Comparison of Three Volatility Forecasting Models

THESIS

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Table of Contents

1. ABSTRACT

Although forecasting volatility is an important component of assessing financial risks, it is difficult for many investors because most methods require advanced mathematical knowledge. However, there are two types of time-series models, generalized autoregressive conditional heteroskedasticity (GARCH) (1,1) and exponentially weighted moving average (EWMA), that can be used by investors with only basic training. Furthermore, the implied volatility indexes launched by the Chicago Board Options Exchange (CBOE) provide investors with a direct assessment of market volatility. However, it is unclear which of these three models is best for individual investors. To find out which of these three is the best forecasting method for investors to use, this research first checks whether implied volatility indexes can provide more accurate forecasts than GARCH (1,1) and EWMA by comparing the predictive ability of 11 implied volatility indexes (namely, VIX, VXST, VIX3M, VXMT, VXO, VXD, RVX, VXN, VFTSE, VHSI, and VHSI) with that of GARCH (1,1) and EWMA for the underlying stock indexes. Second, this research focuses on comparing in detail the volatility forecasting ability of GARCH (1,1) and that of EWMA to find which is the best method when volatility indexes are not available or volatility indexes are not good to use. The root mean-square error (RMSE) is used to examine the predictive ability of the three volatility forecasting methods mentioned and the results show that the implied volatility indexes perform better than the GARCH (1,1) and EWMA models for stock indexes in most situations. Additionally, it is shown that GARCH (1,1) has stronger forecasting powers than EWMA for stock indexes. Overall, most implied volatility indexes can be regarded as good forecasts of future volatility to be used by investors in the markets. If an implied volatility index is unavailable or not suitable for a particular case, averaging the forecasts from GARCH (1,1) and EWMA would be a good way to ensure investors get relatively accurate forecasts.

2. INTRODUCTION

In finance, volatility refers to the amount of uncertainty or risk in the size of changes in a security's value. For example, if the price of an asset changes significantly and frequently over a certain period of time, it has a high volatility during that period. If the price changes only slightly and infrequently over a certain period of time, it has low volatility during that period of time. Forecasting the volatility of the price of an asset accurately over the investment holding period is important for an investor to assess investment risk (Ser-Huang and Clive, 2002). Assessing the investment risk through volatility can help investors to make investment decisions and discover investment opportunities. When investors face a choice between investments that would give them the same expected future returns, usually they would choose the one with a lower future volatility in order to get the same expected returns but with lower risk (Daryl, 2016). Higher volatility investments can provide a good investment opportunity because they can generate much larger gains, but with higher risk. Therefore, finding out which investments have higher future volatility is beneficial to investors who are willing to take a higher risk to get potentially higher returns.

Having spent decades researching methods for predicting volatility, researchers have come up with various ways to estimate future volatility. Currently, there are two general approaches used to do this. The first method is based on using historical data, such as time series models like GARCH-type models. The second is based on option prices, using implied volatility. Although comprehensive research on forecasting volatility has been conducted, this has mainly focused on creating, examining, and comparing complex volatility models. Thus, the implication of results in this research area often are more beneficial to institutional investors who are able to handle the complex models. Very little research has been provided results that could help individual investors to forecast volatility. Therefore, this research focuses on finding accessible and relatively accurate volatility forecasting methods and providing results that could be used by individual investors.

There are many different volatility-forecasting methods available, but only a limited number of these models are accessible to individual investors. Currently, there are two time-series models, GARCH (1,1) and EWMA, that can be easily manipulated by individual investors. These are accessible to individual investors because there are several publicly available tools that can be used to easily calculate the estimated volatility. For example, there is a free way to get these forecasts that only requires a little additional work: an Excel template for building GARCH (1,1) and EWMA models, provided by John C. Hull on his website[1], requires investors to run Solver in Excel to get the estimated parameters of models and calculate the forecasts by themselves. The easiest way to get the forecasts is to use the Hoadley Finance Add-in for Excel[2],

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[1] http://www-2.rotman.utoronto.ca/~hull/ofod/garchexample/.

[2] The add-in tool and tutorial are available on https://www.hoadley.net/options/develtoolsvolcalc.htm.

which can provide forecasts of GARCH (1,1) and EWMA to investors directly. However, this Add-in costs $176. Other than using GARCH (1,1) and EWMA models, individual investors can get the estimated volatility by using implied volatility indexes, which directly represent the expectations of volatilities for future markets.

Although these three volatility forecasting methods can all be used easily by individual investors, which one is best remains unclear. Using implied volatility indexes is most convenient for individual investors to obtain estimated future volatility: because the volatility index represents future volatility, there is no need for investors to calculate future values. However, it is unclear if these volatility indexes outperform GARCH (1,1) and EWMA for forecasting volatility. Additionally, most implied volatility indexes are constructed for a limited number of stock indexes. For most assets, investors cannot obtain volatility directly. Given this limitation, GARCH (1,1) and EWMA are feasible time-series models for individual investors to operate independently. However, it also remains unclear which of these two time-series models gives better volatility forecasting. Therefore, the main goal of this research is to find out which of these three volatility forecasting methods is best for individual investors.

To do so, this research will first check whether implied volatility indexes could provide more accurate forecasts than GARCH (1,1) and EWMA by comparing the predictive ability of 11 implied volatility indexes (VIX, VXST, VIX3M, VXMT, VXO, VXD, RVX, VXN, VFTSE, VHSI, and VHSI) with that of GARCH (1,1) and EWMA for the underlying stock indexes. Only the implied volatility indexes for stock indexes are to be examined in this research, because these are the most widely used and most common type of volatility indexes in the markets. Thus, it is believed that they have the most general meaning to individual investors. If volatility indexes perform better, then they are surely the best method for investors to use, given their accuracy and the ease with which they can be obtained. However, volatility indexes are unavailable for most assets and it might be found that volatility indexes perform worse than the two time-series models. Thus, to help individual investors find a better method to use in these situations, part of this research focuses on comparing in detail the volatility forecasting ability of GARCH (1,1) and EWMA. Since the assets to be examined in this work are only stock indexes, our results for the sole comparison of GARCH (1,1) and EWMA models do not have enough generalized application meanings to individual investors. Rather, these results only are likely to help investors concerned with stock indexes. For other assets, we are unable to conclude which is a better model for individual investors to use. Inspired by Armstrong's suggestion (2001) that averaging forecasts when it is unclear which model is better, instead of examining large amounts of assets in each category, we choose to examine whether averaging the forecasts could provide a generalized way that could be used by individual investors to get good forecasts for most assets when implied volatility indexes are not good to use or unavailable. This work chooses to examine the averaging method not only based on Armstrong's suggestion, but also because this method can be easily operated by individual investors.

The results of this work show implied volatility indexes can provide more accurate forecasts than GARCH (1,1) and EWMA models for stock indexes in most situations. Given that volatility indexes are the easiest way for investors to get estimated volatilities and have relatively good forecasting accuracy in most cases, we recommend individual investors use them in most situations. In terms of comparing the GARCH (1,1) and EWMA models, our results show that GARCH (1,1) could provide more accurate estimates than EWMA in most cases. Finally, we find that averaging forecasts from GARCH (1,1) and EWMA is very likely to be able to help investors to get good forecasts for most assets because, generally, average forecasts are more accurate than single forecasts. Therefore, we recommend individual investors to average the forecasts from GARCH (1,1) and EWMA to get the estimated forecasts when implied volatility indexes are not good to use or are unavailable.

3. **LITERATURE REVIEW**

Before the comparison of implied volatility indexes with GARCH-type models, much research was carried out to compare the implied volatility obtained from the Black-Scholes model with GARCH-type models. In 1973, the Black-Scholes model was introduced by Fischer Black, Myron Scholes, and Robert Merton as an option-pricing formula that provides a way to get the implied volatility of an underlying asset through back-calculation given the option price. Subsequently, in 1982, autoregressive conditional heteroskedasticity (ARCH) models were proposed by Engle. Furthermore, in 1986, GARCH—general autoregressive conditional heteroskedasticity—models were proposed by Bollerslev. Thus, after 1986, research began comparing GARCH-type models with implied volatility. Much research, which focused on examining the information content, found that implied volatility can provide efficient information on future volatility and provide incremental information relative to the forecasts provided by GARCH-type models (e.g. Day and Lewis, 1992; Christensen and Prabhala, 1998; Blair, Poon, and Taylor, 2001). However, in terms of the accuracy of the implied volatility, much early research found that implied volatility could not provide more accurate forecasts than GARCH-type models. For example, Lamoureux and Lastrapes (1993) studied 10 individual stocks and found that a GARCH-type model performed better than implied volatility in most cases under the RMSE and mean absolute error (MAE) tests. This was found to be because the options on individual stocks sometimes do not have high liquidity, which cannot reflect fair option prices. Additionally, there was also a serious maturity mismatch problem when using these options, because it is hard to find options with maturities that exactly match the predicting periods. Thus, calculations of implied volatility can sometimes have large errors. As a result, most papers preferred to use the S&P 500 and S&P 100 as underlying assets because the options on these stock indexes have high trading volumes that can reflect fairer prices. However, the mismatch maturity problem still caused measurement errors in the calculations of implied volatility. For instance, when Day and Lewis (1992) compared the implied volatility for the S&P 100 index options to GARCH-type models, they still found that the implied volatility could not provide more accurate forecasts than the GARCH-type models.

In 1993, the CBOE launched the first volatility index, VIX. Then, CBOE successively launched volatility indexes based on index options other than S&P indexes, such as the VXN for the NASDAQ 100 index in 2001, the VXD for the DJIA index in 2005, and the RVX for the Russell 2000 index in 2006. In 2003, it decomposed the VIX into the VIX for the S&P 500 and the VXO for the S&P 100. At the same time, many foreign countries also created volatility indexes, such as the VFTSE for the London Stock Exchange's FTSE 100, the Indian VIX, and so on. The methodology for constructing volatility indexes solved the mismatch problem to some degree because it estimates the volatility by averaging the price of options that have maturities around the predicting periods. As a result, later research began to examine volatility indexes to check whether they were able to provide more accurate forecasts. Many results show that they did indeed perform better than some GARCH-type models in most situations. For example, Blair, Poon, and Taylor (2001) found that the VIX could provide more accurate forecasts compared to the Glosten-Jagannathan-Runkle GARCH (GJR-GARCH) models under the tests of a linear function of mean square error (MSE). Meanwhile, Bluhm and Yu (2001) found that for the stock index from the German stock market (DAX), the implied volatility index (VDAX) provided better forecasts than some GARCH-type models (such as GARCH and exponential GARCH (EGARCH), etc.) under tests for the mean absolute percentage error (MAPE). Furthermore, Corrado and Miller (2005) found that the VXO, VXN, and VIX could provide more accurate forecasts than the GJR-GARCH (1,1) in most cases. Additionally, Shaikh and Padhi (2013) found that the Indian VIX provided more accurate forecasts than GJR-GARCH in most situations under RMSE and MAE tests.

Most previous papers focus on examining newly-created complex models such as GJR-GARCH that are inaccessible to individual investors. Thus, most of them provide implications that are more accessible to institutional investors and do not discuss the practical applications of how their results could be used by individuals. The research here focuses on generating recommendations according to our results that are more meaningful to individual investors. Therefore, we chose the GARCH (1,1) and EWMA models because they are much easier for use by individual investors than the complex models examined in previous research.

Additionally, most previous research has examined at most two or three stock indexes and usually only focused on one market at a time. To expand the scope of this area of research and to get more generalized results for individual investors, we compare all the volatility indexes (VIX, VXST, VIX3M, VXMT, VXO, VXD, RVX and VXN) on U.S. stock indexes with GARCH (1,1) and EWMA. Also, to examine whether the results are different across different markets, three foreign volatility indexes are examined: the VFTSE of the FTSE 100 from the London Stock Exchange, the HSI from the Hong Kong Stock Exchange, and the JNIV of the Tokyo Stock Exchange. These three volatility indexes track the volatility of the three largest foreign stock exchanges (as ranked by stocktotrade.com, according to market capitalization in April 2017), which are the indexes most watched by investors and for all of which historical data for more than ten years is available. Thus, these three indexes have been chosen because they are significant to many investors and enough data exists to allow thorough research.

With respect to whether EWMA or GARCH is better for volatility forecasting, relatively little research exists and the conclusions of different research papers vary. For example, Guo (2012) used the individual stock data of PetroChina and TCL and found that GARCH (1,1) had better predicting power than EWMA under an MSE test. In contrast, Canturk and Cahit (2014) examined the volatility of exchange rates (GBP/TRY and EUR/TRY) and found that EWMA performed better under an RMSE test. In a test on Bitcoin volatility, Naimy and Hayek (2018) found GARCH (1,1) to perform better than EWMA under an RMSE test. This paper will help diversify this research area by comparing the predictive abilities of EWMA and GARCH (1,1) models for eight stock indexes. Most previous research only provides results restricted to the assets examined and does not try to find a general volatility forecasting method for individual investors to use. Unlike previous research, we aim to provide practical recommendations that will enable individual investors to easily get relatively good estimated volatilities by themselves for most assets. Thus, we do not end our discussions by only providing results restricted to stock indexes, but rather we try to further discuss whether averaging the forecasts from GARCH (1,1) and EWMA could be a generalized method for individual investors to use.

4. REVIEW OF CONCEPTS AND MODELS

4.1 Volatility

Volatility (represented by σ) is defined as the degree of variation of a trading price series over time as measured by the standard deviation of logarithmic returns:

The log return is defined as:

$$u_i = \ln \frac{S_i}{S_{i-1}}$$

(where S_i is the stock price at time i)

Volatility is defined as:

$$\sigma = \sqrt{\frac{1}{n-1}\sum (u_t - \bar{u})^2}$$

4.2 The GARCH (1,1) Model

The GARCH model is a derivation of the ARCH model. The ARCH model, proposed by Engle (1982), is a time-series model that estimates variance (the square of volatility) based on a linear combination of the previous rate of returns and long-running average variance. The following gives the ARCH(q) model where q is the lagging period:

$$\sigma_t^2 = \gamma V_l + \alpha_1 u_{t-1}^2 + \alpha_2 u_{t-2}^2 + \ldots + \alpha_q u_{t-q}^2$$

(where V_l is the long-run average variance and $\gamma + \alpha_1 + \alpha_2 + \ldots + \alpha_q = 1$)

Because this study does not focus on the long-running average variance, ω is substituted for γV_l for convenience.

In this case, the model is written as:

$$\sigma_t^2 = \omega + \alpha_1 u_{t-1}^2 + \alpha_2 u_{t-2}^2 + \ldots + \alpha_q u_{t-q}^2$$

The GARCH model, which provides better prediction than the ARCH model, was proposed by Bollerslev (1986). The GARCH (p, q) model places the p autoregressive items of σ_t^2 into the ARCH(q) model, so the volatility series is written as:

$$\sigma_t^2 = \omega + \alpha_1 u_{t-1}^2 + \alpha_2 u_{t-2}^2 + \ldots + \alpha_q u_{t-q}^2 + \beta_1 \sigma_{t-1}^2 + \beta_2 \sigma_{t-2}^2 + \ldots + \beta_p \sigma_{t-p}^2$$

(where $\gamma + \alpha_1 + \alpha_2 + \ldots + \alpha_q + \beta_1 + \beta_2 + \ldots + \beta_p = 1$)

Since its first conception, researchers have successively proposed many GARCH-type models based on the basic GARCH (p, q) model. These include the nonlinear asymmetric GARCH (NAGARCH), integrated GARCH (IGARCH), and EGARCH models. However, the original derivation remains the most widely applied in finance.

According to the described GARCH (p, q) model, the GARCH (1,1) model can be written as:

$$\sigma_t^2 = \omega + \alpha u_{t-1}^2 + \beta \sigma_{t-1}^2$$

It is the simplest of the GARCH-type models, yet it has good predictive ability compared to others. Hansen and Lunde (2005) compared 330 GARCH-type models and did not find a better performing model than the GARCH (1,1). This evidence strongly supports using the GARCH (1,1) as a representative of GARCH-type models for comparison with implied volatility indexes.

4.3 The EWMA Model

The EWMA model is a time-series model that estimates volatility by assigning exponentially decreasing weights to previous data. The most recent data are assigned the largest weights. The equation for the model is as follows:

$$\sigma_t^2 = (1 - \lambda) u_{t-1}^2 + \lambda \sigma_{t-1}^2$$

(where λ is between 0 and 1)

EWMA is a special case of GARCH (1,1). Compared to GARCH (1,1), the EWMA does not consider the long-run variance. In GARCH (1,1), it is recognized that the variance will return to an average level in the long run. Therefore, theoretically, the GARCH (1,1) model should perform better than EWMA. However, it is unclear whether this addition of the long-run variance does truly improve the prediction results (Hull, p.526, 2009).

4.4 Implied Volatility Indexes

An implied volatility index represents the implied volatility of some underlying assets. Implied volatility is a parameter (the volatility of the stock price) in option pricing formulas that is not directly observed from the market but can be derived from the option price. For example, when calculating the price of an option due in one month, we first estimate the volatility for the future month. Conversely, when the price of this option on the markets is known, we can use the pricing formula to easily obtain future volatility. Therefore, the implied volatility index is a more straightforward method for estimating future volatility compared to the GARCH (1,1) or EWMA models.

The specific methodology created by CBOE to calculate volatility indexes is complex and involves multiple mathematical calculations. Because the goal of this research is not to provide individual investors with an understanding of this methodology, here these mathematical algorithms are not described. However, as an example, for VIX, the CBOE uses options with 23 to 37 days to expiration to maintain a 30-day weighted-average time to expiration. Next, the volatility of each option and subsequent 30-day weighted average of the volatilities are calculated. Finally, the value is multiplied by 100, to give the VIX[3].

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[3] The methodology of VIX can be founded in https://www.cboe.com/micro/vix/vixwhite.pdf

5. DATA AND EMPIRICAL METHODOLOGY

In this research, eight stock indexes were studied: the S&P 500, S&P 100, DJIA, Russell 2000, NASDAQ 100, Nikkei 225, FTSE 100, and HSI. Meanwhile, 11 corresponding volatility indexes were used: the VIX, VXST, VIX3M, VXMT, VXO, VXD, RVX, VXN, JNIV, VFTSE, and VHSI.

The VIX, VXST, VIX3M, and VXMT are the implied volatilities of the S&P 500 for different periods. Specifically, VIX represents the expected annualized implied volatility in the S&P 500 index over the next 30 calendar days initiated today. VXST is for the next nine calendar days, VIX3M is for the next three months, and VXMT is for the next six months. VXO, VXD, RVX, VXN, JNIV, VFTSE, and VHSI represent the expected annualized implied volatility in the S&P 100 index, DJIA index, Russell 2000, NASDAQ 100, Nikkei 225, FTSE 100, and HSI, respectively, over the next 30 calendar days.

The data for implied volatility indexes and stock prices was obtained from http://www.cboe.com, https://www.investing.com/, and https://finance.yahoo.com/.

The stock price data used in this research is from 8.21.2008 to 7.7.2017. The stock price used to forecast the volatility is for the period 1.7.2011 to 12.6.2017.

5.1 Implied Volatility Index

The first step to get the implied volatility index forecasts is to construct the forecasting intervals. This research uses 1.7.2011 as the first day of forecasting. For 30-day forecasting, the forecasting intervals are constructed in the following way:

The first forecasting interval includes 1.7.2011 and the following 29 calendar days. The second 30-day forecasting interval starts from the first trading day after the last day of the first forecasting interval. The following forecasting intervals are constructed in the same way. For constructing the nine-day forecasting intervals, the first forecasting day of the nine-day forecasting is the same as that of the 30-day forecasting for each forecasting interval. The same method is applied to three-month (90 days) forecasting and six-month (180 days) forecasting.

Then, the value of implied volatility indexes at the first day in each forecasting interval is selected as the forecasts for the volatility of stock indexes. Because the value of implied volatility indexes is originally given in percentage terms, it is converted to decimal form by dividing by 100.

5.2 The GARCH (1,1) Methodology

This research uses out-of-sample forecasting for the GARCH (1,1) and EWMA models, that is, using data from 1 to $t\text{-}1^{th}$ trading days as a sample to estimate the volatility from the t^{th}

trading day to the last trading day of a prediction interval. The forecasting intervals are kept the same as those used in the implied volatility index forecasting.

According to Yuan (2013), using 600 to 800 data points is most suitable for constructing a GARCH (1,1) model. This research uses the rolling estimation method, which chooses data from 600 trading days before every prediction interval to build the GARCH (1,1) model. For example, the trading data from 8.14.2013 to 12.30.2015 (600 trading days) is used here to construct the GARCH (1,1) model to forecast the volatility of January 2016. Likewise, for estimating the volatility of February 2016, trading data from 9.12.2013 to 1.29.2016 (600 trading days) is used.

After building the GARCH (1,1) model, the dynamic prediction method is used to estimate the volatility for every trading day in prediction intervals. For example, suppose we use the trading data from August 14th 2013 to December 30th 2015 (600 trading days) to construct a GARCH (1,1) model:

$$\sigma_t^2 = \omega + \alpha u_{t-1}^2 + \beta\sigma_{t-1}^2$$

(σ_t is the volatility on the t^{th} day, S_t is the stock price in t^{th} day)

The estimation process is as follows:

Suppose 1.4.2016 is the t^{th} day in the above model. To estimate the volatility for 1.4.2016 (the first trading day in 2016), we use u_{t-1}^2 and σ_{t-1}^2 from 12.30.2015. Then, in order to estimate the volatility for the $t+1^{th}$ day (1.5.2016), we would encounter a problem because we do not know the real values of u_t^2 and σ_t^2. To solve this problem, we would use $\hat{\sigma}_t^2$ as σ_t^2. Because the expected value of u_t^2 is σ_t^2, we would again use $\hat{\sigma}_t^2$ as u_t^2 (Hull, p.525, 2009). In a similar fashion, we can obtain the volatility for every trading day in January 2016.

Finally, the volatility for January 2016 is:

$$\sqrt{\text{sum of all } \widehat{\sigma^2} \text{ of every trading day in prediction interval}}$$

In order to compare the volatility with other forecasting models, we convert the volatility into annualized volatility by multiplying by $\sqrt{\frac{\text{trading days per year}}{\text{trading days in prediction interval}}}$.

5.3 The EWMA Methodology

The process for using the EWMA model to estimate volatility is similar to that for the GARCH (1,1) model. Again 600 trading days before every prediction interval have been used to build the EWMA model:

$$\sigma_t^2 = (1-\lambda)u_{t-1}^2 + \lambda\sigma_{t-1}^2$$

(σ_t is the volatility in t^{th} day, S_t is the stock price in the t^{th} day)

The estimation process is then as follows:

Suppose the first day in the prediction interval is the t^th trading day. As the expected value of u_{t-1}^2 is σ_{t-1}^2, on the t^th trading day, we can get $\hat{\sigma}_t^2 = E(\sigma_t^2) = (1-\lambda)\sigma_{t-1}^2 + \lambda\sigma_{t-1}^2 = \sigma_{t-1}^2$ (Hull, p.525, 2009). Then on the t+1^th trading day, again we use $\hat{\sigma}_t^2$ as σ_t^2 because we do not know the true value of σ_t^2. Then we have $\hat{\sigma}_{t+1}^2 = \hat{\sigma}_t^2 = \sigma_{t-1}^2$. Therefore, the estimated volatilty is σ_{t-1} for every trading day in a prediction interval.

Finally, the volatility is:

$$\sigma_{t-1} * \sqrt{\text{trading days in prediction interval}}$$

Again, for comparison purposes, we convert the volatility into the annualized volatility by multiplying by $\sqrt{\frac{\text{trading days per year}}{\text{trading days in prediction interval}}}$.

5.4 The Realized Volatility

In this paper, the series of realized volatility is used as the benchmark for comparison. The actual volatility is calculated by the following formula:

$$\sigma^{re} = \sqrt{\frac{1}{n-1}\sum(u_t - \bar{u})^2}$$

(where n is the number of trading days in prediction intervals, and $u_t = \ln\left(\frac{s_t}{s_{t-1}}\right)$ s_t is the stock price on the t^th day)

Because σ^{re} is the standard deviation of the daily return rate, we convert σ^{re} into an annual term for comparison purposes by multiplying by $\sqrt{\text{trading days per year}}$.

6. METHOD OF EVALUATION

To check the accuracy of the forecasts of the three models, this paper uses the most popular methods employed in previous research, namely the RMSE as used by, for example, Lamoureux and Lastrapes, 1993; Hansen and Lunde, 2005; and Shaikh and Padhi, 2013.

RMSE is preferred because it assigns a large punishment for large errors through squaring the errors.

The formula is:

$$\text{RMSE} = \sqrt{\frac{1}{N}\sum_{n=1}^{N}(\sigma_n^{re} - \hat{\sigma}_n)^2}$$

(where N is total number of estimates, σ_n^{re} is realized volatility, and $\hat{\sigma}_n$ is the GARCH (1,1), EWMA and implied volatility forecast)

In evaluation of the error performances, smaller values for RMSE are preferred.

7. EMPIRICAL RESULTS

Table 1: RMSE of forecasts from GARCH (1,1), EWMA, and volatility indexes for eight stock indexes. (A smaller RMSE means the corresponding method has a better predictive power.)/ Average daily trading volume of options for corresponding underlying assets (2011-2017)

Volatility Index	Underlying Asset	Forecasting Period	Garch(1,1)	EWMA	Implied	Average daily trading volume(units) of options*
VXD	DIJA	30 days	0.05244	0.05668	0.05450	21019
VXN	NASDAQ 100	30 days	0.06168	0.06618	0.05908	23386
VXO	S&P 100	30 days	0.06737	0.06407	0.05325	30656
RVX	Russell 2000	30 days	0.06583	0.06374	0.06185	50927
JNIV	Nikkei 225	30 days	0.08575	0.08739	0.08647	126052
VFTSE	FTSE 100	30 days	0.06765	0.06879	0.05739	27703
VHSI	HSI	30 days	0.06006	0.05865	0.05318	29615
VXST	S&P 500	9 days	0.07773	0.07946	0.05777	45479
VIX	S&P 500	30 days	0.05575	0.05881	0.05758	180767
VIX3M	S&P 500	90 days	0.05653	0.06225	0.06875	9491
VXMT	S&P 500	180 days	0.05619	0.06391	0.07657	5776
						*Data was calculated and obtained from:
				Highest RMSE		www.historicaloptiondata.com
				Medium RMSE		https://www.lseg.com/derivatives/lsedm-stats
				Lowest RMSE		http://www.jpx.co.jp/english/markets/statistics-derivatives/trading-volume/index.html
						http://www.hkex.com.hk/-/media/HKEX-Market/News/Research-Reports/HKEX-Surveys

From Table 1, we can see that the implied volatility indexes have lower RMSEs than the two time-series models for the NASDAQ 100, S&P 100, Russell 2000, FTSE 100, HSI, and S&P 500 (nine days). That is, the volatility indexes perform better than the two time-series models in these cases. For the DIJA, Nikkei 225, and S&P 500 (30 days), the implied volatility indexes have lower RMSEs than the EWMA model but have higher RMSEs than the GARCH (1,1) model, which means these three volatility indexes can provide more accurate forecasts than the EWMA model but fail to perform better than GARCH (1,1). For the S&P 500 (90 days and 180 days), the implied volatility indexes have the highest RMSEs and, therefore, the worst predictive abilities.

Notably, in Table 1, compared to other volatility indexes, the RMSEs of VIX3M and VXMT are much higher than those of the two time-series models, which indicates that VIX3M and VXMT have much higher predictive error than the time-series models and thus have lower predictive powers. Compared to other cases, the average daily trading volume of the long-term options on the S&P 500 is much lower (less than 10,000 unit) while the average daily trading volume of options on other underlying assets is at least greater than (20,000 unit) (see the average daily trading volume column in table 1). Thus, the low accuracy of VIX3M and VXMT is probably mainly due to the low liquidity options, which cause the calculations of the implied volatility to become biased.

According to Table 1, if we just consider the short-term (nine days and 30 days) volatility indexes (for which the calculations are less biased because the options on the underlying assets have relatively high trading volumes), then the implied volatility indexes have lower RMSEs in most situations, which means they could provide more accurate forecasts. The EWMA model never have lower RMSEs than the volatility indexes, which means the EWMA never outperforms volatility indexes in short-term cases. Therefore, the overall implied volatility indexes have better predictive abilities than the GARCH (1,1) and EWMA models. This result makes sense, because implied volatility is derived from option prices and option prices represent people's expectations for future markets. According to the research of Chiras and Manaster in 1978, implied volatility usually contains more efficient information about future volatility than the estimated volatility obtained from historical data. So, it is understandable that implied volatility indexes are likely to perform better at volatility forecasting than the GARCH (1,1) and EWMA models, which are constructed using historical data.

To summarize, when options on the underlying assets have low liquidity, like the long-term S&P 500 options, the volatility indexes can include many biases that cause them to underperform compared to the two time-series models. Thus, investors should not use implied volatility indexes in these situations. Instead, individual investors could use GARCH (1,1) to get the volatility of stock indexes because GARCH (1,1) has the lowest RMSEs (highest predictive ability) in situations where volatility indexes are biased. When options on the underlying assets have relatively high liquidity, volatility indexes could provide more accurate forecasts than GARCH (1,1) and EWMA in most cases. Therefore, we think it is better for individual investors to use volatility indexes for stock indexes when options have relatively high liquidity.

In terms of the comparison of the GARCH (1,1) and EWMA models, according to Table 1, the GARCH (1,1) have a higher RMSEs value than EWMA model for S&P 100, Russell 2000 and HIS, but it has a smaller value of RMSEs than EWMA in most cases. This indicates GARCH (1,1) has a better predictive power than EWMA model in most cases. Thus, in general, it can be inferred that GARCH (1,1) is more likely to provide more accurate forecasts then EWMA. This result is understandable: The volatility have the mean reversion property which means the volatility will tend to go back to the average level in long run, since the GARCH (1,1) includes the property of long-run average variance, thus it could help provide more accurate forecasts to some degree (Hull, p.526, 2009). The underperformance of the GARCH (1,1) model in those three cases might be because we include data from the financial crisis (2008–2009) to construct some models, which might cause the estimations of parameters for the GARCH (1,1) and EWMA models to be inaccurate. Compared to the EWMA, the GARCH (1,1) model is likely to be more biased because it has one additional parameter to estimate (the long-run variance rate).

To check this, we tried to exclude the possibly biased data (the estimated volatility from January 2011 to May 2012). The results shown in Table 2, the overall RMSEs value decrease which means the estimated volatility have less biased after we exclude the financial crisis data, but the comparison results do not have much difference. Particularly, for the GARCH (1,1) and EWMA models, there is no difference between the comparison results shown in Table 1 and Table 2. EWMA's RMSEs are still smaller than those of GARCH (1,1) for Russell 2000, HSI, and S&P 100, which means EWMA still outperforms GARCH (1,1) in these three cases. Overall, our results indicate GARCH (1,1) earns an advantage in forecasting accuracy than EWMA because GARCH (1,1) has the additional long-run variance rate factor in its formula. However, this factor does not ensure that GARCH (1,1) is more accurate in every case, because when the actual volatility in predicting interval does not show the property of mean reversion, then the long-run variance rate factor will not have the contribution in improving the forecasting accuracy.

Since the assets examined in this work were stock indexes, it is carefully concluded from the results in this work that GARCH (1,1) is likely to provide more accurate forecasts than EWMA for stock indexes. However, according to results from previous research (e.g. Guo, 2012; Canturk and Cahit, 2014; Naimy and Hayek 2018), the predictive abilities of GARCH (1,1) and EWMA seem to vary according to the different types of assets examined. Thus, it is hard to extend the conclusions for GARCH (1,1) from stock indexes to other types of assets. Additionally, there is not enough previous research available to generalize a reliable conclusion about which model is better to use for each type of asset.

Table 2: RMSEs for forecasts from GARCH (1,1), EWMA, and volatility indexes for eight stock indexes excluding estimates from January 2011 to May 2012.

Volatility Index	Underlying Asset	Forecasting Period	Garch (1, 1)	EWMA	Implied
VXD	DIJA	30 days	0.05093	0.05606	0.05366
VXN	NASDAQ 100	30 days	0.05924	0.06485	0.05776
VXO	S&P 100	30 days	0.05328	0.05309	0.05234
RVX	Russell 2000	30 days	0.05451	0.05152	0.05071
JNIV	Nikkei 225	30 days	0.08469	0.08550	0.08047
VFTSE	FTSE 100	30 days	0.06014	0.06062	0.05571
VHSI	HSI	30 days	0.05573	0.05154	0.05047
VXST	S&P 500	9 days	0.06243	0.06362	0.05356
VIX	S&P 500	30 days	0.05256	0.05623	0.05592
VIX3M	S&P 500	90 days	0.05279	0.05450	0.05794
VXMT	S&P 500	180 days	0.05356	0.05458	0.06544

To help individual investors get good forecasts for most assets, this work examined whether averaging forecasts from GARCH (1,1) and EWMA could be used as a generalized method when volatility indexes are unavailable or biased. The idea of averaging the forecasts derives from Armstrong (2001) who suggested that this approach could be a good way to obtain forecasts when it is unclear which model is best. Armstrong found that averaging improved the accuracy of forecasts in 30 different examples of forecasting research. Based on the evidence from these 30 research cases, we believe Armstrong's suggestion to average forecasts is reasonable for use as a generalized method for individual investors to get volatilities for most assets if volatility indexes are unavailable or biased. However, because none of the previous 30 research works examined by Armstrong involved volatility forecasting, it was necessary to check that the same results would arise for volatility forecasting. Thus, this work first examined whether the average forecasts of GARCH (1,1) and EWMA could provide more accurate forecasts than single forecasts for stock indexes by comparing the RMSEs of average forecasts and the single forecasts from GARCH (1,1) and EWMA.

The average forecasts were obtained by the following formula:

$\sigma_n^{Avg} = \frac{\sigma_n^g + \sigma_n^e}{2}$ (σ_n^{Avg} is average forecast; σ_n^g is the forecast of GARCH (1,1); σ_n^e is the forecast of EWMA)

The RMSEs of the average forecasts were obtained by the following formula:

$\text{RMSE} = \sqrt{\frac{1}{N}\sum_{n=1}^{N}(\sigma_n^{re} - \sigma_n^{Avg})^2}$ (where N is total number of estimates, σ_n^{re} is realized volatility)

The results were shown in Table 3. From Table 3, it can be seen that the RMSEs for average forecasts of GARCH (1,1) and EWMA are smaller than the RMSEs for the single GARCH (1,1) and EWMA forecasts. This indicates that the average forecasts could be more accurate than the single forecasts. This result aligns with the results shown in the previous 30 different examples of forecasting research examined by Armstrong which indicates the averaging method could also work in volatility forecasting. The reason that averaging could improve forecast accuracy is that some forecasts from different forecasting models could have inverse prediction errors that could be offset through averaging (Jing-rong, Yu-Ke and Yan, 2011). Therefore, we believe it is highly possible that the average volatility forecasts from GARCH (1,1) and EWMA for assets other than stock indexes could also deliver more accurate forecasts. Thus, we recommend individual investors to average volatility forecasts from GARCH (1,1) and EWMA to get volatilities for assets other than stock indices when volatility indexes are unavailable or biased.

Since the average forecasts from GARCH (1,1) and EWMA would be more accurate for stock indexes, we also recommend individual investors that could average the forecasts from GARCH (1,1) and EWMA to get more accurate estimated volatility.

Table 3: RMSEs for single forecasts of GARCH (1,1) and EWMA, and as calculated using the average of both methods.

RMSE		
Garch(1,1)	**EWMA**	**GARCH(1,1) &EWMA**
0.05244	0.05668	0.05194
0.06168	0.06618	0.06088
0.06737	0.06407	0.06393
0.06583	0.06374	0.06214
0.08575	0.08739	0.08340
0.06765	0.06879	0.06619
0.06006	0.05865	0.05695
0.07773	0.07946	0.07742
0.05575	0.05881	0.05428
0.05653	0.06225	0.05417
0.05619	0.06391	0.05338

8. CONCLUSION

In this research it was found that implied volatility indexes had smaller RMSEs in most cases tested, so they are likely to be a better guide than the GARCH (1,1) and EWMA models for individual investors who want to know the volatility of stock indexes. However, individual investors need to be more cautious when they face volatility indexes for which the corresponding options have relatively low trading volumes, because this can bias calculations and result in less reliable estimations. In these situations, investors should not use implied volatility indexes. If no implied volatility index is available or the implied volatility index is not good to use, then the GARCH (1,1) model is likely to provide more accurate forecasts for stock indexes because the GARCH (1,1) model tended to have smaller RMSEs than the EWMA in most cases. Since averaging forecasts from EWMA and GARCH (1,1) could improve the accuracy of forecasts for stock indexes compared to the forecasts from GARCH (1,1), thus it is believed that individual investors could average the forecasts from GARCH (1,1) and EWMA to get more accurate forecasts for stock indexes. For other assets that do not have implied volatility indexes or do not have unbiased volatility indexes, we cannot provide the exact results about which model is more likely to better, but we believe averaging forecasts from EWMA and GARCH (1,1) is a possible good solution for individual investors to get good forecasts by themselves based on the evidence that average forecasts are more accurate than single forecasts shown in much previous research.

9. FURTHER RESEARCH

At this stage, we are unable to conclude whether or not to use volatility indexes for other types of assets that have relatively unbiased volatilities. Since this is just a very small portion of assets, we have left this issue as a question for further research. It would be worth checking whether the results are consistent for other volatility indexes with different underlying assets, such as the volatility indexes for non-U.S. stock ETFs, interest rates, and individual stocks. With this information we could give individual investors a more complete and reliable suggestion about whether to use volatility indexes.

Our evaluation results might include some biases because high-frequency data has not been used in this research. The work of Blair, Poon, and Taylor (2001) indicated that low-frequency data (daily-based stock prices) might cause biases in evaluation results and that high-frequency data (taken every 5 minutes) can give a more precise evaluation. No high-frequency data, which might influence the results, was used in this research. Thus, in further research, high-frequency data could be used to see whether the outcomes change.

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