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Forced Vibrations of a Continuous Span Bridge

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VIBRATORY motion shortens the life of a bridge considerably. This is especially true in the case of a continuous span bridge, since the span of the bridge will vibrate with increasing amplitudes when the natural frequency of the span and the stimulus producing vibration approach resonance. In actual cases, the amplitude of vibration will build up until frictional losses are equal to the energy supplied by the stimuli—moving load or loads on the span of the bridge. If frictional losses were not present, the amplitude of vertical vibrations would become infinite. These vertical forced vibrations may be damped out to some extent by the use of a dynamic vibration absorber, invented by Frahm in 1909.

The dynamic vibration absorber, in itself, is a very simple device that operates on the principle of transmission of forces. It consists of a vibratory system having a natural frequency equal to that of the disturbing force. For a general case, take the mass of the bridge as m_1, and that of the dynamic absorber as m_2. Then, let weights m_1 and m_2 be suspended in space by two springs with spring constants k_1 and k_2. If certain proportions are used for k_2 and m_2 it, will be found that when a disturbing force P_0 Sin w t. is applied to a system m_1k_1, system m_2k_2 will vibrate at all times with a force equal in magnitude and opposite in direction to that of P_0 Sin w t. Since system m_2k_2 opposes P_0 Sin w t, there is no force acting on m_1 and system m_1k_1 will not vibrate. From this follows, that the natural frequency of system m_2k_2 must be equal to that of the disturbing force if system m_1k_1 is to be stationary.

In the above analysis, it is assumed that damping of the external force does not occur or that periodic disturbing forces are not applied to system m_1k_1, for this is equivalent to a change of frequency of the disturbing force. As we have already observed, the dynamic damper does not prevent vibration at other frequencies than that of resonance. Applying these principles to a continuous span bridge, we come to the conclusion that the absorber will damp out most of the span vibrations when the forced vibrations are in resonance with the natural frequency of the resonant absorber.

Extensive experimentation to determine the best method of installing a dynamic vibration absorber on a bridge, has proved that maximum efficiency ranges from 60 to 70 per cent. This partial effectiveness is due to the fact that the ratio of the natural frequency of the span of a bridge and the natural frequency of the resonant absorber cannot be maintained constant. This ratio is not a constant because the frequency of the span will vary with each different position of the moving load, which acts as an aggregate to the effective mass of the bridge. Thus, at different intervals of time, the frequency of the span will vary as a function of mass, while the frequency of the absorber will remain constant. It is this change in frequency of the span that will destroy (at least temporarily) the effectiveness of the dynamic absorber for bridges. A damped vibration absorber could be used on bridges to cover a large range of frequencies, but this will make the absorber too complicated and difficult to install; and would require expensive maintenance.

A dynamic absorber can be installed on a bridge. It is to be understood that the mass of the absorber (m_2), is comparatively smaller than that of the span (m_1). Therefore, m_2 will vibrate much more violently than m_1, although m_2 will have a vibrating frequency equal to P_0 Sin w t when preventing vibration. The method of installing the absorber was used by Mr. Louis Vandegrift[1] in his studies of continuous span bridge vibrations, as suggested to him by Professor P. W. Ott, of the Ohio State University Department of Mechanics. This arrangement proved to be about 60 per cent effective in preventing span vibrations, but there is still considerable advantage in the installation of an absorber.

As we have said above, the natural frequency of the absorber must be made equal to the frequency of P_0 Sin w t, by appropriate values of m_2 and k_2. Values of m_2 and k_2 are correctly chosen only when a complete knowledge of the amplitude and frequency of vertical vibrations of

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[1] Designing Engineer of Bridges, Ohio Department of Highways.

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the bridge are known. Here, ingenuity plays a great part in recording data of vibrations, since frequencies of the span are apt to vary with different positions of the moving load. However, by the use of vibrographs placed at different positions on the span, a fairly accurate record of vibration may be obtained.

In connection with the vibrographs used to measure vibration, the advancement of electricity has greatly replaced mechanical methods. Electrical instruments are essentialy seismic instruments. The only difference consists in that the mechanical vibrations are converted into voltage. This is accomplished by allowing a coil to move in a magnetic field, motion being produced by the mechanical vibrations which are to be measured. When the coil vibrates in the magnetic field, there will be an alternating voltage induced in the coil, proportional to the relative motion of the coil. This induced voltage is then amplified and the frequency and amplitude of vibration are recorded by means of an oscillograph. There are other methods of measuring vibration electrically, but this is one of the simplest, and the principle is very much used.

In conclusion, we will add that in designing a vibration absorber, great care must be taken in choosing values of k_2 and m_2, since the absorber may prove to aid vibration if it is not properly tuned to the frequency of the external driving force.