORE

As mentioned earlier, an error in groundwater modeling is defined as the absolute difference at a particular time between the waterhead computed at a given model location and the true water head at the corresponding location in the groundwater system:

$$E_{t,L} = \left|\left|h_{t,L} - \hat{h}_{t,L}\right|\right| \tag{5.14}$$

Where $E_{t,L}$ is the modeling error at location L (the L notation refers to the standard two-dimensional co-ordinate (x,y) system at time t; $h_{t,L}$ is the water level computed by the aquifer model at location L and time t and $\hat{h}_{t,L}$ is the true water level at a corresponding point and time in the groundwater system.

Modeling errors can be classified as those associated with:

(i) computation

(ii) mathematical assumption

(iii) basic data

Generally speaking, the three errors mentioned above include most of those in aquifer modeling. Our work was concerned with prediction errors caused by errors in basic data. We define an error in basic data as the difference between the estimated or measured value of a model variable and the corresponding true value of the groundwater system. Making errors in basic data is probably one of the major sources of errors in modeling.

Errors in basic data are classified as:

a) Errors in aquifer parameters

(i) coefficient of storage

(ii) coefficient of transmissivity

b) Errors in initial and final conditions of waterhead

c) Errors in input and output functions

(i) discharge (including pumpage)

(ii) recharge

d) Errors in boundary configuration

Each of the above includes some errors that lead to further errors in predicting future water levels.

Generally, data errors can be of several types, such as instrumental or measurement, interpolation sampling, and errors due to data not being representative of the aquifer. Measurement errors create minor problems whereas interpolation errors arise when field data are contoured to yeild estimates for all model nodes. Such contouring commonly is done for transmissivity and initial water levels. Sometimes field data may not be representative of or even from the aquifer being modeled. Measurements of water levels in wells affected by local pumping or in wells tapping parched water bodies, for example, will not be representative of aquifer conditions. Errors due to interpolation and nonrepresentative data are significant problems.

For the Miami River Basin in Southern Ohio the coefficient of storage is reasonably well known because adequate measurements of its value have been made over different sections of the aquifer. On the other hand, errors in estimates of transmissivity are present due to the consideration of its (transmissivity) average value over different sections of the aquifer. Finally the average value becomes nonrepresentative of that area due to its variation over space.

Error in initial water level may be due to

(i) measurement error

(ii) interpolation error

(iii) nonrepresentative location in the aquifer at that point in time.

In addition, errors in final water levels for one or more historical periods of time used in calibrating the model lead to modeling errors. Groundwater models commonly are calibrated by adjusting model parameters so that computed water levels match historically measured levels at one or more points in time. These final water levels can be in error for the same reasons that initial levels were in error.

Discharge and recharge estimates used in the model can be in error for several reasons, which can be classified as follows:

(i) errors in quantity

(ii) errors in the assumed location

(iii) errors related to time variations in discharge or recharge not accounted for by the model.

Much of the pumpage data in the Miami Basin are reasonably accurate as far as quantity and location of pumpage is concerned. Most of the recharge in the Miami Basin is caused by induced recharge from boundaries and subsurface flow from the Great Miami River. Adequate data from recharge are available from Speiker.

Errors also are introduced into the model because the model boundaries do not duplicate exactly those of the groundwater system.

The above study gives us some appreciation of different errors involved in groundwater modeling. Later we show by statistical analysis how data errors on transmissivity, storativity, pumpage and water head observation affect the groundwater model output.

5.4 COMPUTATIONAL RESULTS AND SENSITIVITY ANALYSES FOR DECOMPOSED MODEL

5.4.1 Introduction

In this chapter the numerical methods used to accomplish the goals stated in previous chapters will be presented. As an example of using the identification algorithm developed in this chapter to estimate transmissivity values, the Fairfield-New Baltimore aquifer system is considered. The model-estimated parameters for transmissivity functions were then used for model validation to establish the capability of the model to predict real system behavior. This aquifer system was also used previously as a source for hydrogeological data for identifying and validating the model developed in Phases I and II. This facilitates a direct comparison of the results of this work with those models.

The purpose of the sensitivity analysis was to show the effect of errors in observed head, pumpage, transmissivity and storativity on the predicted head values calculated by the mathematical model developed herein.

5.4.2 Identification Model Calibration

The calibration of the model was done for the Fairfield-New Baltimore aquifer system. Spieker [1968] and Miami Conservancy District, Dayton, Ohio furnished the basic hydrogeological data for this system. The time period 1952 to 1962 was chosen for the identification and validation processes and was used in this way:

(1) 1952-1956 for model identification

(2) 1956-1962 for model validation

Observed water heads at different grid points of cells #4, #5 and #6 were generated for 1952 to 1956 using Spieker's mathematical model, parameters and conditions that he determined for the same problem area. This provided water head estimates for the six pumping wells of the region which were used for individual cell parameter identification of this work. Generated water head observations are presented in Tables 5.4(a), 5.4(b) and 5.4(c).

The identification algorithm was started using the initial guess of transmissivity averaged between cells as follows:

$$T_1 = 0.25$$
$$T_2 = 0.51$$
$$T_3 = 0.907$$
$$T_4 = 0.915$$
$$T_5 = 0.649$$
$$T_6 = 0.412$$
$$T_7 = 0.36$$
$$T_8 = 0.201$$
$$T_9 = 0.663$$

$T_{10} = 0.66$

$T_{11} = 0.62$

$T_{12} = 0.209$

Where subscripts 1,......12 of T mentioned above represent the following flow relation between cells (See Fig. 5.5)

Subscripts	Flow Relation Between Cells
1	2←1
2	2←4
3	4←3
4	4←10
5	6←5
6	5←7
7	7←6
8	7←8
9	6←9
10	5←10
11	5←11
12	7←12

The initial guess of transmissivity is based on the geological information of that area. The aquifer was simulated by the multicell model to produce:

(i) the interflow between adjacent cells

(ii) an averaged water level in all cells

For the five-year period (1952-1956) using initial guesses, parameters $(b_1, b_2, b_3, b_4, b_5)$ of transmissivity function

$$T(x,y) = b_1x^2 + b_2y^2 + b_3x + b_4y + b_5$$

of cells #4, #5 and #6 were identified after being subjected to the above information developed in the iterative process.

Computationally, the identification scheme of this work is very effective. However, the initial guess of transmissivity plays a dominant role in computation time. The least-square error function between observed and calculated head of each cell converges quadratically to a minimum even with bad initial values (corresponding to a large initial least-square error). The model-predicted drawdowns for 1952 to 1956 are shown in Tables 5.5(a), 5.5(b) and 5.5(c). A comparison of the real (observed) drawdown values and the model's predicted drawdown (Tables 5.6(a), 5.6(b) and 5.6(c)) shows generally good agreement between them. Results of the identification of transmissivity function parameters are tabulated in Table 5.7.

Observation Point	Pumping Period=1(1952) Drawdowns(FT)	Pumping Period=2(1953) Drawdowns(FT)	Pumping Period=3(1954) Drawdowns(FT)	Pumping Period=4(1955) Drawdown(FT)	Pumping Period=5(1956) Drawdowns(FT)
4 8	-0.801	-1.368	-1.511	-1.542	-1.251
5 7	-0.423	-0.801	-0.965	-1.001	-0.309
5 8	-1.201	-1.913	-2.141	-2.192	-1.586
6 7	-0.204	-0.572	-0.752	-0.801	0.392
7 5	1.056	1.092	1.033	1.016	2.721
7 6	3.273	3.231	3.124	3.092	5.770
8 4	0.722	0.795	0.778	0.761	1.864
8 5	3.541	3.662	3.612	3.599	6.301
8 6	3.839	3.915	3.837	3.805	7.351

Water Head Observations of Cell #4
(generated after Spieker)

Table: 5.4(a)

Observation Point	Pumping Period=1(1952) Drawdowns(FT)	Pumping Period=2(1953) Drawdowns(FT)	Pumping Period=3(1954) Drawdowns(FT)	Pumping Period=4(1955) Drawdowns(FT)	Pumping Period=5(1956) Drawdowns (FT)
7 6	-0.989	-1.528	-1.643	-1.960	-1.979
7 7	-0.305	-0.521	-0.583	-0.705	-0.726
8 6	-1.218	-1.699	-0.142	-2.077	-2.045
9 4	-5.595	-7.247	-7.357	-8.558	-8.330
9 5	-11.385	-13.771	-13.516	-15.852	-15.233

Water Head Observations of Cell #5
(generated after Spieker)

TABLE: 5.4(b)

Observation Point	Pumping Period=1(1952) Drawdowns(FT)	Pumping Period=2(1953) Drawdowns(FT)	Pumping Period=3(1954) Drawdowns(FT)	Pumping Period=4(1955) Drawdowns(FT)	Pumping Period=5(1956) Drawdowns(FT
4 6	-0.198	-0.232	-0.248	-0.245	-0.245
5 6	-0.482	-0.547	-0.576	-0.571	-0.570
6 7	-4.944	-5.038	-5.087	-5.075	-5.072
6 8	-1.134	-1.174	-1.238	-1.214	-1.211
7 5	-0.237	-0.292	-0.320	-0.317	-0.316
7 7	-0.991	-1.075	-1.128	-1.116	-1.113
8 5	-0.126	-0.173	-0.201	-0.119	-0.197
8 7	-0.576	-0.647	-0.702	-0.690	-0.687

Water Head Observations of Cell #6
(generated after Spieker)

TABLE: 5.4(c)

Observation Point	Pumping Period=1(1952) Drawdowns(FT)	Pumping Period=2(1953) Drawdowns(FT)	Pumping Period=3(1954) Drawdowns(FT)	Pumping Period=4(1955) Drawdowns(FT)	Pumping Period=5(1956) Drawdowns(FT)
4 8	-0.791201	-1.418124	-1.821123	-1.552213	-1.461061
5 7	-0.442321	-0.861231	-1.115321	-1.021241	-0.339420
5 8	-1.351347	-2.031471	-2.561246	-2.302120	-1.629146
6 7	-0.413424	-0.552139	-0.962124	-0.841216	-0.512344
7 5	1.021236	1.132134	1.073659	1.166122	-2.741932
7 6	3.373432	3.241416	3.144630	3.292243	6.149243
8 4	1.019234	1.205618	0.808357	1.041642	2.124128
8 5	3.769213	3.922412	3.822124	3.629624	6.311426
8 6	4.091456	3.925243	4.097162	3.905271	7.501460

Cell #4 Water Head Predicted by the Model

TABLE: 5.5(a)

Observation Point	Pumping Period=1(1952) Drawdowns(FT)	Pumping Period=2(1953) Drawdowns(FT)	Pumping Period=3(1954) Drawdowns(FT)	Pumping Period=4(1955) Drawdowns(FT)	Pumping Period=5(1956) Drawdowns(FT)
7 6	-1.011012	-1.538213	-1.652134	-1.981245	-1.991234
7 7	-0.315112	-0.542641	-0.681235	-0.728634	-0.766198
8 6	-1.328431	-1.782145	-0.156143	-2.331240	-2.056231
9 4	-5.825120	-7.366123	-7.567916	-8.577421	-8.531041
9 5	-11.415341	-13.972034	-13.646450	-16.121456	-15.281468

Cell #5 Water Head Predicted by the Model

TABLE: 5.5(b)

Observation Point	Pumping Period=1(1952) Drawdowns(FT)	Pumping Period=2(1953) Drawdowns(FT)	Pumping Period=3(1954) Drawdowns(FT)	Pumping Period=4(1955) Drawdowns(FT)	Pumping Period=5(1956) Drawdowns(FT
4 6	-0.225143	-0.252164	-0.259942	-0.232114	-0.442143
5 6	-0.572261	-0.681432	-0.562143	-0.591241	-0.583264
6 7	-5.213462	-5.224126	-5.386432	-5.171242	-5.291348
6 8	-1.321420	-1.191264	-1.352684	-1.525146	-1.401342
7 5	-0.248168	-0.308148	-0.517941	-0.422136	-0.328116
7 7	-1.213480	-1.086142	-1.153121	-1.125334	-1.724321
8 5	-0.145321	-0.576452	-0.227418	-0.231468	-0.212346
8 7	-0.591242	-0.665432	-1.031402	-0.841531	-0.883451

Cell #6 Water Head Predicted by the Model

TABLE: 5.5(c)

Pumping Period = 1(1952)

Observation Point		Water Head Predicted By Spiekers Model $\hat{h}$ (FT.)	Water Head Predicted By Sarkars Model h(FT.)	$\Delta = h - \hat{h}$
4	8	-0.801	-0.791201	0.01
5	7	-0.423	-0.442321	0.02
5	8	-1.201	-1.351347	0.15
6	7	-0.204	-0.413424	0.21
7	5	1.056	1.021236	0.03
7	6	3.273	3.373432	0.10
8	4	0.722	1.019234	0.30
8	5	3.541	3.769213	0.22
8	6	3.839	4.091456	0.26

Cell #4 Water Head Comparison

TABLE: 5.6(a)

Pumping Period = 2(1953)

Observation Point		Water Head Predicted By Spiekers Model $\hat{h}$ (FT.)	Water Head Predicted By Sarkars Model h(FT.)	$\Delta = h - \hat{h}$
4	8	-1.368	-1.418124	0.05
5	7	-0.801	-0.861231	0.06
5	8	-1.913	-2.031471	0.12
6	7	-0.572	-0.552139	0.02
7	5	1.092	1.132134	0.04
7	6	3.231	3.241416	0.01
8	4	0.795	1.205618	0.41
8	5	3.662	3.922412	0.26
8	6	3.915	3.925243	0.01

TABLE: 5.6(a)
(Continued)

Pumping Period = 3(1954)

Observation Point		Water Head Predicted By Spiekers Model $\hat{h}$ (FT.)	Water Head Predicted By Sarkars Model h(FT.)	$\Delta = h - \hat{h}$
4	8	-1.511	-1.821123	0.31
5	7	-0.965	-1.115321	0.15
5	8	-2.141	-2.561246	0.42
6	7	-0.752	-0.962124	0.21
7	5	1.033	1.073659	0.04
7	6	3.124	3.144630	0.02
8	4	0.778	0.808357	0.03
8	5	3.612	3.822124	0.21
8	6	3.837	4.097162	0.26

Pumping Period = 4(1955)

Observation Point		Water Head Predicted By Spiekers Model $\hat{h}$ (FT.)	Water Head Predicted By Sarkars Model h(FT.)	$\Delta = h - \hat{h}$
4	8	-1.542	-1.552213	0.01
5	7	-1.001	-1.021241	0.02
5	8	-2.192	-2.302120	0.11
6	7	-0.801	-0.841216	0.04
7	5	1.016	1.166122	0.15
7	6	3.092	3.292243	0.20
8	4	0.761	1.041642	0.28
8	5	3.599	3.629624	0.03
8	6	3.805	3.905271	0.10

TABLE: 5.6(a)
(Continued)

Pumping Period = 5(1956)

Observation Point		Water Head Predicted By Spiekers Model $\hat{h}$(FT.)	Water Head Predicted By Sarkars Model h(FT.)	$\Delta = h - \hat{h}$
4	8	-1.251	-1.461061	0.21
5	7	-0.309	-0.339420	0.03
5	8	-1.586	-1.629146	0.04
6	7	0.392	0.512344	0.12
7	5	2.721	-2.741932	0.02
7	6	5.770	6.149243	0.37
8		1.864	2.124128	0.26
8	5	6.301	6.311426	0.01
8	6	7.351	7.501460	0.15

TABLE: 5.6(a)
(Continued)

Pumping Period = 1(1952)

Observation Point		Water Head Predicted By Spiekers Model $\hat{h}$(FT.)	Water Head Predicted by Sarkars Model h(FT.)	$\Delta = h - \hat{h}$
7	6	-0.989	-1.011012	0.02
7	7	-0.305	-0.315112	0.01
8	6	-1.218	-1.328431	0.11
9	4	-5.595	-5.825120	0.23
9	5	-11.385	-11.415341	0.03

Cell #5 Water Head Comparison

TABLE: 5.6(b)

Pumping Period = 2(1953)

Observation Point		Water Head Predicted By Spiekers Model $\hat{h}$(FT.)	Water Head Predicted By Sarkars Model h(FT.)	$\Delta = h - \hat{h}$
7	6	-1.528	-1.538213	0.01
7	7	-0.521	-0.542641	0.02
8	6	-1.699	-1.782145	0.09
9	4	-7.247	-7.366123	0.12
9	5	-13.771	-13.972034	0.2

Pumping Period = 3(1954)

Observation Point		Water Head Predicted By Spiekers Model $\hat{h}$(FT.)	Water Head Predicted By Sarkars Model h(FT.)	$\Delta = h - \hat{h}$
7	6	-1.643	-1.652134	0.09
7	7	-0.583	-0.681235	0.1
8	6	0.142	-0.156143	0.01
9	4	-7.357	-7.567916	0.21
9	5	-13.516	-13.646450	0.13

Pumping Period = 4(1955)

Observation Point		Water Head Predicted By Spiekers Model $\hat{h}$(FT.)	Water Head Predicted By Sarkars Model h(FT.)	$\Delta = h - \hat{h}$
7	6	-1.960	-1.981245	0.02
7	7	-0.705	-0.728634	0.02
8	6	-2.077	-2.331240	0.26
9	4	-8.558	-8.577421	0.01
9	5	-15.852	-16.121456	0.27

TABLE: 5.6(b)
(Continued)

Pumping Period = 5(1956)

Observation Point		Water Head Predicted By Spiekers Model $\hat{h}$(FT.)	Water Head Predicted By Sarkars Model h(FT.)	$\Delta = h - \hat{h}$
7	6	-1.979	-1.991234	0.02
7	7	-0.726	-0.766198	0.04
8	6	-2.045	-2.056231	0.01
9	4	-8.330	-8.531041	0.20
9	5	-15.233	-15.281468	0.05

TABLE: 5.6(b)
(Continued)

Pumping Period = 1(1952)

Observation Point		Water Head Predicted By Spiekers Model $\hat{h}$(FT.)	Water Head Predicted By Sarkars Model h(FT.)	$\Delta = h - \hat{h}$
4	6	-0.198	-0.225143	0.02
5	6	-0.482	-0.572261	0.09
6	7	-4.944	-5.213462	0.30
6	8	-1.134	-1.321420	0.19
7	5	-0.237	-0.248168	0.01
7	7	-0.991	-1.213480	0.22
8	5	-0.126	-0.145321	0.01
8	7	-0.576	-0.591242	0.01

Cell #6 Water Head Comparison

TABLE: 5.6(c)

Pumping Period = 2(1953)

Observation Point		Water Head Predicted By Spiekers Model $\hat{h}$(FT.)	Water Head Predicted By Sarkars Model h(FT.)	$\Delta = h - \hat{h}$
4	6	-0.232	-0.252164	0.02
5	6	-0.547	-0.681432	0.14
6	7	-5.038	-5.224126	0.19
6	8	-1.174	-1.191264	0.02
7	5	-0.292	-0.308148	0.01
7	7	-1.075	-1.086142	0.01
8	5	-0.173	-0.576452	0.40
8	7	-0.647	-0.6654432	0.02

Pumping Period = 3(1954)

Observation Point		Water Head Predicted By Spiekers Model $\hat{h}$(FT.)	Water Head Predicted By Sarkars Model h(FT.)	$\Delta = h - \hat{h}$
4	6	-0.248	-0.259942	0.01
5	6	-0.576	-0.562143	0.01
6	7	-5.087	-5.386432	0.30
6	8	-1.238	-1.352684	0.12
7	5	-0.320	-0.517941	0.19
7	7	-1.128	-1.153121	0.03
8	5	-0.021	-0.227418	0.02
8	7	-0.702	-1.031402	0.33

TABLE: 5.6(c)
(Continued)

Pumping Period = 4(1955)

Observation Point		Water Head Predicted By Spiekers Model $\hat{h}$(FT.)	Water Head Predicted By Sarkars Model h(FT.)	$\Delta = h - \hat{h}$
4	6	-0.245	-0.232114	0.01
5	6	-0.571	-0.591241	0.02
6	7	-5.075	-5.171242	0.10
6	8	-1.214	-1.525146	0.31
7	5	-0.317	-0.422136	0.11
7	7	-1.116	-1.125334	0.01
8	5	-0.119	-0.231468	0.04
8	7	-0.690	-0.841531	0.25

Pumping Period = 5(1956)

Observation Point		Water Head Predicted By Spiekers Model $\hat{h}$(FT.)	Water Head Predicted By Sarkars Model h(FT.)	$\Delta = h - \hat{h}$
4	6	-0.245	-0.442143	0.20
5	6	-0.570	-0.583264	0.01
6	7	-5.072	-5.291348	0.22
6	8	-1.211	-1.401342	0.19
7	5	-0.316	-0.328116	0.01
7	7	-1.113	-1.724321	0.61
8	5	-0.197	-0.212346	0.15
8	7	-0.687	-0.883451	0.20

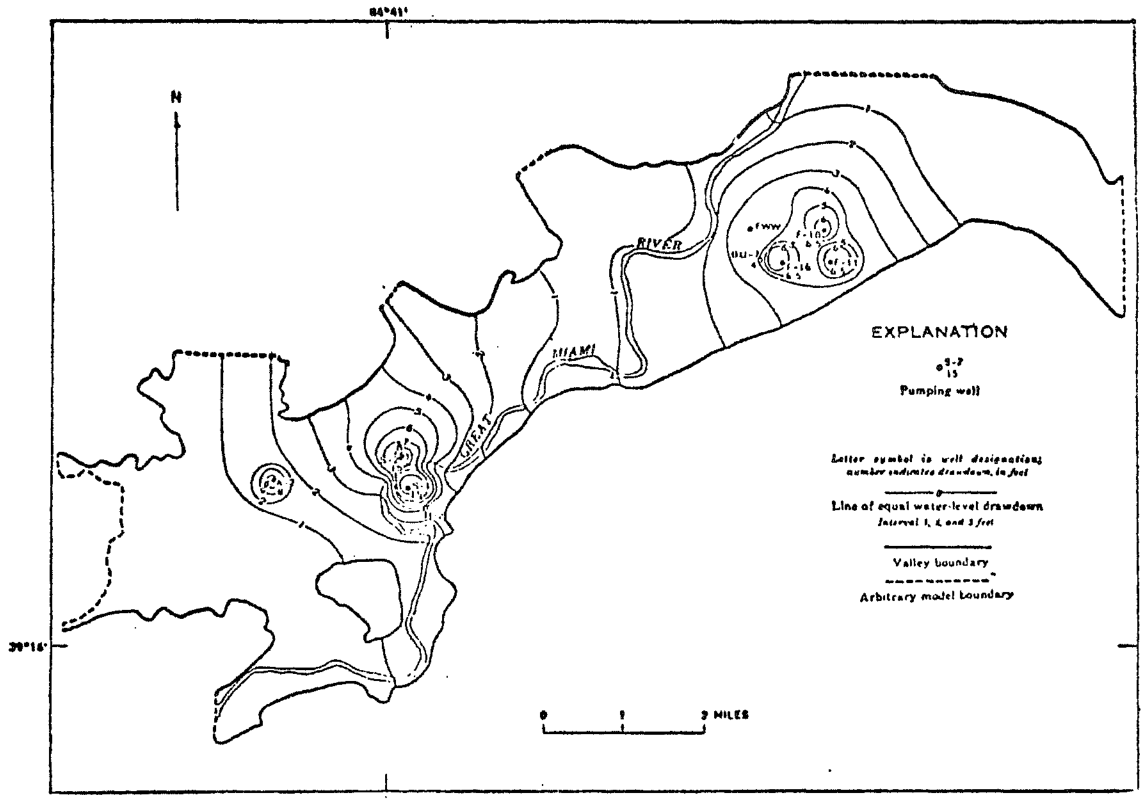
TABLE: 5.6(c)
(Continued)

ranges between 2% and 15%. However the predicted drawdown in Phases I and II varies from 15% to 33% and 5% to 31% respectively for those same well locations. This implies an impressive improvement in predictive ability was obtained in this work due to its decomposed modeling approach of using additional information to obtain an overall better model yielding more accurate results.

PARAMETERS	CELL #4	CELL #5	CELL #6
b_1	$.2132\times10^{-10}$	$.1245\times10^{-11}$	$-.4013\times10^{-11}$
b_2	$.1013\times10^{-11}$	$-.1300\times10^{-11}$	$.2132\times10^{-10}$
b_3	$.4121\times10^{-6}$	$.2140\times10^{-8}$	$.3012\times10^{-7}$
b_4	$.8234\times10^{-7}$	$.1611\times10^{-8}$	$.5034\times10^{-7}$
b_5	.6	.56	.46

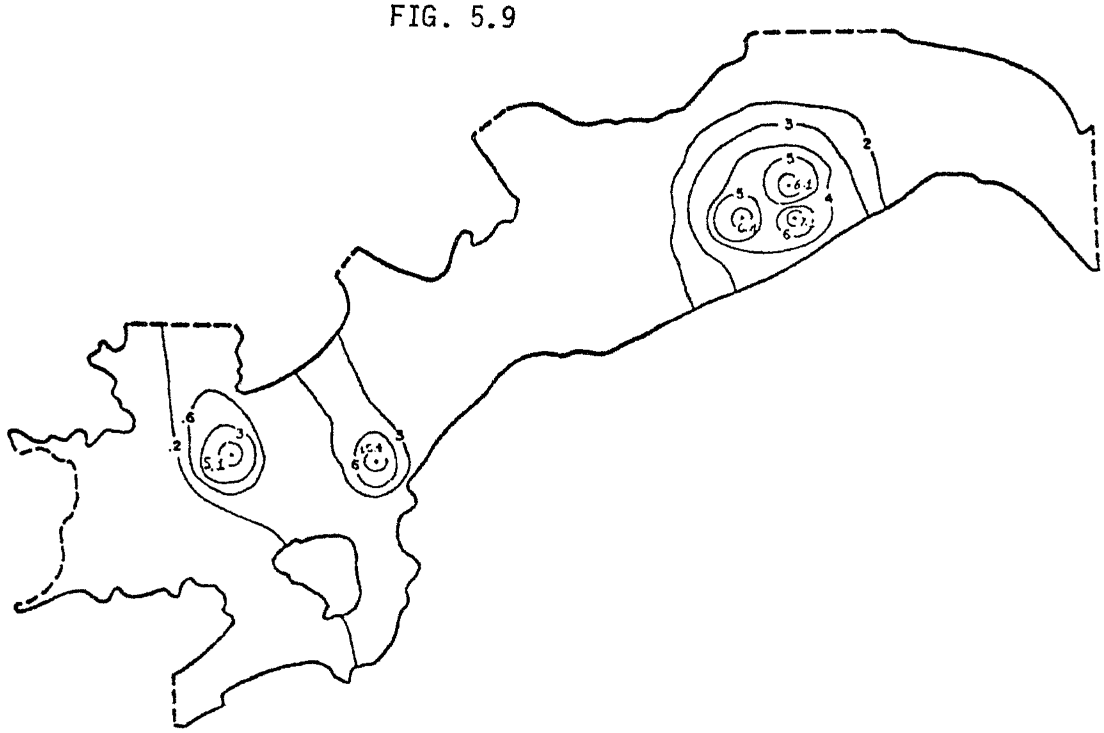
TABLE: 5.7

Results of the Identification of Cell #4, #5 & #6 of the Fairfield-New Baltimore Aquifer System



Drawdowns Caused by Pumping for the
Period 1952-62. Real System Observations made on
November 1962 (After Speieker)

FIG. 5.9



Drawdowns Caused by Pumping for The
Period 1952-62, Based on Decomposed Model

FIG. 5.10

Well Name	Cell Location	Observed Head(ft) After Spieker	Multicell Concept			Singlecell Concept			Singlecell Concept		
			T Quadratic Phase III			T Quadratic Phase II			T Linear Phase I		
			Predicted Head(ft) h	Difference in $h \sim \hat{h}$	Error (%)	Predicted Head(ft) h	Difference in $h \sim \hat{h}$	Error (%)	Predicted Head(ft)	Difference in $h \sim \hat{h}$	Error (%)
A-2	6	6.0	5.09	0.1	15.0	4.15	1.85	31.0	4.0	2.0	33.0
S1-S2	5	15.0	15.4	0.45	3.0	12.0	3.0	20.0	12.0	3.0	20.0
F-16	4	6.5	6.49	0.01	2.0	6.14	0.36	5.6	7.7	1.2	18.4
F-10	4	6.5	6.08	0.42	6.0	6.05	0.45	6.93	7.5	1.0	15.3
F-11	4	6.5	7.08	0.58	10.0	7.40	0.9	14.0	8.7	2.2	30.0

TABLE: 5.8

Results of the Fairfield-New Baltimore Aquifer

Model Forecasted Results

(Water Heads Compared on November 1962)

5.4.3 Sensitivity Analysis

5.4.3.1 *Introduction*

Generally hydrologic phenomena are affected by complex natural events, the details of which cannot be anticipated precisely. Hence the analysis of hydrologic systems is often viewed in terms of stochastic processes. However, the analysis of groundwater flow has traditionally been based on a deterministic approach to the solution of the governing partial differential equation. Natural variability, such as temporal fluctuations in groundwater recharge, storativity, infiltration, evapotranspiration and spatial variation in transmissivity, is usually dealt with only in terms of average conditions. Yet natural variability may be an important feature of groundwater flow in that it may be possible to infer aquifer properties from water table fluctuations.

In the following analysis, effect of temporal variability in various groundwater system parameters on hydraulic head values of the Fairfield-New Baltimore aquifer are examined. Before the development of different optimization methodologies used for ground water parameter identification, this type of analysis was also used for precise estimation of these parameters. In this work, various sensitivity analyses were performed to determine the effect of errors in transmissivity, storativity, observed head and pumpage on model prediction. The resulting sensitivity and statistical analyses as discussed in the following section were found to be useful in finding which parameter must be specified with the greatest accuracy in order to model adequately the groundwater system, and which parameter of the groundwater system is causing most sensitivity on the model water head prediction.

5.4.3.2 *Effect of Errors in Storativity on Model Water Head Prediction*

A sensitivity analysis was performed to determine the effect of error in storativity on the parameter values and its influence on waterhead prediction. The behavior of model waterhead prediction at five well locations due to the small change in storage coefficients of different cells (Cells #4, #5 and #6) was studied. For the bulk of the area covered by Cells #4 and #5, where the groundwater occurs under unconfined conditions, the storativity was perturbed around a value of 0.2 (S_1 = 0.15, S_2 = 0.2, S_3 = 0.25) which is a typical value for an unconfined aquifer. In the area covered by Cell #6, the storativity was perturbed around 0.1 (S_1 = 0.07, S_2 = 0.1, S_3 = 0.15), because here, although the groundwater is largely unconfined, a thin layer of clay locally separates the aquifer into two parts [Spieker, 1968]. This separation is considered to reduce the storativity to slightly less than the normal value of 0.2 associated with unconfined conditions.

Table 5.9(a)-5.9(c) shows the sensitivity analyses for five well locations. This required three solutions of the identification algorithm and three corresponding solutions for computing waterhead prediction. A statistical analysis of error in waterhead prediction due to change in storativity was also performed (See Table 5.9(d)). The analysis indicated that under a varying range of error in storativity (± 25% of average value), the percentage error in waterhead prediction has mean value (μ) in the range of 0 to -12 and standard deviation (σ) 0 to 0.01. This shows that in general the deviation of output at different well locations is not appreciably sensitive to the change in the storativity parameter. It has also been noted that in two well locations (S1-S2 and A-2) the % of error in waterhead prediction is zero even where the percentage of error in storativity lies in the range of -30% to +30%. The conclusion of less sentitive output due to change in storativity holds equally for constant and varying pumping conditions. However the error in predicting output depends not only on storativity exclusively but also on other hydrologic phenomena in an aquifer.

5.4.3.3 *Effect of Errors in Observed Drawdown on Model Waterhead Prediction*

To evaluate the effect on model prediction due to the errors in observed drawdown, a sensitivity analysis was also performed. The identification problem was rerun with error artificially introduced in drawdown at five pumping well locations (F-10, F-11, F-16, S1-S2 and A-2). Table 5.10(a)-5.10(c) demonstrates results of this analysis. H_2 represents the computed head values when no error was introduced in the observed head under optimal conditions, whereas H_1 and H_3 represent the computed head values when different sets of error were introduced into the observed head. It was noted according to a statistical analysis (see Table 5.10(d)) that under various percentages of error ($\pm$ 5%) in observed head, the mean (μ) and standard deviation (σ) of percentage error in waterhead prediction varies from 12 to -14 and 0 - 0.11, respectively. This reveals that computed head values are moderately sensitive to error in observed drawdown. Generally more error in observed head results in more inaccurate waterhead forecasting. Although the results for only two sets of error are shown in Table 5.10(d), many other sets of error were examined and no exceptions to the aforementioned conclusions were found.

5.4.3.4 *Effect of Errors in Pumpage on Waterhead Prediction*

A sensitivity analysis was also performed to evaluate the effect on the parameter values identified and model prediction due to the error in pumpage at different wells in the aquifer. This is especially important since in a water resource system the rate of pumping varies for different reasons. The identification problem was also rerun with changed pumping. This yielded the effect of this change on the optimal parameter values causing different waterhead predictions (See Table 5.11(a)-5.11(c)). A statistical analysis of errors in pumpage (See Table 5.11(d)) indicates that under its various percentage error ($\pm$ 10%), the mean (μ) and standard deviation (σ) of percentage error in waterhead prediction varies in a range of 8 to -17 and 0.02 to 0.08, respectively. The results of this analysis also demonstrate that the computed head values are closely related to the amount of pumpage error. Generally more error in pumpage will result in more drawdown and vice versa. However this relationship does not follow any particular pattern due to the various geological characteristics of the aquifer which affect waterhead drawdown.

5.4.3.5 *Effect of Errors in Transmissivity on Waterhead Prediction*

As mentioned earlier, transmissivity is an important property in a groundwater system. Its accurate estimation plays a dominant part in forecasting groundwater system response to various hydrologic stresses. To evaluate the effect of inaccurate estimation of the transmissivity parameter on waterhead prediction, a sensitivity analysis was done. This analysis was carried out by changing parameters representing transmissivity function T(x,y). As mentioned earlier transmissivity is approximated by a second-order polynomial function

$$T(x,y) = b_1x^2 + b_2y^2 + b_3x + b_4y + b_5$$

since it is known that the parameter b_5 of above equation has more weight in the function than any other parameters, e.g., b_1, b_2, b_3 & b_4.

Hence this parameter (b_5) was slightly changed around its optimal value, keeping other optimal parameters constant. The behavior of model waterhead prediction at five well locations due to this small change in transmissivity coefficient parameters was studied by means of statistical analysis (See Table 5.12(d)). The analysis indicated that under a range of error in transmissivity ($\pm$ 9% of its optimal value), the percentage error in waterhead prediction has mean value (μ) and standard deviation (σ) in the range of 16 to -17 and 0 to 0.03, respectively. This shows that in general: (i) the model waterhead prediction is quite sensitive to change in transmissivity and (ii) as transmissivity increases, the waterhead drawdown tends to decrease and vice versa. This is particularly true within the semiconfined aquifer zone (Well A-2) which is similar to the characteristics shown for the unconfined aquifer zone (Well F-10, F-11, F-16 and S1-S2) of the Fairfield-New Baltimore area.

5.4.3.6 *Comparative Statistical Analysis of Errors*

On the basis of the results of the statistical analyses just examined, a comparative study of the effect of errors in different parameters on waterhead prediction was made by answering the following problem. Let ε_h, ε_s, ε_{oh}, ε_p and ε_T be the percentage error of waterhead response (drawdown), storativity, observed head, pumpage and transmissivity respectively. Show how much ε_h varies for certain values of ε_s, ε_{oh}, ε_p and ε_T

Define

$E(\varepsilon_h/\varepsilon_s)$ = Expected value of error in response to given error in storativity.

$E(\varepsilon_h/\varepsilon_{oh})$ = Expected value of error in response to given error in observed head.

$E(\varepsilon_h/\varepsilon_p)$ = Expected value of error in response to given error in pumpage.

$E(\varepsilon_h/\varepsilon_T)$ = Expected value of error in response to given error in transmissivity.

Considering Well (A-2) for the present study and collecting information from Table 5.9(d), 5.10(d), 5.11(d) and 5.12(d) we have

$E(\varepsilon_h/\varepsilon_s = 30) = 0$

$E(\varepsilon_h/\varepsilon_{oh} = 9) = 1.0$

$E(\varepsilon_h/\varepsilon_p = 10) = 4.0$

$E(\varepsilon_h/\varepsilon_T = 9) = 17.0$

The above statistical statement clearly explains that in the present case 9% of the error in transmissivity has 17% of the error in response while

(i) 30% of error in storativity has no error in response

(ii) 9% of error in observed head has 1% of error in response

(iii) 10% of error in pumpage has 4% of error in response.

Thus above sensitivity and statistical analyses establish the following facts:

(1) In general the modeling technique of this chapter is less sensitive to change in parameters.

(2) Waterhead prediction is more sensitive to change in transmissivity than to change in any other parameters. Hence if transmissivity of a model is not quite accurately known, the model output becomes erroneous.

WELL NAME	STORATIVITY		YEAR	DRAWDOWNS (FT)
F-10	S_1	0.15	1952	2.98
			1953	2.89
			1954	2.79
			1955	2.77
			1956	5.33
	S_2	0.2	1952	3.26
			1953	3.22
			1954	3.11
			1955	3.08
			1956	5.76
	S_3	0.25	1952	3.58
			1953	3.57
			1954	3.39
			1955	3.41
			1956	6.40
F-11	S_1	0.15	1952	3.51
			1953	3.52
			1954	3.44
			1955	3.41
			1956	6.78
	S_2	0.2	1952	3.82
			1953	3.90
			1954	3.82
			1955	3.79
			1956	7.34
	S_3	0.25	1952	4.20
			1953	4.29
			1954	4.16
			1955	4.21
			1956	8.14
F-16	S_1	0.15	1952	3.24
			1953	3.29
			1954	3.24
			1955	3.22
			1956	5.80
	S_2	0.2	1952	3.53
			1953	3.65
			1954	3.60
			1955	3.58
			1956	6.29
	S_3	0.25	1952	3.99
			1953	4.09
			1954	4.03
			1955	4.04
			1956	7.10

CELL #4

Results of Sensitivity Analysis
Effect of Errors in Storativity on Water Head Prediction

TABLE: 5.9(a)

WELL NAME	STORATIVITY		YEAR	DRAWDOWN (FT)
S1-S2	S_1	.15	1952	11.55
			1953	13.86
			1954	13.57
			1955	15.92
			1956	15.27
	S_2	.2	1952	11.41
			1953	13.80
			1954	13.56
			1955	15.89
			1956	15.27
	S_3	.25	1952	11.52
			1953	13.85
			1954	13.58
			1955	15.92
			1956	15.27

Table 5.9(b)

CELL #5

Effect of Errors in Storativity on Water Head Prediction

WELL NAME	STORATIVITY		YEAR	DRAWDOWN (FT)
A-2	S_1	.07	1952	4.99
			1953	5.01
			1954	5.07
			1955	5.07
			1956	5.07
	S_2	.1	1952	4.94
			1953	5.03
			1954	5.08
			1955	5.07
			1956	5.07
	S_3	.15	1952	5.68
			1953	5.04
			1954	5.09
			1955	5.07
			1956	5.07

Table 5.9(c)

CELL #5

Effect of Errors in Storativity on Waterhead Prediction

Well	% Error of Storativity	Year	% Error	Drawdowns % Error Mean (μ)	Standard Deviation (σ)	Variance (σ²)
F-10	-25	1952	- 9.0	-9.0	0.01	δ
		1953	-10.0			
		1954	-10.0			
		1955	-10.0			
		1956	- 8.0			
	25	1952	10.0	11.0	0.01	δ
		1953	11.0			
		1954	9.0			
		1955	11.0			
		1956	11.0			
F-11	-25	1952	- 8.0	-9.0	0.01	δ
		1953	-10.0			
		1954	-10.0			
		1955	-10.0			
		1956	- 8.0			
	25	1952	10.0	10.0	0.01	δ
		1953	10.0			
		1954	9.0			
		1955	11.0			
		1956	11.0			
F-16	-25	1952	- 8.0	-9.0	0.01	δ
		1953	-10.0			
		1954	-10.0			
		1955	-10.0			
		1956	- 8.0			
	25	1952	10.0	12.0	0.01	δ
		1953	12.0			
		1954	12.0			
		1955	13.0			
		1956	13.0			
S1-S2	-25	1952	1.0	δ	δ	δ
		1953	0			
		1954	0			
		1955	0			
		1956	0			
	25	1952	1.0	δ	δ	δ
		1953	0			
		1954	0			
		1955	0			
		1956	0			
A-2	-30%	1952	1.0	δ	δ	δ
		1953	0			
		1954	0			
		1955	0			
		1956	0			
	30%	1952	1.5	δ	δ	δ
		1953	0			
		1954	0			
		1955	0			
		1956	0			

$\delta < 0.001$

Statistical Analysis of Errors in Storativity

TABLE: 5.9(d)

Percentage Of Error Introduced In Water Head Observation		Well Name	Drawdowns (FT) 1952	1953	1954	1955	1956
At Well Location F-11 -7% of 1952 +5% of 1953 +12% of 1954 -9% of 1955 +4% of 1956	H_1	F-10	3.5	3.0	2.86	3.48	5.11
		F-16	3.38	3.25	3.73	3.34	5.77
		F-11	3.5	3.14	3.31	3.0	6.10
No Error	H_2	F-10	3.26	3.22	3.11	3.08	5.76
		F-16	3.53	3.65	3.60	3.58	6.10
		F-11	3.82	3.90	3.82	3.65	7.16
7% of 1952 -5% of 1953 -12% of 1954 9% of 1955 -4% of 1956	H_3	F-10	3.22	3.55	3.2	4.0	4.7
		F-16	3.53	3.77	3.92	3.75	5.5
		F-11	4.25	4.49	4.0	4.24	8.01

TABLE: 5.10(a)

Cell #4

Sensitivity Analysis

Effect of Errors in Observed Drawdown On Water Head Prediction

Percentage Of Error Introduced In Water Head Observation		Well Name	Drawdowns (FT) 1952	1953	1954	1955	1956
At Well Location S1-S-2 -7% of 1952 5% of 1953 12% of 1954 -9% of 1955 -4% of 1956	H_1	S1-S2	12.0	14.42	13.25	16.4	15.52
No Error	H_2	S1-S2	11.41	13.80	13.56	15.89	15.27
7% of 1952 -5% of 1953 -12% of 1954 9% of 1955 -4% of 1956	H_3	S1-S2	11.65	13.4	13.7	15.57	14.8

TABLE: 5.10(b)

Cell #5

Sensitivity Analysis

Effect of Errors in Waterhead Observation on Waterhead Prediction

Percentage Of Error Introduced In Water Head Observation		Well Name	Drawdowns (FT)				
			1952	1953	1954	1955	1956
At Well Location A-2 -7% of 1952 5% of 1953 12% of 1954 -9% of 1955 +4% of 1956	H_1	A-2	4.98	5.07	5.12	5.11	5.04
No Error	H_2	A-2	4.94	5.03	5.08	5.07	5.07
7% of 1952 -5% of 1953 -12% of 1954 9% of 1955 -4% of 1956	H_3	A-2	4.88	4.97	5.06	5.13	5.11

TABLE: 5.10(c)

Cell #6

Sensitivity Analysis

Effect of Errors in Waterhead Observation on Waterhead Prediction

Well	% Error of Observed Head	Year	Drawdowns % Error: % Error	Mean (μ)	Standard Deviation (σ)	Variance (σ^2)
F-10	5%	1952	7.0	-1.0	0.11	0.012
		1953	-7.0			
		1954	-8.0			
		1955	13.0			
		1956	-11.0			
	-5%	1952	-1.0	5.0	0.12	0.014
		1953	10.0			
		1954	8.0			
		1955	14.0			
		1956	-6.0			
F-11	5%	1952	-5.0	-4.0	0.02	δ
		1953	-5.0			
		1954	-4.0			
		1955	-4.0			
		1956	-4.0			
	-5%	1952	2.0	2.0	0.04	δ
		1953	3.0			
		1954	2.0			
		1955	2.0			
		1956	3.0			
F-16	5%	1952	-4.0	-2.0	0.07	δ
		1953	-11.0			
		1954	4.0			
		1955	7.0			
		1956	-5.0			
	-5%	1952	1.0	2.0	0.07	δ
		1953	3.0			
		1954	9.0			
		1955	5.0			
		1956	-10.0			
S1-S2	5%	1952	5.0	2.0	0.03	δ
		1953	4.0			
		1954	-2.0			
		1955	3.0			
		1956	2.0			
	-5%	1952	2.0	-1.0	0.02	δ
		1953	-3.0			
		1954	1.0			
		1955	-2.0			
		1956	-3.0			
A-2	9%	1952	1.0	1.0	0.01	δ
		1953	1.0			
		1954	1.0			
		1955	1.0			
		1956	-1.0			
	-9%	1952	-1.0	δ	δ	δ
		1953	-1.0			
		1954	0			
		1955	1.0			
		1955	1.0			

$\delta < 0.001$

Statistical Analysis of Errors in Observed Head

TABLE: 5.10(d)

Percentage Of Error Introduced In Pumping		Well Name	Drawdowns (FT) 1952	1953	1954	1955	1956
-10% of 1952 -5% of 1953 -15% of 1954 -10% of 1955 -8% of 1956	P_1	F-10	2.68	2.72	2.42	2.45	4.84
		F-16	2.98	3.17	2.96	3.02	5.33
		F-11	3.14	3.33	3.0	3.05	6.19
No Error	P_2	F-10	3.26	3.22	3.11	3.08	5.76
		F-16	3.53	3.65	3.60	3.58	6.10
		F-11	3.82	3.90	3.82	3.65	7.16
10% of 1952 5% of 1953 15% of 1954 10% of 1955 8% of 1956	P_3	F-10	3.39	3.44	3.12	3.18	5.88
		F-16	3.67	3.86	3.88	4.11	7.37
		F-11	3.89	3.92	3.85	3.66	7.37

TABLE: 5.11(a)

Cell #4

Percentage of Error Introduced In Pumping		Well Name	Drawdowns (FT) 1952	1953	1954	1955	1956
-10% of 1952 -5% of 1953 -15% of 1954 -10% of 1955 -8% of 1956	P_1	S1-S2	10.51	13.21	11.95	14.52	14.23
No Error	P_2	S1-S2	11.41	13.80	13.56	15.89	15.27
10% of 1952 5% of 1953 15% of 1954 10% of 1955 8% of 1956	P_3	S1-S2	12.31	14.4	15.14	17.25	16.31

TABLE: 5.11(b)

Cell #5

Percentage of Error Introduced In Pumping		Well Name	Drawdowns (FT) 1952	1953	1954	1955	1956
-10% of 1952 -5% of 1953 -15% of 1954 -10% of 1955 -8% of 1956	P_1	A-2	4.76	4.92	4.8	4.87	4.76
No Error	P_2	A-2	4.94	5.03	5.08	5.07	5.07
10% of 1952 5% of 1953 15% of 1954 10% of 1955 8% of 1956	P_3	A-2	5.12	5.14	5.36	5.27	5.37

TABLE: 5.11(c)

Cell #6

Sensitivity Analysis

Effect of Errors in Pumpage on Waterhead Prediction

Well	% Error of Pumpage	Year	% Error	Drawdowns % Error		
				Mean (μ)	Standard Deviation (σ)	Variance (σ²)
F-10	-10%	1952	-18.0	-18.0	0.03	δ
		1953	-16.0			
		1954	-22.0			
		1955	-20.0			
		1956	-16.0			
	10%	1952	4.0	3.0	0.03	δ
		1953	7.0			
		1954	0			
		1955	0			
		1956	0			
F-11	-10%	1952	-18.0	-17.0	0.03	δ
		1953	-15.0			
		1954	-21.0			
		1955	-16.0			
		1956	-14.0			
	10%	1952	4.0	6.0	0.07	δ
		1953	6.0			
		1954	8.0			
		1955	15.0			
		1956	3.0			
F-16	-10%	1952	-16.0	-15.0	0.02	δ
		1953	-13.0			
		1954	-18.0			
		1955	-16.0			
		1956	-13.0			
	10%	1952	2.0	1.0	0.03	δ
		1953	5.0			
		1954	1.0			
		1955	0			
		1956	3.0			
S1-S2	-10%	1952	- 8.0	- 4.0	0.08	0.01
		1953	- 4.0			
		1954	-12.0			
		1955	- 9.0			
		1956	- 7.0			
	10%	1952	8.0	8.0	0.03	δ
		1953	4.0			
		1954	12.0			
		1955	8.0			
		1956	7.0			
A-2	-10%	1952	- 4.0	- 4.0	0.02	δ
		1953	- 2.0			
		1954	- 6.0			
		1955	- 4.0			
		1956	- 6.0			
	10%	1952	4.0	4.0	0.02	δ
		1953	2.0			
		1954	6.0			
		1955	4.0			
		1956	6.0			

$\delta < 0.001$

Statistical Analysis of Errors in Pumpage

TABLE: 5.11(d)

Well Name		Transmissivity Parameters	Year	Drawdown (FT)
F-10	T_1 = 0.58	b_1 = $.2132 \times 10^{-10}$	1952	3.67
		b_2 = $.1031 \times 10^{-11}$	1953	3.64
		b_3 = $.4121 \times 10^{-6}$	1954	3.52
		b_4 = $.8234 \times 10^{-7}$	1955	3.49
		b_5 = .57	1956	6.44
	T_2 = 0.61	b_1 = $.2132 \times 10^{-10}$	1952	3.26
		b_2 = $.1031 \times 10^{-11}$	1953	3.22
		b_3 = $.4121 \times 10^{-6}$	1954	3.11
		b_4 = $.8234 \times 10^{-7}$	1955	3.08
		b_5 = .6	1956	5.76
	T_3 = 0.64	b_1 = $.2132 \times 10^{-10}$	1952	2.96
		b_2 = $.1031 \times 10^{-11}$	1953	2.90
		b_3 = $.4121 \times 10^{-6}$	1954	2.80
		b_4 = $.8234 \times 10^{-7}$	1955	2.78
		b_5 = . 63	1956	5.23
F-11	T_1 = 0.58	b_1 = $.2132 \times 10^{-10}$	1952	4.29
		b_2 = $.1031 \times 10^{-11}$	1953	4.41
		b_3 = $.4121 \times 10^{-6}$	1954	4.32
		b_4 = $.8234 \times 10^{-7}$	1955	4.28
		b_5 = .57	1956	8.20
	T_2 = 0.61	b_1 = $.2132 \times 10^{-10}$	1952	3.82
		b_2 = $.1031 \times 10^{-11}$	1953	3.90
		b_3 = $.4121 \times 10^{-6}$	1954	3.82
		b_4 = $.8234 \times 10^{-7}$	1955	3.79
		b_5 = .6	1956	7.34
	T_3 = 0.64	b_1 = $.2132 \times 10^{-10}$	1952	3.46
		b_2 = $.1031 \times 10^{-11}$	1953	3.52
		b_3 = $.4121 \times 10^{-6}$	1954	3.45
		b_4 = $.8234 \times 10^{-7}$	1955	3.42
		b_5 = .63	1956	6.66
F-16	T_1 = 0.58	b_1 = $.2132 \times 10^{-10}$	1952	3.96
		b_2 = $.1031 \times 10^{-11}$	1953	4.11
		b_3 = $.4121 \times 10^{-6}$	1954	4.07
		b_4 = $.8234 \times 10^{-7}$	1955	4.04
		b_5 = .57	1956	7.04
	T_2 = 0.61	b_1 = $.2132 \times 10^{-10}$	1952	3.53
		b_2 = $.1031 \times 10^{-11}$	1953	3.65
		b_3 = $.4121 \times 10^{-6}$	1954	3.60
		b_4 = $.8234 \times 10^{-7}$	1955	3.58
		b_5 = .6	1956	6.29
	T_3 = 0.64	b_1 = $.2132 \times 10^{-10}$	1952	3.20
		b_2 = $.1031 \times 10^{-11}$	1953	3.29
		b_3 = $.4121 \times 10^{-6}$	1954	3.25
		b_4 = $.8234 \times 10^{-7}$	1955	3.23
		b_5 = .63	1956	5.25

TABLE: 5.12(a)

Cell #4

Sensitivity Analysis

Effect of Errors in Transmissivity on Waterhead Prediction

Well Name		Transmissivity	Year	Drawdown (FT)
S1-S2	$T_1 = 0.53$	$b_1 = .1245X10^{-11}$	1952	13.21
		$b_2 = .1300X10^{-4}$	1953	16.06
		$b_3 = .2140X10^{-8}$	1954	15.81
		$b_4 = .1611X10^{-8}$	1955	18.51
		$b_5 = .53$	1956	17.82
	$T_2 = 0.56$	$b_1 = .1245X10^{-11}$	1952	11.41
		$b_2 = .1300X10^{-11}$	1953	13.80
		$b_3 = .2140X10^{-8}$	1954	13.56
		$b_4 = .1611X10^{-8}$	1955	15.89
		$b_5 = .56$	1956	15.27
	$T_3 = 0.6$	$b_1 = .1245X10^{-11}$	1952	10.5
		$b_2 = .1300X10^{-11}$	1953	13.0
		$b_3 = .2140X10^{-8}$	1954	12.5
		$b_4 = .1611X10^{-8}$	1955	14.0
		$b_5 = .6$	1956	13.4

TABLE: 5.12(b)

Cell #5

Well Name		Transmissivity Parameters	Year	Drawdown (FT)
A-2	$T_1 = 0.42$	$b_1 = -.4013X10^{-11}$	1952	5.51
		$b_2 = .2132X10^{-10}$	1953	5.55
		$b_3 = .3012X10^{-7}$	1954	5.58
		$b_4 = .5034X10^{-7}$	1955	5.56
		$b_5 = .42$	1956	5.52
	$T_2 = 0.46$	$b_1 = -.4013X10^{-11}$	1952	4.94
		$b_2 = .2132X10^{-10}$	1953	5.03
		$b_3 = .3012X10^{-7}$	1954	5.08
		$b_4 = .5034X10^{-7}$	1955	5.07
		$b_5 = .46$	1956	5.07
	$T_3 = 0.50$	$b_1 = -.4013X10^{-11}$	1952	4.14
		$b_2 = .2132X10^{-10}$	1953	4.20
		$b_3 = .3012X10^{-7}$	1954	4.24
		$b_4 = .5034X10^{-7}$	1955	4.22
		$b_5 = .5$	1956	4.22

TABLE: 5.12(c)

Cell #6

Sensitivity Analysis

Effect of Errors in Transmissivity on Waterhead Prediction

Well	% Error of Transmissivity	Year	% Error	Drawdowns % Error Mean (μ)	Standard Deviation (σ)	Variance (σ²)
F-10	-5	1952	13.0	13.0	δ	δ
		1953	13.0			
		1954	13.0			
		1955	13.0			
		1956	12.0			
	5	1952	- 9.0	-10.0	0.01	δ
		1953	-10.0			
		1954	-10.0			
		1955	-10.0			
		1956	- 9.0			
F-11	-5	1952	12.0	13.0	0.01	δ
		1953	13.0			
		1954	13.0			
		1955	13.0			
		1956	12.0			
	5	1952	-10.0	-10.0	0.01	δ
		1953	-10.0			
		1954	-10.0			
		1955	-11.0			
		1956	- 9.0			
F-16	-5	1952	12.0	13.0	0.01	δ
		1953	13.0			
		1954	13.0			
		1955	13.0			
		1956	12.0			
	5	1952	- 9.0	-11.0	0.03	δ
		1953	-10.0			
		1954	-10.0			
		1955	-10.0			
		1956	-17			
S1-S2	-5	1952	11.0	10.0	0.01	δ
		1953	11.0			
		1954	10.0			
		1955	10.0			
		1956	10.0			
	5	1952	- 9.0	- 9.0	0.03	δ
		1953	- 6.0			
		1954	- 8.0			
		1955	-12.0			
		1956	-12.0			
A-2	-9	1952	17.0	15.0	0.01	δ
		1953	16.0			
		1954	15.0			
		1955	15.0			
		1956	15.0			
	9	1952	-16	-17.0	δ	δ
		1953	-17			
		1954	-17			
		1955	-17			
		1956	-17			

δ < 0.001

Statistical Analysis of Errors in Transmissivity

TABLE: 5.12(d)

5.5 SUMMARY AND CONCLUSIONS

In recent works [Lopez, 1973; Lopez, Haimes and Das, 1974] represented in Phases I and II the parameter identification methodology of groundwater systems is essentially based on the observed input data and the associated response. However these methodologies do not use various existing information from the geological map of the system. This consequently leads to: (i) developing a mathematical model which becomes nonrepresentative of the real physical system and (ii) a slight change from the data base for such a system which results in a substantial fluctuation in model response.

The groundwater model variable for which various existing information is available includes (in addition to transmissivity) storativity, initial water levels, discharge, recharge, boundary conditions and topology. The model developed in this work utilizes the existing information so that the mathematical model is closely representative of the physical system. Its sensitivity to changes in the data is less compared to other models. The model was applied to a real groundwater system in southern Ohio. A systematic way of identifying the transmissivity function was developed by decomposing the system into blocks. This provides the systems analyst with the possibility of making use of the various hydrological information for identifying a parameter of different blocks. Besides being computationally superior to the methods developed by previous authors, this identification and model validation closely approximates the physical system (see Table 5.8). Approximation of the transmissivity function by a second-order polynomial function for each block provides a closer distribution of transmissivity values, since the transmissivity within a cell is somewhat homogeneous. The dynamic nature of the boundary conditions for each cell is more realistic in the modeling of groundwater systems. An error introduced, if due to gross approximation of boundary conditions, is not likely to be present in a multicell model.

Since a mass balance is seen for each cell in each time period, an error introduced by numerical approximation is confined to the system and thereby distributed in model output over the aquifer. This has also been observed by comparing the result of this phase with that of previous phases using the same data base and is shown in Table 5.8.

Identifying groundwater parameters of each cell involves solving a partial differential equation describing the flow in porous media by numerical approximation. Since the area of each cell is comparatively small, it provides us with finer grid points over each cell without increasing computational difficulty. This is because each cell model may be solved independent of the others. Hence the methodology developed in this work becomes computationally more tractable. The finer the grids the more accurate the numerical solutions.

Under the rather simplified decomposition approach of this chapter, the method developed for identifying transmissivity parameters from observed head values proved very accurate. However the accuracy of the results will be affected to a considerable degree by the choice of well locations within a cell at which the waterhead is observed.

The procedure developed for evaluating transmissivity was tested for as many as three cells. There is no apparent reason why the method could not be extended to a greater number of cells. It must be realized, however, that as the number of cells increases, the computer time and analysis time increases. The computer time for the identification algorithm of this chapter also depends on the guess of the average value of transmissivity parameters. Should the optimization process fail to produce a solution, the user will have to supply a new starting point. The information generated in unsuccessful runs can be used to make better initial guesses.

Concerning the core requirement, the program requires about 72K words on the Univac 1108 digital computer. As for computer time, with three cells (see Section 5.5, the Fairfield-New Baltimore Aquifer) and a period of five years (with yearly changes in pumpage rate) the program takes 112 seconds.

Sensitivity and statistical analyses applied to the case study reveal that the model is quite sensitive to changes in identified parameter (transmissivity) while less sensitive to other parameters (storativity, pumpage and observed head). Therefore it was decided that the only parameter to be identified would be transmissivity, which also compensates for errors in identifying other parameters.

CHAPTER 6

AN OPTIMAL CONTROL ANALYSIS FOR THE MANAGEMENT OF A GROUNDWATER AQUIFER-STREAM SYSTEM

6.1 A GENERAL DISCUSSION

The developments introduced in Phase II, Haimes [1974], and Chapter 1 of this study provide the basis for coupling a complex real physical system with any desired control scheme. The system may comprise both aquifers and a stream network, interacting throughout the basin. The control scheme may consider utilizing certain parts of or the entire water resource at the considered area. It may refer to an isolated subsystem, or to an administrative framework which is imposed on the regional structure. The main idea is that a controlled input such as pumpage or artificial recharge is subject to a decision process for its magnitude and distribution. This same input affects the physical system, which responds accordingly. The system response is directly and indirectly considered in the decision process, and hence embedded in this process is the feedback to the input from the system response to the output. Using the response functions in the form developed in Phase II allows for explicitly coupling the physical system response with the decision process. The functions are essentially the acting analytical tool whereby system response and controlled input are interrelated. It is therefore possible to construct a management model in which the input stress imposed is considered as a control variable. This variable is specified by the solution of the

optimal control problem in the decision process.

In the following we intend to examine the management control problem formulation and the solution which should be applied to a system comprising a complex water resources system. In particular, we expect to demonstrate the real advantage of the response functions hierarchy while applied to mathematical models of the conjunctive use of ground and surface water systems.

This analysis is not available in the literature and constitutes a major contribution of this study. At first, an optimal control problem is formulated. The analysis of this problem should serve in better understanding the management model.

The effectiveness of using an optimal control theory for solving management models is well illustrated by Hullett [1974], (for applying distributed parameter control theory to optimal estuary aeration). Unfortunately, the distributed parameter control system which is identified for the conjunctive use of ground and surface water is too complicated for successfully using existing optimal control theory, and hence some simplifications must be made. Analyzing the simplified problem provides some insight into certain features of the original problem, and evaluates some of the necessary conditions for an optimal solution. A numerical solution is proposed. It results from discretizing the distributed parameter control formulation of the mathematical model. Finally, in this chapter, a quadratic program resulting from applying the numerical analysis is discussed. The next chapter is devoted to the application of the mathematical model to the case study which has been analyzed throughout. Not all the features characterizing the management control model are identified in the case study area. However, to be close to reality, no additional generated information is assumed which would make the case more general. The application is restricted to the existing structure, reducing the model to a forecasting tool for future operations. It is found however to be of great interest by itself. Case 2 is then formulated. This is a hypothetical system featuring most of what is characterized by the management control mode. This case is aimed at illustrating the prospects of using that model for a full-scale conjunctive use of ground and surface water systems. Management models of great variety have been applied and used for optimal control in water resources systems. The response functions which are developed in our study should be applied in particular to a short-term planning model. Evidently, the functional relation between inputs such as pumping or recharge, and responses such as drawdown and interflow should mostly affect the operational aspects of the water resources development. The planning for capacity expansion is affected only through the aggregation of the operational effects. Models devoted to the capacity expansion problem are well developed. The coupling of the operational aspects as considered in the forthcoming model with a desired capacity expansion model is a straightforward task; however, this problem is beyond this study's scope. Buras [1963], developed a dynamic programming algorithm to solve the problem of conjunctive use of reservoirs and aquifers. The operating policy considers the physical system in a lump form which introduces a considerable error by neglecting the distributed parameter system characterizing the groundwater system. As opposed to the lumped parameter approach, an analysis is suggested (Yu and Haimes [1974]) whereby a multilevel formulation is used for explicitly coupling the distributed parameter system with a management scheme to optimize conjunctive use of ground and surface water. Maddock and Haimes [1975], use the algebraic technological functions for coupling a groundwater system with a tax-quota management scheme. In the development below, conjunctive use of an aquifer system and a surface water system is considered. At this stage the regional administrative considerations will be included as well. However, regardless of the administrative structure, individual activities such as pumping from wells or consuming water from some common pools (like surface reservoirs), necessitate an information flow between people. Subject to such information, the single user is provided with the tools to make his own water use plans more efficiently and still maintain an independent operation policy.

6.2 THE REGIONAL SYSTEM

A basin comprising aquifers traversed by streams is considered. Users throughout the basin pump water from aquifers by means of operating wells. Each user's desire for water is primarily governed by economics, but may also take into consideration the stream water response, e.g., water level and quality in his vicinity. Surface water may be used directly after proper treatment either for artificial recharge or to create a competing source of supply.

The stochastic nature of stream flows, precipitation, natural recharge to the groundwater, and other such aspects affecting water balance in the system may play an essential role in a real system. The preliminary development here, however, is deterministic, in order to focus on the modeling procedures. A major recommendation to further improve this study's developments would be to include stochastic inputs and reduce deterministic assumptions. Actually, the modeling procedures are not restricted to deterministic systems. If the statistics of the stochastic input are known, mean value, variance, and lags should be considered inherently in the model (Maddock [1974]). Stream flow variations are particularly important for surface water balance and precipitation and evapo-transpiration, for groundwater balance.

We assume that for each single user, there is one aquifer cell from which he pumps his water from one or more wells. A single cell may underlie a number of stream reaches. Note that this definition of an aquifer cell is not restricted to geological or hydrological boundaries, though it may be subject to geographical, legal or political ones.

If a user operates artificial recharge facilities, these are considered aggregated at a single point inside his defined area. Water is transferred to this point from the different streams according to the recharge plan.

In the case of inelastic water demand, the economic criterion is the gross cost of water supply. Each user attempts to minimize the capital, operational, and maintenance and replacement cost of water use and artificial replenishment.

With water demand as a function of water price, the economic criterion is the net benefit obtained from water use.

The method of model superposition applied to either case may show a real advantage in the formulation process as well as in the solution strategy. The optimization problems conducted by each user are coupled to one another through the physical system. The proposed methodology enables the decoupling of these programs. A general responsive model provides each user with the following information:

1) Water levels at different operating wells during the time horizon.
2) The expected time at which drawdown at some wells will exceed casing and screening designs.
3) The quantity of water induced from the stream into an aquifer in the vicinity of the operating wells.

This information may cause the user to change his operational and design plans, in order to either reduce per unit water cost, or increase his net benefit.

These revised plans are not expected to affect total demand patterns for the inelastic case. They may, however, affect the following:

1) The operational plans of particular wells. Quantities pumped from some wells may be transferred to other wells within the aquifer cell.
2) The design plans. The user may redesign the drilling of wells and pipeline construction based on the expected water levels in the aquifer and the stream as determined by the responsive model.

If water demand is a function of water price, the total pumpage pattern and recharge plans of each user may also be subject to changes. In the following chapters, a coordination scheme is imposed on the system to provide the model with regional optimal control considerations. Each user's decisions thus become subject to input directed by the overall regional planning. It should be noted that model formulation is by no means restricted to a particular management problem. As shown later, through introducing new structural concepts in the formulation, the decomposed system functions provide an easy way for the model to successfully handle a variety of problems. Actually, in the forthcoming discussion we first analyze the proposed formulation features which may be common for different problems involving groundwater systems. Then while applying the model to two entirely different structures of case studies, the problems are still formulated and solved by the same principle, which makes use of the decomposed functions.

6.3 MODEL FORMULATION

To provide more insight into the model formulation and solution it is worthwhile to first consider the problem in the context of the optimal control of a distributed parameter system. Assume there are L users in the region. For each user there is a corresponding aquifer cell, and the ℓ^{th} user has m_ℓ wells which are located at the ℓ_{th} cell. There are U_ℓ streams traversing the ℓ^{th} cell area, from which a particular user may choose to transfer water for artificial recharge purposes to the recharge facility located in the ℓ^{th} cell area, and also to supply directly some of his water needs in that area. The ℓ^{th} user considers some or all of the following cost functions that will be discussed in detail subsequently:

1. Construction cost function:

$$Z_1^{\ell} = \int_0^T [e^{-rt} C_{\ell}(t)]dt \qquad (6.1)$$

2. Pumping cost function (operation):

$$Z_2^{\ell} = \int_0^T [e^{-rt} \sum_{k_{\ell}=1}^{m_{\ell}} P_{\ell}(k_{\ell}) \cdot q_{\ell}(k_{\ell},t) \cdot h_{\ell}(k_{\ell},t)]dt \qquad (6.2)$$

3. Surface water supply cost function (operation):

$$Z_3^{\ell} = \int_0^T [e^{-rt} \sum_{u=1}^{U_{\ell}} S_{\ell}(u)\ x_{\ell}(u,t)]dt \qquad (6.3)$$

4. Artificial recharge cost function (operation):

$$Z_4^{\ell} = \int_0^T [e^{-rt} \sum_{u=1}^{U_{\ell}} V_{\ell}(u) \cdot v_{\ell}(u,t)]dt \qquad (6.4)$$

5. Depletion of stream penalty cost function (see case study):

$$Z_5^{\ell} = \int_0^T [\ \sum_{u=1}^{U_{\ell}} Q_{\ell}(u,t) \cdot (x_{\ell}(u,t) + v_{\ell}(u,t) + f^u(\ell,t) - B_{\ell}(u,t)]dt \qquad (6.5)$$

here

r	annual interest rate
$C_\ell(t)$	construction cost for water supply projects considered by user ℓ
$P_\ell(k_\ell)$	pumping cost per acre-ft/ft for the k_ℓ^{th} well
$k_\ell(k_\ell,t)$	total lift at k_ℓ time t
$S_\ell(u)$	cost per acre-ft of water supply to ℓ^{th} area from the u^{th} stream (including treatment cost)
$q_\ell(k_\ell,t)$	pumpage from the k_ℓ^{th} well
$x_\ell(u,t)$	water supply from the u^{th} stream
$v_\ell(u,t)$	recharge from the u^{th} stream
$V_\ell(u)$	recharge cost per acre-ft of water from the u^{th} stream
$Q_\ell(u,t)$	weighting function to amplify the penalty cost corresponding to the depletion of different streams traversing the ℓ^{th} area
$f^u(\ell,t)$	quantity of water induced from the u^{th} stream into the ℓ^{th} aquifer cell due to natural recharge during time period t
$B_\ell(u,t)$	upper limit for quantity of water removed from the u^{th} stream into the ℓ^{th} area by means of artificial or natural recharge and direct supply (see application to case study).

The lift $h_\ell(k_\ell,t)$ in equation (6.2) comprises the steady state lift, $H_\ell(k_\ell)$, the drawdown at k_ℓ due to pumping from wells inside ℓ, $D_\ell(k_\ell, t)$, and the drawdown at cell ℓ due to the aggregated pumping from all other cells, $\hat{D}(\ell,t)$.

Hence

$$h_\ell(k_\ell,t) = H_\ell(t) + D_\ell(k_\ell,t) + \hat{D}(\ell,t) \tag{6.6}$$

The aquifer system equations which are assumed to mathematically approximate these drawdowns are:

1. Inside the particular cell model:

$$S(\hat{x}) \frac{\partial D_\ell(\hat{x},t)}{\partial t} = \frac{\partial}{\partial \hat{x}} [T(\hat{x}) \frac{\partial}{\partial \hat{x}} D_\ell(\hat{x},t)] - \sum_{k=1}^{m_\ell} q_\ell(\hat{x}_k,t)\delta(\hat{x}-\hat{x}_k) \tag{6.7}$$

$$D_\ell(\hat{x},t) \in R \tag{6.8}$$

2. The aggregated multicell model:

$$S(\hat{x}) \frac{\partial \hat{D}(\hat{x},t)}{\partial t} = \frac{\partial}{\partial \hat{x}} [T(\hat{x}) \frac{\partial}{\partial \hat{x}} \hat{D}(\hat{x},t)] - \sum_{r=1}^{L} q_N(\hat{x}_r,t)\delta(\hat{x}-\hat{x}_r) \tag{6.9}$$

$$\hat{D}(\hat{x},t) \in \hat{R} \tag{6.10}$$

3. The steady state model:

$$\frac{\partial}{\partial x}\left[T(\hat{x})\frac{\partial}{\partial x}H(\hat{x})\right] = 0 \tag{6.11}$$

$$H(\hat{x}) \in \bar{R} \tag{6.12}$$

Here

$\hat{x} = (x,y)$	spatial coordinates
$S(\hat{x})$	storativity coefficient
$T(\hat{x})$	transmissivity coefficient
$\delta(\hat{x}-\hat{x}_k)$	Dirac delta function
R_ℓ	the particular ℓ^{th} cell domain, including boundary conditions.
$\bar{R}$	the particular cell domain with boundary conditions associated with steady state conditions
$\hat{R}$	the entire system (multicell) domain including boundary conditions
$q_N(\hat{x}_r,t)$	the net aggregated pumping rate from the r^{th} cell, where $q_N(\hat{x}_r,t) = q(\hat{x}_r,t) - \sum_{u=1}^{U_r} v_r(u,t)$

The flow function $f^u(\ell,t)$ in equation (6.5) comprises the stream aquifer flow function of water induced from the u^{th} stream into the ℓ^{th} aquifer cell due to pumping from inside ℓ, $\overset{*}{f}{}^u(\ell,t)$, and from the other cells, $\hat{f}^u(\ell,t)$, and the steady state flow from the u^{th} stream into the ℓ^{th} aquifer cell, I^u_ℓ. Hence

$$f^u_\ell(\ell,t) = \overset{*}{f}{}^u(\ell,t) + \hat{f}^u(\ell,t) + I^u_\ell \tag{6.13}$$

The functions in (6.13) are discussed in Phase II. They are derived respectively from the system equations (6.7) - (6.12).

At this stage we do not assume explicit solutions to the system equations (in the form of Green's functions). However in Phase II we develop the groundwork for stating the following equations:

$$\overset{*}{f}{}^u(\ell,t) = F^{*u}_\ell(\underline{q}_\ell(\hat{x}_k,t), D(x,t), t) \tag{6.14}$$

$$\hat{f}^u(\ell,t) = \hat{F}^u_\ell(\underline{q}(\hat{x}_r,t), \hat{D}(\hat{x}_r,t), t) \tag{6.15}$$

$$I^u_\ell = \overline{F}^u(H(\hat{x})) \tag{6.16}$$

Explicit form of the functions (6.14) - (6.16) is given in (6.43) - (6.45).

The ℓ^{th} user is evidently considering the benefits of his water use. Through the model formulation, no restriction is imposed on the particular characteristic of the water use, and benefits may be incurred by either agricultural, municipal or industrial interests.

Let $W_\ell(t)$ denote the net return per acre-ft of water supply considered by the ℓ^{th} user during time period t. Economies of scale are not considered, and the value of $W_\ell(t)$ is not affected by the quantity of supply. The benefit which the ℓ^{th} user should expect is directly related to the quantity of water he consumes:

$$W_\ell = \int_0^T [e^{-rt} W_\ell(t) \, (\sum_{k_\ell=1}^{m_\ell} q_\ell(k_\ell,t) + \sum_{u=1}^{U_\ell} x_\ell(u,t))]dt \tag{6.17}$$

Actually, there are two functions which may involve economies of scale. The benefit function is practically determined by the particular user's activities, and economies of scale are introduced by construction of consuming water projects. Benefit is not an explicit function of the quantity of supply. The construction cost function (6.1), however, is eventually subject to economies of scale associated with quantity of water supply. The capacity expansion and/or construction of water supply projects using ground and surface water is developed and presented in Chapter 7. Two basic conjunctive water supply management plans are considered in Chapter 7. These are: (i) short-term operational planning: (ii) long-term expansion and/or construction planning.

Under a benefit-cost analysis [Howe 1971], the ℓ^{th} user is interested in maximizing the criterion functional $\hat{Z}_\ell$:

$$\max_{(\underline{q},\underline{x},\underline{v})_\ell} \quad (\hat{Z}_\ell = W_\ell - \sum_{p=1}^{5} Z_p^\ell) \tag{6.18}$$

where W_ℓ is given by (6.17) and Z_p^ℓ, p=1,...,5, are given by (6.1) - (6.5).

In addition to the system equations (6.6) - (6.16) which must be satisfied by the optimal solution to (6.18) there are restrictions (physical, economic or others) to account for:

1. Minimum water requirements must be met:

$$\sum_{k_\ell=1}^{m_\ell} q_\ell(k_\ell,t) + \sum_{u=1}^{U_\ell} x_\ell(u,t) \geq R_\ell(t) \qquad t \in [0,T] \tag{6.19}$$

2. Drawdown must not exceed designs:

$$h_\ell(k_\ell,t) \leq h_{\ell max}(k_\ell) \qquad t \in [0,T] \quad k_\ell=1,\ldots,m_\ell \tag{6.20}$$

3. Pumping capacity must be restricted:

$$q_\ell(k_\ell,t) \leq Q_{\ell max}(k_\ell) \qquad t \in [0,T] \quad k_\ell=1,\ldots,m_\ell \tag{6.21}$$

4. Recharge facility capacity must be constrained:

$$\sum_{u=1}^{U_\ell} v_\ell(u,t) \leq v_{\ell max} \qquad t \in [0,T] \tag{6.22}$$

5. Surface water supply must have an upper limit:

$$x_\ell(u,t) \le x_{\ell max}(u) \qquad t \in [0,T] \quad u=1,\ldots,U_\ell \tag{6.23}$$

6. Infiltrating rate limit must be constrained:

$$f^u(\ell,t) \le Q^u_{INF,\ell} \qquad t \in [0,T] \quad u=1,\ldots,U_\ell \tag{6.24}$$

here

$R_\ell(t)$	minimal water requirements function
$h_{\ell max}(k_\ell)$	maximum lift allowed at the $k_\ell{}^{th}$ well
$Q_{\ell max}(k_\ell)$	upper limit for pumping from k_ℓ
$v_{\ell max}$	recharge facility capacity limit
$x_{\ell max}(u)$	surface water supply system from the u^{th} stream capacity limit
$Q^u_{INF,\ell}$	maximum infiltrating rate from the u^{th} stream into the ℓ^{th} cell

The mathematical model defined by (6.1) - (6.24) constitutes an optimal control problem in a distributed parameter system. Evidently in its present form the classical control is inadequate for solving the optimal control policy. Fortunately, the application of numerical techniques based on certain assumptions reduces the model to a form where well-known techniques from systems engineering are applicable for optimally solving the system.

In accordance with what we stated in Section 6.1, a better insight into the control problem is achieved by analyzing the system using methodologies from the field of optimal control. A main source of complication which is introduced to the original problem is caused by the distributed parameter system equations and the fact that the waterhead distribution must be coupled with the control variables. Therefore, prior to solving the original problem, a simplified case is considered. Conserving the main features of the original problem, it should provide the analytical tool for studying the nature of the problem and its solution.

6.4 A SIMPLIFIED CASE FOR MANAGEMENT CONTROL STUDY

In the following we develop the ground for stating a necessary condition for optimal solution to the problem formulated in (6.1) to (6.24).

Theorem: A necessary condition for the control problem of a distributed parameter groundwater system, as formulated in (6.1) through (6.24) so as to constitute an optimal control solution, is that the Green's functions of the systems in (6.7) through (6.12) should be in positive times and the constraints in (6.19) through (6.24) should be a convex set.

Proof:

Consider a single aquifer cell which is described by the following system equations:

$$S \frac{\partial D(x,t)}{\partial t} = T \frac{\partial^2 D(x,t)}{\partial x^2} - \sum_{k=1}^{M} q(x_k,t)\delta(x-x_k) \qquad \begin{matrix} x \in [0,L) \\ t \in [0,1] \end{matrix} \tag{6.25}$$

and boundary conditions:

$$D(x,0) = g(x) \tag{6.26}$$

$$D(0,t) = D(L,t) = 0 \tag{6.27}$$

here S and T are storage and transmissivity coefficients, respectively, in the homogeneous one-dimensional space. $D(x,t)$ is the drawdown function, $q(x_k,t)$ is the pumpage from a well located at x_k and there are M wells in the field. δ is the Dirac delta function. $g(x)$ is a known function of initial head distribution. The mathematical model defined by (6.25) - (6.27) has the solution [Roach, 1970]:

$$D(x_k,t) = \sum_{j=1}^{M} \int_0^t G(x_k, x_j, t-\tau) q(x_j,\tau) d\tau \qquad t \in [0,1] \tag{6.28}$$

where G is the Green's Function which is explicitly derived for a given $g(x)$ in terms of the system's eigen-values and eigen-functions, (see Appendix A, Phase II).

Define the planning time horizon T and let $[0,1]$ in (6.26) comprise a unit time step, so that there are exactly N such time steps in the horizon, $n=1,\ldots,N$. The pumping from a well at x_k, $q(x_k,t)$ is assumed to comprise a series over time of discharge rates, where the rate is constant during each single time step, but may vary from time step to time step. Hence

$$q(x_k,t) = q(k,n), \quad n=1,\ldots,N$$

Considering only pumping from wells, and no recharge or surface water supply options, the performance criterion function is:

$$Z = \int_0^T [\hat{P}(t)\underline{q}(t)\underline{D}^T(t)]dt = \sum_{n=1}^{N} \int_{t=n-1}^{n} [\hat{P}(t)\underline{q}(n)\underline{D}^T(t)]dt \qquad (6.29)$$

where $\hat{P}(t) = e^{-rt}P(t)$ and r is the discount rate. Substitute ((6.28) into (6.29) to obtain

$$Z = \sum_{n=1}^{N}\sum_{k=1}^{M} \int_{n-1}^{n} [\hat{P}(t)q(k,n) \sum_{j=1}^{M} \int_{n-1}^{t} G_n(k,j,t-\tau)q(j,n)d\tau]dt \qquad (6.30)$$

$G_n(k,j,t-\tau)$ is the Green's function for the system equations (6.25) - (6.27) where $t \in [n-1,n]$ and $g(x) = D(x,n-1)$ (6.31) is the initial condition.

In a compact form, (6.30) becomes

$$Z = \sum_{n=1}^{N} \int_{n-1}^{n} [\hat{P}(t)\underline{q}(n)\underline{B}_n(t)\underline{q}^T(n)]dt$$

where
$$\underline{B}_n(t) = \int_{n-1}^{t} \underline{G}_n(\tau)d\tau \qquad t \in [n-1,n] \tag{6.32}$$

$\underline{G}_n(t)$ is a matrix of the Green's function whose elements $G_n(k,j,t-\tau)$ state the response at k due to unit pumping at j for the n^{th} time period. $\underline{B}_n(t)$ is a matrix whose elements are

$$B_n(k,j,t) = \int_{n-1}^{t} G_n(k,j,t-\tau)d\tau$$

Finally, as $\underline{q}(n)$ is a time invariant function for each $n, n=1,\ldots,N$:

$$Z = \sum_{n=1}^{N} \underline{q}(n) \cdot \int_{n-1}^{n} \hat{P}(t)\, \underline{B}_n(t)dt \cdot \underline{q}^T(n)$$

$$= \sum_{n=1}^{N} \underline{q}(n) \cdot \underline{B}(n) \cdot \underline{q}^T(n) \tag{6.33}$$

where
$$\underline{B}(n) = \int_{n-1}^{n} \hat{P}(t)\, \underline{B}_n(t)\, dt$$

Equation (6.33) states that the criterion function (6.29) comprises the summation of n decoupled quadratic terms, each depending on the system solution at a particular time period n, $n=1,\ldots,N$. Necessary and sufficient conditions for the criterion function (6.29) to constitute a unique optimal control solution for a convex constraints set is that $\underline{B}(n), n=1,\ldots,N$ should be positive definite matrices (Hadley [1964] Bryson and Ho [1969]).

To understand the immediate application of this result to the management control problem, we now investigate the physical meaning of the $\underline{B}(n)$ matrices. The criterion function essentially consists of a discounted multiplication of flows and the associated lifts. Equating equations (6.29) and (6.33) yields the following:

$$Z = \sum_{n=1}^{N} \underline{q}(n) \cdot \underline{B}(n) \cdot \underline{q}^T(n) = \sum_{n=1}^{N} \hat{P}(n) \cdot \underline{q}(n) \cdot \underline{D}(n) \qquad (6.34)$$

Here $\hat{P}(n)$ is the discount factor for the n^{th} time step, and $\underline{D}(n)$ is the vector of water head drawdown in the pumping wells at the end of the n^{th} period. But $\underline{D}(n)$ is also the solution to the system equation (6.25) for $t \in [0,n]$ and the initial condition $\underline{D}(x,0) = g(x)$, and is given by:

$$\underline{D}(n) = \int_0^n \underline{G}(\tau)\underline{q}(\tau)d\tau \qquad (6.35)$$

where $\underline{G}$ is the Green's function defined for $t \in [0,n)$ and there are n time steps in t. Substitute (6.35) into (6.34) to obtain:

$$Z = \sum_{n=1}^{N} \underline{q}(n)\ \underline{B}(n)\ \underline{q}^{T}(n) = \sum_{n=1}^{N} \hat{P}(n)\ \underline{q}(n) \int_{0}^{n} \underline{G}(\tau)\ \underline{q}(\tau) d\tau \qquad (6.36)$$

Equation (6.36) implies, that for $\underline{B}(n)$ to be a positive definite matrix, the integral on the right-hand side of (6.36) should be positive for all n, given $\underline{q}(t)$ positive function. This is true provided $\underline{G}(t)$, the Green's matrix for the system's mathematical model, is positive for all t. This last conclusion is applicable for stating the necessary and sufficient conditions for optimality in the simplified optimal control case. However for the original problem these conditions may not be sufficient but necessary, as more elements other than pumping wells are considered. By this we conclude the proof. The theorem is simply stating that the management control model can be applied to systems which do not contain certain irregularities. In this sense an irregularity means that it is possible under a certain circumstance that imposed pumpage will induce a negative drawdown at some point in the aquifer. Such a situation would be very rare.

Our next step is to solve the original distributed parameter system optimal control by doing a numerical analysis.

6.5 A NUMERICAL SOLUTION PROCEDURE

6.5.1 Model formulation

There are two basic concepts which we use for properly reformulating the management control as was discussed in Section 6.3. First, discretizing the time dimension allows for converting the time integrals into summations over a series of time steps. The second concept used is the one developed in Phase II of this study. It assumes the existence of Green's functions for the systems which are modeled by equations (6.7) through (6.12). An aquifer simulation model is used for determining these Green's functions for certain points in time and space. Consequently, fraction algebraic functions are derived to approximate infiltration rates through stream beds. The superposition of both the Green's functions (β's, γ's) and the fraction functions (ϕ's, ψ's) is applied. A detailed discussion of these functions is in Part II of this report. Resulting from these two concepts is the following quadratic program:

$$\max_{(\underline{q},\underline{x},\underline{v})_\ell} \left\{ \hat{Z}_\ell = \sum_{n=1}^{T} (1+r)^{-n} \cdot W_\ell(n)\left[\sum_{k_\ell=1}^{m_\ell} q_\ell(k_\ell,n) + \sum_{u=1}^{U_\ell} x_\ell(u,n)\right] \right.$$

$$- \sum_{n=1}^{T} (1+r)^{-n} C_\ell(n)$$

$$- \sum_{n=1}^{T} (1+r)^{-n} \left[\sum_{k_\ell=1}^{m_\ell} P_\ell(k_\ell) \cdot q_\ell(k_\ell,n) \cdot h_\ell(k_\ell,n)\right]$$

$$- \sum_{n=1}^{T} (1+r)^{-n} \left[\sum_{u=1}^{U_\ell} S_\ell(u) \cdot x_\ell(u,n)\right]$$

$$- \sum_{n=1}^{T} (1+r)^{-n} \left[\sum_{u=1}^{U_\ell} V_\ell(u) \cdot v_\ell(u,n)\right]$$

$$- \sum_{n=1}^{T} \sum_{u=1}^{U_\ell} Q_\ell(u,n) \cdot [x_\ell(u,n) + v_\ell(u,n)$$

$$\left. + f^u(\ell,n) - B_\ell(u,n)] \right\} \qquad (6.37)$$

With the system's equations in the form of algebraic technological functions (A.T.F.):

$$h_\ell(k_\ell,n) = H_\ell(k_\ell) + D_\ell(k_\ell,n) + \hat{D}(\ell,n) \qquad (6.38)$$

where
$$D_\ell(K_\ell,n) = \sum_{j=1}^{m_\ell} \sum_{i=1}^{n} [\beta_\ell(k_\ell,j,n-i+1)q_\ell(j,i)]$$

$$-\sum_{i=1}^{n} [\beta_\ell(k_\ell,v_\ell,n-i+1).v_\ell(u,i)] \qquad (6.39)$$

$$\hat{D}(\ell,n)= \sum_{\substack{r=1\\ r\neq\ell}}^{L} \sum_{i=1}^{n} \gamma(\ell,r,n-i+1)\ [q(r,i)-v(r,i)] \qquad (6.40)$$

$$q(r,i) = \sum_{k_r=1}^{m_r} q_r(k_r,i) \qquad (6.41)$$

$$v(r,i) = \sum_{u=1}^{u_r} v_r(u,i) \qquad (6.42)$$

and the stream-aquifer flow functions:

$$\overset{u}{f}(\ell,n) = \overset{*}{f}{}^{u}(\ell,n) + \hat{f}^{u}(\ell,n) + I_\ell^{u} \qquad (6.43)$$

where
$$\overset{*}{f}{}^{u}(\ell,n) = \sum_{k_\ell=1}^{m_\ell} \sum_{i=1}^{n} \overset{u}{\phi}_\ell(k_\ell,n-i+1).q_\ell(k_\ell,i) \qquad (6.44)$$

$$\hat{f}^{u}(\ell,n) = \sum_{\substack{r=1\\ r\neq\ell}}^{L} \sum_{i=1}^{n} \overset{u}{\psi}_\ell(r,n-i+1)\ [q(r,i)-v(r,i)] \qquad (6.45)$$

The notations used in (6.37)-(6.45) are essentially the same as those used for the original distributed parameter control problem formulated in Section 6.3. The discretization of time t into n time periods provided the above objective function formulation. For the system equations, the following terms were used based on the existence of the Green's functions:

$\beta_\ell(k_\ell,j,n-i-1)$ is the algebraic technological term relating the drawdown at the k_ℓ-th well to the pumping of one unit of water from the j-th well during the i-th period. Both k_ℓ and j are located at the ℓ-th cell.

$\gamma(\ell,r,n-i+1)$ is the algebraic technological term relating the average drawdown at the ℓ-th cell to aggregated pumping of one unit of water at the r-th cell, during the i-th period.

$\phi^u_\ell(k_\ell,n-i+1)$ is the quantity of water induced from the u-th stream into the ℓ-th cell during the n-th period due to one unit of pumping at the k_ℓ-th well during the i-th period.

$\psi^u_\ell(r,n-i+1)$ is the quantity of water induced from the u-th stream into the ℓ-th cell during the n-th period due to one unit of pumping at the r-th cell during the i-th period.

I^u_ℓ is the quantity of water induced from the u-th stream into the ℓ-th cell during one time period with no imposed pumpage and the system in steady state.

The system's constraints follow the same order as the constraints set in the original model:

$$\sum_{k_\ell=1}^{m_\ell} q_\ell(k_\ell,n) + \sum_{u=1}^{U_\ell} x_\ell(u,n) \geq R_\ell(n) \qquad n=1,\ldots,T \tag{6.46}$$

$$h_\ell(k_\ell,n) \leq h_{\ell max}(k_\ell) \qquad n=1,\ldots,T \quad k_\ell=1,\ldots m_\ell \tag{6.47}$$

$$q_\ell(k_\ell,n) \leq Q_{\ell max}(k_\ell) \qquad n=1,\ldots,T \quad k_\ell=1,\ldots,m_\ell \tag{6.48}$$

$$\sum_{u=1}^{U_\ell} v_\ell(u,n) \leq v_{\ell max} \qquad n=1,\ldots,T \tag{6.49}$$

$$x_\ell(u,n) \leq x_{\ell max}(u) \qquad n=1,\ldots,T \quad u=1,\ldots,U_\ell \tag{6.50}$$

$$f^u(\ell,n) \leq Q^u_{INF,\ell} \qquad n=1,\ldots,T \quad u=1,\ldots,U_\ell \tag{6.51}$$

6.5.2 Solution Strategies

The quadratic optimization program stated in (6.37) through (6.51) is considered solely by the ℓ-th user in the basin. However, there are other water users in the area, and up to L such distinct optimization programs may be respectively performed and each would correspond to a single user. Each individual program can be solved provided it is decoupled from other activities which are not under the ℓ-th user direct control. The L programs are coupled through the physical system responses, including the $\hat{D}(\ell,n)$ and $f^u(\ell,n)$ functions relating the system effect on the ℓ-th user from pumpage imposed in other parts of the hydrologic system by other users. In addition, stream balance considerations, such as the term $B_\ell(u,n)$, couple the systems' operations which are performed by all users.

1. The coupling through the term $\hat{D}(\ell,n)$.

In equation (6.40) we represented $\hat{D}(\ell,n)$ explicitly:

$$\hat{D}(\ell,n) = \sum_{\substack{r=1 \\ r\neq\ell}}^{L} \sum_{i=1}^{n} \gamma(\ell,r,n-i+1)\ [q(r,i) - v(r,i)] \qquad (6.52)$$

q(r,i), v(r,i), are the aggregated pumpage and artificial recharge, respectively, which are considered by users for different cells. Once these values are specified, the solution for $\hat{D}(\ell,n)$ is explicitly given in (6.52).

2. The coupling through the term $f^u(\ell,n)$

In equation (6.43) $f^u(\ell,n)$ was defined:

$$f^u(\ell,n) = \overset{*}{f}{}^u(\ell,n) + \hat{f}^u(\ell,n) + I_\ell^u \tag{6.53}$$

and
$$\hat{f}^u(\ell,n) = \sum_{\substack{r=1 \\ r\neq\ell}}^{L} \sum_{i=1}^{n} \psi_\ell^u(r,n-i+1)\,[q(r,i)-v(r,i)] \tag{6.54}$$

The same arguments are used here for the coupling term $\hat{f}^u(\ell,n)$ where specification of $q(r,i)$ and $v(r,i)$ provides explicitly the value of $f^u(\ell,n)$.

3. The coupling through the term $B_\ell(u,n)$.

The value of the term $B_\ell(u,n)$ should be assigned externally to the optimum control problem being considered by a particular user. The stream balance evidently concerns each user but is affected by all users' operations and by other things not controlled by any of them such as upstream inflow. It is therefore assumed that stream balance terms like $B_\ell(u,n)$ are specified for each user for each problem setting. In the following chapters at least one possible approach is presented to assign stream balance terms to each user according to an external consideration set.

Some of the conceptual solution strategies are illustrated by analyzing two case studies.

6.6 A QUADRATIC PROGRAM MODEL

This section is concerned with using a standard quadratic programming solution for this study's model. A modification of the procedure developed by Wolfe [1959], is presented. A listing of the source program is in Kuster and Mize [1973]. Originally, the procedure suggested by Wolfe [1959] is for the following:

PROBLEM A:

$$\begin{aligned} \underset{\underline{x}}{\text{Minimize}} \quad Z &= \underline{P}\,\underline{x} + 1/2\,\underline{x}^T\,\underline{C}\,\underline{x} \\ \underline{A}\,\underline{x} &\leq \underline{b} \\ \underline{x} &\geq \underline{0} \end{aligned} \tag{6.55}$$

where

$$\underline{x} = (x_1, x_2, \ldots, x_n)^T$$

$$\underline{P} = (P_1, P_2, \ldots, P_n)$$

$$\underline{b} = (b_1, b_2, \ldots, b_m)^T$$

$$\underline{A} = \begin{bmatrix} a_{11} & a_{1n} \\ a_{m1} & a_{mm} \end{bmatrix} \qquad \underline{C} = \begin{bmatrix} c_{11} & c_{1n} \\ c_{n1} & c_{nn} \end{bmatrix}$$

The requirements for problem 1 to obtain a solution via the proposed procedure are:

a) The matrix $\underline{C}$ is assumed positive definite and symmetric.

b) The constraints are assumed to be of the form:

$$\sum_{i=1}^{n} a_{ij}\, x_i \leq b_j \qquad j=1,\ldots,m$$

and all b_j are non-negative.

If these requirements are fulfilled, a solution is warranted, Hadley, [1964]. The problem formulation as in Section 6.5.1 reduces to the compact vector form of problem B:

PROBLEM B:

$$\begin{aligned} \underset{\underline{x}}{\text{Minimize}} \quad & (Z = \underline{P}\,\underline{x} + 1/2\, \underline{x}^T \underline{C}\, \underline{x}) \\ & \underline{A}^1 \underline{x} \leq \underline{b}^1 \qquad \underline{b}^1 \geq 0 \\ & \underline{A}^2 \underline{x} \geq \underline{b}^2 \qquad \underline{b}^2 \geq 0 \\ & \underline{x} \geq 0 \end{aligned} \tag{6.56}$$

where

$$\underline{A}^1 = \begin{bmatrix} a^1_{11} & a^1_{1n} \\ a^1_{P1} & a^1_{Pn} \end{bmatrix} \qquad \underline{A}^2 = \begin{bmatrix} a^2_{11} & a^2_{1n} \\ a^2_{q1} & a^2_{qn} \end{bmatrix}$$

$$\underline{b}^1 = (b_1^1, \ldots, b_p^1)^T \qquad b_i^1 \geq 0 \qquad i=1,\ldots,P$$

$$\underline{b}^2 = (b_1^2, \ldots, b_q^2)^T \qquad b_i^2 > 0 \qquad i=1,\ldots,q$$

Unfortunately, the constraints $\underline{A}^2 \underline{x} \geq \underline{b}^2$ contradict requirements (b) for the application of the Wolfe algorithm. The following technique is suggested to overcome this problem:

Define vectors

$$\underline{Y} = (y_1, y_2, \ldots, y_q)^T$$

$$\underline{x}^1 = (x_1^1, x_2^1, \ldots, x_n^1)$$

$$\underline{m} = (m_1, \ldots, m_q)$$

x_i^1, y_i are decision variables, m_i is a non-negative and yet unspecified number. A new quadratic programming model is defined.

PROBLEM C:

$$\underset{\underline{x}', \underline{Y}}{\text{Minimize}} \; (Z_m = \underline{P}\,\underline{x}^1 + 1/2\, \underline{x}^{1T} \underline{C}\, \underline{x}^1 - \underline{m}\, \underline{Y})$$

$$\underline{A}^1 \underline{x}^1 \leq \underline{b}^1$$

$$\underline{Y} - \underline{A}^2 \underline{x}^1 \leq \underline{0} \qquad \underline{x}^1 \geq 0 \qquad \underline{Y} \geq 0$$

$$\underline{Y} \leq \underline{b}^2 \tag{6.57}$$

Theorem: If the problem B poses an optimal solution Z^* for $\underline{x}^* = (x_1^*, \ldots, x_n^*)^T$, then Z_m^* is the optimal solution for Problem C with $\underline{x}^1 \equiv \underline{x}^{1*}$ and $\underline{Y}^* \equiv \underline{b}^2$ where $Z_m^* = Z^* - \underline{m}\, \underline{b}^2$.

Proof: One should observe, that if in Problem C the vector of variables $\underline{Y}$ is set to $\underline{Y} = \underline{b}^2$ so that $y_i = b_i^2$, $i=1,\ldots,q$, then Problem C is reduced to the original Problem B with the objective function value differing in a constant scalar $\underline{m}\ \underline{b}^2$. Hence, if we prove that $\underline{Y} = \underline{b}^2 = \underline{Y}^*$ for Z_m^*, then the optimal solution of C for $\underline{x}^1 = \underline{x}^{1*}$ coincides with the optimal solution of B for $\underline{x} = \underline{x}^*$ and $x^{1*} = \underline{x}^*$.

To prove $\underline{Y}^* = \underline{b}^2$ we apply the Kuhn-Tucker (Kuhn and Tucker, [1961]) necessary conditions for optimality to both problems B and C.

Let $\underline{\lambda}^1 = (\lambda_1^1, \lambda_2^1, \ldots, \lambda_p^1)^T$

$\underline{\lambda}^2 = (\lambda_1^2, \lambda_2^2, \ldots, \lambda_q^2)^T$

be the Lagrange multipliers corresponding to the two sets of constraints $\underline{A}\,\underline{x} - \underline{b}^1 \leq \underline{0}$, $\underline{b}^2 - \underline{A}^2\,\underline{x} \leq \underline{0}$, respectively in Problem B and $\underline{A}^1\,\underline{x}^1 - \underline{b}^1 \leq \underline{0}$, $\underline{Y} - \underline{A}^2\,\underline{x}^1 \leq \underline{0}$ in Problem C. Let $\underline{\lambda}^3 = (\lambda_1^3, \ldots, \lambda_q^3)^T$ be the Lagrange multipliers corresponding to the sets of constraints $\underline{Y} - \underline{b}^2 \leq \underline{0}$ in Problem C, thus the application of the Kuhn-Tucker conditions to Problem B yields:

1) $$x_i\left(P_i + \sum_{j=1}^{n} C_{ij}\, x_j + \sum_{j=1}^{p} a_{ij}^1\, \lambda_j^1 - \sum_{j=1}^{q} a_{ji}^2\, \lambda_j^2\right) = 0$$

$i = 1,\ldots,n$

2) $x_i \geq 0 \qquad i = 1,\ldots,n$

3) $\underline{P} + \underline{C}\ \underline{x} + \underline{A}^{1T}\ \underline{\lambda}^1 - \underline{A}^{2T}\ \underline{\lambda}^2 \geq \underline{0}$

4) $\lambda_i^1(\sum_{j=1}^{n} a_{ij}^1\ x_j - b_j^1) = 0 \qquad i=1,\ldots,P$

5) $\lambda_i^1 \geq 0 \qquad i=1,\ldots,P$

6) $\underline{A}^1\ \underline{x} - \underline{b}^1 \leq \underline{0}$

7) $\lambda_i^2(b_i^2 - \sum_{j=1}^{n} a_{ij}^2\ x_j) = 0 \qquad i=1,\ldots,q$

8) $\lambda_i^2 \geq 0 \qquad i=1,\ldots,q$

9) $\underline{b}^2 - \underline{A}^2\ \underline{x} \leq \underline{0}$ (6.58)

Applying the same conditions to problem C yields:

1) $x_i^1(P_i + \sum_{j=1}^{n} C_{ij}\ x_j^1 + \sum_{j=1}^{P} a_{ji}^1\ \lambda_j^1 - \sum_{j=1}^{q} a_{ji}^2\ \lambda_j^2) = 0$

$i = 1,\ldots,n$

2) $x_i^1 \geq 0 \qquad i = 1,\ldots,n$

3) $\underline{P} + \underline{C}\ \underline{x}^1 + \underline{A}^{1T}\ \underline{\lambda}^1 - \underline{A}^{2T}\ \underline{\lambda}^2 \geq \underline{0}$

4) $\lambda_i^1(\sum_{j=1}^{n} a_{ij}^1\ x_j^1 - b_j^1) = 0 \qquad i=1,\ldots,P$

5) $\lambda_i^1 \geq 0 \qquad i = 1,\ldots,P$

6) $\underline{A}^1 \underline{x}^1 - \underline{b}^1 \leq \underline{0}$

7) $\lambda_i^2(y_i - \sum_{j=1}^{n} a_{ij}^2 x_j^1) = 0 \qquad i=1,\ldots,q$

8) $\lambda_i^2 \geq 0 \qquad i = 1,\ldots,q$

9) $\underline{Y} - \underline{A}^2 \underline{x}^1 \leq \underline{0}$

10) $\lambda_i^3(y_i - b_i^2) = 0$

11) $\lambda_i^3 \geq 0 \qquad i = 1,\ldots,q$

12) $\underline{Y} - \underline{b}^2 \leq \underline{0}$

13) $y_i(-m_i + \lambda_i^2 + \lambda_i^3) = 0 \qquad i=1,\ldots,q$

14) $y_i \geq 0 \qquad i = 1,\ldots,q$

15) $-\underline{m}^T + \underline{\lambda}^2 + \underline{\lambda}^3 \geq \underline{0}$ (6.55)

Condition (10) in Problem C states that either $y_i = b_i^2$ or $\lambda_i^3 = 0$, $i = 1,\ldots,q$. Let $y_i = b_i^2$, $i = 1,\ldots,q$ and substitute into equations 7 and 9 in Problem C. The set of equations 1-9 in Problem C is identical to the equations which result from applying the Kuhn-Tucker conditions to Problem B. Assuming that Problem B constitutes a solution then this same solution must hold for the subset of equations 1-9 in Problem C. In order that such a solution holds for the entire set of equations in Problem C, equations 10-15 should also be satisfied. The condition $y_i = b_i^2$ satisfies both equations 10 and 12. Given b_i^2 positive, then equation 13 states $y_i > 0 \rightarrow -m_i + \lambda_i^2 + \lambda_i^3 = 0$. Condition 11 states that $\lambda_i^3 \geq 0$ and hence $m_i - \lambda_i^2 \geq 0$, or $m_i \geq \lambda_i^2$. This should also satisfy condition 15. We may now conclude, that if $\underline{m}$ is set to $m_i \geq \lambda_i^2$, $i = 1,\ldots,q$, the entire set of conditions is satisfied provided Problem B has a solution. This implies that $\underline{Y}^* = \underline{b}^2$ is the optimal value of $\underline{Y}$, and the proof is concluded.

In the following chapters this proposed modification is actually used and provides the utilization of the standard quadratic program of Problem A.

6.7 APPLICATION OF THE MANAGEMENT CONTROL MODEL TO THE FAIRFIELD-NEW BALTIMORE AREA: A CASE STUDY

6.7.1 Introduction

A detailed description of the Fairfield-New Baltimore area in southwestern Ohio is given in Phases I and II, Haimes [1973,1974], and in Chapter 5 of this report. A simulation procedure which is developed in Chapter 4 was applied to the aquifer underlying the area. Consequently, Algebraic Technological Functions (A.T.F.) which are developed in Phase II to relate drawdown to pumping from wells was constructed for wells located at the studied area. Flow fraction functions between streams and aquifer relating to well pumpage were also determined for application to the particular area. The management control model introduced in Chapter 6 comprises in its structure and its formulation most of what was discussed in Phase II for coupling the physical system with the desired control scheme. Thus, the functions determined throughout this study are now available for coupling the Fairfield-New Baltimore system with any imposed control scheme. The water resources in the Fairfield-New Baltimore area are under the supervision of the Miami Conservancy District (M.C.D.) and the U.S. Geological Survey, (U.S.G.S.). However, neither the M.C.D. nor any other authority has the jurisdiction to impose a regional policy for water resources development, (Spieker [1968]; Plummer [1974]). As a result, water users are free to choose their own policies for developing their water supply systems, and only a few restrictions are imposed with respect to water quality, the Clean Water Act [1972], and Water Rights [1968]; Cincinnati Well Field Case). The management model in Chapter 6 may be reduced to handle the Fairfield-New Baltimore case study. Actually, the model does not assume any administrative coordination between activities of individual water users in the area. The only connection between these activities is essentially their common need to take into account the physical system's response. Each user can do this provided his own optimal performance is subject to feed-in of information of others' activities. Such an information flow is actually available from the proposed management formulation using the response functions hierarchy.

We have identified five major users in the Fairfield-New Baltimore area (Plummer [1974]), Figure 6.1:

1) American Cyanamid + Fisher Body (Cell 2)
2) Hamilton South Field + Fairfield (Cell 4)
3) Southwestern Ohio (Cell 5a)
4) Cincinnati (Cell 5b)
5) U.S. Atomic Energy Commission (Cell 6)

Others use relatively small amounts of water and can be ignored for our purposes here.

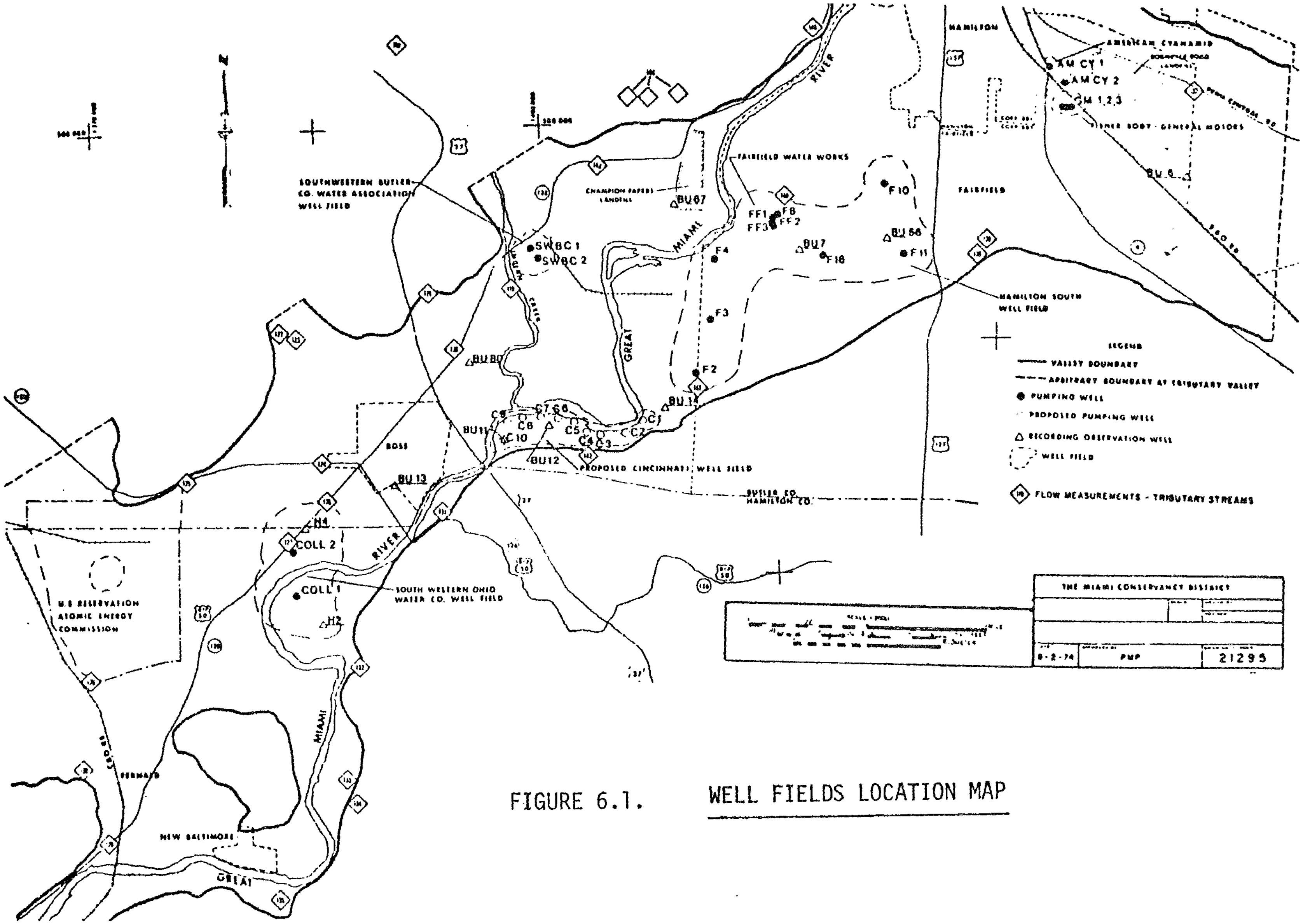


FIGURE 6.1. WELL FIELDS LOCATION MAP

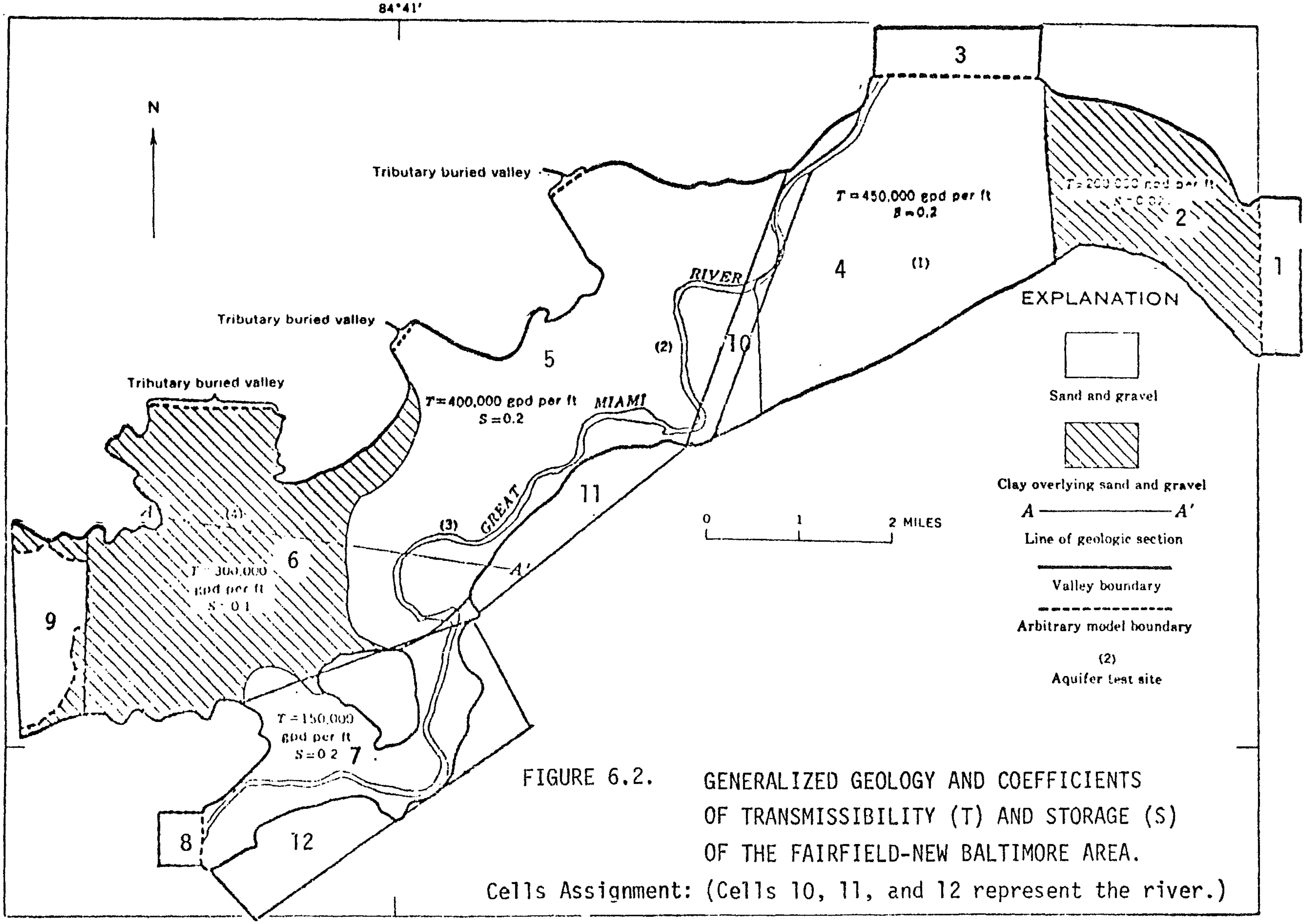


FIGURE 6.2. GENERALIZED GEOLOGY AND COEFFICIENTS OF TRANSMISSIBILITY (T) AND STORAGE (S) OF THE FAIRFIELD-NEW BALTIMORE AREA.

Cells Assignment: (Cells 10, 11, and 12 represent the river.)

Water needs in the Fairfield-New Baltimore area are classified for municipal and industrial use. At present all water requirements are met by groundwater from operating wells. No direct supply from streams is yet considered, due to water availability from the aquifer and quality restrictions on surface water. Also there is no need for artificial recharge, therefore, it has not been introduced. Information is available for identifying the physical system. Also available are some projections of future water needs. It is assumed that these needs will be inelastic and that users will not be concerned about cost of water, only its availability.

The main goal of applying the management model to this case is to come up with a prediction tool to evaluate water use activities and the system's response to them. The resulting policy may be acceptable to the water users because it assumes that they all will seek an optimal operation policy. It should point out some of the most critical developments in the system while supply is increasing, and may probably initiate the desire for a coordinated system providing improved exploitation of the water resources.

6.7.2 APPLICATION TO THE FAIRFIELD-NEW BALTIMORE CURRENT ADMINISTRATIVE STRUCTURE

Unfortunately, the current situation in the Fairfield-New Baltimore area includes only part of the options accounted for in the management model in Chapter 6. Actually, we do not propose that the general model be applied only to cases where all the options encompassed by the model pertain. In the following, only a certain part of the general model formulation is applied to the actual case as defined by the Fairfield-New Baltimore area. We utilize the following information:

Table 6.1 summarizes the projections of water requirements for 1974-1983 (Spieker [1968]; Plummer [1974]). Table 6.2 tabulates the algebraic technological functions (A.T.F.) relating drawdowns in wells to aggregated pumpage, under various boundary conditions along the stream reaches. More detailed data are available for Hamilton South Field (Cell 4). Table 6.3 tabulates the A.T.F. functions corresponding to three wells in that field. Functions of flows between stream and aquifer related to pumping from cells are tabulated in Table 6.4. In Table 6.5 maximum infiltration rates from stream reaches into the aquifer are listed, (based on Spieker [1968]).

TABLE 6.1

WATER REQUIREMENTS PROJECTIONS IN THE FAIRFIELD-NEW BALTIMORE AREA

(Figures are given in acre-ft/day)

Year	Cell					
	2	4	5a	5b	5a & 5b	6
1974	1.5	30.6	55.7		55.7	3.
1975	1.6	31.2	57.2		57.2	3.
1976	1.7	31.8	58.7	122.	180.7	3.
1977	1.8	32.4	60.2	122.	182.2	3.
1978	1.9	33.0	61.7	122.	183.7	3.
1979	2.0	33.6	63.2	122.	185.2	3.
1980	2.1	34.2	64.7	122.	186.7	3.
1981	2.2	34.8	66.2	122.	188.2	3.
1982	2.3	35.4	67.7	122.	189.7	3.
1983	2.4	36.0	69.2	122.	191.2	3.

TABLE 6.2

ALGEBRAIC TECHNOLOGICAL FUNCTIONS $\gamma(\ell,r,i)$ FOR CELLS IN THE FAIRFIELD-NEW BALTIMORE AREA
(Figures are given in ft/millions ft^3/day)
(NCHu=0 reach u acts as a constant head boundary. NCHu=1 reach u acts as a constant flow source)
The sign (-) means that the drawdown at ℓ is not affected by pumpage at r because a constant head boundary is between them.

N C H u		Year	$\gamma(2,r,i)$				$\gamma(4,r,i)$				$\gamma(5,r,i)$				$\gamma(6,r,i)$			
			r				r				r				r			
10	11	i	2	4	5	6	2	4	5	6	2	4	5	6	2	4	5	6
0	0	1	19.6	1.7	--	--	2.0	3.3	--	--	--	--	3.4	1.0	--	--	1.0	11.3
		2	0.1	0.2	--	--	0.3	0.4	--	--	--	--	0.7	0.9	--	--	0.9	3.6
		3	0.	0.	--	--	0.	0.1	--	--	--	--	0.2	0.4	--	--	0.4	1.2
1	0	1	20.5	1.9	0.6	0.2	2.2	4.8	1.3	0.2	0.8	1.3	3.2	1.0	0.2	0.3	1.0	11.5
		2	0.5	0.6	0.5	0.3	0.6	1.7	0.8	0.3	0.5	0.9	0.9	1.0	0.4	0.7	1.0	3.8
		3	0.2	0.2	0.2	0.2	0.2	0.9	0.3	0.2	0.2	0.3	0.3	0.6	0.4	0.5	0.6	1.5
		4	0.1	0.1	0.1	0.1	0.	0.5	0.1	0.1	0.0	0.1	0.1	0.2	0.2	0.3	0.4	0.7
0	1	1	9.6	1.7	--	--	2.0	3.3	--	--	--	--	4.6	1.2	--	--	1.3	12.5
		2	0.1	0.2	--	--	0.3	0.4	--	--	--	--	2.1	1.5	--	--	1.6	3.7
		3	0.	0.	--	--	0.	0.1	--	--	--	--	1.	1.	--	--	1.	1.5
		4	0.	0.	--	--	0.	0.	--	--	--	--	0.5	0.6	--	--	0.6	0.7
		5	0.	0.	--	--	0.	0.	--	--	--	--	0.3	0.3	--	--	0.3	0.4
1	1	1	19.3	2.1	0.2	0.	2.1	3.8	0.6	0.	0.9	1.2	5.2	1.4	0.1	0.3	1.4	11.
		2	0.4	0.6	0.5	0.1	0.8	1.2	1.1	0.4	1.0	1.5	3.3	2.1	0.5	0.7	2.1	3.6
		3	0.2	0.2	0.4	0.2	0.4	0.6	1.0	0.6	0.6	1.1	2.2	1.6	0.4	0.7	1.7	1.7
		4	0.	0.1	0.3	0.1	0.2	0.3	0.6	0.5	0.4	0.6	1.5	1.0	0.2	0.5	1.2	0.9

TABLE 6.3

$\beta(k,j,i)$ Values $\left[Ft/Ft^3/Day\right]$ *1000
Wells in Cell 4

Year I	(F-10,J,I)			(F-11,J,I)			(F-16,J,I)		
	J			J			J		
	F-10	F-11	F-16	F-10	F-11	F-16	F-10	F-11	F-16
1	10.00	4.77	2.99	4.82	11.51	4.74	3.05	4.77	9.82
2	0.98	1.04	0.74	1.01	1.32	0.94	0.73	0.95	0.83
3	0.24	0.27	0.19	0.26	0.31	0.23	0.18	0.22	0.17
4	0.07	0.09	0.06	0.08	0.08	0.06	0.05	0.06	0.04
5	0.02	0.02	0.02	0.02	0.02	0.02	0.01	0.01	0.01

TABLE 6.4

The Fairfield Aquifer Area

$\psi_r^u(\ell,n)$ Values $\left[1000\ Ft^3/Day\right]$

(One Unit Pumpage imposed on ℓ during the $i = 1$ Period)

u . . .	10		10		11		12	
r . . .	4		5		5		7	
ℓ	4	2	6	5	4	5	6	5
n ⋮								
1	557	220	40	190	60	290	60	10
2	52	120	90	120	130	190	90	20
3	5	20	50	30	80	40	35	10
4	-	-	20	10	30	15	40	5

TABLE 6.5

MAXIMUM INFILTRATION RATES $Q_{INF,\ell}^{u}$ AND STEADY STATE INFILTRATION RATES I_{ℓ}^{u} FROM STREAM REACHES INTO AQUIFER CELLS IN THE FAIRFIELD-NEW BALTIMORE AREA

(Figures are given in acre-ft/day and are based on 325,000 GPD/acre stream bed.)

Reach u / Cell ℓ	10	11	12
4	/ -28.		
5	/ 28.	95. / -20.	
4 & 5	90. / 0.		
7			100. / -3.

LEGEND: $Q_{INF,\ell}^{u}$ / I_{ℓ}^{u}

We can now find out the direct effect of all users' pumping plans on the system response and how this will affect a particular user. The coupling terms, see Section 6.5.2 are determined for the inelastic water use projections at each cell. It is therefore possible to isolate any optimal control problem of any user. The drawdown at each cell due to pumping from all other cells (on the basis of the projected pumping of Table 6.1 is given in Table 6.6 These values are obtained by applying the methodologies as described in Chapters 3 and 4 of Phase II. Table 6.7 summarizes infiltration rates from stream reaches into cells due to the projected imposed pumpage throughout the entire area. Notice that at the end of 1978 stream reaches 10 and 11 (Figure 6.2) are expected to induce maximum infiltration rates into the aquifer. (This last result is already accounted for in the drawdown figures in Table 6.6 after 1978.)

TABLE 6.6

DRAWDOWN AT CELLS IN THE FAIRFIELD-NEW

BALTIMORE AQUIFER DUE TO PUMPING FROM OTHER CELLS

(In Feet)

Year n	Cell ℓ			
	2	4	5	6
1974	2.3	0.1	0.13	2.5
1975	2.6	0.1	0.25	4.9
1976	2.6	0.1	0.30	5.5
1977	2.7	0.1	0.30	5.5
1978	2.8	0.1	0.30	5.6
1979	4.6	1.2	0.5	5.6
1980	4.6	1.2	0.6	5.7
1981	4.9	1.2	0.7	5.7
1982	5.1	1.2	0.7	5.8
1983	5.4	1.2	0.7	5.8

TABLE 6.7

INFILTRATION RATES FROM STREAM REACHES INTO AQUIFER CELLS IN THE FAIRFIELD-NEW BALTIMORE AREA CORRESPONDING TO PUMPAGE PROJECTIONS OVER 10 YEARS

(Figures are given in acre-ft/day)

Year i	$f^{10}(4,i)$	$f^{10}(5,i)$	$f^{11}(5,i)$	$f^{10}(\bar{R},i) = f^{10}(4,i)+f^{10}(5,i)$
1974	-10.9	45.	-10.	34.1
1975	-8.6	46.1	7.8	37.5
1976	-8.0	65.5	46.	57.5
1977	-7.6	88.	70.	80.4
1978	-7.0	97.	95.	90.
1979	-7.0	97.	95.	90.
1980	-7.0	97.	95.	90.
1981	-7.0	97.	95.	90.
1982	-7.0	97.	95.	90.
1983	-7.0	97.	95.	90.

Note that $f^u(\ell,i)$ indicates the infiltration in acre-ft/day during period i from the u^{th} stream into the ℓ^{th} cell.

Tables 6.6 and 6.7 provide the terms for decoupling each user's considerations from those of the rest of the users. Table 6.8 the aggregated drawdown at each cell over the years resulting from the projected water requirements of all users.

TABLE 6.8

AGGREGATED DRAWDOWN AT CELLS IN THE FAIRFIELD-NEW BALTIMORE AQUIFER OVER TEN YEARS DUE TO AGGREGATED PUMPAGE BY ALL USERS (In Feet)

Year n	Cell ℓ			
	2	4	5	6
1974	3.6	4.8	9.8	4.5
1975	3.9	5.0	10.	6.9
1976	4.0	5.1	28.	7.5
1977	4.2	5.3	33.	7.5
1978	4.4	5.4	35.	7.6
1979	6.4	5.8	35.	7.7
1980	6.5	7.0	36.	7.8
1981	6.8	6.8	36.5	7.8
1982	7.1	6.9	37.3	7.9
1983	7.5	7.0	38.	7.9

TABLE 6.9

TECHNICAL INFORMATION - WELLS IN THE HAMILTON SOUTH FIELD,

FAIRFIELD-NEW BALTIMORE AREA

Well	Ground Level ft	Steady State Groundwater Level - ft	Depth ft	Maximum Pumpage Capacity acre-ft/day	Initial Lift ft	Maximum Drawdown ft
F-10	581.	548.	200.	13.1	83.	30.
F-11	584.	548.	200.	13.1	86.	30.
F-16	575.	547.	200.	13.1	78.	30.

P(k) cost of pumping 0.0404 $/acre-ft/ft

Currently, the only user who may be concerned with the optimal operation of his wells under the affecting well operations of other users is the City of Hamilton in its South Well Field, Cell 4. It is probably in the interests of other users, in particular the City of Cincinnati, to consider an optimal policy for their water supplies. Nevertheless, the City of Cincinnati Well Field is not yet operating, and in the present state we confine ourselves to the available information, based on the actual situation. In Table 6.9 is some of the model's required technical information for three wells operated by the City of Hamilton in that area. Algebraic technological functions (beta functions) are tabulated in Table 6.3 corresponding to these wells. A listing of infiltration rates from reach 10 into Cell 4 due to well pumpage inside the cell is given in Table 6.10.

TABLE 6.10

$\phi_4^{10}(k,n)$ FLOW BETWEEN STREAM REACH 10 AND CELL 4 AS A FRACTION OF WELL PUMPAGE IN THE HAMILTON SOUTH FIELD, FAIRFIELD-NEW BALTIMORE AREA [(acre-ft/day)/(acre-ft/day)]

Year n	Well		
	F-10	F-11	F-16
1	0.56	0.53	0.60
2	0.06	0.05	0.08
3	0.01	0.01	0.01

The following quadratic mathematical model was solved for the City of Hamilton South Well Field operation:

$$\underset{q(k,n)}{\text{minimize}} \left[Z_4 = \sum_{n=1}^{10} \left\{ (1+r)^{-n} \sum_{k=1}^{3} P(k) \cdot q(k,n)[H(k) + \hat{D}(4,n) + \sum_{j=1}^{3} \sum_{i=1}^{n} \beta(k,j,n-i+1) \cdot q(j,i)] \right\} \right]$$

subject to:

$$\sum_{j=1}^{3} \sum_{i=1}^{n} \beta(k,j,n-i+1) \cdot q(j,i) \leq D(k)_{max} \qquad n=1,\ldots,10; \quad k=1,2,3$$

$$q(k,n) \leq Q(k)_{max} \qquad n=1,\ldots,10; \quad k=1,2,3$$

$$\sum_{k=1}^{3} q(k,n) \geq R(n) \qquad n=1,\ldots,10$$

$$f^{10}(4,10) \leq Q_{INF,4}^{10}$$

$$q(k,n) \geq 0 \qquad n=1,\ldots,10; \quad k=1,2,3$$

The various terms in the above formulation are described in detail in section 6.5.1 The control variables q(1,n), q(2,n) and q(3,n) correspond to pumping from wells F-10, F-11, and F-16, respectively, from 1974 - 1983, see Figure 6.1.

Tables 6.1-6.10 provide all necessary information for solving the particular model. The computer program and the solution procedure follow the discussion in section 6.8. Table 6.11 gives the pumping values which minimize the objective function while satisfying the constraints.

TABLE 6.11

OPTIMAL SCHEDULE OF WELL PUMPAGE IN THE HAMILTON SOUTH WELL FIELD, FAIRFIELD-NEW BALTIMORE AREA

Figures are given in acre-ft/day.

Year n	Well			Water Requirement R(n)
	1(F-10)	2(F-11)	3(F-16)	
1974	13.1	13.1	4.4	30.6
1975	13.1	13.1	5.0	31.2
1976	13.1	13.1	5.6	31.8
1977	13.1	13.1	6.2	32.4
1978	6.8	13.1	13.1	33.0
1979	13.1	13.1	7.4	33.6
1980	13.1	8.0	13.1	34.2
1981	8.6	13.1	13.1	34.8
1982	13.1	13.1	9.2	35.4
1983	13.1	13.1	9.8	36.0

Notice that the binding constraints in this particular case are those associated with the maximal capacity of wells. All Lagrange multipliers associated with constraints considering drawdown limits are zero. If the City of Hamilton would like to improve its well operation and reduce operational expenses, it should consider increasing its wells' capacities -- in particular wells F-10 and F-11. A more profound analysis of conclusions which can be drawn by solving such a capacity problem and an example are in the next chapter.

6.7.3 CONCLUSIONS

This chapter concludes this study's reference to the case study on the Fairfield-New Baltimore area. The following results were achieved by applying the various mathematical developments to this case. A step-by-step illustration of the developing methods and models provided a profound insight into the various functions, procedures and formulations. This chapter constitutes a complete model structure, whereby this study's developments are put together in one structure illustrating the important potential for complex groundwater systems modeling, planning and managing. Once a suitable physical simulation model is available, response functions may be determined. For any set of inputs, these functions provide an explicit computation of the system's time varying response. These functions may thus practically replace the original simulation model. Certainly predictions of water table throughout the aquifer are possible via these functions. Furthermore, these functions allow for the coupling of the system response to pumping with any computational framework such as a management model. The benefit to the Fairfield-New Baltimore area from this study's applications is a by-product which should be studied directly by those who are interested in developing this area's water resources. In particular the M.C.D. has access to both the physical system by means of data acquisition and to the administrative structure by means of the mandate it has to monitor this particular area for reasons described by Spieker [1968] and Plummer [1974]. The application of the management model to the studied area restricted the model formulation to the extent that the real present situation defined it. To further illustrate this study's contributions, an imaginary case is considered in the next chapter. This case features most options accounted for in the general model formulation where conjunctive use of ground and surface water are considered.

CHAPTER 7

EXAMPLE PROBLEM
A CONJUNCTIVE USE OF GROUND AND SURFACE WATER SYSTEMS

7.1 INTRODUCTION

In this chapter we formulate a hypothetical system featuring most of what is characterized by the management control model of Chapter 6. The hypothetical system is aimed at showing the prospects of using that model for conjunctive use of ground and surface water systems. In particular are shown the options of water supply from a surface reservoir and artificial recharge from a stream into an aquifer. These options, which are not considered in the previous case study introduce (in addition to the aquifer operation) a new dimension to the problem of water resources optimal control. The physical description (Haimes and Macko [1973]), requires cooperation among users for effecting drawdowns, and among aquifer, stream and surface reservoir water balance. The goal description requires co-ordination between surface reservoir control and aquifer cells control for the optimum allocation of surface water. The management model objective function as well as the constraints are well adapted to such a problem. The forthcoming discussion should illustrate the applicability and practicability of the model. It shows the variety of conditions under which the model can be successfully utilized, in particular it emphasizes the coupling of a complex groundwater system with a desired management scheme.

A long-term capacity expansion planning model for conjunctively supplying water from ground and surface water systems is then presented in Section 6.7. It includes capital cost of construction and/or expansion of ground and surface water supply projects along with the operational costs, so that the demand for water may be met for the entire planning period.

7.2 SHORT-TERM SUPPLY PLANNING

The problem investigated herein involves a basin comprising aquifers traversed by streams. Water supply systems are assumed to be already developed, consisting of two major elements: pumping wells and surface reservoir. There are L users in the region, to each of whom there corresponds an aquifer cell. The ℓ^{th} user has m_ℓ wells located at the ℓ^{th} cell. There is a single stream traversing all cells. A variable inflow, $\overline{Y}(n)$, enters the basin upstream, and after interacting with aquifer and recharge facilities along its flow, it enters a reservoir of maximal capacity C_m. A surface supply system constructed and operated by a regional agency, pumps water from the reservoir for direct use after proper treatment. Surface water therefore competes with water from wells, and users consider each on a practical economic basis. Finally, each user has the option of transferring water from the stream to the artificial recharge facility in his area so as to recover drawdowns in his aquifer cell.

The problem is formulated and solved on two levels of interactive procedure: The first comprises L optimization programs corresponding to L users in the basin. A particular optimization problem is considered by the ℓ^{th} user for maximizing his net benefit. The gross benefit is due to the quantities of water he consumes over a period of time from both ground and surface water supplies. The costs associated with his water supply are incurred by his using well operations and artificial recharge facilities, and by his consuming quantities of water from the surface water allocated to him. Water use provides him with benefits. For each time period his projected water use activities determine the benefit in dollars per unit of water supply. Technical constraints define the feasible set of decisions the user can make. To execute his optimal policy, the ℓ^{th} user needs information on variables and parameters which are not exclusively under his control. These include drawdown caused by other users, pumping wells, quantities of water available from the stream for artificial recharge, and price and quantities of water available from the surface water system. This information is available on the second level which is comprised of two stages. At the first, the physical system's coupling functions are determined. Resulting from pumping and recharge plans are drawdowns in aquifers and interactions with the stream. The effects of overall activities in the basin on each particular system response can thus be calculated. The second stage of the second level takes care of the surface reservoir operation. An optimization program is carried out. This is aimed at determining the optimal utilization of the surface water supply system. The program is solved subject to reservoir water balance considerations. This balance results from stochastic flow inputs and required outputs of supply. Stage two of the second level provides the first level with the quantities of surface water available for each time period and the associated cost per unit. It is assumed that the cost per unit

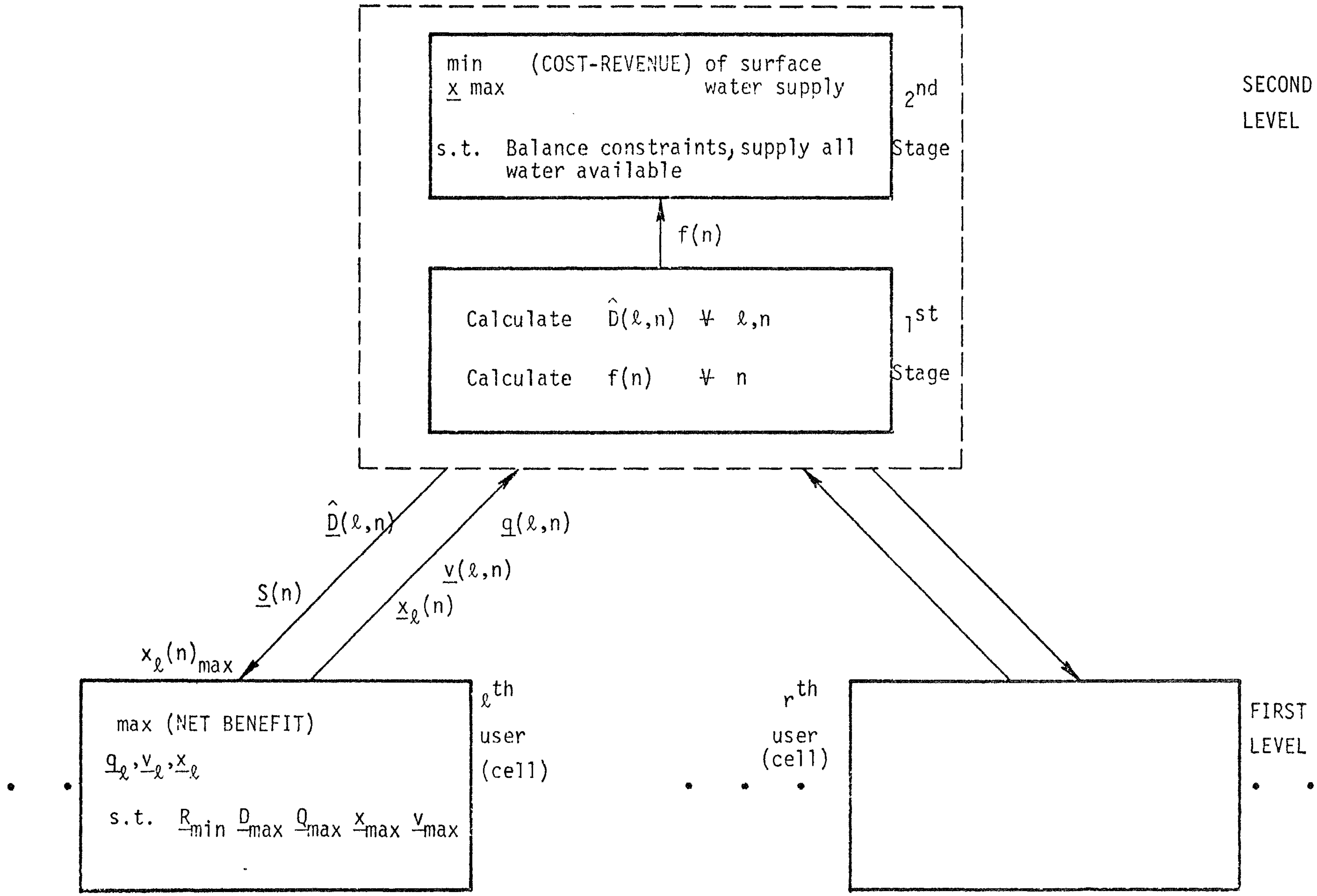


FIGURE 7.1 EXAMPLE PROBLEM MODEL HIERARCHY

of surface water is the same for all users, even if some may affect it more than others. Figure 7.1 shows the model hierarchy. Specific definitions and the different functions involved are discussed further on.

7.3 PROBLEM FORMULATION

7.3.1 First level Optimization Model

Consider the following quadratic model for the ℓ^{th} user:

$$\underset{\underline{q}_\ell, \underline{v}_\ell, \underline{x}_\ell}{\text{minimize}} \left[Z_\ell = \sum_{n=1}^{T} \left[(1+r)^{-n} \left\{ \sum_{k=1}^{m_\ell} P_\ell(k) \cdot q_\ell(k,n) \left(H_\ell(k) + \hat{D}(\ell,n) + \sum_{j=1}^{m_\ell} \sum_{i=1}^{n} \beta_\ell(k,j,n-i+1) q_\ell(i,j) - \sum_{i=1}^{n} \gamma(\ell,\ell,n-i+1) v(\ell,i) \right) + S(n) x(\ell,n) + V_\ell(n) \cdot v(\ell,n) - W_\ell(n) \left(\sum_{k=1}^{m_\ell} q_\ell(k,n) + x(\ell,n) \right) \right\} \right] \right] \quad (7.1)$$

subject to: $$\sum_{k=1}^{m} q_\ell(k,n) + x(\ell,n) \geq R_\ell(n)_{min}$$

$$n=1,\ldots,T \qquad (7.2)$$

$$\hat{D}(\ell,n) - \sum_{i=1}^{n} \gamma(\ell,\ell,n-i+1)v(\ell,i)$$

$$+ \sum_{j=1}^{m_\ell}\sum_{i=1}^{n} \beta_\ell(k,j,n-i+1)q_\ell(j,i) \leq h_\ell(k)_{max}$$

$$n=1,\ldots,T \quad k=1,\ldots,m_\ell \qquad (7.3)$$

$$q_\ell(k,n) \leq Q_{max}(k) \qquad n=1,\ldots,T \quad k=1,\ldots,m_\ell \qquad (7.4)$$

$$v(\ell,n) \leq v_{\ell max} \qquad n=1,\ldots,T \qquad (7.5)$$

$$x(\ell,n) \leq x_\ell(n)_{max} \qquad n=1,\ldots,T \qquad (7.6)$$

Where

T is the number of time periods that comprise the planning horizon

r is the interest rate

m_ℓ is the number of wells located at the ℓ^{th} cell

$P_\ell(K)$ is the pumping cost per acre-ft per ft for the k^{th} well

$q_\ell(k,n)$ is the quantity of water pumped from the k^{th} well during the n^{th} period

$H_\ell(k)$ is the lift under steady state conditions at the k^{th} well

$\hat{D}(\ell,n)$ is the drawdown in the ℓ^{th} cell at the end of the n^{th} time period due to aggregate pumpage and recharge in all other cells (by other users) in the region

$\beta_\ell(k,j,n-i+1)$ is the algebraic technological term relating the drawdown at the k^{th} well to the pumping of one unit of water from the j^{th} well during the i^{th} period, and both k and j are located at the ℓ^{th} cell

$\gamma(\ell,r,n-i+1)$ is the algebraic technological term relating the average drawdown at the ℓ^{th} cell to aggregated pumping of one unit of water at the r-th cell during the i^{th} period

$v(\ell,n)$ is the quantity of water used for artificial recharge at the ℓ-th recharge facility during period n

$S(n)$ is the periodical price per acre-ft of surface water supply from the reservoir

$x(\ell,n)$ is the quantity of water supply to the ℓ-th user from the surface reservoir during time period n

$V_\ell(n)$ is the operating cost of recharge per acre-ft in the ℓ-th area with water from the stream

$W_\ell(n)$ is the return per acre-ft of water supply for the ℓ-th user during the n-th period

$R_\ell(n)$ is the minimum water requirements for the ℓ-th user in the n-th period

$h_\ell(k)_{max}$ is the maximum lift allowed at the k-th well due to well design

$Q(k)_{max}$ is the upper limit for pumping from the k^{th} well

$v_{\ell max}$ is the recharge facility capacity limit

$x_\ell(n)_{max}$ is the allocation of surface water supply to the ℓ-th user for the n-th period

The input to the first level from the second level includes

$\hat{D}(\ell,n)$ the drawdown at the ℓ-th cell due to pumpage and recharge in other cells; $S(n)$, the price per unit of water supply from the surface reservoir; $x_\ell(n)_{max}$, the upper limit for quantities of water allocated for the surface water supply. The output from the first level to the second level includes $q_\ell(k,n)$, the pumping plan; $v(\ell,n)$, the artificial recharge plan; and $x(\ell,n)$, the surface water requirement plan.

7.3.2 Second Level - First Stage

Two sets of functions are considered:

$$\hat{D}(\ell,n) = \sum_{\substack{r=1 \\ r\neq\ell}}^{L} \sum_{i=1}^{n} \gamma(\ell,r,n-i+1) \cdot \left[\sum_{k=1}^{m_r} q_r(k,i) - v(r,i)\right] \quad (7.7)$$

$\hat{D}(\ell,n)$ is the drawdown observed in the ℓ-th cell area due to the net pumping throughout the rest of the system.

$$f(n) = \sum_{r=1}^{1} \sum_{i=1}^{n} \psi(r,n-i+1) \left[\sum_{k=1}^{m_r} q_r(k,i) - v(r,i)\right] + \sum_{r=1}^{1} I_r \quad (7.8)$$

where f(n) is the total amount of water induced from the stream into the different aquifer cells during the n-th period.

The values of $\hat{D}(\ell,n)$ are available for updating the first level while f(n) values are used by the second stage of the second level to determine the stream balance.

7.3.3 Second Level-Second Stage

At this stage the operation of the surface reservoir is considered. The following steps are included:

1. Determine the net flow from the stream actually entering the reservoir, y(n):

$$y(n) = \bar{Y}(n) - \sum_{\ell=1}^{L} [f(\ell,n) + v(\ell,n)] - E(n) \quad (7.9)$$

here $\overline{Y}(n)$ is the stream flow entering the basin upstream, and is naturally a stochastic variable. Assuming variables $Y_1(n), Y_2(n) \ldots Y_M(n)$ with probabilities $p_1, p_2, \ldots, p_M$ then the expected value of $Y(n)$ is $\overline{Y}(n)=E(Y(n))=\sum_{j=1}^{M} p_j Y_j(n)$.

Similar discussion can be found in Buras [1963].

$f(\ell,n)$ is the quantity of water induced from stream into aquifer in the ℓ-th area, and is determined by the first stage.

$v(\ell,n)$ is the quantity of water from stream transferred into the ℓ-th artificial recharge facility, and results from the first level ℓ-th optimization program.

$E(n)$ is the water loss due to evaporation from stream, reservoir and other facilities, not including overflows due to floods. This quantity, like the upstream flow, is assumed known.

2. Check for the reservoir over-flow. Let C_o and C_m denote the reservoir capacity at the outset of the planning period and the maximum reservoir capacity, respectively. Let

$$\left.\begin{array}{l} \hat{x}(n)=\sum_{\ell=1}^{L} x(\ell,n) \\ \text{If } y(n)>C_m-C_o+\sum_{i=1}^{n} \hat{x}(n) - \sum_{i=1}^{n-1} y(i) \\ \text{then} \quad y(n)=C_m-C_o+\sum_{i=1}^{n} \hat{x}(n) - \sum_{i=1}^{n-1} y(i) \end{array}\right\} \quad n=1,\ldots,T \qquad (7.10)$$

3. Consider the cost function for surface reservoir operation: Let the periodic fixed expenses be α_1 \$/period and the operational cost be $\alpha_2\hat{x}(n)+\alpha_3\hat{x}(n)^2$ where $\hat{x}(n)=\sum_{\ell=1}^{L} X(\ell,n)$. The per

unit cost considered for time period n is S(n):

$$S(n)= (\alpha_1+\alpha_2\hat{x}(n) + \alpha_3 \hat{x}(n)^2)/ \hat{x}(n) \qquad (7.11)$$

The users want the system to provide them with surface water supply while maintaining the most efficient operation. Restrictions are the physical limits and the input-output balance considerations. The agency operating this system does not control the requirements for the water it allocates. It does, however, provide the users with an optimal plan of allocations and the associated cost per unit supplied. A particular plan for surface water allocation is $(\bar{x}(1), \bar{x}(2), \ldots, \bar{x}(T))$, where $\bar{x}(n)= \sum_{\ell=1}^{L} x_\ell(n)_{max}$ is the sum of surface water allocations for all users at period n. Recall, however, that the actual use $x(\ell,n)$ is not necessarily fixed for a given $x_\ell(n)_{max}$, but is limited from above by this allocation, that is $x(\ell,n)\leq x_\ell(n)_{max}$. As a result, $\hat{x}(n)<\bar{x}(n)$ introduces the possibility that an optimal surface water allocation does not necessarily imply full utilization of the available water. Being more realistic, it is possible that some users may wish to consume other users' , unused water. Define $\overset{*}{x}(n)=\bar{x}(n)-\hat{x}(n)$ as the amount of water which should be reallocated to these users where the Lagrange multiplier corresponding to the constraint $x(\ell,n) \leq x_\ell(n)_{max}$ is non-zero (meaning that the allocation of surface water $x_\ell(n)_{max}$ is restricting the ℓ^{th} user plans). Overall optimal considerations require that the surplus $\overset{*}{x}(n)$ be allocated according to the values of the associated Lagrange multipliers. But such considerations are not assumed binding in this particular case (each user is interested solely in his own profits). Hence, surplus is shared equally among users who may use it regardless of the marginal benefits. The optimal surface water allocation program is:

$$\underset{\bar{x}}{\text{Minimize}} \sum_{n=1}^{T} (1+r)^{-n} \left\{ (\alpha_1+\alpha_2\bar{x}(n)+\alpha_3\bar{x}(n)^2) - S(n)\bar{x}(n) \right\} \tag{7.12}$$

Subject to:

1. Quantities available may not exceed the reservoir maximal capacity being also the upper limit for the surface water system supply capacity:

$$\bar{x}(n) \leq C_m \qquad n=1,\ldots,T \tag{7.13}$$

2. Periodic allocations may not exceed available water in the reservoir:

$$\sum_{i=1}^{n} \bar{x}(i) \leq C_o + \sum_{i=1}^{n} y(i) \qquad n=1,\ldots,T-1 \tag{7.14}$$

3. Allocations should allow for full utilization of all surface water available over the entire time horizon:

$$\sum_{i=1}^{T} \bar{x}(i) = C_o + \sum_{i=1}^{T} y(i) \tag{7.15}$$

4. The amount stored in the surface reservoir at the end of each period should not exceed the maximal storage capacity:

$$\sum_{i=1}^{n} \bar{x}(i) \geq \begin{Bmatrix} 0 & n: \left\{ \sum_{i=1}^{n} y(i) + C_o < C_m \right. \\ \sum_{i=1}^{n} y(i) + C_o - C_m & \text{Otherwise} \end{Bmatrix} \quad n=1,\ldots,T-1 \qquad (7.16)$$

The model formulation in (7.1) - (7.16) is a program for optimal conjunctive use of ground and surface water. It follows the conceptual model represented in Chapter three, with these modifications:

Construction cost is not considered.

The penalty cost function for depletion of the stream is originally stated explicitly as a factor in the performance criterion. Here it is given a meaningful application. The surface reservoir operation considers the stream balance. The upper limit $B_{\ell}(u,n)$, (see Eqn. 7.5), is interpreted through a set of reservoir balance constraints. The penalty term $Q_{\ell}(u,n)$ is assigned a large value, converting the cost criterion to a set of strict constraints. The infiltration limit constraint in the original model is interpreted here in the second level commonly for all users through the stream balance calculations.

In Figure 7.2 a flow-chart of the model given in (7.1) through (7.16) summarizes the different computations involved.

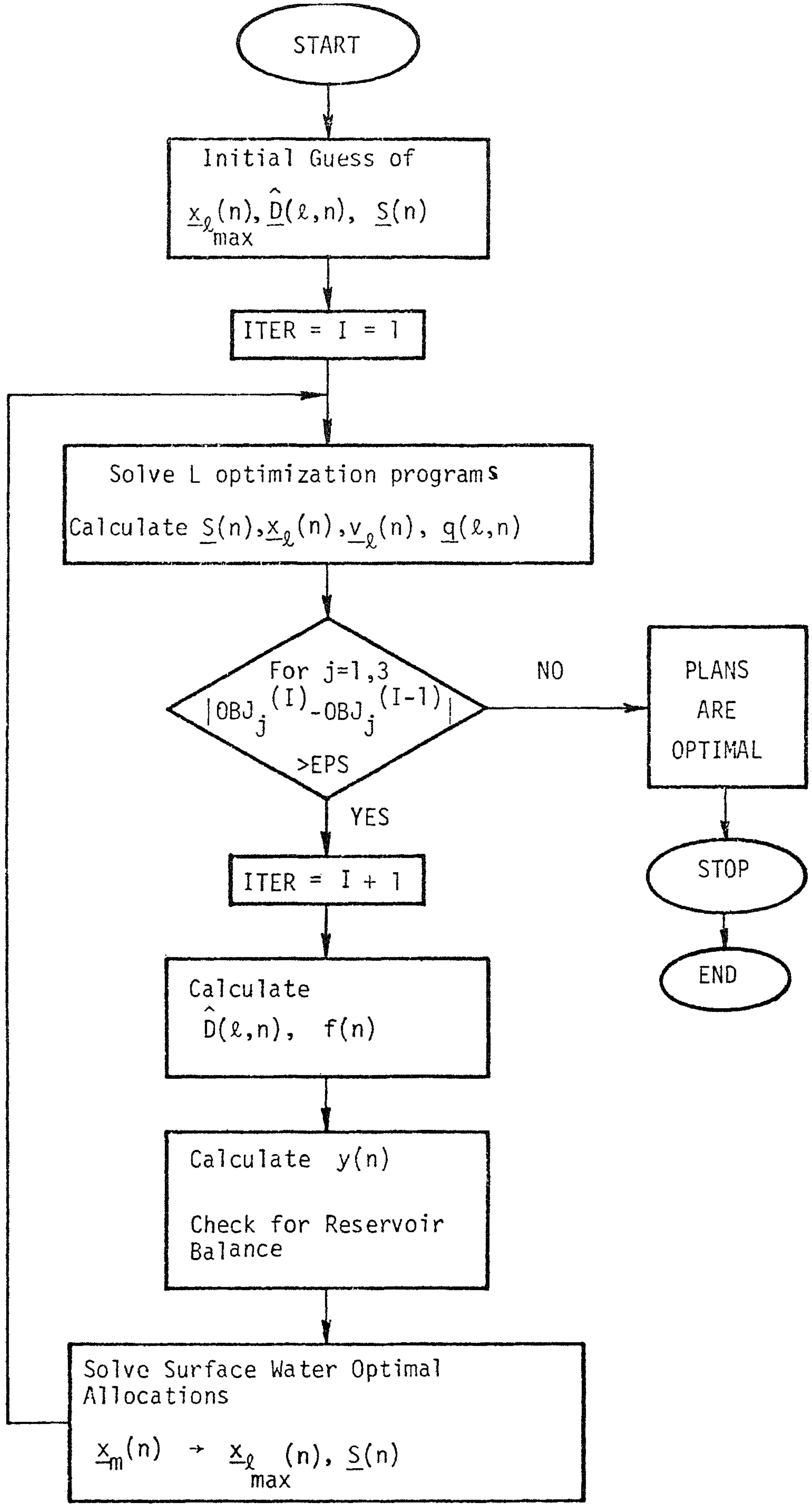


FIGURE 7.2 EXAMPLE PROBLEM PROGRAM FLOW-CHART

7.4 HYPOTHETICAL CASE INPUT DATA AND COMPUTATIONAL RESULTS

In constructing a hypothetical case aimed at illustrating, verifying and refining the model, we believe the data we have generated reflect reality. Realism of information and functions utilized is our main concern. The results obtained from using the generated data and functions are expected to convince the reader as to the model's actuality and prospective applicability.

Three users, L = 3, are in the area. Each operates three wells to meet his water needs, and in addition may choose to buy surface water from the reservoir. Each of two users owns an artificial recharge facility with a limited capacity. The time horizon of planning is six years; application to a longer period is discussed later. Tables 7.1 - 7.7 give the information on the various users. Tables 7.8 - 7.9 show the information applied to the surface reservoir system.

NOTE: The response functions are assumed in effect for three years. Effects of pumpage on the system response after the third year are negligible in this case.

TABLE 7.1

SIX YEARS' PROJECTIONS OF MINIMUM WATER REQUIREMENTS

IN THE HYPOTHETICAL CASE STUDY

(Figures are given in acre-ft/day)

Year n \ User (cell)	1	2	3
1	70.	60.	15.
2	70.	65.	15.
3	75.	70.	15.
4	75.	70.	15.
5	75.	70.	15.
6	80.	70.	15.

TABLE 7.2

ALGEBRAIC TECHNOLOGICAL FUNCTIONS $\beta(k,j,i)$ FOR WELLS AT EACH OF THE CONSIDERED CELLS IN THE HYPOTHETICAL CASE STUDY

(Figures are given in ft/acre-ft/day)

User (Cell) ℓ	Year i	$\beta_\ell(1,j,i)$ j 1	2	3	$\beta_\ell(2,j,i)$ j 1	2	3	$\beta_\ell(3,j,i)$ j 1	2	3
1	1	.436	.208	.13	.21	.502	.207	.133	.208	.428
	2	.043	.045	.032	.044	.058	.041	.032	.041	.036
	3	.01	.012	.008	.011	.014	.010	.008	.010	.007
2	1	.392	.174	.109	.196	.458	.187	.131	.196	.414
	2	.039	.044	.031	.044	.052	.039	.031	.039	.035
	3	.009	.011	.008	.011	.013	.009	.007	.009	.007
3	1	.349	.153	.006	.183	.436	.179	.122	.183	.392
	2	.037	.041	.030	.039	.048	.037	.028	.037	.033
	3.	.009	.010	.007	.010	.013	.008	.006	.008	.006

TABLE 7.3

ALGEBRAIC TECHNOLOGICAL FUNCTIONS $\gamma(\ell,r,i)$

FOR CELLS IN THE HYPOTHETICAL CASE STUDY

(Figures are given in ft/acre-ft/day)

Year i	$\gamma(1,r,i)$ r			$\gamma(2,r,i)$ r			$\gamma(3,r,i)$ r		
	1	2	3	1	2	3	1	2	3
1	.044	.009	.004	.009	.039	.007	.002	.013	.035
2	.005	.003	.003	.002	.005	.001	.001	.001	.003
3	.001	.0	.0	.0	.002	.0	.0	.0	.001

TABLE 7.4

$\psi_r(\ell,i)$ FLOW BETWEEN STREAM AND CELLS AS A FRACTION OF THE PUMPAGE IN THE HYPOTHETICAL CASE STUDY

Year	$\psi_r(1,i)$			$\psi_r(2,i)$			$\psi_r(3,i)$		
	r			r			r		
i	1	2	3	1	2	3	1	2	3
1	.55	.19	.09	.25	.40	.10	.30	.10	.30
2	.05	.12	.01	.01	.10	.01	.05	.01	.10
3	.0	.03	.04	.0	.02	.01	.0	.0	.05

TABLE 7.5

TECHNICAL INFORMATION - WELLS IN THE HYPOTHETICAL CASE STUDY

User (cell) ℓ	Well k_ℓ	Maximum Capacity $Q_\ell(k)_{max}$ [acre-ft/day]	Initial Lift $H_\ell(k)$ [ft]	Maximum Drawdown $D_\ell(k)_{max}$ [ft]	Recharge Facility Maximum Capacity $v_{\ell\ max}$ [acre-ft/day]
	1	20.	70.	25.	
1	2	30.	75.	25.	20.
	3	40.	80.	25.	
	1	30.	100.	25.	
2	2	40.	100.	25.	25.
	3	40.	100.	25.	
	1	7.	150.	20.	
3	2	7.	120.	20.	0.
	3	7.	170.	20.	

P(k) Cost of pumping 0.0404 dollar/acre-ft/ft,

k=1,2,3

TABLE 7.6

EXPECTED BENEFIT PER ACRE-FT OF WATER USE IN THE HYPOTHETICAL CASE STUDY

(In Dollars/acre-ft)

User (cell) ℓ / Year i	1	2	3
1	54.	56.	60.
2	57.	58.	60.
3	61.	60.	60.
4	64.	62.	60.
5	67.	64.	60.
6	71.	66.	60.

TABLE 7.7

COST OF ARTIFICIAL RECHARGE OPERATIONS IN THE HYPOTHETICAL CASE STUDY

(In dollars/acre-ft)

User (Cell) ℓ / Year i	1	2	3
1	1.	.7	0.
2	1.	.7	0.
3	1.	.7	0.
4	1.	.7	0.
5	1.	.7	0.
6	1.	.7	0.

TABLE 7.8

EXPECTED VALUES OF FLOWS ENTERING UPSTREAM $\overline{Y}(n)$ AND ANNUAL EVAPORATION RATE $E(n)$ FIGURES FOR THE HYPOTHETICAL CASE STUDY

(In acre-ft/day)

Year n	Upstream Flow $\overline{Y}(n)$	Evaporation Rate $E(n)$	$\overline{Y}(n) - E(n)$
1	300.	80.	220.
2	300.	80.	220.
3	300.	80.	220.
4	300.	80.	220.
5	300.	80.	220.
6	300.	80.	220.

TABLE 7.9

SURFACE RESERVOIR TECHNICAL INFORMATION FOR THE HYPOTHETICAL CASE STUDY

Initial Reservoir Capacity CAPo = 130. acre-ft/day

Maximal Reservoir Capacity CAPm = 150. acre-ft/day

Operation Cost Coefficients:

α_1 = 20. α_2 = 1. α_3 = .01

Interest Rate = .08

Figure 7.3 is the optimal solution corresponding to the input data in Tables 7.1 - 7.9. The convergence criterion (Figure 7.2) is $\varepsilon = 100$. The solution is achieved after the fourth iteration. Figure 7.4 represents the solution convergence rate. The computation time on the UNIVAC 1108 digital computer at Case Western Reserve University is 652 seconds and file usage is 114442 words. The solution for the six-year operation period proves that the model constitutes an optimal solution. However, there are at least two difficulties which should be discussed.

First is the difficulty associated with the convergence rate. Two different iterative loops are embedded in the model. One is in the quadratic program subroutine where Wolfe's Algorithm, Wolfe [1959], is used. This algorithm requires iterative procedure for solving Phase one of the Simplex Tableaux and convergence conditions are well established. The second iterative loop corresponds to the coordination scheme between the two levels (Figure 7.1). It comprises both the physical description and a computational algorithm of transferred parameters and functions between the two levels. The resulting procedure is actually not related to any known coordinating algorithm (Haimes and Macko [1973]; Lasdon [1970]). The coordination is merely an information flow among users and between them and the surface water supply system. Each user sets his own policy, but there is no overall regional management policy. We could not find any analytical approach by which to prove conditions for convergence. We can only say that all ten different runs of the program utilizing different input data showed consistency with regard to the convergence rate. No run iteration number exceeded 5. The other difficulty is the dimensionality of the program. In particular the planning horizon plays a critical role in the program's size. A one-unit increase in the planning period introduces to each program at the first level 3 + k decision variables, where k is the number of wells associated with a particular user. The number of constraints is increased by 4 + 2k. In the second level it adds two decision variables and four constraints to the surface reservoir optimization model. Figure 7.5 is a graph of the computation time versus the planning time for this case study.

We conclude this discussion by stating that the model is available for use and is capable of solving larger-sized problems. Of course, the trade-off between computation time and computer capacity should be considered.

To complete this model analysis, a sensitivity analysis was carried out. It should provide some guidance for any future developments based on this model, in particular with respect to information and data acquisition.

User	Year	Well Pumping Plan 1	Well Pumping Plan 2	Well Pumping Plan 3	Recharge Plan	Surface Water Use Plan
1	1	15,038	30,000	40,000	20,000	53,333
	2	20,000	13,462	40,000	20,000	30,278
	3	20,000	12,897	40,000	20,000	15,216
	4	20,000	12,864	40,000	20,000	14,444
	5	16,040	30,000	15,146	20,000	18,649
	6	20,000	30,000	12,122	20,000	18,789
2	1	4,456	40,000	40,000	25,000	53,333
	2	,000	40,000	19,921	25,000	30,278
	3	30,000	12,382	40,000	25,000	15,216
	4	30,000	10,176	40,000	25,000	14,444
	5	30,000	9,655	40,000	25,000	18,649
	6	30,000	9,769	40,000	25,000	18,789
3	1	7,000	7,000	7,000		43,333
	2	7,000	7,000	7,000		30,278
	3	7,000	7,000	7,000		15,216
	4	7,000	7,000	7,000		14,444
	5	7,000	7,000	7,000		18,649
	6	7,000	7,000	7,000		18,789

Surface Water Per Unit Cost

Year	1	2	3	4	5	6
$/acre-ft	2.616	2.105	1.854	1.852	1.883	1.885

FIGURE 7.3. EXAMPLE PROBLEM - THE SIX-YEAR OPTIMAL SOLUTION

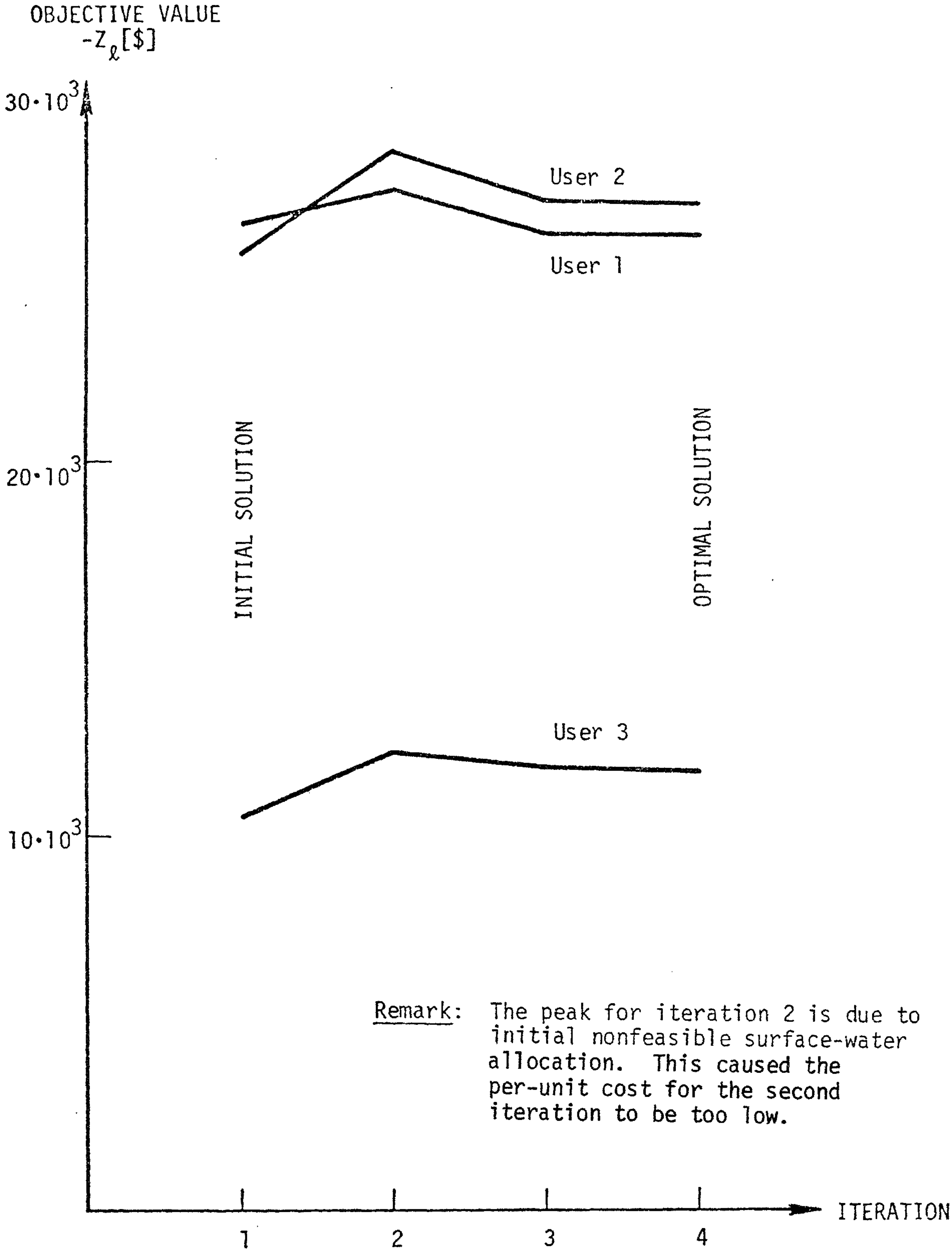


FIGURE 7.4. CONVERGENCE RATE OF OPTIMAL SOLUTION. CASE II, SIX-YEAR OPERATION.

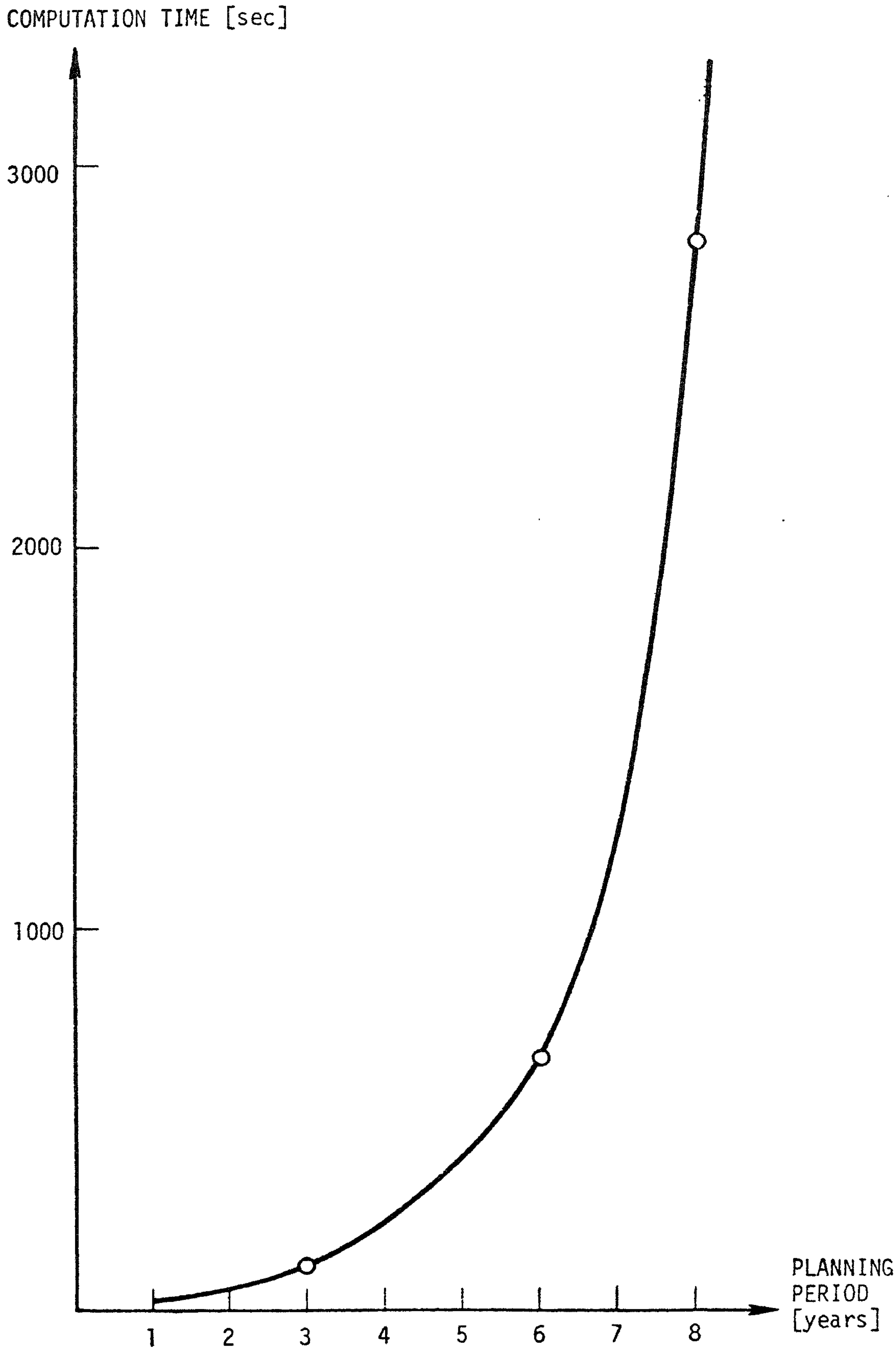


FIGURE 7.5. COMPUTATION TIME VERSUS PLANNING PERIOD, EXAMPLE PROBLEM, UNIVAC 1108.

7.5 SENSITIVITY ANALYSIS

The main purpose of the forthcoming discussion is to point out some elements of concern associated with this model. A sensitivity analysis of different aspects in the model should assist in that task. To save computer time, the sensitivity analysis was performed for a three-year planning period.

7.5.1 The objective's value and the upstream flow

Particular care should be given to the input data. This is especially true because probabilistic data introduce uncertainty into the basic results. Figure 7.6 represents the sensitivity of the optimal solutions by means of the objective value to the probabilistic stream flow. It is clear that each user's performance is linearly dependent on the net upstream flow. This flow is essentially the measure of surface water availability. An interesting comparison is made in Table 7.10 where the slopes of the curves in Figure 7.6 are compared with the Lagrange multipliers associated with the constraints (7.6). These constraints limit the available surface water for each time period. The multipliers are interpreted as the cost per unit of upstream flow. Its relation to actual operational plans is discussed in Section 7.5.2.

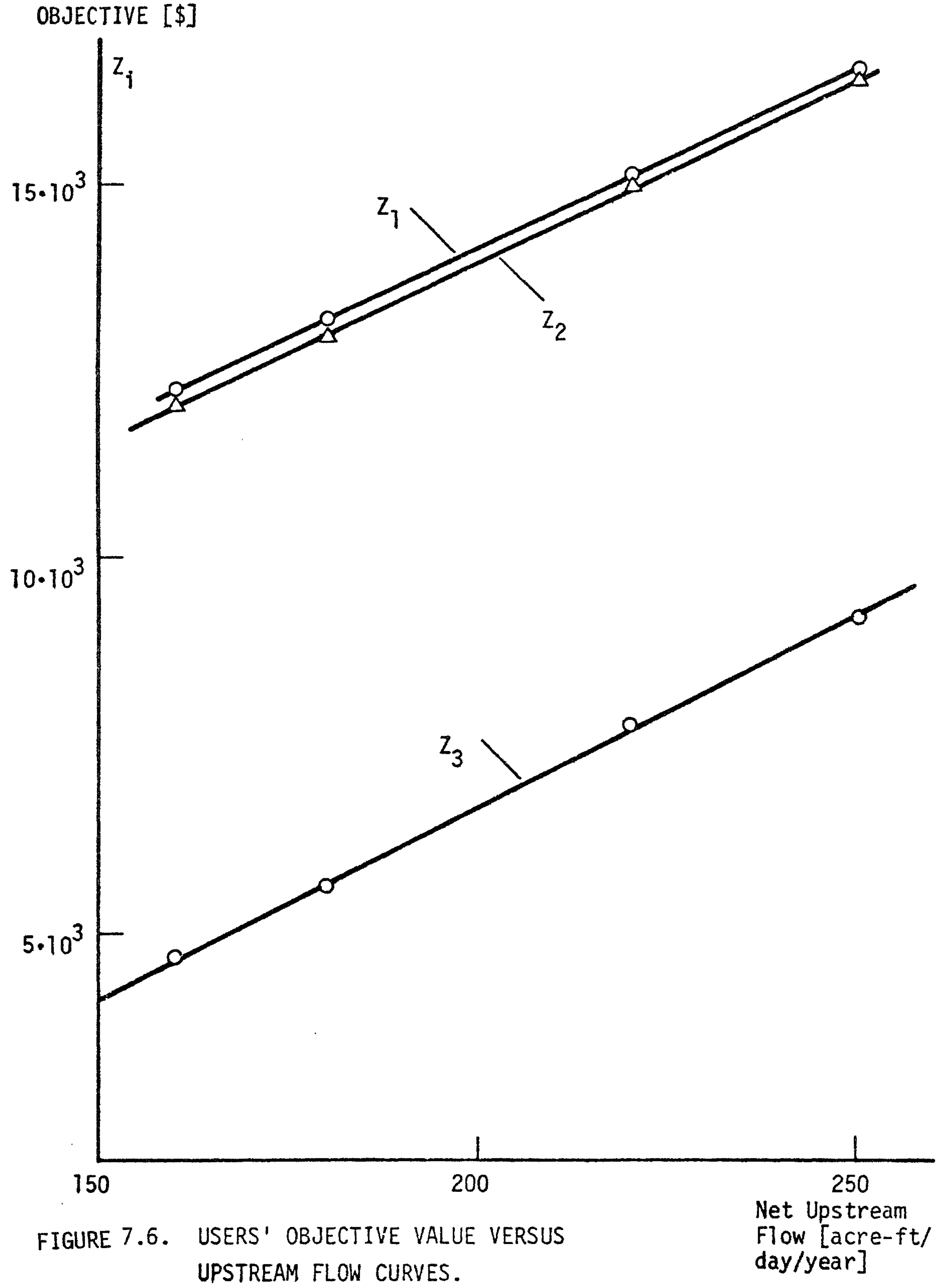


FIGURE 7.6. USERS' OBJECTIVE VALUE VERSUS UPSTREAM FLOW CURVES.

TABLE 7.10

A COMPARISON BETWEEN THE SLOPES OF THE OBJECTIVE VERSUS STREAM FLOW CURVES AND THE LAGRANGIANS ASSOCIATED WITH SURFACE WATER AVAILABILITY CONSTRAINTS

User	Year i	Lagrange Multiplier λ_i	$\bar{\lambda} = (\sum_{i=1}^{3} \lambda_i)/3$	Slope of Sensitivity Curve
1	1	47.5	47.2	47.
	2	47.0		
	3	46.9		
2	1	49.4	47.8	48.
	2	47.8		
	3	46.1		
3	1	53.1	49.2	50.
	2	49.5		
	3	46.1		

7.5.2 The Operational Plans and the Upstream Inflow

The information generated for the hypothetical case assigns high priorities to water use. This should spur the optimal operation planners to utilize all available water sources. Hence, a decrease in one source such as surface water availability should not affect pumping plans. However, it will affect the surface water use plans. This effect is illustrated in Figure 7.7. The probabilistic nature of stream flow in this case is eventually a factor in considering surface water use. Another component which is dependent upon stream flow is artificial recharge. This activity certainly competes with surface water supply for quantities from the stream. In this model each user's independent policy causes him to disregard any possible benefit to him from having more surface water to use if he uses less water for artificial recharge. The various users could realize immediate benefits if they would coordinate their artificial recharge activities.

7.5.3 The Effect of Aggregated Drawdown

A particular user's program considers the drawdown caused by other users (the term $\hat{D}(\ell,n)$ both in the objective cost function (7.1) and in the upper limit for drawdown constraint (7.3). The sensitivity of the objective value to changes in $\hat{D}(\ell,n)$ is well defined by the Lagrange multipliers associated with the constraints (7.3). In Table 7.11 are the corresponding multipliers' values for the three users' optimal plans. These are interpreted as the dollar value of a unit drawdown.

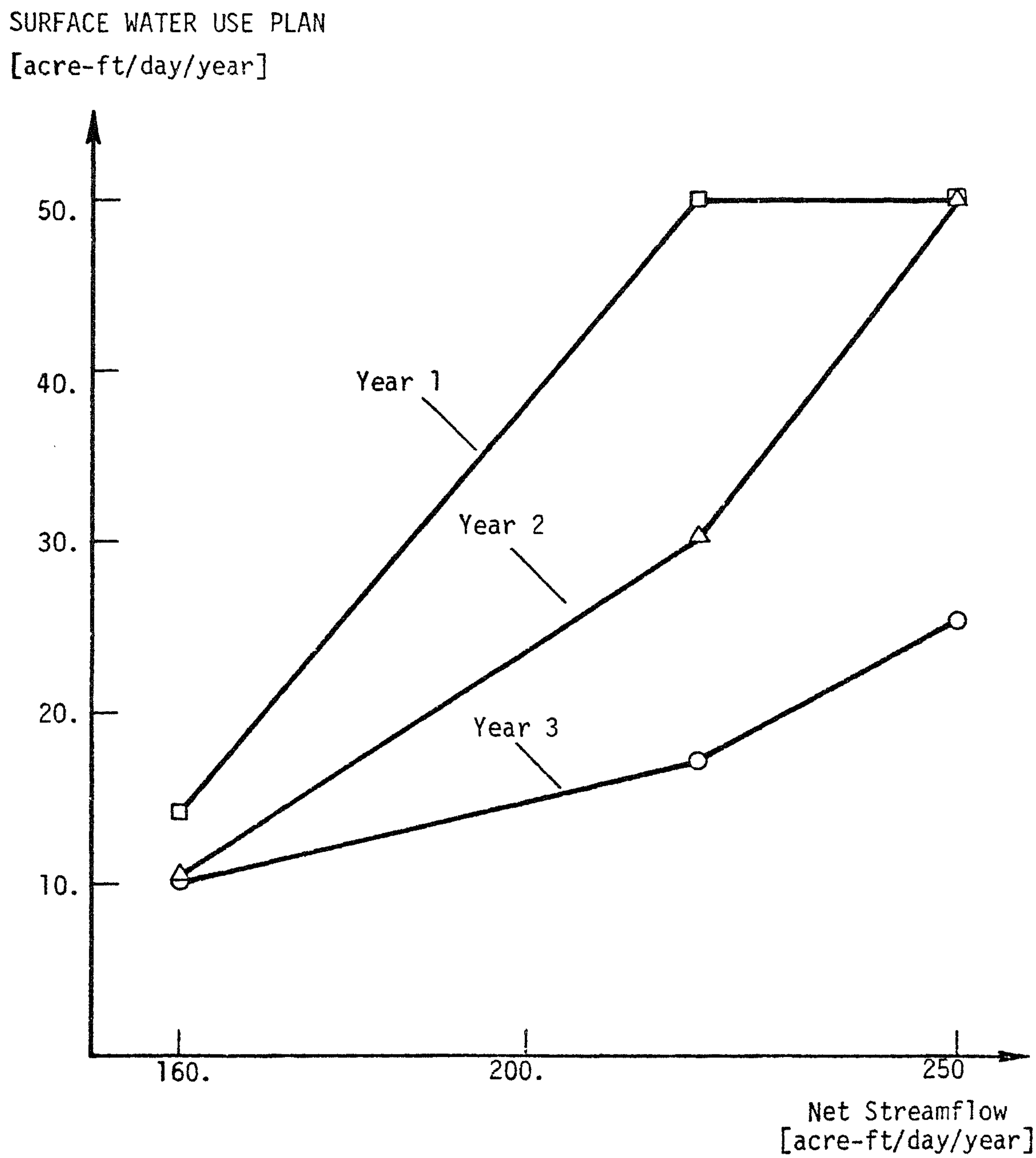


FIGURE 7.7. SURFACE WATER PLAN VERSUS UPSTREAM FLOW.

TABLE 7.11

LAGRANGE MULTIPLIERS ASSOCIATED WITH LIMITING DRAWDOWN CONSTRAINTS UNDER AN OPTIMAL OPERATION PLAN

Year	User (cell) 1		User (cell) 2		User (cell) 3	
i	Well k	$\lambda_1(k,i)$ \$/ft	Well k	$\lambda_2(k,i)$	Well k	$\lambda_3(k,i)$
1	1	0.	1	0.	1	0.
	2	75.8	2	85.1	2	0.
	3	0.	3	0.	3	0.
2	1	0.	1	0.	1	0.
	2	47.8	2	52.9	2	0.
	3	69.3	3	72.8	3	0.
3	1	0.	1	0.	1	0.
	2	57.9	2	62.1	2	0.
	3	76.6	3	76.6	3	0.

TABLE 7.12

THE OPERATIONAL PLANS AND PERTURBATIONS IN THE UPPER LIMIT FOR DRAWDOWN

User (Cell)	Year i	Well k	$D_2(2)_{max}$ = 25. ft Surface Water Use [acre-ft/day/year]	Well Pumpage	$D_2(2)_{max}$ = 24. ft Surface Water Use [acre-ft/day/year]	Well Pumpage
		1		20.		20.
	1	2	50.	30.	50.	30.
		3		34.96		34.74
		1		20.		20.
1	2	2	30.26	13.58	32.96	13.36
		3		40.		40.
		1		20.		20.
	3	2	15.22	12.87	15.42	12.88
		3		40.		40.
		1		4.45		0.
	1	2	50.	40.	50.	40.
		3		40.		39.32
		1		0.		0.
2	2	2	30.26	40.	32.96	40.
		3		19.93		15.76
		1		30.		30.
	3	2	15.22	12.38	15.42	13.29
		3		40.		40.

The effect of perturbations in $\hat{D}(\ell,n)$ on the operation plans is much more complicated and may not be explicitly derived from the optimal solution information. These perturbations are more significant in affecting the drawdown constraints (7.3). (The effect of these in the objective function can be measured by conducting a sensitivity analysis on the initial lift, $H_\ell(k)$. This is found to have no effect on the operational plan.)

Perturbations in $\hat{D}(\ell,n)$ with respect to the constraints (7.3) are essentially equivalent to perturbations in the upper limit for drawdown $D_\ell(k)_{max}$. In Table 7.12 a sensitivity analysis of the operational plan to limit drawdown is summarized. A unit change in $D_2(2)_{max}$ for the second user is introduced. Eventually the operational plan is unpredictably sensitive to such perturbations.

7.6 EXAMPLE PROBLEM SUMMARY AND CONCLUSIONS

Applying the management model developed in this study to the hypothetical case achieved these goals:

1. A full utilization of the model for a realistic hypothetical case.
2. A step-by-step analysis of conjunctive use of ground and surface water systems.
3. A profound analysis of advantages, drawbacks and prospective uses of the proposed model formulation, solution and implementation.

The Example problem analysis completes the development of the management control model analyzed in Chapter 6. It illustrates the potential inherent in the model for an even more detailed analysis of the conjunctive use of ground and surface water. A solution to the problem for short-term operation planning is given and is proved to be stable and satisfactory.

The trade-off between computer time and capacity should be further studied. The model results in optimal operational plans for water use and the associated value of the performance criterion. It illustrates one more time how the response functions can actually be used to couple a groundwater system model with a large-scale management model. The sensitivity analysis points out that if the performance function depends on input information, changes in the objective value caused by input variations can be predicted and evaluated once a given deterministic input is solved. On the other hand, optimal operative plans are heavily dependent upon some of the model's parameters in an unpredictable way. It is therefore necessary to first identify physical parameters of the system as accurately as possible. These include transmissivity and storage coefficients upon which the algebraic technological functions are dependent. Also the physical coefficients related to the stream bed are needed for accurate estimation of infiltration rates. The stream flow probabilistic features are important if surface water is the main source of supply for regional development. This model is a possible tool for evaluation of this factor and for possible compensation of groundwater supply in case surface water is lacking.

7.7 LONG-TERM CONJUNCTIVE SUPPLY MODEL

7.7.1 Introduction

The growth in population and increasing demand for industrial and agricultural outputs require an additional supply of fresh water in the Basin. An expansion of urban centers will increasing population requires more municipal water for public consumption, whereas an increase in industrial activities places an increased requirement for water suitable for industrial use. Similarly, intensive agriculture suggests an increased demand for water to irrigate farm lands. The purpose of developing this model is to determine the optimum quantities of water which are conjunctively utilized from ground and surface water sources within the Basin to meet a growing future demand for water for municipal, industrial and agricultural use.

The advantages of jointly utilizing surface facilities and groundwater basins have been known for many years. Burt [1964], and Leonard [1964] have presented a comprehensive discussion on the economic advantages of utilizing ground and surface water conjunctively. The concept of conjunctive use is simply that of jointly utilizing surface facilities and groundwater basins to supply the desired water at minimum cost. The efficient use of ground and surface water resources can be achieved only when both ground and surface water are integrated as to the size, location and date of construction of surface reservoirs, aqueducts, wells and pumps, and replenishment facilities. In general, economic, budgetary and other practical limitations rule out the development of a total water supply system that is responsive to present and predicted demands over the planning period. Hence a systematic planning is needed in order to determine when and how large to construct supply projects. The proposed projects are drawn from a set of feasible groundwater and surface water projects so that their total utilization lies within the limitations of the Basin's hydrologic resources. The term "total utilization" is used here to imply the satisfaction of demands for multiple purposes of water such as municipal, industrial and agricultural.

It is also assumed that the stream hydrology is adequately characterized by their average flow records. However, the stochastic nature of streamflow, precipitation, and other such aspects affect the water balance in the surface stream. For our analysis the model is formulated by assuming these variables as deterministic. If the average monthly or annual flows in streams are determined by monitoring the flow at several sampling points over a long time horizon, then these values can be used without introducing any appreciable error in model output.

7.5.2 Supply Model

A river basin planning area comprised of groundwater and surface water is considered. It is assumed that the basinal requirements for water for municipal, industrial, and agricultural consumption are fulfilled from both these sources. A surface water project may include reservoirs, desalting plants, diversions, etc. A groundwater project can be wells and pumps, an artificial recharge facility, and a distribution system. The use of reservoirs for surfacewater storage is a well established practice in water supply and flood control. However, increasing demand with a potentially limited amount of water emphasizes the need for including groundwater in planning a river basin development. The demand for municipal, industrial and agricultural uses can also be partially satisfied through inter- and intra-basin transfers. However, because of interstate effects, curtailment of uses in the exporting region and costs imposed on the area of origin can become a part of the real cost of water transfer. Also, the present laws on water rights make the interbasin, even intra-basin, transfer of water very difficult. An example may be cited. Under the present laws of most states, it is very difficult for the owner of an irrigation water right to transfer it to industrial uses in a nearby location, even though the water may be more valuable for industrial than agricultural use and the economy may benefit far more from the industrial use [National Water Commission, 1973]. In this study, water transfer projects are not included. The potential role of groundwater basins can be realized only if the management of ground and surface water supplies is effectively integrated. This can be done only through a coordinated approach to allocating these two resources with regard to conservation of local water, importation, replenishment, extractions, and distribution. The complexity of integrated management arises not only from the large dimension of the problem but also due to the physical coupling of these resources.

We will assume that there exists a regulatory authority who has the say as to how much water can be withdrawn from surface reservoirs and how much can be pumped from each groundwater basin. There may be a number of pumping wells in each groundwater basin. In this analysis, a group of wells may be collectively termed as a single source of supply. The cost of pumping water from an aquifer is directly related to the groundwater level, which in turn is responsive to recharges and withdrawals. Thus, in any cost minimization problem related to groundwater withdrawal and recharge, an explicit knowledge of the state of its water level is desirable. The groundwater flow can be described by a two-dimensional partial differential equation, Bear [1972]. This flow equation can be solved numerically to obtain the state of the waterhead in time and space as it responds to withdrawal and recharge. However, there are two important physical parameters such as transmissivity and storage coefficient which are distributed in nature and their values are usually not known. In many instances, the value of the storage coefficient can be adequately determined through field tests. However, this cannot be said for transmissivity. Transmissivity is a highly variable discrete distributed parameter. The partial differential equation describing the groundwater flows can be simulated only when the physical properties are known. Once the physical properties and characteristics of an aquifer are known, it is possible to apply appropriate physical laws and to predict its response.

Thus, it is clear that identifying unknown aquifer parameters is essential for making an optimal operational decision in the planning of a water resources system where groundwater or conjunctive effects of ground and surface water hydrology are considered. Once the parameters are identified, the groundwater model can be simulated to determine its response to pumping and recharge. However, in the management model, the quantity of water withdrawn from the selected groundwater projects is treated as decision variables. Hence it is difficult to couple the groundwater operational policy explictly with the management models which seek to optimize an economic objective. The difficulty can however be overcome by utilizing an algebraic technological function, Maddock [1972]. The groundwater parameter identification scheme is presented in Chapter 5. The use of an algebraic technological function in integrating a groundwater simulation model with management model is described in Phase I and II of this project and also in previous chapters.

The quantity of groundwater withdrawn through pumpage and the amount of recharge are both treated as decision variables in the planning model. Hence it is necessary to couple the simulated groundwater response to an optimization management model. The algebraic form of a technological function allows the groundwater system to be explicitly included in an optimization model be it a linear, nonlinear, or dynamic program. The algebraic technological function relates pumping and recharge in the system to drawdown (or water level) at those pumping and recharge locations [Maddock, 1972].

Once again it is assumed that the aquifer is homogeneous and uniform in thickness, and drawdown is small with respect to the saturated thickness with wells fully penetrating the artesian field. For convenience, the differential equation describing such an aquifer is presented:

$$\frac{\partial}{\partial x}[T(x,y)\ \frac{\partial s}{\partial x}(x,y,t)] + \frac{\partial}{\partial y}[T(x,y)\ \frac{\partial s}{\partial y}(x,y,t)] = S(x,y)\frac{\partial s}{\partial t}(x,y,t)$$
$$+ \sum_{j=1}^{W} Q(j,t)\delta(x-x_j)\delta(y-y_j) \tag{7.17}$$

where $s(x,y,t)$ is drawdown, $T(x,y)$ and $S(x,y)$ are transmissivity and storage coefficient respectively, $\delta(x-x_j)$ is a Dirac delta function, j indicates the j^{th} pumping well, $Q(j,t)$ is the rate of withdrawal from groundwater basin at the j^{th} well, and W is the total number of wells, $j = 1,2,\ldots,W$.

The numerical solution of (7.17) produces the response of drawdown over time and space for given initial and boundary conditions. The drawdown can be expressed as:

$$s(x,y,t) = \sum_{j=1}^{W} \int_{0}^{t} G(x,y,j,t-\tau)Q(x,y,\tau)d\tau \tag{7.18}$$

where $G(x,y,j,t-\tau)$ is the Green's function for (7.17) satisfying the initial and boundary conditions. The details of Green's function are available in Maddock [1972], Dreizin [1975], and Kreyszig [1965].

Consider a time interval of one year for the purpose of analysis, where t indicates the year of pumping. Then, the time integral in equation (7.18) can be replaced by a discrete sum and drawdown can be represented by a technological function as follows:

$$s(k,t) = \sum_{j=1}^{W} \sum_{\tau=1}^{t} \beta(k,j,t-\tau+1)\, Q(j,\tau) \qquad (7.19)$$

where t represents the year when drawdown is calculated, $s(k,t)$ is the drawdown at well location k in the year t due to the pumping of W wells, $\beta(k,j,t-\tau+1)$ is a response coefficient for the year t relating drawdown at the k^{th} well to unit pumping at the j^{th} well in the year τ, and $Q(j,\tau)$ is the amount of water pumped from the j^{th} well in the year τ.

The coefficients β are not given explicitly by their derivation, but by using a digital computer simulation model proposed in the previous section, the algebraic technological coefficients can be achieved [Maddock, 1969].

The advantages of developing the parameters β can be realized now. Essentially the value of $\beta(k,j,t-\tau+1)$ at a location k over the groundwater basin indicates the drawdown in the year t due to unit withdrawal or recharge at any other location j in the year τ. The net drawdown $s(k,t)$ is then expressed as a sum of drawdowns due to decisions on withdrawals and recharges $Q(j,\tau)$ for all $j = 1,2,\ldots,W$ and $\tau = 1,2,\ldots,t$. By using (7.19) one can couple the behavior of groundwater system to the operational decisions of a groundwater project. Maddock [1969] showed that an algebraic technological function exists for an inhomogeneous aquifer with irregular boundary conditions, which are included in this analysis. The use of (7.19) in determining the most economic and physically feasible operating policies of groundwater projects is demonstrated in the next section. A conjunctive ground and surface water supply management problem is developed in the following section, where economic construction and the expansion schedule of the projects, as well as their operational policies over the planning horizon, are examined.

7.7.3 Mathematical Modeling of Supply Objective

Water demands may be satisfied from groundwater and surface water sources. Also a considerable expense may be saved by utilizing secondary treated waste effluent in artificial groundwater recharge. The cost of operating groundwater projects may increase quadratically as the water level in the groundwater basin depletes. However, by utilizing treated wastewater in recharging the basin in a relatively minimum cost leads to an effective savings in overall operational costs of the supply projects. It is assumed that the existing facilities for water supply for industrial, municipal and agricultural uses are not capable of meeting the growing needs in the Basin over the planning period. Thus, the goal of the Basin management authority is to meet the future needs for water most economically. This includes the expansion of existing projects, construction of new supply projects, and operation and maintenance of these projects. The objective of the supply model is thus to determine the optimal schedule for expansion and construction of supply facilities along with the optimum operating level of each of these facilities in each year at minimum present-value cost over the entire planning period. The demands for municipal, industrial, and agricultural water are assumed to be known for the whole Basin. Various demands are calculated based on the OBERS Series E population projection and future industrial growth and agricultural activities of the Basin, and are treated as an exogeneous function in the supply model. There have been many physical constraints in the system which may not be violated. The important economic characteristics of a supply system relate to the following costs:

(i) Capital cost of groundwater supply projects.

(ii) Capital cost of surface water supply projects.

(iii) Operational cost of groundwater projects.

(iv) Operational cost of surface water projects.

In order to solve the above planning and management problem, a dynamic programming optimization scheme is utilized. The solution will provide the optimal timing and sequencing of the construction of new projects and/or the expansion of existing projects, along with the optimal operational policies of each project [Haimes and Nainis, 1974; Haimes, 1973e].

A dynamic programming for the optimal sequencing of water supply projects was developed by Butcher, Haimes, and Hall [1969]. They, however, included capital cost of projects but neglected the operation and maintenance costs. It was subsequently modified and extended by Haimes and Nainis [1974], Nainis and Haimes [1975], Kolo and Haimes [1973], and Kaplan and Haimes [1975]. The extensions included the consideration of variable operation costs along with fixed capital costs.

Craig [1976], extended the solution procedure of conventional dynamic program for capacity expansion problems by applying a decomposition technique. He proposed a two-level decomposition structure. At the first level, sub-Lagrangian corresponding to each subsystem's cost function is minimized with respect to its decision variables. The overall system is coupled through the total demand functions. The subsystem demands are transferred to the second level coordinator which then adjust the shadow prices in order to satisfy the coupling equation. The optimal solution for the entire system is obtained only when the coupling equations are satisfied. The advantages of the approach can be attributed to the reduction in computational time and the elimination of dimensionality problem inherent to dynamic programming.

Morin and Esogbue [1972] modified the solution approach by using the embedded state space approach. This approach considerably reduces the computational time and computer storage requirements. Here it is further extended to include multiple demands and multiple project capacities. The multiple demands are municipal, industrial and agricultural. Each project is assumed to have supply capacities with respect to each type of water requirement.

Assume that there has been a total of U number of feasible supply projects, including groundwater as well as surface-water projects. The projects are distinct, with different location and sizes. The projects when utilized are capable of meeting the demands up to the end of the planning horizon, and they are within the hydrologic limitations of the Basin.

The parameters and the decision variables for the supply model are described. Let C_u represent the capital cost of constructing project u, where $u = 1,2,\ldots,U$. By using a vector notation, let $\underset{\sim}{Q}_u$ represent the supply capacity of the project u for municipal, industrial, and agricultural water. Therefore, $\underset{\sim}{Q}_u$ is a (3x1) dimensional column vector, represented as:

$$\underset{\sim}{Q}_u = [Q_u^1, Q_u^2, Q_u^3]^T$$

where Q_u^i, $i = 1,2,3$ represents the supply capacity of project u in the i^{th} requirement. When i = 1, it represents municipal water supply, for i = 2, it is industrial supply and for 1 = 3, it indicates agricultural supply capacity. The decisions include the schedule of construction and expansion of supply projects and their operating levels with respect to each type of water, i.e., municipal, industrial and agricultural. Let $\underset{\sim}{v}_1$ be a (TUx1) column vector representing the quantities of water supplied by all projects for municipal use over the planning period.

$$\underline{v}_1 = \begin{bmatrix} \underline{v}_{11} \\ \hdashline \underline{v}_{12} \\ \hdashline \vdots \\ \hdashline \underline{v}_{1u} \\ \hdashline \vdots \\ \hdashline \underline{v}_{1U} \end{bmatrix}$$

where $\underline{v}_{1u}$ is a (Tx1) dimensional column vector representing the quantities of municipal water supplied by project u over the planning period. Thus,

$$\underline{v}_{1u} = \begin{bmatrix} v_{1u1} \\ \hdashline v_{1u2} \\ \hdashline \vdots \\ \hdashline v_{1ut} \\ \hdashline \vdots \\ \hdashline v_{1uT} \end{bmatrix}$$

where v_{1ut} is the quantity of municipal water supplied by project u in year t, in millions of gallons per day. In a similar way, we can define v_{2ut} and v_{3ut} as the amount of industrial and agricultural water supplied respectively by project u in year t in MGD. Let D_t^i be the gross water withdrawal for the i^{th} requirement in year t, i = 1,2,3. For i = 1, it represents municipal water, i = 2 represents industrial water, whereas i = 3 indicates an agricultural water supply requirement.

The dynamic programming model is formulated by considering a number of projects as stages. For a total of U number of projects, a total of U stages are involved. The state is represented by the permutation schedules of cumulative capacities for a number of projects under consideration at each stage.

Let $\hat{q}^1$ be the municipal water supply state variable, and, $\hat{q}^2$ and $\hat{q}^3$ be the state variables for industrial and agricultural water supplies. An inverse demand function, $\psi(\hat{q}^1, \hat{q}^2, \hat{q}^3)$ can be defined as follows:

$$\psi(\hat{q}^1, \hat{q}^2, \hat{q}^3) = \{\inf t: \hat{q}^i \leq D_t^i \text{ , for some } i, i = 1,2,3\}$$

The inverse demand function $\psi(\hat{q}^1, \hat{q}^2, \hat{q}^3)$ can be interpreted as the smallest integral time in which a supply capacity $\hat{q}^i$ for some i is insufficient to supply at least one demand, where ψ can be obtained from demand functions D_t^i for $i = 1,2,3$ and $t = 1,2,\ldots,T$. Thus, ψ gives the time as an explicit function of accumulative supply capacities.

The present-value cost of constructing and/or expanding ground and surface water supply projects, as well as the annual operation and maintenance cost in a region over the planning period is given by $\hat{f}_3(\underset{\sim}{Q}, \underset{\sim}{v})$, where $\underset{\sim}{Q}$, a vector of decision variables of expansion sequence of projects over the planning period, $\underset{\sim}{v}$ is a vector of operational policies, representing the amount of water withdrawn from the projects over the planning period,

$$\hat{f}_3(\underset{\sim}{Q},\underset{\sim}{v}) = C_u(1+\rho)^{-t_u} + \sum_{t=1}^{T}(1+\rho)^{-(t-1)}\left\{\sum_{u\varepsilon I_g}\Pi_u^g(v_{1ut},v_{2ut},v_{3ut})\right.$$

$$\left. + \sum_{u\varepsilon I_s}\Pi_u^s(v_{1ut}, v_{2ut}, v_{3ut})\right\} \tag{7.20}$$

In (7.20), t_u represents the time when a project u is completed, ρ is the annual interest rate, C_u is the fixed cost of project u, $\Pi_u^g(v_{1ut}, v_{2ut}, v_{3ut})$ is the annual operation and maintenance cost of groundwater supply projects, for supply levels of v_{1ut}, v_{2ut}, v_{3ut} for municipal, industrial, and agricultural water used in the year t, and I_g is a subset indicating the projects of groundwater resources. Similarly, $\Pi_u^s(v_{1ut}, v_{2ut}, v_{3ut})$ represents the annual operating and maintenance cost function of surface water supply project u for supply levels of v_{1ut}, v_{2ut}, v_{3ut} for municipal, industrial, and agricultural uses in year t, and I_s is a subset indicating the projects of surface water resources.

In equation (7.19), s(u,t) represents the drawdown at location u in the year t. The total lift in pumpage, $\hat{s}(u,t)$, is expressed as the sum of steady state or initial lift $\hat{\ell}(u)$ at the u^{th} project location and drawdown s(u,t).

$$\hat{s}(u,t) = \hat{\ell}(u) + s(u,t) \tag{7.21}$$

Expressing in terms of algebraic technological function, total lift in groundwater withdrawal is:

$$\hat{s}(u,t) = \hat{\ell}(u) + \sum_{j=1}^{W} \sum_{\tau=1}^{t} \beta(u, j, t - \tau + 1) Q(j,\tau) \qquad (7.22)$$

The wastewater plants treatment problem developed in Chapter 2 considers the use of secondary effluent for artificial groundwater recharge. Thus, in objective function (7.20) $\pi_u^g(v_{1ut}, v_{2ut}, v_{3ut})$, which represents the sum of variable operating costs of groundwater supply projects $u \varepsilon I_g$ in year t, depends on the artificial recharge decisions $\underline{x}_5^*$ of the wastewater treatment model. It has been assumed that the secondary effluent from the set of wastewater treatment plants nearest to recharge facility is utilized for the purpose. Therefore, (7.22) can be modified to incorporate the decisions concerning artificial recharge to the withdrawal requirements from groundwater projects.

$$\hat{s}(u,t) = \hat{\ell}(u) + \sum_{j=1}^{W} \sum_{\tau=1}^{t} \beta(u,j,t - \tau + 1) [Q(j,\tau) - \sum_{j \varepsilon I_u} x_{5j\tau}^*] \qquad (7.23)$$

where, $x_{5j\tau}^*$ represents the optimum quantity of secondary effluent from plant j, utilized in groundwater recharge at supply location u, and I_u represents a set of wastewater treatment plants j, the secondary effluents of which are utilized for groundwater recharge at location u.

The variable costs of the surface water projects are assumed to be linear in their operating levels. The operating cost of groundwater supply projects are jointly related to the state of water level in the Basin and the quantity of withdrawal. Thus, the variable operating cost functions are:

(i) Variable Cost of Surface Water Projects:

$$\Pi_u^s(v_{1ut},v_{2ut},v_{3ut}) = \sum_{i=1}^{3} c_{iu}v_{iut}, \quad u\epsilon I_s \tag{7.24}$$

$$t = 1,2,\ldots,T$$

(ii) Variable cost of Groundwater Projects:

$$\Pi_u^g(v_{1ut},v_{2ut},v_{3ut}) = \sum_{i=1}^{3} \hat{c}_{iu}\hat{s}(u,t)v_{iut}, \quad u\epsilon I_g \tag{7.25}$$

$$t = 1,2,\ldots,T$$

where c_{iu} is the per unit operational cost of surface water supply project u utilized in requirement i. Once again, $i = 1$, indicates municipal water supply and $i = 2$, industrial supply, whereas $i = 3$, represents agricultural supply of water. Similarly, $\hat{c}_{iu}$ represents the per-unit operational cost of groundwater withdrawal from project $u\epsilon I_g$, utilized to meet the requirement i.

The supply system is subjected to a set of physical constraints which must not be violated.

(i) Constraint on allowable lift:

$$\hat{\ell}(u) + \sum_{j=1}^{W}\sum_{\tau=1}^{t} \beta(u,j,t-\tau+1)[Q(j,\tau) - \sum_{j\epsilon I_u} x^*_{5j\tau}] \leq \hat{s}^{max}_{ut} \tag{7.26}$$

(ii) Resource Demand Constraints:

$$\sum_{u \in I_s} v_{iut} + \sum_{u \in I_g} v_{iut} \geq D_t^i \tag{7.27}$$

$$i = 1,2,3; \quad t = 1,2,\ldots,T$$

(III) Project's Supply Capacity Constraints:

$$v_{iut} \leq Q_u^i \,, \quad i = 1,2,3; \tag{7.28}$$

where, $\hat{s}_{ut}^{max}$ is the maximum allowable lift in pumpage from groundwater project u. Other variables were defined earlier in the chapter.

For the development of the dynamic programming model, consider a sequence of sets $\Omega_1, \Omega_2, \ldots, \Omega_U \varepsilon \Omega$, where Ω_u represents the point set of $\begin{bmatrix} U \\ u \end{bmatrix}$ possible cumulative capacities of permutation schedules consisting of u projects. In order to simplify the notations, the state variables of supply capacities are expressed in vector notation as:

$$\hat{\underline{q}} = [\hat{q}^1, \hat{q}^2, \hat{q}^3]^T$$

Therefore, Ω_1 is the set of possible capacities which can be reached by utilizing one project only.

$$\Omega_1 = \{\hat{\underline{q}}_1, \hat{\underline{q}}_2, \ldots, \hat{\underline{q}}_u, \ldots, \hat{\underline{q}}_U\}$$

where, now, $\hat{\underline{q}}_u$ is a (3x1) column vector of state variables associated with municipal, industrial, and agricultural supply capacities from project u.

$$\hat{\underline{q}}_u = [\hat{q}_u^1, \hat{q}_u^2, \hat{q}_u^3]^T$$

Let Ω_2 be the set of feasible capacities that can be reached by a combination of two projects only.

$$\Omega_2 = \{\hat{q}_1 + \hat{q}_2, \hat{q}_1 + \hat{q}_3, \ldots, \hat{q}_1 + \hat{q}_U, \hat{q}_2 + \hat{q}_3, \ldots, \hat{q}_2 + \hat{q}_U, \ldots, \hat{q}_{U-2} + \hat{q}_{U-1}, \hat{q}_{U-2} + \hat{q}_U, \hat{q}_{U-1} + \hat{q}_U\}$$

Similarly, at the u^{th} stage in the dynamic program recursive equation, rather than considering an entire set of fixed increments defined over Ω, we can consider only those $\hat{q}\varepsilon\Omega_u$, since only the permutation schedule of u projects has to be considered at stage u.

Define a new cost function $h_u^{k_u}(\hat{q})$ as the present value cost of building a set of u number of projects in a sequence k_u. Project u is built in a year t_u which can be expressed as an explicit function of capacity, as $t_u = \psi(\hat{q}-\hat{q}_u)$, and which will optimally satisfy the demand until the year $\psi(\hat{q})$. Hence,

$$h_u^{k_u}(\hat{q}) = C_u(1+\rho)^{\psi(\hat{q}-\hat{q}_u)} + \sum_{t=\psi(\hat{q}-\hat{q}_u)}^{\psi(\hat{q})} (1+\rho)^{-(t-1)} \left\{ \sum_{u\varepsilon I_g} \Pi_u^g(v_{1ut}, v_{2ut}, v_{3ut}) + \sum_{u\varepsilon I_s} \Pi_u^s(v_{1ut}, v_{2ut}, v_{3ut}) \right\}$$

The first stage recursive equation considering only one project, is given by:

$$g_1^{k_1}(\hat{q}) = \min_{\substack{u\varepsilon k \\ q\varepsilon\Omega_1}} h_u^{k_u}(\hat{q}) \tag{7.29}$$

s.t.

$$0 \leq \hat{q} \leq \sum_{u\varepsilon k_1} Q_u \tag{7.30}$$

The constraint (7.30) above indicates that the quantity of supply from project in a sequence k_1 must be within the capacity of the project.

The recursive equation at the second stage is given by:

$$g_2^{k_2}(\underline{\hat{q}}) = \min_{\substack{u \in k \\ u \notin k_1 \\ \underline{\hat{q}} \in \Omega_2}} \{h_u^{k_u}(\underline{q}_2, \underline{\hat{q}}, k_1) + g_1^{k_1}(\underline{\hat{q}} - \underline{\hat{q}}_2)\} \quad (7.31)$$

$$\underline{0} \leq \underline{\hat{q}}_2 \leq \underline{\hat{q}} \leq \sum_{u \in k_2} \underline{Q}_u$$

Two projects are considered from set Ω_2. The optimum two-project sequence can be written as:

$$k_2^*(\underline{\hat{q}}) = k^*_1(\underline{\hat{q}} - \underline{\hat{q}}_u^*) \oplus u^*$$

where, $k_2^*(\underline{\hat{q}})$ is an optimal two-project schedule in order to supply $\underline{\hat{q}}$ amount of water, u^* is the project built second in schedule for a two-stage dynamic program, and $k_1^*(\underline{\hat{q}} - \underline{\hat{q}}_u^*)$ is the optimal one project schedule to supply $\underline{\hat{q}} - \underline{\hat{q}}_u^*$ amount of water.

The general recursive equation of u^{th} stage can be written as:

$$g_u^{k_u}(\underline{\hat{q}}) = \min_{\substack{u \in k \\ u \notin k_{u-1} \\ \underline{\hat{q}} \in \Omega_u}} \{h_u^{k_u}(\underline{\hat{q}}_u, \underline{\hat{q}}, k_{u-1}) + g_{u-1}^{k_{u-1}}(\underline{\hat{q}} - \underline{\hat{q}}_u) \quad (7.32)$$

$$0 \leq \underline{\hat{q}}_u \leq \underline{\hat{q}} \leq \sum_{u \in k_u} \underline{Q}_u$$

A total of u projects is considered from a set of Ω_u, in order to meet a supply of $\underline{\hat{q}}$. The optimum u project sequence is then,

$$k_u^*(\underline{\hat{q}}) = k_{u-1}^*(\underline{\hat{q}} - \underline{\hat{q}}_u^*) \oplus u^*$$

where $k_u^*(\underline{\hat{q}})$ is an optimal schedule of u projects providing exactly a capacity of $\underline{\hat{q}}$ to meet the demand, u^* is the project built in the last stage of a u-stage dynamic program, and $k^*_{u-1}(\hat{\underline{q}} - \hat{\underline{q}}^*_u)$ is an optimal (u-1) project schedule. Note that for a total of U projects, $k^*_U(\hat{\underline{q}})$ is a schedule of construction of supply projects formed by taking a permutation of numbers u, u = 1,2,...,U, a project is completed only when the cumulative capacities of all previously built projects are totally utilized. The completion time of a project can be expressed in terms of inverse demand function ψ described earlier, by,

$$t^*_{[j]} = \min\{\psi(\sum_{u=1}^{j} Q^i_{[u]}) \quad \text{for some } i,\ i = 1,2,3\}$$

where the brackets denote order in the sequence. In other words, [j] = u denotes project u is in j^{th} position of the sequence. Consider a single demand function D_t and cumulative supply capacity of projects being denoted by $\hat{q}$. Then a permutation schedule $k^*_u(\hat{q})$ can be illustrated graphically in Figure 7.7.

The dynamic program solution (7.32) is the optimum present-value cost of water supply projects' construction and operation over the planning period. Thus,

$$\hat{f}^*_3(\underline{Q}^*, \underline{v}^*) = g_u^{k^*_u}(\hat{\underline{q}}^*)$$

where $\underline{Q}^*$ and $\underline{v}^*$ are the optimal construction and operating variables respectively; $g_u^{k^*_u}(\hat{\underline{q}}^*)$ is the optimal cost obtained from the dynamic programming model for an optimal sequence of construction k^*_u.

Constraint (7.26) shows that the allowable total lift due to pumpage from groundwater reservoirs may not exceed its maximum limit. Constraint (7.27) is related to the operating variables of supply projects. It indicates that the total supply of fresh water conjunctively from ground and surface water projects must be at least equal to the demand. In (7.28) the capacity constraints are presented, which means that the supply from any project u must not exceed the available capacity of that project.

In summary, a long-term water supply model utilizing both ground and surface water sources is developed in this chapter. The model includes both construction and/or expansion of supply projects to meet the demand for the entire planning period, and an operational policy to determine the level of supply from each project. An embedded state space dynamic programming model is employed to determine the optimal sequence and time of construction of ground and surface water supply projects and for optimal allocation of amount of water supply from each project each year so that the total present value cost is minimum. The embedded state space approach of Morin and Esogbue [1972] is modified to include operation and maintenance cost functions along with capital costs of the projects, and to include the multiple demand functions. In this study the dynamic program model is employed to supply requirements consisting of municipal, industrial and agricultural uses, as presented by individual demand functions.

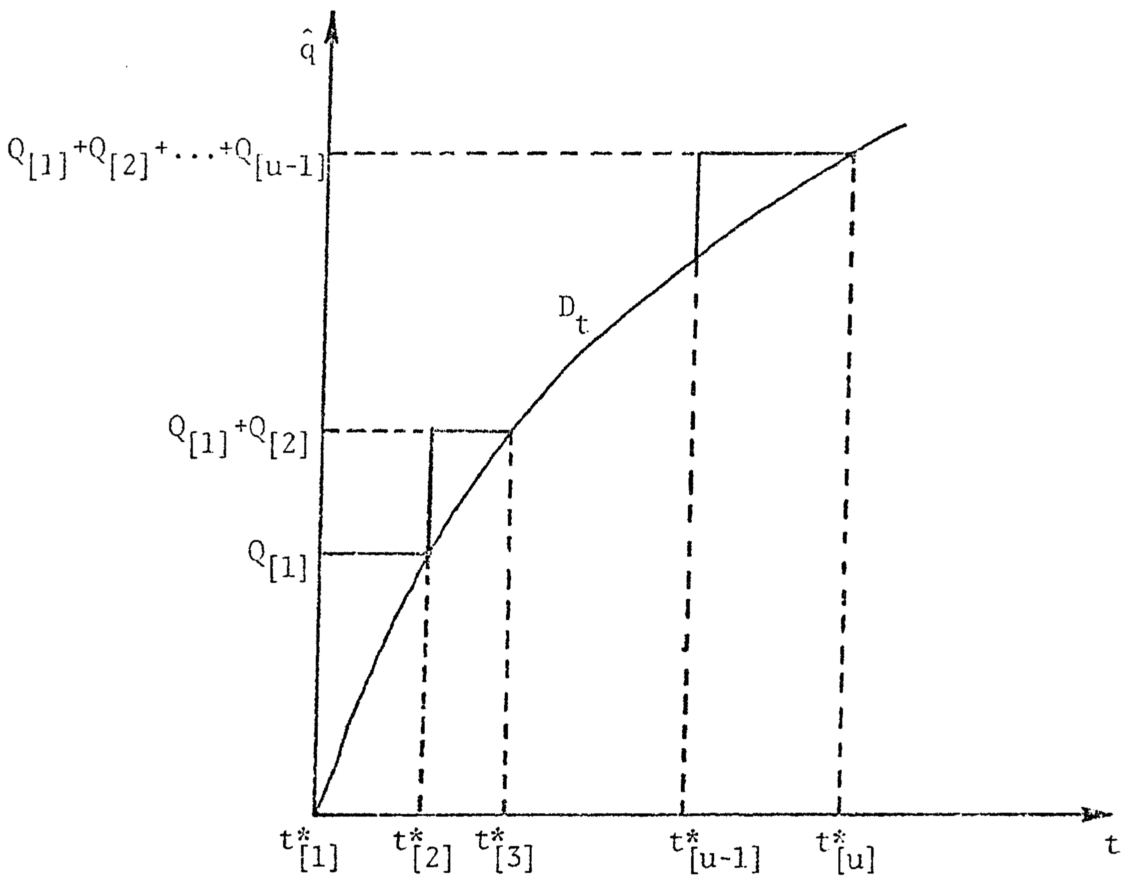


Figure 7.8. Permutation Points on a Single-Purpose Demand Curve

The groundwater withdrawal cost is developed as a function of groundwater head through the use of algebraic technological function. This not only allows to employ a realistic operating cost function for groundwater supply, but makes it possible to couple a groundwater simulation model explicitly with an optimization problem. Thus, this charter presents a short-term and a long-term water supply model by conjunctively using a ground and surface water system. A case study problem for short-term operational planning model is also presented in Section 7.4.

CHAPTER 8

A TAX-QUOTA MODEL IN A MULTICELL-MULTISTREAM SYSTEM

8.1 GENERAL DISCUSSION

The objective of the following discussion is to clarify and verify the application of decomposed water resources response functions in the formulation and solution stages of a management model, where administrative framework is assumed.

Many different management schemes have been utilized in the literature to formulate management models in water resources. Considering a groundwater system traversed by a stream-network, the management mechanism suggested by Maddock and Haimes [1974], is adopted here.

In particular, the tax-quota management scheme of Maddock and Haimes can be applied with only minor changes to the water resources system defined in our previous development. The water system in the original study comprised a single aquifer (dry alkaline valley), assuming that no other water sources existed in the region. The mathematical model used for simulating the aquifer is a linear groundwater model in a compact form. Since individual decisions are made for pumping patterns the management model formulation was forced to decompose the decision-making process. However, the physical system model representation (resulting from a compact scheme--the single simulation program) causes each user to have to consider the detailed

pumping policy of all the other users. As a result, the management model formulation of the original study requires a great deal of data and computer storage, either of which is not always available. Also, when applied to a real system the modeling efforts are expected to be very difficult.

In the present study we propose the extension of the original approach in two directions:

1. To consider a more complex water resources system comprising multi-aquifer cells traversed by a multi-stream network with artificial recharge and water import options, and a regional performance criterion applied to ground and surface water measurements.

2. To apply the modeling procedure (developed in this study) to the physical system, including the decomposed formulation of the response technological functions.

The management model formulation is expected to be much simplified. The decomposition of the decision-making process is followed by a suitable representation of the decomposed physical system response, which can be easily coupled with the management model formulation.

8.2 MODEL FORMULATION

There are L users in the region. To each user corresponds an aquifer cell, and the ℓ^{th} user has m_ℓ wells located at the ℓ^{th}

cell. Each user $\ell \in L$ maximizes his own net revenues $\hat{Z}_\ell$:

$$\underset{(\underline{q},\underline{x},\underline{v})_\ell}{\text{Max}} \left\{ \hat{Z}_\ell = \sum_{n=1}^{T} \left((1+r)^{-n} \left[W_\ell(n) \left(\sum_{k_\ell=1}^{m_\ell} q_\ell(k_\ell,n) + x(\ell,n) \right) - \sum_{u=1}^{U_\ell} V_\ell(u) \cdot v_\ell(u,n) - S_\ell(n) \cdot x(\ell,n) \right] \right) - Z_\ell \right\} \quad (8.1)$$

T is the number of time periods that comprise the design horizon.

r is the interest rate.

m_ℓ is the number of wells located at the ℓ^{th} cell and operated by the ℓ^{th} user.

$W_\ell(n)$ is the return per acre-ft of water supply for the ℓ^{th} user during the n^{th} period. $W_\ell(n)$ can be a constant relating only to ℓ and n, or a function of

$$\sum_{k_\ell=1}^{m_\ell} q_\ell(k_\ell,n) + x(n),$$

the total water supply to user ℓ during period n.

$q_\ell(k_\ell,n)$ is the quantity of water pumped from the k_ℓ^{th} well during the n^{th} time period.

$V_\ell(u)$ is the operating cost of recharge per acre/ft in the ℓ^{th} area with water from the u^{th} stream.

$Y_\ell(u,n)$ is the quantity of water from the u^{th} stream used for artificial recharge at the ℓ^{th} recharge facility during period n, and there are u_ℓ reaches of streams traversing the ℓ^{th} cell area.

$S_\ell(n)$ is the cost per acre/ft of water imported into the ℓ^{th} area during the n^{th} period.

$x(\ell,n)$ is the quantity of water imported into the ℓ^{th} area during the n^{th} period for direct use by the ℓ^{th} user.

Z_ℓ is the cost function:

$$Z_\ell = \sum_{n=1}^{T} (1+r)^{-n}\Big[C_\ell(n) + \sum_{k_\ell=1}^{m_\ell} P_\ell(k_\ell)q_\ell(k_\ell,n)(H_\ell(k_\ell) + D_\ell(k_\ell,n) + \hat{D}(\ell,n))\Big] \tag{8.2}$$

$C_\ell(n)$ is the constructing cost to the ℓ^{th} user at the n^{th} period according to his particular plans.

$P_\ell(k_\ell)$ is the pumping cost per acre/ft ft for the $k_\ell{}^{th}$ well.

$H_\ell(k_\ell)$ is the lift under steady state conditions at the $k_\ell{}^{th}$ well.

$\hat{D}_\ell(k_\ell,n)$ is the drawdown in the $k_\ell{}^{th}$ well at the end of the ℓ^{th} period due to the aggregated pumpage and recharge in all other cells (by other users) in the region.

$D_\ell(k_\ell,n)$ is the drawdown in the $k_\ell{}^{th}$ well at the end of the n^{th} period due to the aggregated pumpage and recharge in the ℓ^{th} cell.

$D_\ell(k_\ell,n)$ is given by:

$$D_\ell(k_\ell,n) = \sum_{j=1}^{m_\ell} \sum_{i=1}^{n} \beta_\ell(k_\ell,j,n-i+1)q_\ell(j,i) \qquad (8.3)$$

$$- \sum_{i=1}^{n} \left[\beta_\ell(k_\ell,v_\ell,n-i+1) \cdot \sum_{u=1}^{u_\ell} v_\ell(u,i)\right]$$

where $\beta_\ell(k_\ell,j,m-i+1)$ is the algebraic technological term relating the drawdown at the $k_\ell{}^{th}$ well to the pumping of one unit of water from the j^{th} well during the i^{th} period, and both k_ℓ and j are located at the ℓ^{th} cell.

The second term on the right of (10.3) stands for the negative drawdown at well k_ℓ caused by the artificial recharge at point v_ℓ.

$\hat{D}(\ell,n)$ is given by:

$$\hat{D}(\ell,n) = \sum_{\substack{r=1 \\ r\neq\ell}}^{L} \sum_{i=1}^{n} \gamma(\ell,r,n-i+1) \cdot (q(r,i) - v(r,i)) \qquad (8.4)$$

where $\gamma(\ell,r,n-i+1)$ is the algebraic technological term relating the average drawdown at the ℓ^{th} cell to aggregated pumping of one unit of water at the r^{th} cell, during the i^{th} period. $q(r,i)$ is the quantity of water pumped from the r^{th} cell by the r^{th} user during the i^{th} period,

$$q(r,i) = \sum_{k_r=1}^{m_r} q_r(k_r,i) \qquad (8.5)$$

for the r-th cell:

$$v(r,i) = \sum_{u=1}^{U_r} v_r(u,i) \qquad (8.6)$$

Equation (8.2) contains the products of $q_\ell(k_\ell,n)$ and $q(r,i)$, $r \neq \ell$, i.e. the products of pumping values on the ℓ^{th} cell area at particular wells and aggregated pumping values on all other cells in the region. The coupling of $q_\ell(k_\ell,n)$ and $v(r,i)$, the aggregated artificial recharge at other cells, is similar.

Two vectors of pseudo-variables $\sigma_1(r,n)$, $\sigma_2(r,n)$ are introduced into equation (8.2) These vectors will uncouple the pumping values on the ℓ^{th} area wells from all other cells' pumpages and recharges.

Let

$$\left.\begin{aligned} \sigma_1(r,n) &= q(r,n) \\ \sigma_2(r,n) &= v(r,n) \end{aligned}\right\} \quad \begin{aligned} r&=1,\ldots,L \\ n&=1,\ldots,T \end{aligned} \qquad (8.7)$$

Then equation (8.2) becomes:

$$Z_\ell = \sum_{n=1}^{T} (1+r)^{-n} \Bigg\{ C_\ell(n) + \sum_{k_\ell=1}^{m_\ell} P_\ell(k_\ell) q_\ell(k_\ell,n) \Bigg[H_\ell(k_\ell)$$

$$+ \sum_{j=1}^{m_\ell} \beta_\ell(k_\ell,j,n-i+1) \cdot q_\ell(j,i) - \sum_{i=1}^{n} (\beta_\ell(k_\ell,v_\ell,n-i+1) \qquad (8.8)$$

$$\cdot \sum_{u=1}^{U_\ell} v_\ell(u,i)) + \sum_{\substack{r=1 \\ r\neq\ell}}^{L} \gamma(\ell,r,n-i+1) \cdot \Big[\sigma_1(r,i) - \sigma_2(r,i)) \Big] \Bigg]\Bigg\}$$

Notice here that the dimension of the vector of pseudo-variables is reduced with respect to the original scheme. The pseudo-variables account for the aggregated activities of each user. The possible 'estimation' by one user of pumpage and recharge planned by others is much more feasible for aggregated operation than it is for a detailed plan applied to each well. Hence, this solution strategy thus provides both a conceptual and methodological advantage.

If the ℓ^{th} user estimates a set of (L-1) net aggregated pumping values $(\sigma_1(r,i) - \sigma_2(r,i))$ for the L-1 users, then these estimates become the set of pseudo-variables.

The ℓ^{th} user is interested in maximizing $\hat{Z}_\ell$, subject to such constraints as:

1. Non decreasing water supply

$$\sum_{k_\ell=1}^{m_\ell} q_\ell(k_\ell,n) + x(\ell,n) \leq \sum_{k_\ell=1}^{m_\ell} q_\ell(k_\ell,n+1) + x(\ell,n+1) \qquad n=1,\ldots,T-1 \tag{8.9}$$

2. Drawdowns must not exceed casing and screening designs

$$D_\ell(k_\ell,n) + \hat{D}(\ell,n) \leq d_{\ell max}(k_\ell) \qquad \begin{array}{l} n=1,\ldots,T \\ k_\ell=1,\ldots,m_\ell \end{array} \tag{8.10}$$

3. Pumping capacity must not be exceeded

$$q_\ell(k_\ell,n) \leq Q_{\ell max}(k_\ell) \qquad \begin{array}{l} n=1,\ldots,T \\ k_\ell=1,\ldots,m_\ell \end{array} \tag{8.11}$$

4. Upper limit for imported quantities

$$x(\ell,n) \leq x_{\ell\,max}(n) \qquad n=1,\ldots,T \qquad (8.12)$$

5. Recharge facilities capacity must not be exceeded

$$\sum_{u=1}^{u_\ell} v_\ell(u,n) \leq v_{\ell\,max} \qquad n=1,\ldots,T \qquad (8.13)$$

$d_{\ell max}(k_\ell)$ is the maximum drawdown allowed for the k_ℓ^{th} well located at the ℓ^{th} cell, which must not be exceeded because of casing and screening design.

$Q_{\ell max}(k_\ell)$ is the design upper limit on the quantity of water pumped from the k_ℓ^{th} well.

$x_{\ell max}(n)$ is the externally imposed restriction of an upper limit on the quantity of water to be imported into the region for the direct use of the ℓ^{th} user during the n^{th} period.

$v_{\ell max}$ is the designed upper limit on the quantity of water to be artificially recharged in the ℓ^{th} cell recharge facilities.

The regional objective is to enhance the regional net return from water use. As such, the regional optimization problem definition is:

$$\max \ \overline{Z}_\ell = \sum_{\ell=1}^{L} \hat{Z}_\ell \qquad (8.14)$$

Subject to:

1. A lower limit for each user's net benefit

$$\hat{Z}_{\ell} \geq \hat{Z}_{\ell min} \qquad \ell=1,\ldots,L \tag{8.15}$$

2. A set of mass balance constraints

$$\left.\begin{array}{l} \sigma_1(r,n) - q(r,n) = 0 \\ \sigma_2(r,n) - v(r,n) = 0 \end{array}\right\} \quad \begin{array}{l} n=1,\ldots,T \\ r=1,\ldots,L \end{array} \tag{8.16}$$

3. A set of interference constraints

$$\hat{D}(\ell,n) \leq D_{\ell max} \qquad \begin{array}{l} n=1,\ldots,T \\ \ell=1,\ldots,L \end{array} \tag{8.17}$$

4. Water Balance must be maintained in certain streams

$$\sum_{\ell=1}^{L} \left[v_{\ell}(u,n) + f^{u}(\ell,n)\right] \leq B(u,n) \qquad \begin{array}{l} n=1,\ldots,T \\ u=1,\ldots,U \end{array} \tag{8.18}$$

5. All previous individual user constraints (Equations (8.9) through (8.13)).

$\hat{Z}_{\ell min}$ is the minimum expected net benefit associated with water use by the ℓ^{th} user over the planning period.

$D_{\ell max}$ is the upper limit to the drawdown induced by other users activities on the ℓ^{th} user.

$B(u,n)$ is an upper limit on the quantity of water to be removed from the u^{th} stream for natural and artificial recharge.

$f^{u}(\ell,n)$ is the quantity of water induced from the u^{th} stream into the ℓ^{th} cell during the n^{th} period:

$$f^u(\ell,n) = \sum_{k_\ell=1}^{m_\ell} \sum_{i=1}^{n} \phi_\ell^u (k_\ell, n-i+1) q_\ell(k_\ell, i)$$

$$+ \sum_{\substack{r=1 \\ r\neq\ell}}^{L} \sum_{i=1}^{n} \psi_\ell^u(r,n-i+1)\left[q(r,i)-v(r,i)\right] \tag{8.18}$$

$$- \sum_{i=1}^{n} \left[\phi_\ell^u(v_\ell, n-i+1) \cdot \sum_{u=1}^{u_\ell} v_\ell(u,i)\right] + I_\ell^u$$

$\phi_\ell^u(k_\ell,n-i+1)$ is the quantity of water induced from the u^{th} stream into the ℓ^{th} cell during the n^{th} period due to one unit of pumping at the $k_\ell{}^{th}$ well during the i^{th} period.

$q_\ell(k_\ell,i)$ is the quantity of water pumped from the $k_\ell{}^{th}$ well during the i^{th} period.

$\psi_\ell^u(r,n-i+1)$ is the quantity of water induced from the u^{th} stream into the ℓ^{th} cell during the n^{th} period due to one unit of pumping at the r^{th} cell during the i^{th} period.

$q(r,i) - v(r,i)$ is the net quantity of water pumped from the r^{th} cell during the i^{th} period.

I_ℓ^u is the quantity of water induced from the u^{th} stream into the ℓ^{th} cell during one time period with no imposed pumpage and the system in steady state.

The primal solution of the program constituting equations (8.17) through (8.18) provides the quotas for each well and recharge from the stream for each user. The dual solution provides the costs and savings associated with changes in the values of pumpage and recharge. In particular, $q_\ell(k_\ell,n)$ is the quota for the k_ℓ^{th} well of the ℓ^{th} user for the n^{th} time period, and $v_\ell(u,n)$ is the quota for the quantity of water to be used for artificial recharge at the ℓ^{th} area from the u^{th} stream during the n^{th} time period.

The Lagrangian for the maximum regional return program (eqs. (8.9) through (8.18)) is formed as follows (where $\hat{Z}_\ell$ is given by equation (8.1) and (8.2):

$$L = \sum_{\ell=1}^{L} \hat{Z}_\ell + \sum_{\ell=1}^{L} \sum_{n=1}^{T-1} \mu_\ell^{(1)}(n) \Big[\sum_{k_\ell=1}^{m_\ell} q_\ell(k_\ell,n) + (x(\ell,n)$$

$$- (\sum_{k_\ell=1}^{m_\ell} q_\ell(k_\ell,n+1) + x(\ell,n+1)) \Big]$$

$$+ \sum_{\ell=1}^{L} \sum_{k_\ell=1}^{m_\ell} \sum_{n=1}^{T} \mu_\ell^{(2)}(k_\ell,n) \Big[D_\ell(k_\ell,n) + \hat{D}(\ell,n) - d_{\ell max}(k_\ell) \Big]$$

$$+ \sum_{\ell=1}^{L} \sum_{k_\ell=1}^{m_\ell} \sum_{n=1}^{T} \mu_\ell^{(3)}(k_\ell,n) \Big[q_\ell(k_\ell,n) - Q_{\ell max}(k_\ell) \Big]$$

$$+ \sum_{\ell=1}^{L} \sum_{n=1}^{T} \mu_\ell^{(4)}(n) \Big[x(\ell,n) - x_{\ell max}(n) \Big]$$

$$+ \sum_{\ell=1}^{L} \sum_{n=1}^{T} \mu_{\ell}^{(5)}(n) \left[\sum_{u=1}^{U} V_{\ell}(u,n) - V_{\ell max} \right]$$

$$+ \sum_{\ell=1}^{L} \sum_{k_\ell=1}^{m_\ell} \sum_{n=1}^{T} \mu_{\ell}^{(6)}(k_\ell,n) \left[D_\ell(k_\ell n) \right.$$

$$- \left\{ \sum_{j=1}^{m_\ell} \sum_{i=1}^{n} \beta_\ell(k_\ell,j,n-i+1)\ q_\ell(j,i) \right.$$

$$\left. \left. - \sum_{i=1}^{n} \left[\beta_\ell\ (k_\ell, v_\ell, n-i+1) \sum_{u=1}^{u_\ell} v_\ell(u,i) \right] \right\} \right]$$

$$+ \sum_{\ell=1}^{L} \sum_{n=1}^{T} \mu_{\ell}^{(7)}(n) \left[\hat{D}(\ell,n) - \sum_{\substack{r=1 \\ r\neq\ell}}^{L} \sum_{i=1}^{n} \gamma\ (\ell,r,n-i+1)\ (q(r,i) \right.$$

$$\left. - v(r,i)) \right]$$

$$+ \sum_{\ell=1}^{L} \sum_{n=1}^{T} \mu_{\ell}(n)^{(8)} \left[q(\ell,n) - \sum_{k_\ell=1}^{m_\ell} q_\ell(k_\ell,n) \right]$$

$$+ \sum_{\ell=1}^{L} \sum_{n=1}^{T} \mu_{\ell}^{(9)}(n) \left[v(\ell,n) - \sum_{u=1}^{u_\ell} v_\ell(u,n) \right]$$

$$+ \sum_{\ell=1}^{L} \mu_{\ell}^{(10)} \left[\hat{Z}_{\ell min} - \hat{Z}_\ell \right]$$

$$+ \sum_{\ell=1}^{L} \sum_{n=1}^{T} \mu_{\ell}^{(11)}(n) \left[\hat{D}\ (\ell,n) - D_{\ell max} \right]$$

$$+ \sum_{u=1}^{U} \sum_{n=1}^{T} \mu^{(12)}(u,n) \left[\sum_{\ell=1}^{L} \left(v_\ell(u,n) + f^{u}(\ell,n) - B(u,n) \right) \right]$$

$$+ \sum_{\ell=1}^{L} \sum_{u=1}^{u_\ell} \sum_{n=1}^{T} \mu_\ell^{(13)}(u,n) \Big[f^{u}(\ell,n)$$

$$- \Big\{ \sum_{k_\ell=1}^{m_\ell} \sum_{i=1}^{n} \phi_\ell^{u}(k_\ell, n-i+1)\, q_\ell(k_\ell, i)$$

$$- \sum_{i=1}^{n} \phi_\ell^{u}(v_\ell, n-i+1) \cdot \sum_{u=1}^{u_\ell} v_\ell(u,i)$$

$$+ \sum_{\substack{r=1 \\ r \neq \ell}}^{L} \sum_{i=1}^{n} \psi_\ell^{u}(r, n-i+1)\left(q(r,i) - v(r,i)\right)$$

$$+ I_\ell^{u} \Big\} \Big]$$

$$+ \sum_{\ell=1}^{L} \sum_{n=1}^{T} \lambda_\ell^{(1)}(n) \left[\sigma_1(\ell,n) - q(\ell,n) \right]$$

$$+ \sum_{\ell=1}^{L} \sum_{n=1}^{T} \lambda_\ell^{(2)}(n) \left[\sigma_2(\ell,n) - v(\ell,n) \right] \qquad (8.20)$$

Applying the multilevel-decomposition approach, the Lagrangian $\mathcal{L}$ is decomposed into L independent subsystems where all pseudo-variables are assumed to be known parameters at the first level (i.e., to the users) quadratic program optimization:

$$\mathcal{L} = \sum_{\ell=1}^{L} \mathcal{L}_\ell \tag{8.21}$$

and $\mathcal{L}$ is the Lagrangian for the ℓ^{th} subsystem.

The decision variables of subsystem ℓ at the first level optimization are

$q_\ell(k_\ell,n)$'s, $v_\ell(u,n)$'s, $x(\ell,n)$'s

$\hat{D}(\ell,n)$'s and $\mu_\ell^{(p)}$, $p = 1,\ldots,6,8,9$.

The global optimum of the problem is guaranteed when the quadratic functions are convex.

The decision variables for the second level coordination are

$\sigma_1(\ell,n)$'s, $\sigma_2(\ell,n)$'s, $\mu_\ell^{(P)}$, $P=7,10,11,12,13$.

and $\lambda_\ell^{(1)}(n)$, $\lambda_\ell^{(2)}(n)$

Applying some of the Kuhn-Tucker necessary conditions (for stationarity) at the second level optimization yields:

$$\frac{\partial L}{\partial \sigma_1(r,i)} = 0$$

$$= - \sum_{n=i}^{T} \left[(1+r)^{-n} \sum_{\substack{\ell=1 \\ \ell \neq r}}^{L} \sum_{k_\ell=1}^{m_\ell} P_\ell(k_\ell) q_\ell(k_\ell,n) \ . \ \gamma(\ell,r,n-i+1) \right]$$

$$+ \lambda^{(1)}_{r(i)} \qquad (8.22)$$

$$\frac{\partial L}{\partial \sigma_2(r,i)} = 0$$

$$= \sum_{n=i}^{T} \ (1+r)^{-n} \sum_{\substack{\ell=1 \\ \ell \neq r}}^{L} \sum_{k_\ell=1}^{m_\ell} P_\ell(k_\ell) q_\ell(k_\ell,n) \ . \ \gamma(\ell,r,n-i+1)$$

$$+ \lambda^{(2)}_{r\,(i)} \qquad (8.23)$$

$$\frac{\partial L}{\partial \lambda_\ell^{(1)}(n)} = 0 = \sigma_1(\ell,n) - q(\ell,n) \qquad (8.23)$$

$$\frac{\partial L}{\partial \lambda_\ell^{(2)}(n)} = 0 = \sigma_2(\ell,n) - v(\ell,n) \qquad (8.24)$$

which results in:

$$\lambda_r^{(1)}(i) = - \lambda_r^{(2)}(i) = \sum_{n=i}^{T} (1+r)^{-n} \sum_{\ell=1}^{L} \overline{P}_\ell \ . \gamma\ (\ell,r,n-i+1) \ . \ q(r,n)$$

$$(8.26)$$

where

$$\overline{P_\ell} = \left[\sum_{n=1}^{T} \sum_{k_\ell=1}^{m_\ell} P_\ell (k_\ell)\ q_\ell(k_\ell,n)\right] \Big/ \left[\sum_{n=1}^{T} \sum_{k_\ell=1}^{m_\ell} q_\ell(k_\ell,n)\right] \qquad (8.27)$$

At the second level of the hierarchy equations (8.24), (8.25) (8.26) are determined by inserting the 'optimal' values of q(r,n) and v(r,n) produced by the first level optimization. An iterative procedure between the first level and the second level is initiated. The first level supplies the second level with q's and v's and the second level supplies the first level with $\sigma^{(1)}$'s, $\sigma^{(2)}$'s and λ's.

At this point the advantage of the above formulation in comparison with the original study can be appreciated. The iterative procedure originally required the pumping values as well as corresponding the pseudo-variables to originate between the two levels for each well. Using the concepts developed previously in our study, only aggregated activities (pumpage and recharge) and their corresponding pseudo-variables (σ_1, σ_2) are iterated. The dimensionality of the procedure is obviously reduced and convergence is expected to be achieved more rapidly.

The Lagrange multipliers $\lambda^{(1)}$ given by (8.26) are the dual variables corresponding to the constraints: $\sigma_1(\ell,n) - q(\ell,n) = 0$. These represent a cost per unit excess of over-pumping the quota by each user. Notice that in contrast to the original study's scheme, the quota system in the above formulation corresponds to the aggregated pumpage by each user. (Originally quotas were determined for each particular well. This raises sensitivity problems due to the possibility of mechanical failures in well equipment (or such other difficulties) which might not allow the user to operate his well system exactly as the quota system would impose.)

The Lagrange multipliers $\lambda^{(2)}$ are the dual variables corresponding to the constraints $\sigma_2(\ell,n) - v(\ell,n) = 0$. These represent a saving per unit excess of over-recharge or a cost per unit of under recharge, relative to the recharge quota.

A more detailed discussion on the quota system and the different assumptions is given in the original paper.

8.3 TAX COMPUTATION

In the following, a modification of the taxation scheme suggested by Maddock and Haimes is developed. The basic assumption used is that under a feasible tax scheme applied to groundwater pumping, users may cooperate for a tax collection system on an aggregated basis. In other words, each user desires to operate his own wells and recharge facilities according to his own considerations given the aggregated quotas imposed on him. He may reject any attempt to impose a pumping plan for his wells not in correspondence with his own planned operations.

Let $\Delta q(\ell,i)$, $\Delta v(\ell,i)$ (if possitive) be the respective expressions for the ℓ^{th} user pumping and recharging more than his quotas. Then

$$C_o(i) = \sum_{\ell=1}^{L} \lambda_\ell^{(1)}(i) \cdot \left[\Delta q(\ell,i) - \Delta v(\ell,i)\right] \tag{8.28}$$

is the cost (saving if negative) of additional energy that all users have to expend over the remainder of the planning periods to produce their quotas. Expression (8.28) stands for the total tax collected from all users at year i.

Let $$C_I(\ell,i) = \sum_{\substack{r=1 \\ r\neq\ell}}^{L} \sum_{n=i}^{T} (1+r)^{-n}\, \overline{P}_\ell \cdot \gamma(\ell,r,n-i+1)q(r,n) \cdot \left[\Delta q(r,i) - \Delta v(r,i)\right] \tag{8.29}$$

denote the total cost to the ℓ^{th} user due to over activities by other users, then the total cost to all users is

$$C_I^T(i) = \sum_{\ell=1}^{L} C_I(\ell,i) \tag{8.30}$$

Since equation (8.30) is equivalent to (8.28):

$$C_o(i) - C_I^T(i) = \sum_{\ell=1}^{L} \sum_{n=i}^{T} (1+r)^{-n} \sum_{\substack{r=\ell \\ r\neq\ell}}^{L} \overline{P}_r$$

$$\cdot\ \gamma(r,\ell,n-i+1)q(\ell,n)\left[\Delta q(\ell,i) - \Delta v(\ell,i)\right]$$

$$- \sum_{\ell=1}^{L} \sum_{\substack{r=1 \\ r\neq\ell}}^{L} \sum_{n=i}^{T} (1+r)^{-n} P_\ell \gamma(\ell,r,n-i-1)q(r,n)$$

$$\cdot \left[\Delta q(r,i) - \Delta v(r,i)\right] = 0 \qquad (8.31)$$

The ℓ^{th} user is assessed the tax

$$T_x(\ell,i) = \sum_{n=i}^{T} (1+r)^{-n} \left\{ \sum_{\substack{r=1 \\ r\neq\ell}}^{L} \overline{P}_r\ \gamma(r,\ell,n-i+1)q(\ell,n)\left[\Delta q(\ell,i) - \Delta v(\ell,i)\right] \right. \qquad (8.32)$$

$$- \sum_{\substack{r=1 \\ r\neq\ell}}^{L} \overline{P}_\ell\ \gamma(\ell,r,n-i+1)\ .\ q(r,n)\left[\Delta q(r,i) - \Delta v(r,i)\right]$$

The net collected tax for the i^{th} time period is zero:

$$\sum_{\ell=1}^{L} T_x(\ell,i) = 0 \qquad (8.33)$$

8.4 SUMMARY AND CONCLUSIONS

The tax-quota scheme developed by Maddock and Haimes for a simple, isolated aquifer system has been modified in this study for a more general complex groundwater system. The application of the concept of decomposed response functions to the problem formulation makes it possible to account for a vast range of variables affecting decisions.

In our development two aspects of usefulness of the decomposed response functions are illustrated:

1. Simplification of the mathematical formulation and the solution strategy;

2. Extension of the model to handle more of those items affected by the activities considered (e.g., artificial recharge options and stream network response).

In the context of our study, the modified tax-quota system model may be viewed as an illustration of the application of a management scheme for a region in the hopes of initiating an implementation of a management mechanism. The regional performance criterion under the proposed mechanism is expected to considerably improve results obtained from the basic non-management mechanism structure.

CHAPTER 9

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

9.1 SUMMARY AND CONCLUSIONS

This research offers a new approach to the planning and management of complex, large-scale water resources systems. It utilizes the concepts and methodologies from systems engineering theory for the advanced structuring, formulating and solving of mathematical models. These models are aimed at the profound analysis of short- and long-term planning aspects of water resources.

A planning and management methodology for regional water quality control is presented. The planning framework is developed based on a multiobjective analysis in order to take into consideration the conflicting objectives of surface water quality and the cost of expansion and operation of wastewater treatment plants (both secondard and tertiary). Multiobjective analysis in water resources systems has become particularly important in the context of the federal principles and standards for the planning of water and land resources. The objective of the guidelines is to place environmental concerns on a basis equal to economic development.

A regional water resources system may be a complex, large-scale system and may include many elements. In this study, the components included are ground and surface water and wastewater treatment plants.

The water quality objectives represent the levels of water parameters in different segments of the stream, over the entire planning horizon. The resulting levels of pollutants depend on the net effluent

discharges of various pollutants under consideration, as well as, on the hydrologic characteristics of the stream.

Since the cost objective is in terms of dollars, while the water quality objectives are in terms of the pollutant levels (concentration), these objectives are noncommensurable, and a multiobjective optimization approach is desirable. The decision-maker is an individual or an agency desires to simultaneously minimize the cost of wastewater treatment, along with the levels of water quality parameters.

A nonlinear programming is employed to determine the optimal schedule of construction and/or expansion secondary and tertiary processes at each plant location, meeting estimated effluent discharge levels at minimum present value cost. The cost function includes, capital cost of secondary and tertiary units and variable operating cost of each process.

Water quality objectives represent the level of pollutant parameters(or other indicators) in the stream reaches over the planning period, and are developed by using mass balance equation for conservative pollutants and the Streeter-Phelps equation for nonconservative pollutants. Two additional indices of assurance of satisfying the quality objectives and violation norm are also developed.

The cost and quality objectives are integrated to form a multiobjective planning problem. With cost as primary objective and water quality as secondary objectives, the latter objectives are reformulated in the epsilon-constraints form. The epsilon-constraint problem is solved for different levels of pollutants in the stream, corresponding to different discharge policies. The noninferior solutions, including the trade-offs along with optimal cost and corresponding levels of achievement of each objective may be submitted to the decision-maker for his evaluation of the Surrogate Worth function. Preferred solution are obtained by staisfying the optimality criteria of the Surrogate Worth Trade-off method. The above developments are presented in Chapter 2 and Chapter 3.

Chapters 4 to 8 are devoted to a comprehensive modeling of groundwater system, and to developing planning and management methodologies for efficient use of groundwater in general and conjunctive management of ground and surface water in particular. Both short- and long-term planning models of ground and surface water use are presented. In particular is suggests procedures and methodologies for a comprehensive mathematical analysis of hydraulically connected multi-cell aquifer and multi-stream systems. The models consist of hierarchies of response functions relating the system's response to various activities affecting it.

Appropriate response functions are developed which exclusively allow for coupling a complex large-scale water resources system with a management model. This is appreciable step ahead in the state-of-the-art of analyzing conjunctive use of ground and surface water resources and is a major contribution of this study.

In Phase I and II of this study, groundwater parameter identification models are developed and their usefulness is demonstrated. However, in those studies unknown parameters were assumed to be a continuous function of space, without taking into account the heterogeneous property of most aquifers. In this study an approach is adopted which takes into consideration the distributed nature of aquifer properties, by decomposing them into various cells, whose geometric configurations are selected according to the geological characteristics of the aquifer. A sensitivity analysis of model output for errors introduced by input data and parameters is also carried out.

The multicell-particular cell simulation procedure is discussed in Chapter 4 of this report. It provides the construction of mathematical models for numerically solving complex groundwater systems. The basic used is to decompose the system into a number of cells according to certain considerations. These considerations may involve geographical, geological and hydrological characteristics; administrative and operational judgments; or any other requirements associated with the particular need for the groundwater simulation model. The multicell mathematical model is used to approximate cells' boundary conditions associated with a given stress. These boundary conditions are used to isolate each particular cell's mathematical model. The following advantages are realized:

(1) The proposed procedure allows for applying mathematical simulation models to a large-scale and complex system, where the application of a regular compact simulation model on a digital computer is evidently inadequate.

(2) The restriction of computer capacity often needed in simulating a large aquifer system is best overcome by decomposing the model.

(3) The proposed procedure is evidently advantageous in cases where the interest is directed toward an isolated subsystem for a particular response. The modeling efforts can concentrate on the particular subsystem cell, while the rest of the system is accounted for through the aggregated multicell model.

(4) Data acquisition efforts are directed by the model's needs. This is an important factor in evaluating the model.

(5) The flexibility of the model's structure is an appreciable advantage in particular if an administrative scheme is considered. This characteristic is well illustrated by applying the management model to the tax-quota system in Chapter 8.

(6) Most developments later discussed are essentially based on the availability of the decomposed aquifer simulation model. It allows for production of response functions under any desired hierarchy.

The importance of the algebraic technological functions (A.T.F.) in a linear system is realized when the coupling of the physical system with a management framework is desired. Some real and meaningful advantages are associated with the hierarchy of the response functions as described below:

(1) It provides the systme analyst with a methodology by which to handle a large-scale and complex groundwater system within a management framework. The response functions superposition may be easily constructed in agreement with administrative or other considerations, not restricting the management model formulation.

(2) The amount of preparation work associated with the production of response functions for later use in management model formulation is considerably reduced.

(3) If a large number of wells is considered in a management model, then the associated response functions matrices require an extensive computer capacity unless a certain weighting of the response is applied. This is possible via the proposed technique.

The stream-aquifer interactions add a most important aspect to this research. An important contribution is the analysis which considers a multi-stream system interacting with a complex groundwater system. Of particular interest is the superposition of functions relating infiltration form different streams to different aquifer cells. It provides a new analytical tool for coupling infiltration from stream with management framework. The A.T.F. and the stream-aquifer response functions combined in the form developed in this study are the basis for analyzing a complex water resources system within a management framework.

The management model deve-opment and analysis presented in Chapter 6 constitutes a major contribution of this study. The quantitative analysis is made possible by utilizing the mathematical models previously developed.

The following aspects are actually appreciated:

(1) The analysis provides a full demonstration of the advantages associated with previous developments in application to water resources management model formulation and solution prospectives.

(2) An important contribution is made to the analysis of conjunctive use of ground and surface water systems. The proposed model is a first step in taking into account the distributed parameter characteristics of the systems involved in a water resources management model formulation.

The following conclusions may be drawn from this research work.

(i) A multiobjective framework is developed for the long-range planning and management of water and related land resources, including the conflicting objectives of water quality and the costs of point source pollutants. The multiobjective analysis and its implications to the planning is a major step in the direction of the federal guidelines -- the "principles and standards" for the planning of water and related land resources.

(ii) The modeling technique provides a procedure by which an accurate map of drawdown is predicted at different parts of a complex and large-scale groundwater system. The error for the Fairfield-New Baltimore area case study associated with the multi-cell-particular cell approach and the conventional one is found to be of the same order.

(iii) The digital computer time consumption was for the overall simulation model computation more than four times the computer time consumed by solving the same response via the proposed two-stage simulation model. This is only a particular measure indicating the efficiency and worthiness of the newly developed simulation technique. It is expected to be of a more value if applied to a large and complex groundwater system.

(iv) The multicell mathematical model was implemented on the mini-computer owned by the M.C.D. for the Fairfield-New Baltimore aquifer. It is expected that particular cells' mathematical models could also be implemented on that computer because of the reduced size of their associated mathematical model computer program.

(v) The proposed technique is most efficient for data aquisition. This we conclude from the experience gained through the various applications. Accurate and detailed data are most likely needed for particular cells of interest. However, other parts of the system may require limited and aggregated data as employed by the multicell mathematical model. In many cases this model's characteristic is very important.

(vi) The procedure for determining the hierarchy of response functions is well established. In general much fewer computations (computer runs) are required as opposed to running one-level response functions.

(vii) The applications of the stream-aquifer response functions to the various case studies illustrate the usefulness of this study's approach in extending these important functions' applicability. The may be used either to predict infiltration from streams due to pumpage (which is an important factor for stream balance as well) or to be utilized in a management model. It is a powerful tool but is restricted to linear aquifer systems where the stream acts as a constant head boundary.

Results obtained from applying the management model to the Fairfield-New Baltimore area indicate that:

(viii) Even if the particular conditions identified for some cases do not exactly coincide with the conceptual water resources management model framework, it is still possible to sucessfully use some of its fundamentals. The model is not restricted to certain system's structure, and actually may be applied to any mathematical analysis involving groundwater linear systems' control.

(ix) The applications of this study to the studied area provide the water users and the agencies interested in water resources in this area with refined and useful information. This includes all different response functions which may be used for various needs. The effect of pumpage on drawdow is also given, aggregated in cells resulting from ten-year requirement projections. The drawdowns predicted here for the Cincinnati well field exceed the figures predicted by the use of the analog model. This result should be carefully considered. The future infiltration rates from the stream provide the M.C.D. with much needed information for future evaluation of stream flow balance under low flow conditions in this area.

(x) The management control mathematical model is well-established and provides a comprehensive analysis of the most complicated problem of conjunctive use of ground and surface water. Case Study illustrates the model's applicability and practicability in solving problems involving groundwater system control conjunctively with other systems.

(xi) The decomposition of system's response provides for handling large-sized problems. However, it does not automatically solve dimensionality problems such as computer time and capacity.

(xii) The main conclusion from the model's sensitivity analysis in that accurately identifying the physical system's parameters is a major prerequisite for appreciating the management model solution.

The tax-quota model presented in Chapter 8 provides an improved solution strategy and also extends the capability of management models to handle groundwater recharge and surface water supply in addition to well pumpage.

9.2 RECOMMENDATIONS FOR FUTURE RESEARCH

In order to improve and further develop the methodologies presented in this study, the following recommendations are made:

(i) The surface water quality is analyzed by considering two pollutants such as BOD and DO deficit levels. However, to make the analysis more meaningful, other pollutants, such as phosphorus, total dissolved solids, toxic material, pH, thermal load, etc., should be included.

(ii) The decomposed aquifer simulation model comprises a hierarchy of aquifer mathematical models. The error associated with the multicell model aggregation is analyzed. However, the numerical methods used to solve the different models introduce another source of inaccuracy to the final solution.

(iii) The multicell concept allows for considerations other than hydrological - geological to take place in defining cells' boundaries in the model's structure. This introduces uncertainties with respect to the various structural parameters (distance between centers of cells, etc.) in addition to the systems' parameters (storage and transmissivity coefficients). It is therefore necessary to validate the mathematical model accounting for all parameters taking place in the model's formulation. A suitable identification model is desired.

(iv) Apart from the multicell-particular cell modeling procedure which is not restricted to linear systems, all other devleopments in this study are based on the assumption that the aquifer system can be approximated by a linear mathematical model. Extending the developments to handle-non-linear systems, if at all possible, may further contribute to water resources studies. The first effort in that direction should be devoted to computing the error associated with the application of linear models to a non-linear system.

(v) Water resources systems are particularly characterized as being affected by a stochastic input. Precipitation, evaporation, evapotranspiration, stream-flow and other such probabilistic phenomena play essential roles in surface and groundwater systems. The developments in this study consider deterministics model implying that the different probabilistic inputs are represented by their mean. Further research should be devoted to include the affecting stochastic parameters in the various developments. Stochastic control theory may be used to cope with the management of the systems under the stochastic input.

(vi) Coordination technique and multilevel approach may eventually help to cope with a problem involving other water systems, such as water distribution systems. The construction of an overall management model combining all different aspects of water related systems is a major task in water resources planning and management.

REFERENCES

1. Bates, H., "Computer Code for Wolfe Algorithm," Kansas State University, 1970.

2. Bear, J., Dynamics of Fluids in Porous Media. American Elsevier Publishing Company, N.Y., 1972.

3. Bear, J., U. Shamir and E.A. Hefez, "Numerical Modelling of Groundwater Systems." P.N. 180 Technion, Haifa, Israel, June 1972.

4. Bredehoeft, J.D. and G.F. Pinder, "Digital Analysis of Areal Flow in Multiaquifer Groundwater Systems: A Quasi Three-Dimensional Model," Water Resources Research, 6(3), 883-888, 1970.

5. Bryson, A.E., Jr., and Y.C. Ho, Applied Optimal Control, Ginn and Company, Waltham, Mass., 1969.

6. Buras, N., "Conjunctive Operation of Dams and Aquifers." ASCE-J. Hydraulics Division, 89(6): 111-131, 1963.

7. Burt, O.R., "The Economics of Conjunctive Use of Ground and Surface Water," Hilgardia, Vol. 36, No. 2, 1964.

8. Butcher, W.S., Y.Y. Haimes, and W.A. Hall, "Dynamic Programming for the Optimal Sequencing of Water Supply Projects," Water Resources Research, Vol. 5, No. 6, 1969.

9. Cohon, J.L. and D.H. Marks, "A Review and Evaluation of Multiobjective Programming Techniques," Water Resources Research, 11(2), 208-220, 1975.

10. Craig, James, "Decomposition Approach to the Capacity Expansion Problem." M.S. Thesis, Case Western Reserve University, Cleveland, Ohio, 1976.

11. Crawford, N.H. and R.K. Linsley, "Digital Simulation in Hydrology, Stanford Model IV," Technical Report No. 39, Stanford University, California, 1966.

12. Deredec, Alain, "A Systematic Approach to the Water Supply of a Large Urban Area," M.S. Thesis, Colorado State University, Fort Collins, Colorado, 1972.

13. Dreizin, Y.C., "Applications of the Superposition Approach to the Modeling and Management of Ground and Surface Water Systems," Ph.D. Dissertation, Case Western Reserve Univ., Cleveland, Ohio, 1971.

14. Eckenfelder, W.W., Jr., "Water Quality Engineering for Practicing Engineers," Barnes and Noble, Inc., New York, 1970.

15. Engineering News-Record. "Sewage Construction Costs Indexes in 20 Cities," Environmental Protection Agency, 194(2), 1975.

16. Falkenbarg, D.R., "The Identification of Distributed Parameter Systems," Ph.D. Dissertation, Case Western Reserve University, Cleveland, Ohio, 1971.

17. Federal Water Pollution Control Act. "Public Law 92-500" as Amended, October 18, 1972.

18. Federal Register. 1973. Principles and Standards for Planning Water and Related Land Resources. Vol. 38, No. 174, Part III, September 10.

19. Frankel, R.J., "Economic Evaluation of Water Quality: An Engineering-Economic Model for Water Quality Management," SERL Report No. 65-3, University of California, Berkeley, California, 1965.

20. Hadley, G., Nonlinear and Dynamic Programming, Reading, Mass., Addison-Wesley, 1964.

20a. Haimes, Y.Y., "Hierarchical Management of Ground and Surface Water Systems Via the Multicell Approach," Interim Report, July 1975, p. 228. NTIS - P.B. Number 247100.

21. Haimes, Y.Y., "The Integrated System Identification and System Optimization," Ph.D. Dissertation, School of Engineering and Applied Science, University of California, Los Angeles, CA, 1970.

22. Haimes, Y.Y., "Integrated System Identification and Optimization for Conjunctive Use of Ground and Surface Water, Phase I," Office of Water Res. Research, U.S. Dept. of Interior, Wash., D.C., PB 226-869, 1973.

23. Haimes, Y.Y., "Multiobjective Approach for Regional Water Resource Planning and Management. Vol. I." Report submitted to the National Science Foundation, Research Applied to the National Need Program, Case Western Reserve University, Cleveland, Ohio, 1973.

24. Haimes, Y.Y., "Integrated System Identification and Optimization for Conjunctive Use of Ground and Surface Water, Phase II," Office of Water Resources Research, U.S. Dept. of the Interior, Wash.,D.C., NTIS- PB 238-891, 1974.

25. Haimes, Y.Y., P. Das, K. Sung and K.S. Lee, "Modeling and Management of Water and Related Land Resources for Phosphorus Control and Ecolibrium," NTIS, PB-226419. Also report of Systems Engineering Dept., Case Western Reserve University, Cleveland, Ohio, 1973.

26. Haimes, Y.Y., W.A. Hall and H.T. Freedman, Multiobjective Optimization in Water Resources Systems: The Surrogate Worth Trade-off Method, Elsevier Scientific Publishing Company, The Netherlands, 1975.

27. Haimes, Y.Y. and W.A. Hall, "Multiobjectives in Water Resources Systems Analysis: The Surrogate Worth Trade-off Method," Water Resources Research, Vol. 10, No. 4, pp. 615-624, August, 1974.

28. Haimes, Y.Y. and D. Macko, "Hierarchical Structures in Water Resources Systems Management." IEEE-SMC, SMC-3(4), 396-402, July, 1973.

29. Haimes, Y.Y. and W.S. Nainis, "Coordination of Regional Water Resource Supply and Demand Planning Models," Water Resources Research, Vol. 10, No. 6, 1974.

30. Haimes, Y.Y., R.L. Perrine and D.A. Wismer, "Identification of Aquifer Parameters by Decomposition and Multilevel Optimization." Israel Journal of Technology, 6(5), 322-329, 1968.

31. Haimes, Y.Y. and D.A. Wismer, "A Computational Approach to the Combined Problems of Optimization and Parameter Identification," Automatica, 8, 1972.

32. Haimes, Y.Y., HIERARCHICAL ANALYSES OF WATER RESOURCES SYSTEMS: Modeling and Optimization of Large-scale Systems. McGraw-Hill International Book Company, New York, 1976, (in press).

33. Hall, W.A. and J.A. Dracup, Water Resources System Engineering, McGraw-Hill Book Company, New York, 1970.

34. Howe, C.W., Benefit-Cost Analysis for Water Systems Planning, American Geophysical Union, Washington, D.C., 1971.

35. Jacob, C.E., "Flow of Ground Water," Chapter V in Engineering Hydraulics, Ed., H. Rouse, John Wiley and Sons, Inc., New York, pp. 321-386, 1950.

36. Kaplan, M., "The Planning and Operation of a Regional Water Quality Management System: A Multilevel Approach," M.S. Thesis, Systems Engineering Department, Case Western Reserve University, Cleveland, Ohio, 1972.

37. Kaplan, M.A., "Multiobjective Analysis in Water Resources Planning and Analysis: A Framework for Policy Evaluation," Ph.D. Dissertation, Case Western Reserve University, Cleveland, Ohio 1975.

38. Kaplan, M.A. and Y.Y. Haimes, "Dynamic Programming for the Optimal Capacity Expansion of Wastewater Treatment Plants," Report No. SED-WRG-74-5, Systems Engineering Dept., Case Western Reserve University, Cleveland, Ohio. Paper presented at the Tenth American Water Resources Conference, Las Croabas, Puerto Rico, Nov. 18-22, 1974.

39. Karplus and Kawameto, "Identification Problem in Distributed Parameter Systems," Short Course Notes at UCLA, January, 1966.

40. Kleinecke, D.R., "Use of L.P. for Estimating Geohydrological Parameters of Groundwater Basins," Water Resources Research, Vol. 7, No. 2, 1971.

41. Kolo, D.E. and Y.Y. Haimes, "Capacity Expansion and Operational Planning for Regional Water Resource Systems," SRC. Tech. Memo. 48, Case Western Reserve University, Cleveland, Ohio 1973.

42. Kreyszig, E., Advances Engineering Mathematics, John Wiley and Sons, Inc., New York, 1965.

43. Kuester, J.L. and J.H. Mize, Optimization Techniques with Fortran, McGraw-Hill Book Company, New York, 1973.

44. Lasdon, L., Optimization Theory for Large Systems, Macmillan Co., London, 1970.

45. Lasdon, L.S., et al, "Nonlinear Optimization Using the Generalized Reduced Gradient Method," Tech. Memo No. 325, Dept. of Operations Research, Case Western Reserve Univeristy, Cleveland, Ohio, 1973.

46. Leonard, R.L., "Integrated Management of Ground and Surface Water in Relation to Water Importation, the Experience of Los Angeles County," California Agri. Exp. Str., University of California, Los Angeles, 1966.

47. Lions, J.L., Optimal Control of Systems Governed by Partial Differential Equations, Springer-Verlag, Berlin, 1971.

48. Lopez, H., "Modeling and Identification of Groundwater Systems," M.S. Thesis, Case Western Reserve University, Cleveland, Ohio, 1973.

49. Lopez, H., Y.Y. Haimes, and P. Das, "Nonlinear Estimation of Distributed Parameter of Groundwater Systems," Journal of Hydrology, Vol. 30, No. 1, pp. 47-61, May 1976.

50. Maddock, T. III, "A Program to Simulate an Aquifer Using Alternating Direction Implicit-Iterative Procedure," U.S. Dept. of Interior, Geological Survey, 1969.

51. Maddock, T.,III, "Algebraic Technological Function from a Simulation Model," Water Resources Research, Vol. 11, No. 1, 1972.

52. Maddock, T., III, "Management Model as a Tool for Studying the Worth of Data," Water Resources Research, 9(2), 270-280, 1973.

53. Maddock, T., III, "The Operation of a Stream-Aquifer System under Stochastic Demands," Water Resources Research, 10(1), February, 1974.

54. Maddock, T., III, and Y.Y. Haimes, "A Tax-Quota System for the Planning and Management of Groundwater," Water Resources Research, Vol. 11, No. 1, P. 7, February 1975.

55. Marino, M.A. and Yeh, W. W-G, "Identification of Parameters in Finite Leaky Aquifer System," Journal of Hydraulics Division, ASCE, 99(2), pp. 319-336, February, 1973.

56. Marquardt, D.W., "An Algorithm for Least-Square Estimation of Non-Linear Parameters," J. Soc. Indus. Appl. Math., Vol. II, No. 2, June 1963.

57. Michael, R.L., "Costs of Manpower for Municipal Wastewater Treatment Operation and Maintenance," Journal of WPCF, 42(11), 1970.

58. Mikhlin, S.G., Mathematical Physics, An Advanced Course. North Holland Publishing Co., pp. 418-420, 1970.

59. Morin, T.L. and A.M.D. Esogbue, "Some Efficient Dynamic Programming Algorithms for the Optimal Sequencing and Scheduling of Water Supply Projects," Water Resources Research, Vol. 7, No. 3, 1972.

60. Nainis, W.S. and Y.Y. Haimes, "A Multilevel Approach to Planning for Capacity Expansion in Water Resources Systems," IEEE Systems, Man and Cybernetics, SMC-5, No. 1, 1975.

61. National Water Commission. "New Direction in U.S. Water Policy." Washington, D.C., U.S. Government Printing Office, 1973.

62. Owen, L.W., "Groundwater Management and Reclaimed Water," Journal of American Water Works Association, February, 1968.

63. Parizek, R.R. and E.A. Meyers, "Recharge and Groundwater from Renovated Sewage Effluent by Spray Irrigation," Proceedings of AWRA, Fourth Annual Conference, 1968.

64. Peaceman, D.W. and H.H. Rachford, Jr., "The Numerical Solution of Parabolic and Elliptical Difference Equations," J. Soc. Indust. Appl. Math., 3(11), 28-41, 1955.

65. Pinder, G.F. and J.D. Bredehoeft, "Application of the Digital Computer for Aquifer Evaluation," Water Resources Research, 4(5), 1069-1109, 1968.

66. Phillipson, G.A., "Identification of Distributed Systems," American Elsevier, New York, 1971.

67. Plummer, P.M., "Observation and Evaluation of Water Resources in the Hamilton to New Baltimore Area, 1971-1973," The Miami Conservancy District, Dayton, Ohio, August 1974.

68. Prickett, T.A. and C.G. Lonquist, "Selected Digital Computer Techniques for Groundwater Resource Evaluation," Illinois State Water Survey Division, Urbana, Illinois, Bulletin 55, 1971.

69. Ricca, V.T., "The Ohio State University Version of the Stanford Streamflow Simulation Model," Report to the Office of Water Resources Research, U.S. Dept.of Interior, May 1972.

70. Roach, G.F., Green's Functions, Introductory Theory with Applications, Van Norstad Reinhold Co., London, 1970.

71. Shah, K.L. and G.W. Reid, "Techniques for Estimating Construction Costs of Waste Treatment Plants," Journal of WPCF., 42(5), Part I, 1970.

72. Smith, Robert, "Cost of Conventional and Advanced Treatment of Wastewater," Journal of WPCF, 40(9), 1968.

73. Sopper, W.E., "Waste Water Renovation for Reuse: Key to Optimum Use of Water Resources," Water Research, Peragammon Press, 1968.

74. Spieker, A.M., "Groundwater in the Lower Great Miami River Valley, Ohio," U.S. Government Printing Office, No. 605-A,B,C,D., Wash. D.C. 1968.

75. Streeter, H.W. and E.B. Phelps, "Study of the Pollution and Natural Purification of the Ohio River," Public Health Bulletin No. 146, U.S. Public Health Service, Washington, D.C., 1925.

76. Todd, D.K., "The Future Prospects of Artificial Groundwater Recharge," Proceedings, Artificial Groundwater Recharge Conference, Water Research Administration, 1970.

77. Tyson, H.N. and E.M. Weber, "Groundwater Management for the Nation's Future. Computer Simulation at Groundwater Basins," ASCE, Hydraulics Division, vol. 90, pp. 59-78, 1964.

78. Wolfe, P., "The Simplex Method for Quadratic Programming," Econometrica, 27, 382-398, 1959.

79. Yeh, W.G. and G.W. Tauxe, "Quasilinearization and Identification of Aquifer Parameters," Water Resource Research, vol. 7, no. 2, April 1971.

80. Yu, W. and Y.Y. Haimes, "Multilevel Optimization for Conjunctive Use of Ground and Surface Water," Water Resources Research, 10(4), 625-636, August 1974.