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A Precipitation Probability Computer Program

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INTRODUCTION

The following computer program was written and compiled in PDQ FORTRAN (1620 General Program Library 2.0.031) for the purpose of computing amounts of precipitation associated with various probability levels. It is similar to that written by McKay (4) but with several modifications to allow it to operate over a wider variety of values.

The input data consists of a number of years of record of precipitation. This particular program operates on monthly and annual precipitation. The assumption is made that precipitation amounts follow the incomplete gamma distribution (5). Two parameters of the gamma distribution, gamma and beta, are estimated from the data. For gamma values of 100 or less, precipitation corresponding to particular percentage points of the incomplete gamma distribution are computed. For gamma values of more than 100, precipitation corresponding to percentage points of the normal distribution are used (5).

The incomplete gamma distribution may be expressed:

$$P(m,x) = \frac{1}{\Gamma(m)} \int_0^x b^{m-1} e^{-b} \, d\, b,$$

where m= the gamma parameter, $b = X/\beta$ (β = beta parameter), and x the value of the precipitation associated with the probability P (m, x).

Thom (6) showed that x for a given probability may be estimated by a Newton approximation as follows:

Let $j = i - 1$

$$x_i = x_j - \frac{x_j}{m} \left[1 + \frac{x}{m+1} + \frac{x^2}{(m+1)(m+2)} + \frac{x^3}{(m+1)(m+2)(m+3)} - \frac{P\Gamma(m) e^{x_j}}{x_j^{m-1}} \right],$$

where symbols represent values as defined above. Thom (6) also prepared direct and inverse tables of the gamma integral.

In the computer program, the initial estimate of X_0 equals the gamma parameter minus one. The approximation begins at that point. The quantity in the brackets is defined in the program as S and is estimated to the desired accuracy by an iterative procedure.

The IBM 1620, which was used for development of this program, is limited to numbers of the magnitude 10 ± 49. Gamma functions of numbers over 36 resulted in operations beyond the capability of the computer. Therefore, the terms $\Gamma(m)$ and X^{m-1} were expressed as log base e. Tables of the gamma function were used and linear interpolation between gamma, gamma + 1/3, gamma + 1/2, and gamma + 2/3 was used if the gamma parameter fell between 36 and 100. (Tables of log base 10 of the gamma function (1) were converted to log base e.)

If the gamma parameter was less than 36, an algorithm due to Collinge (2) was used to compute the gamma function.

OPERATING PROCEDURE

Input Cards

1. A card with 13 A3 monthly name abbreviations. These abbreviations appear on the output.
2. A card with five F8.5 values of standard deviation corresponding to the normal curve probabilities which represent the last six rainfall probability levels.

[1]The authors wish to express appreciation to G. F. McKay, Environmental Science Services Administration, National Weather Records Center, Asheville, North Carolina, for providing the Fortran IV program from which this routine was developed.

[2]Statistician, Ohio Agricultural Research and Development Center; and State Climatologist, Environmental Science Services Administration, Weather Bureau.

3. Sixty-five cards containing X-35, the value of log base e of the gamma functions of X = 36 to 100, X + 0.33, X + 0.5, and X + 0.67 (format is I3,4E16.8).
4. For each year of record: a) 12 cards with station, year, month, and monthly precipitation; and b) one card with station, year, and annual precipitation. The format is
   cc 1-6 = Station; 8-9 = Year; 10-11 = Month; 43-46 = Total precipitation on monthly cards; 65-68 = Total precipitation on anuual cards; 80 = 3 on annual cards.
5. Following the data one card with —1 in cc 79-80.
6. During the input, the following checks are performed on the tables and the data.
   a. X-35 (X = 36 to 100) is checked for sequence. A sequence error requires that the entire table be re-entered after sequence correction.
   b. During the data reading, the station number of each monthly and annual card is checked against the first month. No year to year check of station is made.
   c. A check of the sequence of the monthly cards is also made. Any error in b or c sends the program back to read the entire year which is in error.

The input data cards being used in this program are produced by two weather summary programs developed by Lytle and Martens (3). The FORTRAN program could easily be modified to utilize other input formats. Examples of the inputs are listed on page 5.

Program Checks and Alternatives

1. Two iterative loops may cause difficulty. The first is connected with the hypergeometric series which is used to evaluate S. The change in the estimate of S is called T in the program and is tested against TEST. The value of TEST is arbitrarily set at 0.0001. The limit for the number of iterations in computing S is 1000. In the data processed to date, 10 to 30 iterations have been sufficient.

SWITCH 1 on will print out current values for the month, the probability level index, the number of iterations, the estimate of S, the estimate of T, and TEST.

SWITCH 2 on allows a manual entry on the typewriter of a new value of TEST. An F10.0 format is used so one must be careful to type in 10 characters to assure replacement of other digits which may be in memory.

2. The second iterative estimate is performed on rainfall divided by the beta parameter. This estimate is called X_1 in the program. It is possible for this iteration to proceed indefinitely. After 10 iterations, a message to that effect is automatically typed out.

SWITCH 3 on will print out current values for the month, the probability level index, the number of iterations, the estimate of X_1, the value of the change in X_1, and DEST which is the criterion against which DX (change in X_1) is tested.

SWITCH 4 on allows entry of a new value of DX.

In approximately 30 trials of actual Ohio weather data, the program never exceeded 10 iterations.

LITERATURE CITED

1. Abramowitz, Milton and Irene A. Stegun. 1965. Handbook of mathematical functions with formulas, graphs, and mathematical tables. National Bureau of Standards, Applied Mathematics Series 55.
2. Collinge, Robert M. 1961. Gamma function. Association of Computing Machinery, Algorithm 31.
3. Lytle, W. F. and Harry A. Martens. 1964. SPS weather summary SDSU. Personal communication.
5. Thom, H. C. S. 1958. A note on the gamma distribution, Monthly Weather Review 86 (4):117-122.
6. Thom, H. C. S. 1967 (in press). Direct and inverse tables of the gamma distribution. ESSA-EDS Tech. Memo No. 6.

EXAMPLES OF DATA INPUT

Monthly Name Card

JANFEBMARAPRMAYJUNJULAUGSEPOCTNOVDECANN

Normal Curve Parameter Card

0.25335 0.52440 0.84162 1.28155 1.64485

Log Base e of Gamma Functions

X-35	GAMMA(X)	GAMMA(X+1/3)	GAMMA(X+1/2)	GAMMA(X+2/3)
1	.92136373E 02	.93327790E 02	.93924663E 02	.94522307E 02
2	.95719898E 02	.96920533E 02	.97521983E 02	.98124184E 02
3	.99330824E 02	.10054043E 03	.10114633E 03	.10175297E 03
4	.10296842E 03	.10418675E 03	.10479700E 03	.10540795E 03
	.	.	.	.
	.	.	.	.
	.	.	.	.
	.	.	.	.
65	.35913496E 03	.36066892E 03	.36143631E 03	.36220397E 03

Data Follows

Example of 1 Year's Data

This Is a Test Deck and Does Not Contain All of the Information That Is Output by Lytle's Program.

```
331234 3101                                0102
331234 3102                                0232
331234 3103                                0213
331234 3104                                0543
331234 3105                                0320
331234 3106                                0376
331234 3107                                0427
331234 3108                                0488
331234 3109                                0452
331234 3110                                0211
331234 3111                                0277
331234 3112                                0375
331234 31                                        4016              3
```

Several Years of Data Would Follow.

A Trailer Card with a —1 in cc 79-80 is Required After the Data Set.

EXAMPLE OF OUTPUT

		PRECIPITATION WITH PROBABILITY EQUAL OR LESS THAN										
STATION		.05	.10	.20	.30	.40	.50	.60	.70	.80	.90	.95
333029	JAN	1.14	1.48	2.00	2.44	2.86	3.30	3.78	4.35	5.08	6.22	7.27
333029	FEB	.86	1.18	1.66	2.09	2.52	2.97	3.46	4.05	4.82	6.03	7.16
333029	MAR	1.56	1.95	2.50	2.96	3.39	3.83	4.32	4.87	5.58	6.67	7.66
333029	APR	1.40	1.75	2.27	2.69	3.10	3.52	3.97	4.49	5.16	6.19	7.14
333029	MAY	1.68	2.00	2.45	2.81	3.15	3.49	3.85	4.26	4.78	5.57	6.28
333029	JUN	1.52	1.87	2.35	2.75	3.13	3.52	3.93	4.41	5.02	5.94	6.79
333029	JUL	1.76	2.16	2.71	3.17	3.60	4.03	4.50	5.04	5.73	6.77	7.72
333029	AUG	1.34	1.67	2.16	2.56	2.95	3.34	3.77	4.26	4.89	5.86	6.75
333029	SEP	1.13	1.41	1.81	2.15	2.47	2.79	3.14	3.55	4.07	4.86	5.59
333029	OCT	.39	.58	.89	1.18	1.47	1.79	2.15	2.58	3.16	4.08	4.96
333029	NOV	.83	1.10	1.51	1.87	2.21	2.57	2.97	3.44	4.05	5.01	5.89
333029	DEC	.96	1.22	1.61	1.93	2.24	2.56	2.91	3.31	3.83	4.63	5.37
333029	ANN	33.14	34.75	36.77	38.28	39.59	40.85	42.14	43.54	45.22	47.63	49.69

FORTRAN SOURCE PROGRAM

```
C         PRECIPITATION PROBABILITIES
      DIMENSION GF(65),GF3(65),GF5(65),GF6(65),SSS(13),CN(11)
      DIMENSION PL(11),U( 35),R(11),PC(13)
      DIMENSION SS(13),SL(13),AV(13),AL(13),ALA(13),AM(13)
C           SETS PROBABILITY LEVELS
      PL(1)=0.05
      PL(11)=0.95
      DO 72 J=2,10
      PL(J)=J-1
   72 PL(J)=PL(J)/10.
C           SETS GAMMA FUNCTION COEFFICIENTS
      A=.16063118E-02
      V=.51589951E-02
      C=.44511400E-02
      D=.72110256E-01
      E=.82111740E-01
      F=.41177419
      G=.42278746
      H=.99999997
C           GAMMA FUNCTION TABLE INTERPOLATION POINTS
      Z1=.16666667
      Z3=.33333333
      Z5=.5
      Z6=.66666667
C           MONTH IDENTIFICATION
      READ 610,AM(1),AM(2),AM(3),AM(4),AM(5),AM(6),AM(7),AM(8),AM(9),
     1AM(10),AM(11),AM(12),AM(13)
  610 FORMAT(13A3)
```

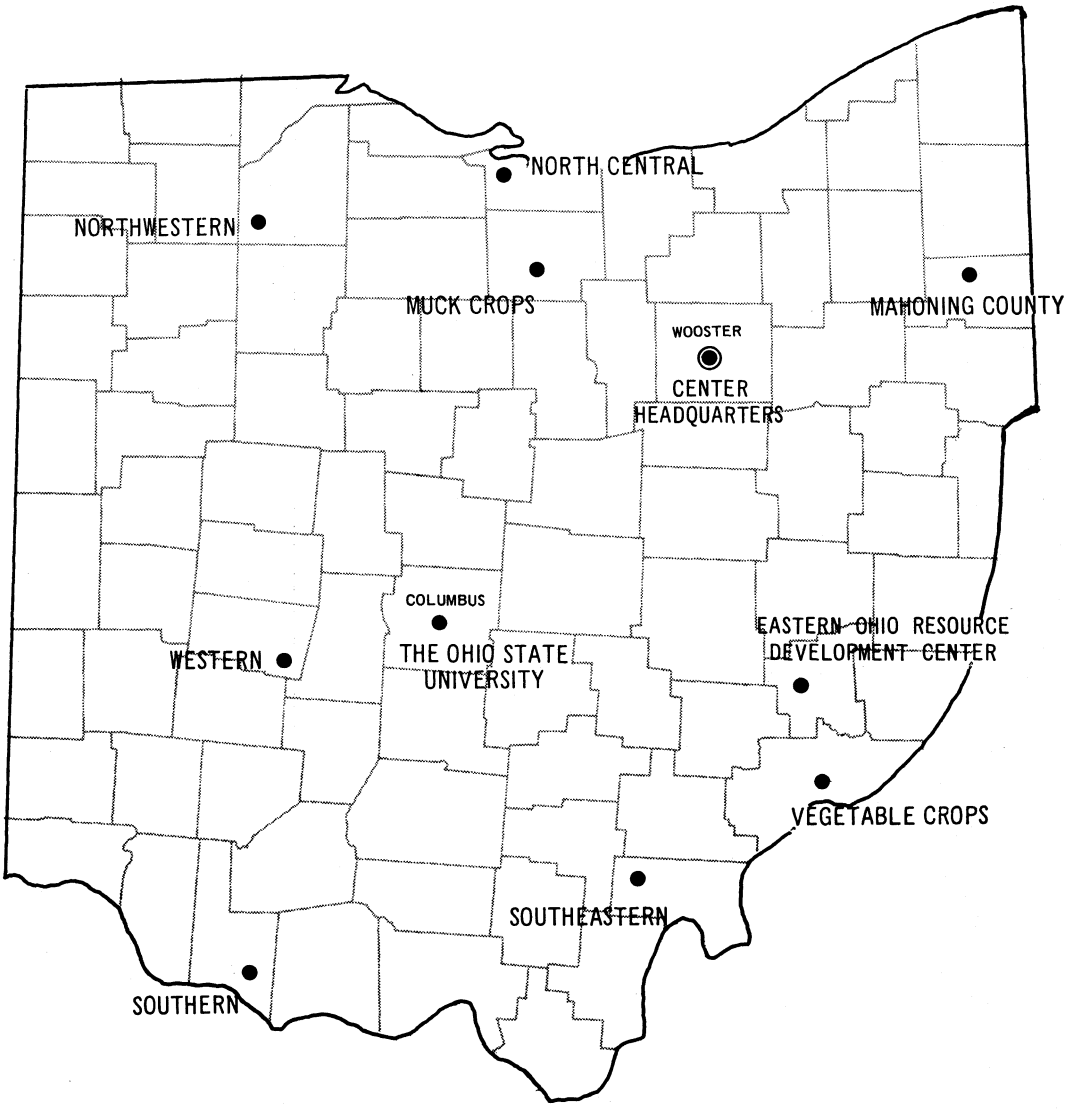
```
C          NORMAL CURVE PARAMETERS
      READ 615,CN(7),CN(8),CN(9),CN(10),CN(11)
  615 FORMAT(5F8.5)
      DO 205 J=1,5
      K=12-J
  205 CN(J)=CN(K)*(-1.0)
      CN(6)=0.0
C          READS GAMMA FUNCTION TABLES
  208 DO 206 J=1,65
      READ 616,L,GF(J),GF3(J),GF5(J),GF6(J)
  616 FORMAT(I3,4E16.8)
      IF(L-J)97,206,97
   97 PRINT 617
  617 FORMAT(17HGF SEQUENCE ERROR)
      PAUSE
      GO TO 208
  206 CONTINUE
    1 DO 2 J=1,13
      SS(J)=0.0
      SSS(J)=0.0
    2 SL(J)=0.0
      CT=0.0
C          READ 12 MONTHLY AND ONE ANNUAL DATA
C          INPUT ERROR MESSAGE REQUIRES READING ALL CARDS FOR YEAR IN
C          ERROR AGAIN AFTER CORRECTING SEQUENCE
    9 DO 7 J=1,12
      READ 601,STA,STB,IY,IM,PC(J),IB
  601 FORMAT(2A3,1X,2I2,31X,F4.2,32X,I2)
      IF(J-1)3,3,4
    3 IF(IB)5,6,6
C          TRAILER WITH -1 IN CC 79-80 CAUSES BREAK TO STATEMENT 5
    6 STAS=STA
      STBS=STB
      IYS=IY
      GO TO 7
    4 IF(IM-J)99,8,99
   99 PRINT 602
  602 FORMAT(9HMONTH ERR)
      PAUSE
      GO TO 9
    8 IF(STAS-STA)98,60,98
   60 IF(STBS-STB)98,61,98
   61 IF(IYS-IY)98,7,98
   98 PRINT 603
  603 FORMAT(11HSTATION ERR)
      PAUSE
      GO TO 9
    7 CONTINUE
      READ 604,STA,STB,IY,PC(13)
  604 FORMAT(2A3,1X,I2,55X,F4.2)
      IF(STAS-STA)98,62,98
   62 IF(STBS-STB)98,63,98
   63 IF(IYS-IY)98,64,98
```

```
C          ADDS SQUARES AND LOGS
   64 DO 65 J=1,13
      SS(J)=SS(J)+PC(J)
      SSS(J)=SSS(J)+(PC(J)*PC(J))
      IF(PC(J))82,82,65
   82 PC(J)=0.001
   65 SL(J)=SL(J)+LOGF(PC(J))
      CT=CT+1.0
      GO TO 9
C          PUNCHES HEADERS
    5 PUNCH 608
  608 FORMAT(18X,49HPRECIPITATION WITH PROBABILITY EQUAL OR LESS THAN,
     1 13H              )
      PUNCH 609,PL(1),PL(2),PL(3),PL(4),PL(5),PL(6),PL(7),PL(8),PL(9),
     1 PL(10),PL(11)
  609 FORMAT(7HSTATION,3X,11F6.2)
      PUNCH 605
C          THIS LOOP DOES 12 MONTHLY AND AN ANNUAL SET OF PROBABILITIES
      BEGIN TRACE
      DO 66 J=1,13
      AV(J)=SS(J)/CT
      AL(J)=LOGF(AV(J))
      ALA(J)=SL(J)/CT
      Z=12.0*(AL(J)-ALA(J))
C          CALCULATES GAMMA PARAMETER
      GG=(6.0+SQRTF(36.0+(4.0*Z)))/(2.0*Z)
C          USES NORMAL CURVE IF GAMMA OVER 100
      IF(GG-100.)200,200,201
  201 SSS(J)=SSS(J)-(AV(J)*SS(J))
      SD=SQRTF(SSS(J)/(CT-1.0))
      DO 203 I=1,11
      R(I)=AV(J)+(CN(I)*SD)
      IF(R(I))202,203,203
  202 R(I)=0.0
  203 CONTINUE
      GO TO 207
  200 X=AV(J)
      B=X/GG
      P=GG-1.0
      L=P+1.0
      IF(L-1)13,13,10
   10 U(1)=GG
C          INTERPOLATES GAMMA FUNCTION OF GAMMA FOR GG BETWEEN 36 AND 100
      IF(GG-36.)109,110,110
  110 K=GG-35.
      SB=K
      DEC=GG-SB-35.
      IF(DEC-Z6)111,113,113
  113 M=K+1
      GAM=GF6(K)+(((DEC -Z6)/Z3)*(GF(M)-GF6(K)))
      GO TO 103
  111 IF(DEC-Z5)114,116,116
  116 GAM=GF5(K)+(((DEC-Z5)/Z1)*(GF6(K)-GF5(K)))
      GO TO 103
  114 IF(DEC-Z3)117,120,120
```

```
  120 GAM=GF3(K)+((DEC-Z3)/Z1)*(GF5(K)-GF3(K)))
      GO TO 103
  117 GAM=GF(K)+((DEC/Z3)*(GF3(K)-GF(K)))
      GO TO 103
C          NUMERICAL APPROXIMATION OF GAMMA FUNCTION OF GAMMA FOR
           GG LESS THAN 36
  109 DO 11 K=1,L
      KK=K+1
   11 U(KK)=U(K)-1.0
      PD=1.0
      DO 12 K=2,L
   12 PD=PD*U(K)
      Y=U(L)
      PD=PD/Y
      GO TO 14
   13 Y=GG
      PD=1.0/Y
   14 GAM=PD*(((((((A*Y+V)*Y+C)*Y+D)*Y+E)*Y+F)*Y+G)*Y+H)/(Y+1.0))
C          ESTIMATES THE VALUE OF XI WHICH MULTIPLIED BY BETA GIVES
C          RAINFALL PROBABLITY
  103 DO 20 I=1,11
      NEST=0
      TEST=0.0001
      DEST=0.0004
      MEST=10
      XO=P
      IF(GG-1.0)51,100,100
  100 IF(XO)50,50,15
   51 XO=0.0001
      GO TO 15
   50 XO=0.01
   15 Z=2.0
      NEST=NEST+1
      T=XO/(P+Z)
      S=T+1.0
C          ITERATIVE ESTIMATE OF S
      DO 16 L=1,1000
      Z=Z+1.0
      T=(T*XO)/(P+Z)
C          SWITCH 1 INTERROGATES VALUE OF T
      IF(SENSE SWITCH 1)74,75
   74 PRINT 611,J,I,L,S,T,TEST
  611 FORMAT(3I5,3E16.8)
      PAUSE
C          SWITCH 2 ALLOWS ENTRY OF NEW T CRITERION (TEST)
      IF(SENSE SWITCH 2)76,75
   76 ACCEPT 612,TEST
  612 FORMAT(F10.0)
   75 S=S+T
      IF(T-TEST)17,17,16
   16 CONTINUE
   17 DL=PL(I)
      IF(DL)41,101,101
  101 IF(GG-36.)121,122,122
  122 ZUD=EXPF(GAM-(P*LOGF(XO)))
      GO TO 105
```

```
  121 ZUD=EXPF(LOGF(GAM)-(P*LOGF(XO)))
  105 X1=XO- (XO/GG)*S +(DL*ZUD*EXPF(XO))
      DX=X1-XO
C          SWITCH 3 INTERROGATES VALUE OF XO
      IF(SENSE SWITCH 3)77,78
   77 PRINT 611,J,I,NEST,X1,DX,DEST
      PAUSE
C          SWITCH 4 ALLOWS ENTRY OF NEW CRITERION FOR DX (DEST)
      IF(SENSE SWITCH 4)79,78
   79 ACCEPT 612,DEST
   78 XO=X1
      IF(XO)41,18,19
   19 IF(ABSF(DX)-DEST)18,18,80
   80 IF(NEST-MEST)15,81,81
C          PRINTS WARNING EVERY TEN XI ITERATIONS SO THAT OPERATOR CAN
           INTERCEDE
   81 PRINT 614,J,I,NEST
  614 FORMAT(2I5,17H   XI ITERATIONS=,I5)
      MEST=MEST+10
      GO TO 15
   41 X1=0.0
   18 R(I)=X1*B
   20 CONTINUE
      END TRACE
  207 IF(J-13)70,71,71
   71 PUNCH 605
  605 FORMAT(80X)
   70 PUNCH 606,STAS,STBS,AM(J),R(1),R(2),R(3),R(4),R(5),R(6),R(7),R(8),
     1 R(9),R(10),R(11)
  606 FORMAT(2A3,1X,A3,11F6.2)
   66 CONTINUE
      PUNCH 607
  607 FORMAT(79X,1H-)
      GO TO 1
       END
```

The State Is the Campus for Agricultural Research and Development



Ohio's major soil types and climatic conditions are represented at the Research Center's 11 locations. Thus, Center scientists can make field tests under conditions similar to those encountered by Ohio farmers.

Research is conducted by 13 departments on more than 6000 acres at Center headquarters in Wooster, nine branches, and The Ohio State University.

Center Headquarters, Wooster, Wayne County: 1918 acres

Eastern Ohio Resource Development Center, Caldwell, Noble County: 2053 acres

Mahoning County Experiment Farm, Canfield: 275 acres

Muck Crops Branch, Willard, Huron County: 15 acres

North Central Branch, Vickery, Erie County: 335 acres

Northwestern Branch, Hoytville, Wood County: 247 acres

Southeastern Branch, Carpenter, Meigs County: 330 acres

Southern Branch, Ripley, Brown County: 275 acres

Vegetable Crops Branch, Marietta, Washington County: 20 acres

Western Branch, South Charleston, Clark County: 428 acres