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Investment Performance of Public Commodity Pools: 1979 to 1989

by

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Abstract

This study investigates performance of public commodity pools both as a single randomly-selected pool and a market portfolio of pools over the 1979-1989 period. A market portfolio of public commodity pools provides superior investment performance relative to a randomly-selected pool. However, in general, this study provides no evidence that even a market portfolio of commodity pools is an attractive stand-alone investment. Nevertheless, there is some evidence that public commodity pools may improve the risk-return performance of a stock-bond portfolio. This evidence is conditional on the time period analyzed. Furthermore, a portfolio analysis is conducted using the lower brokerage, management, and incentive fees paid by institutional investors in commodity pools. The analysis reveals a substantial increase in the diversification benefits of adding commodity pools to a stock-bond portfolio. This suggests that the costs of public commodity pools form a significant deterrent to wider inclusion in investment portfolios.

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I. Introduction

Publicly-traded commodity pools have grown rapidly from a total equity of $7.2 million in one pool during January, 1975 to $1.7 billion in 118 pools during December, 1988 (Irwin and Brorsen, 1985; Basso, 1989).[1] With rapid growth has come increased focus on investment performance. However, results of academic studies of investment performance differ substantially. Brorsen and Irwin (1985), Murphy (1986), and Elton, Gruber, and Rentzler (1987, 1989, 1990) concluded that public commodity pools were inferior investment vehicles compared to other financial instruments. In contrast, Lintner (1983), Irwin and Brorsen (1985), and Irwin and Landa (1987) suggested that public commodity pools produce favorable investment returns.

An important difference between the studies which found inferior performance and the studies which found favorable performance is the methodology used. Brorsen and Irwin, Murphy, and Elton, Gruber, and Rentzler measured returns for a single random pool while Lintner, Irwin and Brorsen, and Irwin and Landa measured returns for a portfolio of pools. In addition, the length of the sample period and number of pools analyzed have varied substantially among the various studies.

This study investigates performance of public commodity pools both as a single randomly-selected pool and as a market portfolio of pools over the 1979-1989 period. The sample is the longest used in a study of commodity pool performance. Four aspects of investment performance are examined: 1) the attractiveness of public commodity pools as stand-alone investments, 2) the role of commodity pools in investment portfolios, 3) the predictability of commodity pools returns, and 4) the impact of costs on the portfolio performance of public commodity pools.

II. Data

Data was collected for all public commodity pools traded from January 1979 through December 1989. The pools include domestic U.S. pools which collect money predominantly from U.S. citizens, as well as off-shore commodity pools which invest in U.S. futures markets but are open only to foreign investors. The initial year was chosen because an analysis (presented in detail in the next section) revealed that ten pools are needed to approximately replicate market performance of all public commodity pools. Ten public commodity pools were traded in January 1979.[2] In contrast, during January 1978, only three public commodity pools were traded.

End of month commodity pool unit values and distributions per unit were collected for each public commodity pool. Sources included: 1) monthly reports by Norwood Securities from January 1979 to April 1982, 2) the "Funds Review" section published monthly in Futures (formerly Commodities) magazine from May 1982 through December 1989, 3) Managed Accounts Reports and 10-Q pools reports from the Securities and Exchange Commission, and 4) direct communication with commodity pool managers to obtain data otherwise not available.

Elton, Gruber, and Rentzler's (1987) procedures were followed for pools entering the data set and for pools that dissolved during the year.[3] A pool did not enter the calendar year's data set until its first January of trading. When a pool liquidated during the year, the dissolution value was reinvested in the market portfolio (average commodity pool) until the end of the calendar year of dissolution. This allowed the usually lower rate of return of a dissolving pool to be included in calculating average returns. Thus, an upward bias due to not including dissolved pools was avoided.

If a pool suspended trading, the unit value from the last month of trading was brought forward until trading resumed. This produced a zero percent monthly rate of return for as long as trading was suspended. Once trading began again, the usual calculations were resumed.

Monthly values of a broad range of financial investments were collected to provide comparisons with public commodity pools. They included buy-and-hold portfolios of common stocks, small stocks, U.S. Treasury-bills, intermediate government bonds, long-term government bonds, and long-term corporate bonds. Data for these instruments were taken from Stocks, Bonds, Bills, and Inflation: 1989 Yearbook by Ibbotson Associates, Inc. In addition, using the Commodity Research Bureau Composite Index of 27 commodity futures prices, returns were calculated to a passive futures buy-and-hold strategy.[4]

III. Public Commodity Pool Returns

Consistent with earlier studies, the total monthly return of a public commodity pool is defined as the change in unit value over a month plus cash distributions per unit during the month divided by the unit value at the end of the preceding month minus one. The formula assumes cash distributions are reinvested into the pool during the month it was distributed. This is consistent with the securities industry's handling of dividends (Stocks, Bonds, Bills and Inflation: 1989 Yearbook).

Two different strategies for investing in public commodity pools were examined: 1) a randomly-selected pool, and 2) a market portfolio of pools. A randomly-selected pool contains both the systematic and unsystematic risk associated with holding only one pool.[5] A market portfolio of pools contains only systematic risk.

To produce the rate of return for a randomly-selected commodity pool, it is assumed that funds are invested in a single randomly-selected pool at the beginning of the month.[6] Then, all available funds at the end of the month are invested in another randomly-selected commodity pool at the beginning of the following month. To produce the rate of return for a market portfolio of commodity pools, an equal amount of money is assumed to be invested in all pools at the beginning of the month. Available funds at the end of the month are then equally invested in all pools at the beginning of the next month.

Following Elton, Gruber, and Rentzler (1987), monthly and annual holding period investment horizons are used in this study. The two holding periods are used to reflect different time horizons which trader's may use when making investments. The average rate of return for a monthly holding period is generated by the average monthly arithmetic rate of return. The average rate of return over an annual holding period is generated by the average monthly geometric rate of return.[7]

As shown in Table 1, public commodity pool returns were highly variable across years for both the monthly and annual holding periods. For example, monthly holding period average returns for a random pool ranged from a high of 4.221 percent per month in 1979 to a low of -0.876 percent in 1986. Furthermore, average returns over a period of years was quite sensitive to the sample period selected. For a monthly holding period, random pool returns averaged 1.125 percent per month over 1979-1989, but decreased to 0.599 percent per month over 1982-1989 and 0.751 percent per month over 1985-1989.

Average return of the random pool and market portfolio diverged when considering annual holding period returns. Over the entire 1979-1989 sample period and the 1982-1989 and 1985-1989 sub-periods, the market portfolio outperformed the randomly-selected pool. This divergence is expected due to the fact that a geometric average will always be less than an arithmetic average, assuming the variance of the series is greater than zero (Grossman, 1987).

Standard deviation of a random pool is calculated as the standard deviation of monthly returns of a pool for a given year, averaged across all pools included in the sample year. A dissolved pool, which ended trading any month other than December was not included in calculating that year's standard deviation of a randomly-selected pool. The reason is that lack of trading during part of the year could bias the standard deviation calculation downward. For the market portfolio, its standard deviation is calculated by first averaging the monthly returns of all pools which traded during the month, and then calculating the standard deviation of the twelve monthly portfolio returns.

A randomly-selected commodity pool's monthly standard deviation ranged from 15.015 percent per month in 1980 to 7.413 percent per month in 1989 (Table 1). Over the entire 1979-1989 period, average monthly standard deviation was 9.972 percent. As expected, standard deviation for the market portfolio of commodity pools was substantially smaller. Its standard deviation for 1979-1989 averaged 6.678 percent per month, a one-third reduction in risk compared to holding a single randomly-selected commodity pool. The smaller standard deviation reflects the less than perfect positive correlation between the various commodity pools in the market portfolio.

The standard deviation comparisons suggest that the relationship between the number of pools held and portfolio risk may be valuable information. To investigate this relationship, note that portfolio variance may be expressed as follows if equal-weighting of pools is assumed (Elton and Gruber, 1987, p.30),

$$\sigma_p^2 = \frac{1}{N} \overline{\sigma_j^2} + \frac{N-1}{N} \overline{\sigma_{jk}} \qquad (1)$$

where

σ_p^2 = portfolio variance,

$\overline{\sigma_j^2}$ = average variance of the j pools (j=1,..,N),

$\overline{\sigma_{jk}}$ = average covariance between the j pools (j=1,..,N, k=1,..,N, j≠k),

N = number of pools.

Further, note that as N becomes large in equation (1), portfolio variance approaches the average covariance between the j pools. Thus, for an equally-weighted market portfolio of commodity pools, variance of the market portfolio approximately equals average covariance of the individual pools, assuming a sufficiently large N.

In order to analyze the relationship between number of pools held and portfolio risk, 1989 was selected as the base year for calculations. The 149 pools active in 1989 is a sufficiently large sample to ensure that the average covariance of individual pools can be accurately approximated by the variance of the market portfolio. Hence, average variance of the individual pools in (1) was assumed to equal variance of a random pool in 1989 (54.952 percent squared). Further, average covariance between the individual pools in (1) was assumed to equal variance of the market portfolio in 1989 (28.730 percent squared). With these inputs, N was varied between 1 and 100, and the resulting portfolio variance calculated.

As shown in Figure 1, portfolio standard deviation dropped quickly as the number of pools increased. Compared to a single pool, combining two pools reduced portfolio standard deviation from 7.413 to 6.460 percent. Combining five pools reduced the standard deviation to 5.820 percent, a decrease of 21.5 percent. Most of the risk reduction was achieved by holding ten pools, and risk of the market portfolio was closely replicated by holding thirty pools.

IV. Stand-Alone Performance

For comparative purposes, average returns and standard deviations of the alternative investments over 1979-1989 are reported in Table 2. Several observations are noteworthy. First, the standard deviation of commodity pool returns was greater than the standard deviation of returns for alternative investments. This was especially true for a randomly-selected pool. Second, returns for commodity pools were not favorable relative to alternative stock and bond investments over both 1982-1989 and 1985-1989. In contrast, over the entire 1979-1989 period, monthly and annual holding period returns for the market portfolio of pools, as well as the monthly holding period returns for a randomly-selected pool, exceed returns for bills and bonds, but not for common and small stocks. Third, over none of the sample periods did the annual holding period return of a randomly-selected commodity pool exceed the return of treasury bills or of the buy-and-hold futures strategy.

Given the well-known tradeoff between the return and risk of investments, a more formal test of stand-alone investment performance is needed. A widely-used method of ranking individual investment alternatives is the Sharpe ratio,

$$\frac{R_c - R_f}{\sigma_c} \tag{2}$$

where

R_c = the expected return of commodity pool c,

R_f = the risk-free return,

σ_c = the standard deviation of commodity pool c.

Sharpe ratios and the corresponding rankings of investments for the three sample periods are presented in Table 3. The most striking result is that under no scenario did a futures investment outrank a stock or bond investment, even for the longest time period. Among the alternative futures investments, except for the monthly holding period over 1982-1989, the market portfolio of commodity pools was either the highest ranked investment or tied for the highest rank.

V. Portfolio Performance: Breakeven Analysis

Elton, Gruber, and Rentzler (1987) show that a commodity pool should be added to a portfolio as long as,

$$\frac{R_c - R_f}{\sigma_c} > \frac{R_p - R_f}{\sigma_p} \cdot \rho_{cp} \tag{3}$$

where

R_c = the expected return of commodity pool c,

R_f = the risk-free return,

σ_c = the standard deviation of commodity pool c,

R_p = the expected return of portfolio p,

σ_p = the standard deviation of portfolio p,

ρ_{cp} = the correlation coefficient between commodity pool c and portfolio p.

Solving (3) for R_c yields the required, or breakeven, rate of return that a commodity pool must generate to enter the portfolio. If commodity pool returns exceed the breakeven return, then addition of commodity pools to the portfolio will improve the return-risk tradeoff of the portfolio.

A key component of the breakeven condition is the correlation between commodity pool returns and portfolio returns. Correlation coefficients between a random commodity pool and the alternative investments are shown in Table 4.[8] The correlation between commodity pool returns and stock and bond returns was near zero on average, as was the correlation between commodity pools and buy-and-hold futures. The average correlation coefficient of 0.643 between random pool returns and market portfolio returns indicates that the degree of co-movement in individual commodity pool returns was relatively high. In addition, monthly commodity pool returns did not show any evidence of correlation with the rate of inflation.[9]

For this study, public commodity pools were considered candidates to enter two common securities portfolios: one consisting of 100 percent common stocks and a second consisting of 60 percent common stocks and 40 percent long-term corporate bonds. Breakeven returns are presented in Table 5. Over the 1979-1989 sample period, returns for a randomly-selected pool exceeded breakeven returns for the monthly holding period, but not for the annual holding period. Average returns for the market portfolio of pools were greater than breakeven returns for both the monthly and annual holding periods. In contrast, when the sample is limited to 1982-1989 or 1985-1989, pool returns were substantially less than breakeven returns for all scenarios.

The breakeven results are helpful in explaining the different conclusions of earlier studies. First, studies that included data from the high return years of the late 1970s tended to find positive portfolio results (e.g. Irwin and Brorsen, 1985). Positive portfolio results were found in the current study only if 1979 was included in the sample. Second, studies that used samples solely from the 1980s have uniformly reported negative portfolio results (e.g. Elton, Gruber, and Rentzler, 1990). Similar results were found in this study for the 1982-1989 and 1985-1989 sample periods.

VI. Portfolio Performance: Optimal Portfolios

The breakeven analysis presented in the previous section showed that public commodity pools were beneficial additions to securities portfolios, if the analysis was based on the full 1979-1989 sample. However, the breakeven analysis did not generate the magnitude of improvement in portfolio return-risk that resulted from including commodity pools. To generate this information, optimal portfolios with and without commodity pools were estimated for the 1979-1989 period.

Elton and Gruber (1987, p.71) show that optimal portfolio proportions can be obtained by solving the following constrained optimization problem:[10]

$$\text{Maximize } \gamma_p = \frac{R_p - R_f}{\sigma_p} \tag{4}$$

Subject to

$$\sum_{i=1}^{N} X_i = 1$$

$$X_i \geq 0 \quad \text{for all } i$$

where

γ_p = the Sharpe Ratio of optimal portfolio p,

R_p = the expected return of optimal portfolio p,

σ_p = the standard deviation of optimal portfolio p,

R_f = the risk-free return,

X_i = the proportion of asset i in optimal portfolio p.

Since the objective function of (4) is non-linear, the optimization problem must be solved using numerical techniques. For this study, solutions were obtained using a numerical algorithm in the GAMS software package.

Recent research suggests that constraining portfolio proportions reduces estimation error when solving optimal portfolio problems (Frost and Savarino, 1988). Hence, optimal portfolios are found under an unconstrained and a constrained scenario. In the constrained scenario, the minimum and maximum portfolio proportions for stocks and bonds are set to equal the minimum and maximum U.S. capital market value weights over 1970-1984 (Ibbotson, Siegel and Love), while public commodity pool proportions may range from 0 to 10 percent.[11]

Results of the portfolio optimization for 1979-1989 are presented in Tables 6 and 7. A randomly-selected commodity pool is held only under the monthly holding period and no constraint scenario. In this case, pools represent five percent of the optimal portfolio, but the addition of pools improves the optimal portfolio's Sharpe Ratio a modest 1.18 percent. The market portfolio of pools is added to the portfolio under all four scenarios, including the maximum allowable proportion of 10 percent for the monthly holding period and constrained portfolio. Addition of the market portfolio of pools improves the optimal portfolio's Sharpe Ratio a maximum of 2.45 percent.

VII. Predictability of Returns

If returns and risks can be predicted, then this information can be used to improve the investment performance of public commodity pools. The tests proposed by Elton, Gruber, and Rentzler (1987) are employed in the analysis. The first test determines whether pools that have high returns or risks in one period also tend to have high values in the following period. This is accomplished by calculating correlation coefficients between average returns or risks for all adjacent years for all commodity pools that are present in the paired years. The second test is similar to the first, except that the population of pools is stratified into those with high, low, or average returns or risks for a given year.

Results of the correlation analysis are similar to those reported in previous studies (Table 8). If all pools are considered, only the correlation for the standard deviation, 0.45, appears to be large enough to be economically meaningful. The other correlations are between -0.10 and +0.10, levels not suggestive of the possibility of selecting better performing pools. The correlations are slightly larger if the sample is stratified into top, middle, and bottom thirds for a given year. However, given the small magnitude of the correlations, it is debatable whether any strategy to select public commodity pools can be used to obtain an economically meaningful increase in performance.

VIII. The Impact of Cost on Portfolio Performance

Performance problems of public commodity pools frequently have been attributed to high operating costs (e.g. Elton, Gruber, and, Rentzler, 1987). Estimates of the total operating costs of public commodity pools range from about 18 to 20 percent of annual equity (Irwin and Brorsen, 1985; Murphy, 1986; Basso, 1989).[12] By comparison, investment costs of stock mutual funds are about one percent of annual equity (Sharpe, 1981).

An analysis of the potential performance impacts of lower costs can be made using evidence from institutional pension fund investments in commodity pools. Institutions have negotiated much lower commission and management costs than those paid by public investors (Hecht, 1989 and Table 9). Costs for institutional commodity pools are 10 to 12 percent of annual equity, approximately eight percentage points less than costs for public commodity pools. The biggest cost reduction is in commissions, which are reduced from nine to two percent of annual equity. This reflects a much lower brokerage charge per trade.[13]

The analysis was conducted by adjusting monthly returns on the market portfolio of pools over 1979-1989 to reflect the lower costs paid by institutional investors. The adjustment required two steps. First, gross returns of public commodity pools were estimated. This entailed subtracting treasury bill returns from net public pool returns and then adding back the public pool costs. Second, the net return to institutional commodity pools was estimated by subtracting the costs of institutional investors from the estimated gross returns and adding back treasury bill returns. Complete details of the procedure are reported in the Appendix.

Lowering costs substantially impacted portfolio performance. As shown in Table 10, average returns of commodity pools after the cost adjustment exceed portfolio breakeven returns for all three sample periods. Moreover, average returns are generally considerably larger than the breakeven returns. These results stand in sharp contrast to the original breakeven results (Table 5), which indicated that public commodity pools were attractive additions to stock and bond portfolios only over 1979-1989.

Optimal portfolio proportions of commodity pools for 1979-1989 increased to about 30 percent in the unconstrained scenarios and to the maximum level of 10 percent in the constrained scenarios. Over 1982-1989 and 1985-1989, proportions ranged from about 2 to 8 percent of optimal portfolios. The earlier analysis found that adding pools increased the optimal portfolio's Sharpe Ratio a maximum of 2.45 percent (Tables 6 and 7). After adjusting for lower costs, the improvement ranged between 13.14 and 27.22 percent for the 1979-1989 sample period. Sharpe Ratios improved between 0.41 and 4.22 percent for the two sub-periods.

These results provide strong evidence of the impact of costs on the investment performance of public commodity pools. It appears that reductions in cost are important for the future of public commodity pools as competitive investments.

IX. Summary and Conclusions

The rapid growth of commodity pools has directed attention toward their investment performance. A number of academic studies have examined their performance; however, conclusions differ substantially. One explanation for the conflicting results is the use of different methodology, notably the use of the returns to a random commodity pool in studies which have found inferior performance versus the returns to a market portfolio of commodity pools in studies which have found acceptable performance. A second explanation is the sensitivity of results to the wide variety of data periods investigated.

This study uses monthly commodity return data for all public commodity pools active over January 1979 - December 1989 to compare results for both a randomly-selected pool and a market portfolio of pools. The sample is the largest sample used in a study of commodity pools.

Public commodity pool returns were sensitive to the period examined. For a monthly holding period, pool returns averaged 1.125 percent per month over 1979-1989, but decreased to 0.599 percent per month over 1982-1989 and 0.751 percent per month over 1985-1989.

In general, the market portfolio of pools outperformed the randomly-selected public pool as a stand alone investment. However, under no scenario did the market portfolio of pools outrank a stock or bond investment based on Sharpe Ratios. Thus, stand alone investment performance of public commodity pools was poor.

Not surprisingly, given the variation in public commodity pool returns over different time periods, the portfolio performance of commodity pools also was highly sensitive to the sample period considered. Over 1979-1989, returns for a randomly-selected pool exceeded portfolio breakeven returns for the monthly holding period only, while average returns for the market portfolio of pools were greater than breakeven returns for both the monthly and annual holding periods. In contrast, over the 1982-1989 and 1985-1989 samples, returns for both a randomly-selected pool and the market portfolio of pools were substantially less than breakeven returns.

To summarize the performance of commodity pools, a market portfolio of pools generally outperformed a randomly-selected commodity pool. However, the most important determinant of portfolio performance of commodity pools was the time period analyzed. In particular, the inclusion of 1979 was critical. The prudent conclusion is that additional years of observation are needed to confirm which period of analysis is consistent with long-term performance of public commodity pools.

The cost of investing in public commodity pools is often mentioned as a reason for their poor performance. When costs were reduced to the level which large institutional pension funds have been able to obtain, commodity pools entered stock and bond portfolios in all three sub-periods. Further, the return-risk tradeoff of stock-bond portfolios was improved as much as 27 percent. Therefore, it would appear that reductions in cost are important for the future of public commodity pools as competitive investments.

References

Basso, T.F. "A Review of Public and Private Futures Funds - 1988," working paper, Trendstat Capital Management, 1989.

Brorsen, B.W. and S.H. Irwin. "Examination of Commodity Fund Performance," Review of Research in Futures Markets, 4(1985): 84-94.

Brorsen, B.W. and S.H. Irwin. "Futures Funds and Price Volatility," Review of Futures Markets, 6(1987): 119-135.

Edwards, F.R. and C. Ma. "Commodity Pool Performance: Is the Information Contained in Pool Prospectuses Useful?" Journal of Futures Markets, 8(1988): 589-616.

Elton, E.J. and M.J. Gruber. Modern Portfolio Theory and Investment Analysis. New York, NY: John Wiley and Sons, 1987.

Elton, E.J., M.J. Gruber and J.C. Rentzler. "Professionally Managed, Publicly Traded Commodity Funds," Journal of Business, 60(1987): 175-199.

Elton, E.J., M.J. Gruber and J.C. Rentzler. "New Public Offerings, Information, and Investor Rationality: The Case of Publicly Offered Commodity Funds," Journal of Business, 62(1989): 1-15.

Elton, E.J., M.J. Gruber and J.C. Rentzler. "The Performance of Publicly Offered Commodity Funds." Financial Analysts Journal, (1990): 23-30.

Frost, P.A. and J.E. Savarino. "For Better Performance: Constrain Portfolio Weights." Journal of Portfolio Management, 14(1988): 29-34.

Futures Dimension Fund II L.P., Prospectus, Merrill Lynch, Pierce, Fenner and Smith, February 1989.

Grossman, S.J. "A Note on Elton, Gruber, and Rentzler's: "Professionally Managed Publicly Traded Commodity Funds," working paper, Department of Economics, Princeton University, October 1987.

Hecht, L. "The Commodities Conundrum." Institutional Investor, December 1989, pp. 191-195.

Hilliard, J. "Hedging Interest Rate Risk with Futures Portfolios Under Term Structure Effects." Journal of Finance, 39(1984): 1547-1570.

Ibbotson, R.G., L.B. Siegel, and K.S. Love. "World Wealth: U.S. and Foreign Market Values and Returns," Journal of Portfolio Management, 11(1985): 1-21.

Irwin, S.H. and B. W. Brorsen. "Public Futures Funds," Journal of Futures Markets, 5(1985): 463-485.

Irwin, S.H. and D. Landa. "Real Estate, Futures, and Gold as Portfolio Assets," Journal of Portfolio Management, (1987): 29-34.

Lintner, J. "The Potential Role of Managed Commodity-Financial Futures Accounts (and/or Funds) in Portfolios of Stocks and Bonds," paper presented at the annual conference of the Financial Analysts Federation, Toronto, Canada, May, 1983.

Murphy, J. A. "Futures Fund Performance: A Test of the Effectiveness of Technical Analysis," Journal of Futures Markets, 6(1986): 175-185.

Sharpe, W. Investments, Englewood Cliffs, N.J.: Prentice-Hall, 1981.

Stocks, Bonds, Bills and Inflation: 1989 Yearbook, Chicago: Ibbotson Associates, Inc., 1990.

Endnotes

1. Commodity pools also are known as commodity funds and futures funds. The official term in all regulatory matters is commodity pool, and hence, will be used throughout the paper.

2. Twelve pools reported monthly public data in January 1979 to Norwood Securities. However, The Talisman Fund and The Dunn Corporation Limited Partnership ceased reporting monthly data in April 1979 and January 1981, respectively. These pools were not included in the data set.

3. Most commodity pools are created to trade for a specific length of time (eg. The Futures Dimension Fund II L.P., Prospectus). However, a pool will cease trading before the specified time if the total equity or unit value falls below the prescribed minimum in the prospectus or an amount needed to trade effectively. The pool may also stop trading if performance is less than acceptable. In the eleven year period from 1979 through 1989, 49 pools ceased trading. Dissolution net asset values were obtained for 42 pools. The net asset value at the end of the last reported month of trading is used as the dissolution value for the remaining 7 pools. For a detailed examination of commodity pool dissolution, see Elton, Gruber, and Rentzler (1990).

4. Futures margins may be deposited in the form of interest-bearing instruments. Hence, buy-and-and hold futures returns are calculated as the sum of the change in the CRB Index and treasury bill returns (Hilliard, 1984).

5. Note that risk is defined relative to a "market" of all public commodity pools.

6. Equity-weighted returns were also calculated and were not significantly different than equal-weighted returns, so the latter were used in this study.

7. An attempt was made to replicate the commodity pool returns reported in Elton, Gruber, and Rentzler (1987). Over the period of July 1979 - June 1985, the following results were found:

	Average Returns MHP	Average Returns AHP	Standard Deviation
	- percent per month -		
EGR Study	0.73	-0.07	10.86
IKZ Study	0.79	0.10	10.48

Comparison of the above results suggests that the data and procedures used in this study closely replicate those of Elton, Gruber, and Rentzler.

8. Monthly returns of the market portfolio of pools exhibited nearly identical correlations with the alternative investments. Hence, only correlations for a randomly selected pool are presented.

9. However, if correlations are estimated using annual returns, then a positive relationship is found. For example, the annual correlation between the market portfolio of public commodity pools and the inflation rate over 1979-1989 is 0.712. This suggests that commodity pool returns are positively correlated with longer-run movements in inflation, but not short-run movements.

10. This formulation assumes riskless borrowing and lending is possible at the same rate and that short sales are not allowed.

11. The actual ranges are:

Common Stocks	45.5 to 64.3%
Small Stocks	4.3 to 7.3%
Intermediate-term Gov't Bonds	8.9 to 19.8%
Long-term Gov't Bonds	7.1 to 19.0%
Long-term Corporate Bonds	9.9 to 17.0%.

Note, in calculating the proportions it was assumed that the market portfolio consisted of only the above five securities.

12. These estimates do not account for initial "load" charges, which may be as high as 12 percent of invested funds.

13. Irwin and Brorsen (1985) reported that investors in their sample of public commodity pools often were charged full retail commission rates. The data in Table 9 imply that institutional investors have negotiated for brokerage rates nearly 80 percent lower than that paid by public investors (assuming similar trading strategies across the two investments).

Appendix

The adjustment of public commodity pool returns to the lower costs of institutional investors was done in two steps. First, gross public commodity pool returns for month t were calculated as follows,

$$GPCP_t = (NPCP_t - TB_t + CCPCP_t + MMPCP_t) \quad \text{if } (NPCP_t - TB_t + CCPCP_t + MMPCP_t) \leq 0 \tag{5a}$$

$$GPCP_t = (NPCP_t - TB_t + CCPCP_t + MMPCP_t) \, / \, (1 - (IPCP_t \, / \, 100)) \quad \text{if } (NPCP_t - TB_t + CCPCP_t + MMPCP_t) > 0 \tag{5b}$$

where

$GPCP_t$ = gross return of the market portfolio of public pools (percent per month),

$NPCP_t$ = net return of the market portfolio of public pools (percent per month),

TB_t = treasury bill return (percent per month),

$CCPCP_t$ = public pool commission cost (percent per month),

$MMPCP_t$ = public pool management cost (percent per month),

$IPCP_t$ = public pool incentive cost (percent of gross returns).

Note that calculation of the gross public commodity pool return is conditional on gross returns before incentive costs. If the latter return is less than or equal to zero, then no incentive costs are assumed to be incurred. Fixed values for commission, management, and incentive costs were assumed, and were based on data in the first row of Table 9. Monthly commission (0.775 percent per month) and management (0.417 percent per month) costs were calculated by dividing the annual figures by twelve. The incentive cost (20 percent of gross return) was applied directly.

The second step was the calculation of net commodity pool returns based on lower institutional costs. This return was calculated as follows,

$$NICP_t = GPCP_t - CCICP_t - MMICP_t + TB_t \quad \text{if } GPCP_t \leq 0 \qquad (6a)$$

$$NICP_t = GPCP_t\,(1 - (IICP_t/100)) - CCICP_t - MMICP_t + TB_t \quad \text{if } GPCP_t > 0 \qquad (6b)$$

where

$NICP_t$ = net return of the market portfolio of institutional pools (percent/month),

$GPCP_t$ = gross return of the market portfolio of public pools (percent/month),

TB_t = treasury bill return (percent/month),

$CCICP_t$ = institutional pool commission cost (percent/month),

$MMICP_t$ = institutional pool management cost (percent/month),

$IICP_t$ = institutional pool incentive cost (percent of gross returns).

Note that calculation of the net institutional commodity pool return is conditional on gross public commodity pool returns. If the latter return is less than or equal to zero, then no incentive costs are assumed to be incurred. Again, fixed values for commission, management, and incentive costs were assumed, and were based on data in the second row of Table 9. Monthly commission (0.167 percent per month) and management (0.208 percent per month) costs were calculated by dividing the annual figures by twelve. The incentive cost (25 percent of gross return) was applied directly.

An example will help illustrate the generation of the institutional commodity pool returns. Assume a net public commodity pool return and treasury bill return of 1.0 and 0.5 percent, respectively, for month t. Then, the gross public pool return is calculated as,

$$GPCP_t = (1.000 - 0.500 + 0.775 + 0.417) / (1 - (20/100)$$
$$= 2.115 \text{ percent,}$$

and the net institutional commodity pool return is,

$$NICP_t = 2.115 (1 - (25/100) - 0.167 - 0.208 + 0.5$$
$$= 1.711 \text{ percent.}$$

Table 1: Rates of Return and Standard Deviation for Public Commodity Pools, 1979-1989.

Year	Number of Pools	Randomly-selected Commodity Pool			Market Portfolio of Commodity Pools		
		Average Return			Average Return		
		MHP[1]	AHP[2]	Standard Deviation	MHP	AHP	Standard Deviation
		- percent per month -			- percent per month -		
1979	10	4.221	3.138	13.713	4.221	3.912	8.352
1980	15	2.520	1.716	15.015	2.520	2.284	7.320
1981	22	0.838	0.399	8.845	0.838	0.670	6.124
1982	43	0.518	0.053	9.436	0.518	0.327	6.450
1983	62	-0.577	-1.177	10.155	-0.577	-0.818	7.303
1984	78	1.098	0.585	9.741	1.098	0.863	7.381
1985	94	1.358	1.006	8.154	1.358	1.212	5.672
1986	98	-0.876	-1.350	9.299	-0.876	-1.066	6.498
1987	111	2.854	2.495	8.441	2.854	2.696	6.038
1988	128	0.715	0.202	9.482	0.715	0.481	7.415
1989	149	-0.297	-0.622	7.413	-0.297	-0.428	5.360
Average:[3]							
1979-89		1.125	0.586	9.972	1.125	0.911	6.678
1982-89		0.599	0.149	9.015	0.599	0.402	6.413
1985-89		0.751	0.346	8.558	0.751	0.571	6.166

[1] Monthly holding period.

[2] Annual holding period.

[3] The average return and standard deviation for a randomly-selected commodity pool are calculated as the averages of the individual year statistics. The average return and standard deviation for the market portfolio of commodity pools are calculated over the entire period.

Table 2: Rates of Return and Standard Deviation for Alternative Investments, 1979-1989.

Investment[1]	Sample Period								
	1979-1989			1982-1989			1985-1989		
	Average Return		Standard Deviation	Average Return		Standard Deviation	Average Return		Standard Deviation
	MHP[2]	AHP[3]		MHP	AHP		MHP	AHP	
	- percent per month -			- percent per month -			- percent per month -		
RS Comm. Pool	1.125	.586	9.972	.599	.149	9.015	.751	.346	8.558
MP Comm. Pools	1.125	.911	6.678	.599	.402	6.413	.751	.571	6.166
B&H Futures	.786	.732	3.291	.572	.529	2.983	.489	.412	2.792
Common Stocks	1.471	1.362	4.652	1.569	1.455	4.766	1.694	1.559	5.107
Small Stocks	1.564	1.396	5.671	1.195	1.051	5.209	.989	.823	5.491
T-Bills	.723	.723	.230	.635	.635	.163	.551	.551	.108
IT Gov't Bonds	.911	.887	2.249	1.057	1.043	1.681	.913	.901	1.548
LT Gov't Bonds	.973	.894	4.023	1.324	1.267	3.401	1.265	1.207	3.467
LT Corp. Bonds	.966	.899	3.704	1.365	1.322	2.980	1.205	1.173	2.544

[1] RS Comm. Pool: Randomly-Selected Commodity Pool; MP Comm. Pools: Market Portfolio of Commodity Pools; B&H Futures: Buy-and-Hold Futures; T-Bills: Treasury Bills; IT Gov't Bonds: Intermediate-term Government Bonds; LT Gov't Bonds: Long-term Government Bonds; LT Corp. Bonds: Long-term Corporate Bonds.
[2] Monthly holding period.
[3] Annual holding period.

Table 3: Sharpe Ratio and Rank for Alternative Investments, 1979-1989.

Investment[1]	Sample Period					
	1979-1989		1982-1989		1985-1989	
	MHP[2]	AHP[3]	MHP	AHP	MHP	AHP
	- Sharpe ratio -					
RS Comm. Pool	.040	-.014	-.004	-.054	.023	-.024
MP Comm. Pools	.060	.028	-.006	-.036	.032	.003
B&H Futures	.019	.003	-.021	-.036	-.022	-.050
Common Stocks	.161	.137	.196	.172	.224	.197
Small Stocks	.148	.119	.108	.080	.080	.050
IT Gov't Bonds	.084	.073	.251	.243	.234	.226
LT Gov't Bonds	.062	.043	.203	.186	.206	.189
LT Corp. Bonds	.066	.048	.245	.231	.256	.244
	- Sharpe ratio rank -					
RS Comm. Pool	7	8	6	8	7	7
MP Comm. Pools	6	6	7	6	6	6
B&H Futures	8	7	8	6	8	8
Common Stocks	1	1	4	4	3	3
Small Stocks	2	2	5	5	5	5
IT Gov't Bonds	3	3	1	1	2	2
LT Gov't Bonds	5	5	3	3	4	4
LT Corp. Bonds	4	4	2	2	1	1

[1] RS Comm. Pool: Randomly-Selected Commodity Pool; MP Comm. Pools: Market Portfolio of Commodity Pools; B&H Futures: Buy-and-Hold Futures; IT Gov't Bonds: Intermediate-term Government Bonds; LT Gov't Bonds: Long-term Government Bonds; LT Corp. Bonds: Long-term Corporate Bonds.

[2] Monthly holding period.

[3] Annual holding period.

Table 4: Correlation Between a Randomly-selected Commodity Pool and Other Financial Investments, 1979-1989.[1]

Year	Number of Pools	Investment[2]								
		MP of Comm. Pools	B&H Futures	Common Stocks	Small Stocks	T-bills	IT Gov't Bonds	LT Gov't Bonds	LT Corp. Bonds	Inflation
		- correlation coefficient -								
1979	10	.641	.395	.086	.139	-.144	.319	.379	.357	.094
1980	13	.389	.121	.098	.137	-.217	-.105	-.265	-.229	-.056
1981	22	.624	-.359	-.056	-.066	.161	.092	.154	.204	.091
1982	43	.676	-.287	-.115	-.228	-.027	-.261	-.067	-.130	.145
1983	60	.708	.389	-.120	-.093	.334	-.193	-.363	-.257	.176
1984	77	.672	-.425	-.320	-.324	-.073	.153	.140	.203	-.110
1985	88	.635	-.155	.299	.404	.127	-.120	-.033	.008	-.255
1986	94	.655	-.195	.310	.305	.343	.385	.462	.485	-.378
1987	106	.705	.496	.143	.134	-.309	-.221	-.192	-.174	.008
1988	124	.744	.486	.240	.071	.085	.156	.172	.239	.019
1989	144	.622	-.334	.383	.368	.111	.162	.205	.240	.143
Average:										
1979-89		.643	.012	.086	.052	.036	.033	.054	.086	-.011
1982-89		.677	-.003	.103	.046	.074	.008	.041	.077	-.032
1985-89		.672	.060	.275	.202	.072	.072	.123	.160	-.093

[1] All correlations are based on the monthly returns of the investments.
[2] MP of Comm. Pools: Market Portfolio of Commodity Pools; B&H Futures: Buy-and-Hold Futures; IT Gov't Bonds: Intermediate-term Government Bonds; LT Gov't Bonds· Long-term Government Bonds; LT Corp. Bonds: Long-term Corporate Bonds.

Table 5: Portfolio Breakeven Analysis for Public Commodity Pools, 1979-1989.

Sample Period Investment Portfolio	Randomly-selected Commodity Pool				Market Portfolio of Commodity Pools			
	Monthly Holding Period		Annual Holding Period		Monthly Holding Period		Annual Holding Period	
	Breakeven Return	Average Return	Breakeven Return	Average Return	Breakeven Return	Average Return	Breakeven Return	Average Return
	- percent per month -				- percent per month -			
1979-1989:								
100% Stock[1]	0.861	1.125*	0.841	0.586	0.872	1.125*	0.850	0.911*
60% Stock, 40% Bonds[2]	0.878	1.125*	0.861	0.586	0.867	1.125*	0.850	0.911*
1982-1989:								
100% Stock	0.817	0.599	0.795	0.149	0.757	0.599	0.743	0.402
60% Stock, 40% Bonds	0.880	0.599	0.864	0.599	0.796	0.599	0.785	0.402
1985-1989:								
100% Stock	1.078	0.751	1.016	0.346	0.974	0.751	0.924	0.571
60% Stock, 40% Bonds	1.196	0.751	1.156	0.346	1.093	0.751	1.059	0.571

Note: A star indicates that the average return of public commodity pools exceeds the breakeven return necessary for entry into an investment portfolio.

[1] 100% common stocks.

[2] 60% common stocks and 40% long-term corporate bonds.

Table 6: Optimal Portfolio Results for A Randomly-Selected Public Commodity Pool, 1979-1989.

Optimal Portfolio	Unconstrained Portfolio[1]		Constrained Portfolio[2]	
	MHP[3]	AHP[4]	MHP	AHP
Proportions:[5]				
RS Commodity Pool	0.050	0.000	0.000	0.000
Common Stocks	0.398	0.536	0.617	0.632
Small Stocks	0.173	0.067	0.073	0.073
IT Gov't Bonds	0.378	0.397	0.140	0.125
LT Gov't Bonds	0.000	0.000	0.071	0.071
LT Corp. Bonds	0.000	0.000	0.099	0.099
Expected Return (percent/month)	1.258	1.176	1.314	1.226
Standard Deviation (percent/month)	3.131	3.170	3.623	3.674
Sharpe Ratio of Optimal Portfolio:				
With Commodity Pools	0.171	0.143	0.163	0.137
Without Commodity Pools	0.169	0.143	0.163	0.137
Change	+ 1.18%	0.00%	0.00%	0.00%

[1] No constraints on optimal portfolio proportions.
[2] Minimum and maximum optimal portfolio proportions for stocks and bonds are set to equal the minimum and maximum U.S. capital market proportions over 1970-1984 (Ibbotson, Siegel and Love). These are: Common Stocks, 45.5 to 64.3%; Small Stocks: 4.3 to 7.3%; IT Gov't Bonds: 8.9 to 19.8%; LT Gov't Bonds: 7.1 to 19.0%; LT Corp. Bonds: 9.9 to 17.0%. Commodity Pool proportions range from 0 to 10%.
[3] Monthly holding period.
[4] Annual holding period.
[5] RS Commodity Pool: Randomly-Selected Commodity Pool; IT Gov't Bonds: Intermediate-term Government Bonds; LT Gov't Bonds: Long-term Government Bonds; LT Corp. Bonds: Long-term Corporate Bonds.

Table 7: Optimal Portfolio Results for the Market Portfolio of Public Commodity Pools, 1979-1989.

Optimal Portfolio	Unconstrained Portfolio[1] MHP[3]	Unconstrained Portfolio[1] AHP[4]	Constrained Portfolio[2] MHP	Constrained Portfolio[2] AHP
Proportions:[5]				
MP Commodity Pools	0.096	0.028	0.100	0.021
Common Stocks	0.397	0.524	0.544	0.617
Small Stocks	0.142	0.060	0.073	0.073
IT Gov't Bonds	0.365	0.388	0.113	0.119
LT Gov't Bonds	0.000	0.000	0.071	0.071
LT Corp. Bonds	0.000	0.000	0.099	0.099
Expected Return (percent/month)	1.247	1.167	1.295	1.219
Standard Deviation (percent/month)	3.036	3.105	3.427	3.624
Sharpe Ratio of Optimal Portfolio:				
With Commodity Pools	0.172	0.143	0.167	0.137
Without Commodity Pools	0.169	0.143	0.163	0.137
Change	+ 1.77%	0.00%	+ 2.45%	0.00%

[1] No constraints on optimal portfolio proportions.
[2] Minimum and maximum optimal portfolio proportions for stocks and bonds are set to equal the minimum and maximum U.S. capital market proportions over 1970-1984 (Ibbotson, Siegel and Love). These are: Common Stocks, 45.5 to 64.3%; Small Stocks: 4.3 to 7.3%; IT Gov't Bonds: 8.9 to 19.8%; LT Gov't Bonds: 7.1 to 19.0%; LT Corp. Bonds: 9.9 to 17.0%. Commodity Pool proportions range from 0 to 10%.
[3] Monthly holding period.
[4] Annual holding period.
[5] MP Commodity Pool: Market Portfolio of Commodity Pools; IT Gov't Bonds: Intermediate-term Government Bonds; LT Gov't Bonds: Long-term Government Bonds; LT Corp. Bonds: Long-term Corporate Bonds.

Table 8: Correlation of Commodity Pool Performance Between Year t and Year t-1, 1979-1989.

Sample	Number of Paired Years	Average Return		Standard Deviation	Sharpe Ratio	
		MHP[1]	AHP[2]		MHP	AHP
		- correlation coefficient -				
All Pools	596	-.057	-.104	.451	.068	.056
Top 1/3 of Pools	204	-.077	-.126	.191	-.080	-.062
Middle 1/3 of Pools	194	-.202	-.240	.075	-.211	-.233
Lower 1/3 of Pools	198	-.143	-.229	.295	.098	.071

[1] Monthly holding period.

[2] Annual holding period.

Table 9: Costs of Futures Investments.

Type of Futures Investment	Cost Category			
	Commissions (annual percent of equity)	Management (annual percent of equity)	Incentive (annual percent of gross trading profits)	Total (annual percent of equity)
Public Commodity Pools	9.3	5.0	20.0	18 to 20
Institutional Commodity Pools	2.0	2.5	25.0	10 to 12

Sources: Irwin and Brorsen (1985), Murphy (1986), Basso (1989), Hecht (1989)

Table 10: Portfolio Breakeven Analysis for the Market Portfolio of Public Commodity Pools after Cost Adjustment, 1979-1989.

Sample Period Investment Portfolio	Monthly Holding Period		Annual Holding Period	
	Breakeven Return	Average Return	Breakeven Return	Average Return
	- percent per month -			
1979-1989:				
100% Stock[1]	0.867	1.725*	0.846	1.539*
60% Stock, 40% Bonds[2]	0.862	1.725*	0.846	1.539*
1982-1989:				
100% Stock	0.753	1.219*	0.739	1.044*
60% Stock, 40% Bonds	0.789	1.219*	0.779	1.044*
1985-1989:				
100% Stock	0.961	1.364*	0.912	1.207*
60% Stock, 40% Bonds	1.075	1.364*	0.982	1.207*

Note: A star indicates that the average return of public commodity pools exceeds the breakeven return necessary for entry into an investment portfolio.

[1] 100% common stocks.

[2] 60% common stocks and 40% long-term corporate bonds.

Table 11: Optimal Portfolio Results for the Market Portfolio of Public Commodity Pools after Cost Adjustment, 1979-1989.

Optimal Portfolio	Unconstrained Portfolio[1]		Constrained Portfolio[2]	
	MHP[3]	AHP[4]	MHP	AHP
Proportions:[5]				
MP Commodity Pools	0.304	0.294	0.100	0.100
Common Stocks	0.323	0.405	0.459	0.472
Small Stocks	0.077	0.000	0.073	0.060
IT Gov't Bonds	0.297	0.301	0.198	0.198
LT Gov't Bonds	0.000	0.000	0.071	0.071
LT Corp. Bonds	0.000	0.000	0.099	0.099
Expected Return (percent/month)	1.388	1.271	1.306	1.209
Standard Deviation (percent/month)	3.095	3.048	3.143	3.140
Sharpe Ratio of Optimal Portfolio:				
With Commodity Pools	0.215	0.180	0.186	0.155
Without Commodity Pools	0.169	0.143	0.163	0.137
Change	+ 27.22%	+ 25.87%	+ 14.11%	+ 13.14%

[1] No constraints on optimal portfolio proportions.
[2] Minimum and maximum optimal portfolio proportions for stocks and bonds are set to equal the minimum and maximum U.S. capital market proportions over 1970-1984 (Ibbotson, Siegel and Love). These are: Common Stocks, 45.5 to 64.3%; Small Stocks: 4.3 to 7.3%; IT Gov't Bonds: 8.9 to 19.8%; LT Gov't Bonds: 7.1 to 19.0%; LT Corp. Bonds: 9.9 to 17.0%. Commodity Pool proportions range from 0 to 10%.
[3] Monthly holding period.
[4] Annual holding period.
[5] MP Commodity Pool: Market Portfolio of Commodity Pools; IT Gov't Bonds: Intermediate-term Government Bonds; LT Gov't Bonds: Long-term Government Bonds; LT Corp. Bonds: Long-term Corporate Bonds.

Table 12: Optimal Portfolio Results for the Market Portfolio of Public Commodity Pools after Cost Adjustment, 1982-1989.

Optimal Portfolio	Unconstrained Portfolio[1]		Constrained Portfolio[2]	
	MHP[3]	AHP[4]	MHP	AHP
Proportions:[5]				
MP Commodity Pools	0.081	0.060	0.078	0.044
Common Stocks	0.158	0.141	0.455	0.455
Small Stocks	0.000	0.000	0.043	0.043
IT Gov't Bonds	0.761	0.799	0.183	0.198
LT Gov't Bonds	0.000	0.000	0.071	0.090
LT Corp. Bonds	0.000	0.000	0.170	0.170
Expected Return (percent/month)	1.151	1.101	1.380	1.298
Standard Deviation (percent/month)	1.744	1.711	2.931	2.938
Sharpe Ratio of Optimal Portfolio:				
With Commodity Pools	0.296	0.272	0.254	0.226
Without Commodity Pools	0.284	0.266	0.250	0.225
Change	+ 4.22%	+ 2.25%	+ 1.60%	+ 0.44%

[1] No constraints on optimal portfolio proportions.
[2] Minimum and maximum optimal portfolio proportions for stocks and bonds are set to equal the minimum and maximum U.S. capital market proportions over 1970-1984 (Ibbotson, Siegel and Love). These are: Common Stocks, 45.5 to 64.3%; Small Stocks: 4.3 to 7.3%; IT Gov't Bonds: 8.9 to 19.8%; LT Gov't Bonds: 7.1 to 19.0%; LT Corp. Bonds: 9.9 to 17.0%. Commodity Pool proportions range from 0 to 10%.
[3] Monthly holding period.
[4] Annual holding period.
[5] MP Commodity Pool: Market Portfolio of Commodity Pools; IT Gov't Bonds: Intermediate-term Government Bonds; LT Gov't Bonds: Long-term Government Bonds; LT Corp. Bonds: Long-term Corporate Bonds.

Table 13: Optimal Portfolio Results for the Market Portfolio of Public Commodity Pools after Cost Adjustment, 1985-1989.

Optimal Portfolio	Unconstrained Portfolio[1]		Constrained Portfolio[2]	
	MHP[3]	AHP[4]	MHP	AHP
Proportions:[5]				
MP Commodity Pools	0.071	0.049	0.053	0.016
Common Stocks	0.226	0.208	0.455	0.455
Small Stocks	0.000	0.000	0.043	0.043
IT Gov't Bonds	0.222	0.285	0.089	0.143
LT Gov't Bonds	0.000	0.000	0.190	0.172
LT Corp. Bonds	0.480	0.458	0.170	0.170
Expected Return (percent/month)	1.262	1.177	1.412	1.301
Standard Deviation (percent/month)	2.283	2.190	3.168	3.092
Sharpe Ratio of Optimal Portfolio:				
With Commodity Pools	0.311	0.286	0.272	0.243
Without Commodity Pools	0.307	0.284	0.270	0.242
Change	+ 1.30%	+ 0.70%	+ 0.74%	+ 0.41%

1 No constraints on optimal portfolio proportions.
2 Minimum and maximum optimal portfolio proportions for stocks and bonds are set to equal the minimum and maximum U.S. capital market proportions over 1970-1984 (Ibbotson, Siegel and Love). These are: Common Stocks, 45.5 to 64.3%; Small Stocks: 4.3 to 7.3%; IT Gov't Bonds: 8.9 to 19.8%; LT Gov't Bonds: 7.1 to 19.0%; LT Corp. Bonds: 9.9 to 17.0%. Commodity Pool proportions range from 0 to 10%.
3 Monthly holding period.
4 Annual holding period.
5 MP Commodity Pool: Market Portfolio of Commodity Pools; IT Gov't Bonds: Intermediate-term Government Bonds; LT Gov't Bonds: Long-term Government Bonds; LT Corp. Bonds: Long-term Corporate Bonds.

Figure 1. Effect of the Number of Commodity Pools on Portfolio Risk