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THE INDIVIDUAL'S INTERNAL MARKET FOR KNOWLEDGE

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The objective of this paper is to develop a micro-economic framework for knowledge. Initial attention of economists to the knowledge industry has been directed to the study of the relationship between knowledge, usually measured as years of formal schooling, and productivity. Two broad bodies of literature which continue to develop can be distinguished. First, there is a body of literature within production theory where knowledge is considered as a factor of production and the relationship between knowledge and output is studied [13, 14, 26, 39]. Second, there is a cost-benefit literature where changes in expected future income streams from additional knowledge are compared to the costs of obtaining that knowledge [1, 4, 5, 15, 17, 22, 23, 24, 25, 40, 42]. Both bodies of literature have furthered the concept of knowledge as a form of capital, i.e., human capital [36, 37, 38]. However, this work has been limited to the returns to knowledge from market employment, and has not been extended to returns from consumption uses.

More recently, two additional bodies of literature are developing, both directed largely at the study of the formal school system. One of these is the study of the distribution impacts of public subsidies to schools [3, 16, 18, 21, 33]. A major problem with this work is the lack of a social benefit function for knowledge. There is disagreement among public subsidies for the stimulation of knowledge acquisition to the extent that externalities exist, public subsidies for income redistribution, and public subsidies for more general welfare redistribution.

A second body of literature is developing around the study of production or transformation functions and cost relationships between knowledge producing factors of production and knowledge [6, 7, 8, 9, 10, 11, 12, 19, 20, 27, 23, 30, 31, 35, 41]. Two major problems emerge in this work. One is the definition of knowledge in quantitative terms. In a knowledge production function, years of schooling is not an adequate definition of knowledge. Measures which allow variations among students or groups of students in quantity of knowledge for given years of schooling, such as achievement test scores and attendance rates, have been developed and used with limited success. A second problem arises from the use of student characteristics in some form as controls or inputs in most production function estimates. Knowledge acquisition requires a direct input on the part of the individual. Even knowledge which is stored, such as that in written form, is only potential knowledge to the individual. It is of no direct value to him until he takes the time to transfer it from storage into his own understanding, at which point he can realize the benefits of it. However, the inclusion of individual characteristics in the knowledge production function leads to a major identification problem between the demand for and the supply of knowledge.

This identification problem leads directly to the subject of this paper: the specification of the individual's internal market for knowledge. The needs for such a specification are several. First, such a specification is needed to further understand and explain the behavior of individuals in the knowledge industry, specifically to provide a framework which includes non-market uses of knowledge. Second, implications of the model for the empirical specification of demand, cost, or production functions for knowledge will be of significant value. Third, any implications which such a framework has for the conceptual definition of knowledge and other factors involved in the knowledge industry as well as the linking of these conceptual definitions to empirical counterparts will be of equal value. Finally, included in the specification is an attempt to specify and justify a social benefit function for the individual which incorporates externalities and income redistribution. If policy is to be evaluated, knowledge of the existence and behavior of such a function is vital.

Several developments in economic theory provide the basis of a micro-economic model for knowledge. First, there is the concept of a consumption activities production function developed by Becker [2], Lancaster [29], and Linder [32]. The individual uses goods and time to produce consumption activities; consumption activities are the elements in the utility function. Second is the concept of time as a scarce resource to the individual, Becker [2] and Linder [32]. Third is the concept, developed by Ben-Porath [4], that the individual has a production function for knowledge. 1/

These concepts are combined in the next section to define a set of production and identity constraints to which the individual is subject. A major simplifying assumption is that the utility function predetermines an ordering of consumption activities so that consumption activities are maximized directly. Further, knowledge does not affect the ordering of consumption activities, although it may affect time preference. Consumption activities can be made endogenous by attaching the utility maximization models of Becker, Lancaster, and Linder to the present model.

1/ This idea was first suggested to me by W. Keith Bryant.

The following section uses the constraints to develop a demand-supply model for knowledge. A third section develops the basis for a social benefit function from externalities and income redistribution and incorporates this into the individual's supply-demand model. The model is extended with two types of knowledge in a fourth section. A final section discusses the implications of the model for current and needed research.

Production Function and Identity Constraints

The production and identity constraints to which the individual is subject are

(1) $P\ (C, K, G, T_C) = 0$ (Consumption Activity Production)

(2) $H\ (E, K, T_E, W, 1/r, Z) = 0$ (Expenditure Production)

(3) $Q\ (k, F, T_K) = 0$ (Knowledge Production)

(4) $T = T_C + T_E + T_K$ (Time Constraint)

(5) $E = P_G'G + P_F'F$ (Expenditure Identity)

where C is a vector of consumption activities, K is the stock of knowledge, k is knowledge produced during the period, E is expenditures, G is a vector of goods and services and P_G its price vector, T, T_C, T_E, and T_K are, respectively, total time, consumption time, expenditure time, and knowledge production time, F is a vector of knowledge producing factors of production and P_F its price vector, W is the income stream from non-human wealth, 1/r is the price of a unit change in the income stream (or r is the market rate of return on non-human wealth and also the social discount rate), and Z is a vector of other exogenous market forces. All equations are for a given time unit and all variables are for the current period. The finite time horizon of the individual is incorporated in the next section.

An expenditure production or generation function is added to the consumption activities and knowledge production functions. The individual maximizes consumption activities subject to three production possibility functions and two identities. However, consumption activities, expenditures, and new knowledge are simultaneously determined within each period.

Equation (4) is a time constraint. The time allocated to the production of consumption activities, expenditures, and new knowledge is equal to the total time available within any period. Equation (5) is an expenditure identity. Total expenditures on goods and knowledge producing factors of production within any period are equal to expenditures generated by equation (2).

Equation (1) is the consumption activities production function. It modifies the Becker, Lancaster, Linder models by the addition of knowledge as an explicit factor of production.

The expenditure production function, equation (2), is derived from

(2') $E = I + W - \frac{\Delta W}{r}$,

where I is income and $\frac{\Delta W}{r}$ is savings, i.e., the purchase of additional income streams. Further

(6) $I = T_I w = T_I f' (K, Z) = f (T_I, K, Z)$

(7) $W - \frac{\Delta W}{r} = g (1/r, W, T_W, K, Z)$,

where T_I is earnings time, T_W is time used for changes in non-human wealth, and w is the market wage rate.[2/] Substituting equations (6) and (7) into (2') yields

(2") $E = f (T_I, K, Z) + g (1/r, W, T_W, K, Z)$,

2/ Psychic or nonpecuniary benefits are included in w. These benefits are direct non-market purchases of G or F.

which is generalized in equation (2), where T_E is the sum of T_I and T_W. Knowledge is a factor in the production of expenditures through its input in the production of earnings and changes in the income stream from non-human wealth.

In the knowledge production function, equation (3), new knowledge (k) is produced by a combination of factor inputs and time. It is assumed that the knowledge production function is homogeneous of degree one in factor inputs and time.

A full definition of knowledge is not attempted, but some of the characteristics of knowledge as used in this paper are explored. First, knowledge is not information; information enters the model as a good (G), an exogenous market force (Z), or a factor input (F). Knowledge is more like acquired ability or acquired skills to the extent that these do not involve the memorization of facts or routine procedures. For example, memorizing the procedure to repair a particular carburetor involves little, if any, knowledge, while learning the principles of carburetor operation so that one can analyze and determine what is wrong with a carburetor does involve knowledge. Knowledge is most closely identified with the analytical framework through which an individual approaches and resolves problems. It is the logical or reasoning ability of the individual plus the communications skills through which this ability is transmitted. In a sense, knowledge is the ability to deal with the unknown, i.e., the ability to reach conclusions which are not routine. Within this context an increase in knowledge is an increase in the complexity of the individual's analytical framework, i.e., the ability to handle more complicated problems. Specialization of knowledge or different types of knowledge can be identified with the application of the analytical framework to different groups of problems. An individual may use general analytical ability in the production of consumption activities, and use a verv specialized analytical ability, e.g., the analysis of faulty carburetors or the analysis of economic problems, in his occupation.

Second, the use of knowledge does not reduce the stock of knowledge. Knowledge may depreciate from loss of memory or technological obsolescence, but the use of knowledge does not reduce the remaining service flow of knowledge. In fact, the opposite may be true, i.e., the use of knowledge to solve a problem may increase the stock of knowledge as a result of learning involved in the experience. Finally, the full stock of knowledge can be brought to bear on any problem at any time. Even specialized knowledge is available for the production of consumption activities since the individual carries it with him at all times, i.e., the individual owns the stock.

In the present model, knowledge is homogeneous. In the knowledge production function, the existing stock of knowledge is assumed to have no effect on the production of new knowledge. In the context of knowledge as analytical ability, this means that knowledge has no effect on the amount of increase in sophistication of the individual's analytical framework, although it does determine the starting point from which this increase occurs.

The total stock of knowledge is used in the production of consumption activities and expenditures. Knowledge is assumed to be a direct factor of production; it enters the production functions independently of other factors of production, and its marginal productivity is determined by relative factor shares.[3/] Under the assumption of homogeneous knowledge and the characteristic

3/ The alternative assumption is that knowledge is factor augmenting. In the present model either assumption leads to the same conclusions.

of knowledge that the full stock can be used in any activity, there is no allocation problem in the use of knowledge. The only problem is how much knowledge to produce at what rate. This allows simplifying the model by combining equations(1) and (2) into a single multiproduct production function

(8) $PH\ (C, E, K, G, T_C + T_E, W, 1/r, Z) = 0.$

In a later extension of the model, two mutually exclusive types of knowledge are introduced: one which can be used to produce only consumption activities and the other to produce only expenditures. In the context of knowledge as analytical ability, this is a gross oversimplification, but does illustrate in the extreme case what happens when knowledge must be allocated between use for consumption activities and use for production of expenditures.

Demand-Supply Model for Knowledge

In this section, the production and identity constraints are used to develop a demand-supply model of knowledge for the individual.

Derived Demand

From equation (8), the within period derived demand for knowledge is

(9) $d\ (mp, K, C, E, P_G, P_T, W, 1/r, Z) = 0$

(10) $mp = MU_C \frac{(dC)}{(dK)} = MU_C \left(\frac{\partial C}{\partial K} + \frac{\partial C}{\partial E}\frac{\partial E}{\partial K}\right),$

where mp is the marginal value product of knowledge, P_T is the marginal value of time in the demand function,[4/] and MU_C is the marginal utility of consumption activities.[5/] In equation (10), mp is defined as the product of MU_C and the

4/ From the production function (8), the marginal value of time is

$$P_T = MU_C \frac{\partial C}{\partial T} = MU_C \frac{\partial C}{\partial E}\ \frac{\partial E}{\partial T}.$$

In the demand function above and later in the supply function, P_T as determined by this relationship is used.

5/ For simplicity, MU_C is assumed to be constant.

marginal physical product of knowledge. The physical product is composed of the direct effect of K on C and an indirect effect of K on C through expenditures (E). The expected partial relationships among mp, K, and exogenous factors are

$$\frac{\partial mp}{\partial K} < 0, \ \frac{\partial mp}{\partial Z} > 0, \ \frac{\partial K}{\partial P_G} > 0, \ \frac{\partial K}{\partial (1/r)} > 0.$$

The partial relationship between the stock of knowledge and its marginal value product is negative. As K increases, given other factors, mp declines. The relationship between mp and Z depends on the definition of Z. For simplicity, Z is defined as the set of market wage rates facing the individual for varying levels of K.[6/] Accordingly, an increase in Z, an upward shift in the wage rate structure, results in an upward shift in the demand for knowledge, i.e., for any K, mp increases.

The demand curve shifts to the right with an increase in the price of substitutes (P_G, 1/r). Goods are a net substitute for knowledge, although some goods may be complements. The income stream from non-human wealth is a substitute for K through its effects on expenditures and 1/r is the per period price of a unit of W. The discussion of the behavior of time is delayed until the full model is developed; time is endogenous and its behavior depends on supply as well as demand.

Equation (9) is the per period relationship among the stock of knowledge, its marginal product, and other factors. However, knowledge is an investment good; it yields a return over more than one period. The demand for knowledge is derived from the current return and all expected future returns. For simplicity in discounting, it is assumed that the within period demand function

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6/ See note 2. This simplifies the wage rate determination in equation (6) to w = Z (K). However, this definition of Z is a summarization of many possible exogenous wage determining factors. Further, it does not include factors affecting non-human wealth.

is a permanent or expected demand function and that all exogenous factors are at expected levels, that time periods are continuous, and that the discount parameters are permanent or expected values.[7/] Under these assumptions, equation (9) can be transformed into an investment demand for knowledge by transforming mp into its marginal present value product

$$(11)\ MP = mp \int_0^{(N - A)} e^{-(\rho + \delta)t}\, dt$$

$$= \frac{mp}{(\rho + \delta)} \left[1 - e^{-(\rho + \delta)(N - A)}\right],$$

where ρ is the private or individual rate of discount, not necessarily equal to r, δ is the rate of depreciation of knowledge, A is the age of the individual in the current period, and N is the expected age at death. The expected remaining life is (N - A) periods.

Three partial relationships of significance are

$$\frac{\partial MP}{\partial \rho} < 0,\ \frac{\partial MP}{\partial \delta} < 0,\ \frac{\partial MP}{\partial (N - A)} > 0.$$

As the individual's rate of discount increases, the marginal present value product of any stock of knowledge declines because the individual places less value on future returns. More rapid depreciation of knowledge also reduces MP. Knowledge does not depreciate from use, i.e., obtaining a flow of services from knowledge in one period does not reduce the remaining flow of services as it does for a machine, where use of the machine reduces the remaining service flow through deterioration. However, knowledge may

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7/ There are many possible variations in discounting. In a country with mandatory retirement, one of the more obvious variations is a drastic movement along equation (9) at the point of retirement due to a change in Z, i.e., a reduction in the set of market wage rates. A significant reallocation of time is likely to occur from expenditure generation to the production of consumption activities or knowledge. What happens to expenditure generation depends on retirement pay and non-human wealth.

depreciate from loss of memory or technological obsolescence in a world witn specialized knowledge. As the individual ages, or his remaining expected life declines, MP declines because the number of periods over which knowledge yields a return is declining.[8/]

Using the transformation equation (11), the investment demand function for knowledge is

(12) $D\ (MP,\ K,\ C,\ E,\ P_G,\ P_T,\ W,\ 1/r,\ Z,\ \rho,\ \delta,\ N - A) = 0.$

All previous partial relationships have the same sign, but differ in magnitude. Figure 1 illustrates the relationship between equations (9) and (12) in the marginal product-stock of knowledge dimension by the curves d and D, respectively.

Supply

The within period supply function for knowledge, the derivative of the dual of the production function, equation (3) is

(13) $S\ (MC,\ k,\ P_F,\ P_T) = 0,$

where MC is the marginal cost of additional units of knowledge. The value of time is determined from the demand side.[9/] The partial relationships are

$$\frac{\partial MC}{\partial k} = 0,\ \frac{\partial MC}{\partial P_F} > 0.$$

A production function homogeneous of degree one in F and T_K implies constant marginal costs, given prices.[10/] However, this does not imply that the individual ever operates along a horizontal cost curve, because P_T is

8/ See Raney [34] for an incorporation of a finite time horizon into the capital stock accumulation decisions of a firm with finite life.

9/ See note 4.

10/ Ben-Porath [4] assumes a production function homogeneous of degree less than one because his demand for knowledge is horizontal. However, in the present model this assumption is not necessary because the value of time changes as the allocation varies, which in turn changes marginal cost (as well as MP).

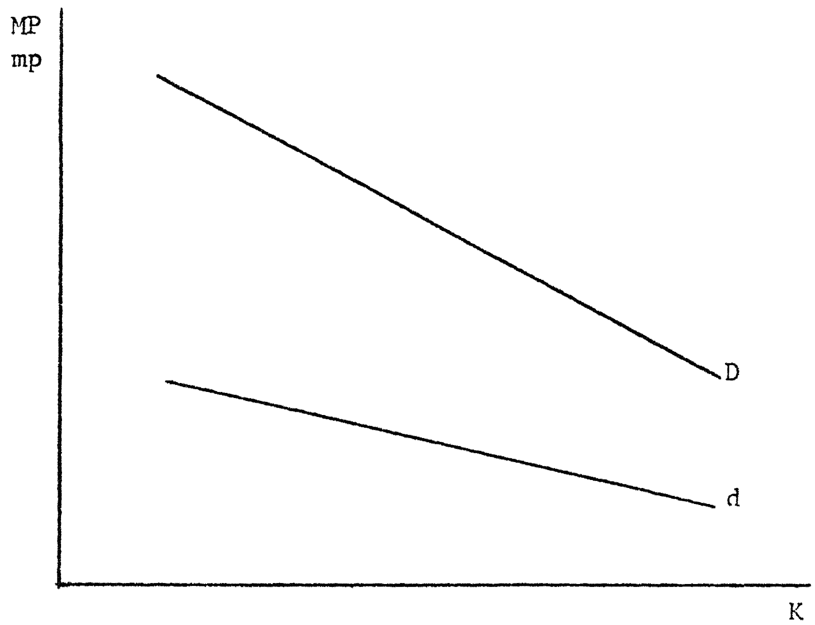

Figure 1--Per Period and Investment Demand for Knowledge

endogenous and continually changes as the time allocation changes. As T_K increases, $T_C + T_E$ decreases, resulting in an increase in P_T, which induces an increase in the F/T_K ratio from the increase in P_T/P_F. Further, the function is bounded absolutely by the time constraint, i.e., the function becomes vertical when all time is allocated to knowledge production for each P_T/P_F ratio. The prices of purchased factors of production have a positive impact on MC.

Equation (13) is the relationship between cost and the quantity of knowledge which can be acquired during a period. However, the total supply of knowledge consists of new knowledge and the stock of knowledge remaining from the previous period

(14) $K = (1 - \delta)K_0 + S,$

where K_0 represents last period's stock of knowledge. When there is depreciation, the stock carried over from the previous period is less than the previous period's stock.

Knowledge supply when there is depreciation is illustrated in Figure 2. Last period's stock is K_0, the carry-over is $(1 - \delta)K_0$. Each $S\ (P_T/P_F)$ is a trace of the marginal cost function for a given P_T/P_F; they become vertical when all time is allocated to knowledge production. An increase in P_T, which is an increase in P_T/P_F, induces a substitution of F for T_K, raises marginal cost, but also increases the total quantity of knowledge which can be produced, $S\ (P_T/P_F)_2$ as compared to $S\ (P_T/P_F)_1$. The curve $T = T_K$ represents the boundary of the marginal cost function. It is positively sloped because a movement along this boundary is an increase in the use of F for given T_K.

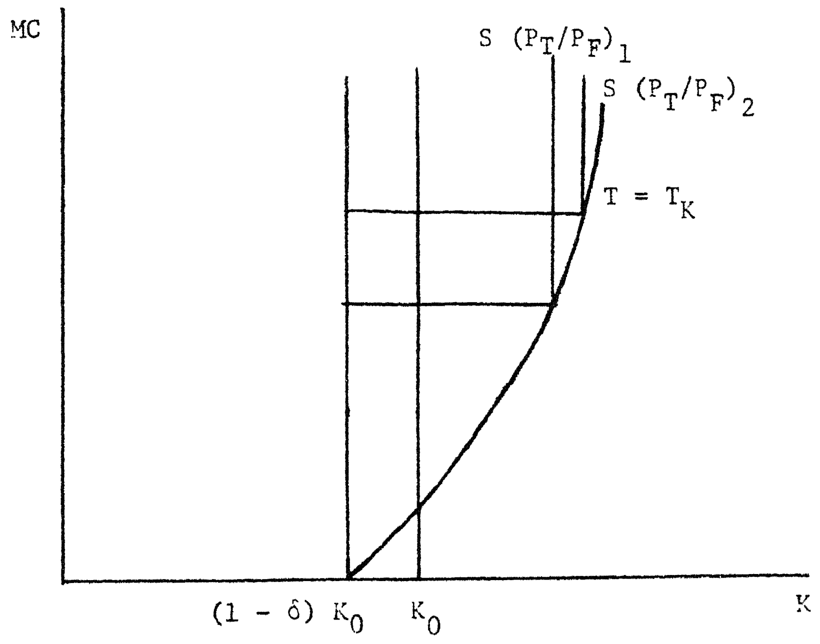

Figure 2--Supply of Knowledge

An individual will never be in equilibrium on the vertical portion of any S curve because he can reduce cost by substituting F for T_K, i.e., he will move along the boundary ($T = T_K$). Further, it is unlikely that an individual will ever be on the boundary of the marginal cost function ($T = T_K$). First, some time is probably needed to produce necessary consumption activities. Second, and more important, as the individual substitutes F for T_K, more time is required to produce expenditures for the purchase of F. In essence, corner solutions are unlikely; each individual is likely to operate internally on the marginal cost function where marginal allocations are made on all resources.

A Digression

The inclusion of depreciation in both the demand and supply functions is not double counting of depreciation. To illustrate, the case of fertilizer is considered, a good with 100 percent depreciation within any production period if carry-over is ignored. On the demand side, the marginal present value product is equal to current period marginal product, i.e., a marginal unit of fertilizer provides no return beyond the current period. This is equivalent to depreciating fertilizer by 100 percent from the current to the next production period.

On the supply side, a stock of fertilizer is purchased in each period to the point where MC = MP. By the next production period, this stock depreciates to zero so that the carry-over stock is zero. This is equivalent to $(1 - \delta)K_C$ coincident with the vertical axis in Figure 2 because $\delta = 1$.

The Model

Equations (12), (13), and (14) are the demand and supply functions for knowledge. However, they do not form a determinate system as there are more endogenous variables than equations. The endogenous variables in these equations are MP, K, MC, k, and P_T, five variables with three equations. The additional equilibrium conditions are

(15) MP = MC

and the time constraint, equation (4), which are sufficient to determine equilibrium. Consumption activities are predetermined through the utility function. Expenditures and the expenditure identity, equations (2) and (5), are internalized in equation (8) and are no longer independent constraints.

The interrelationship between knowledge and the value of time in the model is composed of two effects. The value of time may increase because of an increase in K, Z, or some other factor. First, the increase in P_T increases MP for any given K, i.e., an upward shift in the demand curve for knowledge (the partial relationship between MP and K), which induces substitution of knowledge for time (what happens to goods depends on what induced the change in P_T). At the same time, the marginal cost curve of knowledge (the partial relationship between MC and k), shifts upward because P_T has increased, which induces the substitution of other factors (F) for time, unless an increase in P_F induced the change in P_T. The result of the upward shifts of the demand and supply curves is an excess demand for or an excess supply of knowledge, except where the shifts balance.

The second effect, the reallocation of time, along with expenditures, offsets any excess demand or supply. If an excess demand results from the first order shifts, time and expenditures are reallocated to the production of knowledge, shifting the demand curve downward and the supply curve further upward through a further increase in P_T. If there is an excess supply, the opposite occurs; there is a reallocation away from the production of knowledge shifting the demand curve further upward and the supply curve downward through a partial reduction of P_T.

Expenditures on goods (G) and F are reallocated simultaneously with the reallocation of time, and the expenditure constraints are operative although they have been internalized, i.e., they are not explicit, in the present model. Further, although the effects of a change in P_T on the model have been discussed, the allocation of time is endogenous and any changes in exogenous factors cause a reallocation of time within the demand and supply functions for knowledge.

The model is illustrated in Figure 3. It is assumed that the individual is initially at a within period equilibrium on D_1 and S_1, where he would produce k_1 units of new knowledge. If the demand curve shifts upward, e.g., from an increase in P_G or Z, there is also a first order upward shift in the marginal cost curve from an increase in P_T; the increase in P_T is either caused by or causes the increase in demand. A reallocation of time and expenditures occurs to eliminate any excess supply or demand, with a new equilibrium on D_2 and S_2, with k_2 units of new knowledge produced. Although k_2 is greater than k_1, whether more time or expenditures or both are allocated to the production of knowledge depends on what induced the change in demand. More of at least one, but not necessarily both, must be allocated to knowledge production. There is also an income or expenditure effect which increases or offsets the shifts in Figure 3 depending on what causes the increase in demand.

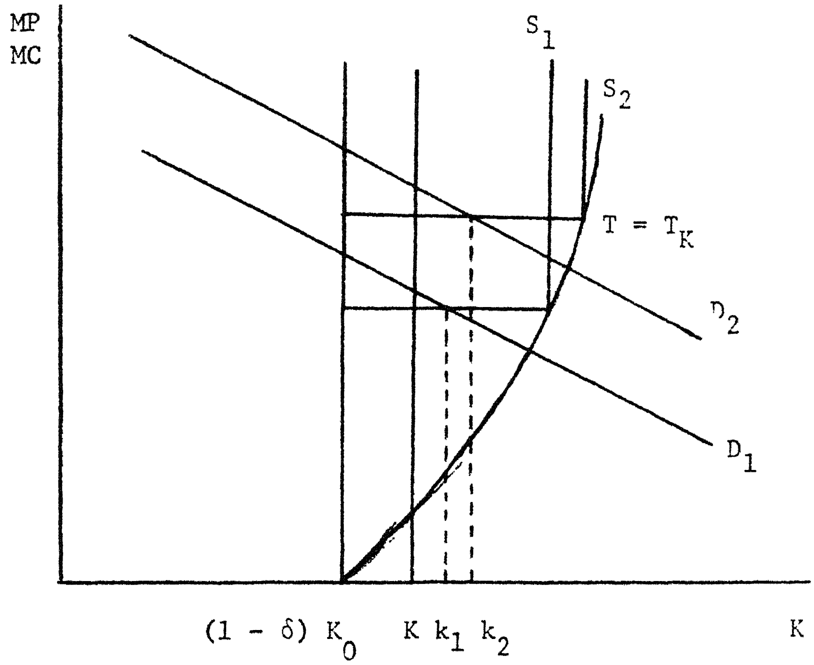

Figure 3--Demand-Supply Model for Knowledge

The illustration is extended in more concrete terms by tracing the effects on the model of a fall in the price of television services, e.g., a fall in the price of television sets or an improvement in television transmissions which increase the service flow of a television set. To the individual, television services are a substitute for the services of radio, magazines, newspapers, and other communications media. More important, television services are a substitute for language skills, i.e., knowledge. Television allows the use of pictures and other graphic illustrations as a substitute for word descriptions on radio; it substitutes a combination of oral and graphic descriptions for written and graphic descriptions in newspapers and magazines. Less sophisticated language skills are needed to obtain the same amount of information because pictures transmit messages directly, which on radio require a word description and subsequent interpretation by the individual, i.e., the mental formation of an image of the picture itself.

In the demand function, a fall in the price of television services is a fall in the price of a good; television services provide entertainment and information as inputs to the production of consumption activities and expenditures. The demand curve shifts downward and the marginal value of time decreases as television services are substituted for both knowledge and time in the production of a given level of consumption activities. Within the demand function, there is a reallocation of knowledge and time from the direct production of consumption activities to the generation of expenditures. The increase in expenditure generation internally offsets part of the reduction in demand for knowledge from production of consumption activities. The marginal cost of producing new knowledge falls with the reduction in the value of time.

Time and expenditures are reallocated to eliminate any excess demand or supply. If the individual were initially in equilibrium on D_2 and S_2 in Figure 3, he might end at equilibrium on D_1 and S_1 after the fall in the price of television services.

The real income or expenditure effect is now considered. It is assumed that the price of television services is the only variable to change. The substitution effect is a substitution of television services for knowledge and time. In addition, the fall in the price of television services results in an increase in the total quantity of consumption activities which the individual can produce per period; each unit of expenditures now buys more goods. This generates an increase in demand for knowledge and an increase in the value of time and results in an adjustment similar to that for the substitution effect alone. If knowledge is a normal good, the income effect will not fully offset the substitution effect and there will be a net reduction in the quantity of new knowledge produced within a period from a fall in the price of television services.

There are several other effects which might result from a reduction in the price of television services which would offset the net reduction in the quantity of new knowledge produced per period as a result of expected income and substitution effects. Two are discussed. First, the production of more television services in substitution for other communications media is likely to require more knowledge, i.e., the production of television services is more knowledge intensive. This results in an increase in Z, the wage rate structure with respect to knowledge. Although this increase would be marginal to most individuals from television alone, if this is the kind of change which occurs broadly with development, then Z is likely to rise significantly with development. This increases the demand for knowledge to generate expenditures, with the result that more goods are purchased generating a further increase in demand for knowledge for consumption activities. At the same time, the marginal value of time rises causing still further substitution of knowledge and goods for time. The marginal cost of producing new knowledge also rises with the value of time, but an increase in Z from the fall in the price of television services alone could be strong enough to cause a net increase in new knowledge produced per period.

Second, television can also be used to produce knowledge as well as to transmit information and entertainment. The fall in the price of television services, a factor of production, causes a substitution of television services for other knowledge producing factors and time, resulting in a downward shift in the marginal cost curve. The full implications of such a change can be traced through the model. The result is a probable further net increase in the per period production of new knowledge.

Social Benefit Function

Up to this point, public intervention in the individual's knowledge market has not been considered. Public intervention is incorporated by the specification of a social benefit function for knowledge which includes externalities (returns to knowledge not captured by an individual) and "socially desirable" changes in the income or expenditure distribution.[11/] An externality with respect to knowledge exists when an increase in the stock of knowledge of individual A increases the total production of consumption

11/ Policy evaluation or development appears more fruitful when based on externalities and expenditure redistribution than when based on a general welfare function where it is necessary to establish that the marginal utility of expenditures of the poor exceeds that of the rich.

activities of B. Expenditure redistribution is an externality in the sense that the case for expenditure redistribution exists when an increase in A's expenditures results in an increase in B's production of consumption activities. These are cases of interdependent utility functions where the knowledge and expenditure distributions are elements of individual utility functions. These interdependencies may be direct, B obtains direct consumption activities from A's increase in knowledge or expenditures; or indirect, A's increase in knowledge or expenditures generates changes such as reductions in the prices of goods, including fewer unattractive neighborhoods, or more tax revenues.

In this paper, the social benefit function is defined for the individual. It is the relationship between the individual's stock of knowledge, the present social marginal value of that knowledge net of present private marginal value, and other factors

(16) $B\ (MB,\ K,\ P_G,\ E/E^*,\ r,\ \rho,\ \delta,\ N - A) = 0$

(17) $MP = \frac{mb}{r + \delta}\ [1 - e^{-(r + \delta)\ (N - A)}],$

where mb is within period social marginal benefits, MB the present social marginal value benefits of knowledge, and E* is mean expenditures or some social norm against which the individual's expenditures are evaluated. The partial relationships of concern are

$$\frac{\partial MB}{\partial K} < 0,\ \frac{\partial MB}{\partial P_G} > 0,\ \frac{\partial MB}{\partial\ (E/E^*)} < 0,\ \frac{\partial\ (E/E^*)}{\partial\ (\rho/r)} < 0.$$

The partials of MB with respect to r, δ, and (N - A) are obvious from equation (17).

The partial between MB and K is negative. The communications industry provides some conceptual support for the existence of a social benefit function and a basis for speculation about its behavior. For a given communications network, it appears reasonable to argue that the ability of the network to distribute information increases as the language skills of each individual increase, i.e., language skills are an input in the production of communications. Society has an incentive to stimulate the acquisition of language skills, i.e., the MB of language skills is positive at some levels of knowledge, because the individual cannot capture the full return of his input. However, as certain levels of language skills are reached by each individual, further increases yield declining marginal social benefits.

There are substitutes for knowledge in the production of communications, e.g., television services, as expressed by the partial of MB and P_G. If television services require fewer language skills than radio services, a fall in the price of television services reduces the marginal social benefits of knowledge in the communications industry. Society has an incentive to shift subsidies from knowledge to television services because it can increase the social benefits of the communications industry by purchasing more of the now lower priced television services for the same total expenditure.

The argument for income or expenditure transfers is direct and simple. It is expressed by the negative partial between MB and E/E*. If society wants to transfer current expenditures, the case for direct cash transfers is as strong as ever. However, if the goal is to shift the expenditure generation function, equation (2), i.e., to increase the permanent expenditures stream, there is a case for subsidizing one or more inputs in that function. Knowledge is the only input which can be subsidized. Time, the social discount rate, and exogenous market forces are beyond control. Income streams or cash can be transferred to subsidize the wealth position. However, as expressed by the partial between E/E* and ρ/r, individuals with low expenditures have low expenditures because they have high private discount rates. Cash or wealth transfers are likely to be spent largely on consumption activities.

Knowledge cannot be sold; it can only be used by an individual. More knowledge is likely to reduce the private discount rate. Finally, the social benefits of knowledge on expenditures production should probably not be discounted over a finite life. They are likely to be permanent. Increased demands for knowledge from a lower private discount rate and experience with the benefits of knowledge carry over to future generations, i.e., they are transferred over generations.

In summary, this argument does not eliminate the need for other subsidies where social benefits exist. Neither does it eliminate the need for cash transfers as a permanent program for some individuals. However, cash transfers and subsidies aimed at solving the poverty problem will not eliminate the problem if they do not change the basic set of inputs in the constraint functions of the individual in any permanent way. A cash grant increases resources during one period, but there is little carry-over to subsequent periods. Knowledge transfers appear to be the most promising method of permanently increasing the resources of the individual.

The social benefit function as defined here can either be added to the private demand function or subtracted from the marginal cost function. If subsidies are paid on the basis of the social benefit function, the quantity of new knowledge produced per period is greater than without the subsidy, although if an individual is on or near the boundary of his marginal cost function, the increase will be marginal.

An Extension With Two Types of Knowledge

There are many types of knowledge and distinguishing various types of knowledge may be of major importance in the model. An overly simplistic extension of this framework to two types of knowledge is briefly explored.

The two types of knowledge are consumption knowledge (K_C) and expenditures knowledge (K_E). Consumption knowledge enters only the consumption activities production function and expenditures knowledge enters only the expenditures production function, modifications of equations (1) and (2), respectively. Corresponding derived demand functions for consumption knowledge and expenditures knowledge are implied by these modified production functions.

There are now two knowledge production functions, one for K_C and another for K_E. The transformation of F and T_K into K_C and K_E is assumed to be identical for both types of knowledge. Depreciation may differ between the two types of knowledge. All constraints, appropriately modified, are still operative; the expenditures knowledge demand function and expenditures identity are now explicit. The social benefit function remains essentially as in equation (16), except that K_C is the only type of knowledge which enters the function directly.

The previous example of a fall in the price of television services in the communications network is extended to obtain a narrow and specific focus. First, with respect to the social benefit function, the fall in the price of television services has the same effect as previously, except that the substitution of television services is now for consumption knowledge and not for knowledge in general.

The individual's derived demand for consumption knowledge shifts to the left because of the reduction in the price of a substitute, television services. The individual no longer needs to maintain the same language ability to maintain his flow of information from the communications network. This decrease in demand with a corresponding decrease in the value of time may be partially offset by shifts in the demand for expenditures knowledge.

Shifts in the derived demand for expenditures knowledge result from changes in market forces coincident with the reduced price of television services. There is a probable increase in demand for expenditures knowledge to produce television services, offset, partially at least, by a reduction in the demand for K_E to produce radio services. This increase in demand for K_E results in higher earnings for individuals who have or acquire this type of knowledge, increasing their derived demand for K_E. The changes in P_T in the two demand functions cause a substitution of K_E for K_C. The two income effects, the fall in a P_G and the increase in Z, cause increases in the demand for K_C and K_E.

As the marginal value of time increases, both marginal cost functions shift upward. This combined with downward shifts in the demand functions, as resources are reallocated to produce more knowledge (K_C or K_E), decrease the quantities of both kinds of knowledge produced per period. The net result of all shifts is a probable increase in production of K_E per period and a decrease in K_C.

Knowledge depreciation, δ, is likely to be greater for K_E than for K_C. Technical change reduces the expected life of K_E. This reduces the demand for K_E and increases the quantity of K_E which must be produced each period for any given stock of K_E.

The effect of television alone in changing the relative quantities of consumption and expenditures knowledge would be marginal at most. But if this is the kind of change which has been occurring broadly in the economy, then the rather significant shifts in emphasis from general knowledge to specialized knowledge are the response to market forces through changes in relative prices, since general knowledge and specialized knowledge correspond approximately to K_C and K_E, respectively.

Implications for Research

Several implications of the model are briefly discussed in terms of the four general bodies of literature initially outlined. The model has not been extended in the direction of determining the market demand for and supply of knowledge. Such an extension is possible, however. The derived demand for goods and services can be obtained from the consumption activities production function. The supply of knowledge to the market is obtained from the expenditures generation function. The knowledge production function must also be incorporated, but this depends on the model (in the model with two types of knowledge, the incorporation is straightforward: consumption knowledge production is combined into the consumption activities function and expenditures knowledge into the expenditures function). The derived demand for goods and services and the supply of knowledge to the market combined with the goods and services production function(s) are basic relationships from which a market supply-demand model for knowledge could be derived. Such an extension can broaden and extend the results of aggregate production research.

The expenditures generation function takes a somewhat different view of income streams. The expenditures function makes explicit the relationship between human and non-human wealth, which is overlooked in the income stream literature. Income may be zero while an individual attends school, but expenditures are not. With zero income, the student (or his parents) decreases his non-human wealth while increasing human wealth, i.e., he sells income streams to produce knowledge.

In a world of perfect "knowledge" and perfect capital markets, this relationship can probably be ignored. But in a world of imperfection, the willingness of an individual to sell income streams to produce knowledge is likely to depend on the non-human income stream (non-human wealth). Further, the willingness of the capital market to purchase non-human wealth income streams differs from its willingness to purchase income streams based on projected returns from human wealth. The discontinuity to the individual from the capital market occurs at zero non-human wealth. As long as the individual owns non-human income streams, he can sell these at market rates and internally transfer the funds to knowledge production. When the individual owns no non-human income streams, he must sell expected future returns from knowledge. Expenditures generated in this way involve a higher discount rate and more constraints on how they can be used. The incorporation of non-human wealth and other sources of expenditures (social security at retirement) may provide significant advances in the results of empirical work on income streams.

With respect to public policy, it is important to distinguish among externalities from knowledge, permanent income or expenditure redistribution through knowledge, and general welfare redistribution, all of which may be goals of knowledge subsidies. The social benefit function establishes a basis for knowledge subsidies from externalities and permanent expenditure redistribution. The social benefits of knowledge have no direct relationship to the costs of producing knowledge.

Further, schooling is not the only source of factors of production for knowledge. Policy should take a broader view of the social benefits of knowledge than those provided by schools. An implication of the social benefit function is that the subsidy should be paid to individuals from the level at which the social benefits occur. It is the individual who acquires knowledge and the most efficient subsidy is one which allows the individual the greatest freedom to efficiently acquire knowledge as contrasted to the present system where the subsidy is only paid for acquiring knowledge in certain institutions. The public may be justified in subsidizing certain kinds of knowledge and not others, but this does not imply that the public must control the institutions. It must only control the kind of knowledge acquired by the individual as a result of the subsidy. Finally, if the social benefits of knowledge are national, public subsidy programs should be national. The major social benefits of knowledge to the communications network, for example, are probably national.

The knowledge production function yields several implications with respect to cross-sectional production functions of both individuals and of schools. The attempt to relate achievement test results to school inputs and student characteristics falls short on several accounts. Achievement test results are relatively narrow measures of knowledge, but measures of total achievement to the extent that they measure the stock of knowledge. School inputs measure only a subset of the total set of factors which enter the individual's knowledge production function. Student characteristics modify this criticism to the extent that they are proxies for other inputs and that they measure the time input of the individual. Broader measures of knowledge are needed along with more precise specification of either the individual's total knowledge production function or his "sub-production" function for that part of his knowledge acquired from the school system.

Another problem arises if there are different types of knowledge. Any school is geared to produce only certain types of knowledge, i.e., the school has production functions for certain types of knowledge into which the student is fitted or which the student fits into his own production function. With the present constraints of the school system on student choice of school, the student is not able, in many cases, to select a school which allows him to optimize his knowledge acquisition, not because he does not attempt to maximize but because the constraints change factor prices to the individual. The student will, of course, substitute other sources of knowledge factors for those of the school to the extent possible. But this invalidates the maximization assumption under which production functions are estimated because a school or the school system can increase its output of knowledge by changing its production functions or its constraints, but without increasing factor inputs. The only way to overcome this problem in the present school system is to find a way of measuring the extent to which the school's factor inputs are used by individuals and to adopt such a use measure rather than an availability measure for inputs.

A final, more general implication of the model is the importance of time. Each individual and each group of individuals are absolutely constrained by time. An individual can purchase time intensive services as substitutes for his time, but he cannot hire time as a direct input. An industry is not so constrained; it directly obtains more time by hiring people for more hours or more people, but only at the expense of consumption and knowledge production time or of some other industries. The difference is that an industrial firm produces goods and services which are not specific to any individual, while the individual produces consumption activities which are specific in the sense that only the individual can produce his own consumption activities. A change from the conceptual idea that an industrial firm hires raw labor and skills to one that it hires time and skills, the basic dichotomy behind the present model, may yield further insights. All individuals have a stock of knowledge. A firm hires the individual with the stock of knowledge in combination with time most nearly fitting the skills required by the position.

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