

## Multiple Ways to Persevere: *Liar's Bingo*

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### Abstract

Some readers may already be familiar with the mathematical task of solving *Liar's Bingo*. In this article, the authors will share the different ways *Liar's Bingo* has provided both the authors and their students the opportunity to persevere on multiple levels as they and students try to explain the different mathematical patterns that emerge in the strips. The authors will share some extensions to *Liar's Bingo* that readers can use in their classrooms and some of the observed patterns and some sample explanations. Spoiler alert: the authors have not found explanations for all the observed patterns. (Yet!)



“That made my heart go really fast!  
It was like an adrenaline rush to figure that out!”

As educators, we are probably familiar with the excitement and, perhaps, sense of pride that comes from completing a difficult task, or from discovering that there is more to a problem than was originally thought. Intriguing mathematical problems might snowball from one question to another with the student never quite sure where it will lead next. Some exciting mathematical problems can be viewed as a web with the original question in the middle and lots of corollary questions stemming from it. These are tasks that might not necessarily be completed in a single sitting, but involving students in tasks that both require and encourage perseverance are powerful tools for developing young mathematicians.

### Perseverance

Providing students with engaging and immersive mathematical experiences goes far beyond simply selecting and posing novel or non-routine problems. While such problems often provide the occasion for deep mathematical thinking, the practice of mathematics is actually more elaborate. The National Council of Teachers of Mathematics (1991) describe the essence of such practice as “exploring, conjecturing, examining, and testing all aspects of problem solving” (p. 95). Given that this manner of mathematical practice is often predicated on periodic experiences of disequilibrium (Carter, 2008), it follows that fostering a sense of perseverance among students is quite desirable. Perseverance has been described as a “student's sense (i.e., intuitive and experiential) in knowing when to continue with, and not to give up too soon on a chosen strategy or action, and at the same time, knowing when to abandon a particular strategy or action in the search for a more effective or useful one” (Thom & Pirie, p. 2). Similarly, the Common Core State Standards for Mathematics (National Governors Association, et al. 2010) addresses perseverance in the first Standard for Mathematical Practice, *Make sense of problems and persevere in solving them*. The authors of the CCSS-M write, “[students] analyze givens, constraints, relationships, and goals. . . consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course, if necessary” (p. 6). Central to these descriptions is the persistence of mathematical thinking and activity in the face of disequilibrium or adversity.

Turning to the notion of persistence as inherent to students' perseverance through a problematic situation, in some instances, classroom and school structures impose boundaries around the extent to which students

are given the opportunity to persist. Indeed, the duration for persistence may be limited to a particular class period as students negotiate a particular problem for approximately 45 minutes to an hour, and then move on to something else regardless of whether or not a viable solution or understanding has been achieved. In some instances, students may return to such vexing mathematical situations, but in other instances, they may not.

*Indeed, teachers may take a long-term view on the development of perseverance among students and purposefully design experiences organized around highly connected opportunities of problematic practice that occur over longer durations, and within such durations, provide the opportunity for students to step away, reflect, and return to the problem space.*

In the case of truly problematic mathematical experiences, we conjecture that the development of perseverance is predicated not only on persistence with practices described above (e.g., conjecturing, monitoring, evaluating, etc.) within some fixed duration of a single lesson, but rather intermittently over a much longer time period. Indeed, teachers may take a long-term view on the development of perseverance among students and purposefully design experiences organized around highly connected opportunities of problematic practice that occur over longer durations, and within such durations, provide the opportunity for students to step away, reflect, and return to the problem space. This manner of developing perseverance, though, likely requires the construction and/or identification of truly rich problematic tasks that allow for longitudinal periods of engagement across multiple class sessions.

### ***Liar's Bingo Task Background***

Catherine Lane was first introduced to *Liar's Bingo* when she attended a meeting of the Cincinnati Math Circle in January of 2013. Steve Phelps facilitated the activity and explained he was introduced to *Liar's Bingo* in a methods course taught by Chuck Shonenshine. Attendees were given strips of paper that had six numbers on it, each colored red or black (see Figure 1). The first task given was to put the strips in some order. This was an open-ended task in an attempt to draw everyone into the activity. After discussing how different people had chosen to order their strips, Steve led the game and acted as the number predictor. Participants would choose a strip, read the colors in order and aloud, but lie about one color. Steve would then correctly identify the number about which they lied. For example, on the strip depicted in Figure 1, a participant may elect to lie about the number 10. The participant would then say the following color pattern, “black” (54), “black” (34), “red” (4), “black” (10 – lie), “black” (16), “black” (15). The task given to the group was to determine how he knew which number was lied about (Klien & Phelps, 2014).



Figure 1. Liar's Bingo number strip

### **Patterns**

The activity itself and the process of ‘solving’ *Liar's Bingo* lured Catherine in, who consequently shared it with Shelly Harkness and Jonathan Thomas. Since our first introduction to the activity, the authors have used it in a wide variety of courses such as *Quantitative Reasoning*, *College Algebra*, and elementary and secondary methods courses.

One of the captivating aspects of *Liar's Bingo* is that students seem to naturally engage in mathematical reasoning as they try to determine how the leader is able to identify the number about which has been lied. Time and time again, we observed students spontaneously discuss patterns they saw in the colors and

numbers on the strips. In an attempt to convince them that the patterns they noted were valid, they would begin to ask other groups of students if they had noticed the same pattern on their strips. We heard students making conjectures and then revising them based on further examination of the strips.

Some patterns that popped up with every class or group with which we have used this activity are:

- Only the digits 0 – 7 are used on the strips.
- The more red numbers on a strip, the higher the number about which you lied.
- The digit in the ones place match on the first three numbers and the digit in the tens place match on the last three numbers.
- If there is only one red number on the strip, then that number is zero.
- There is only one all-black strip and only one all-red strip.
- Three reds in a row (either at the beginning or end of the strip) always make a seven.

Additionally, we often saw participants drawing various charts or tree diagram in an effort to flush-out the patterns, such as the one in Figure 2. These observations served as a launching point for furthering the activity beyond the initial task of finding a solution for *Liar's Bingo*. We found students were impelled to answer why these patterns exist. They (and we) wanted to justify that these patterns were necessary.

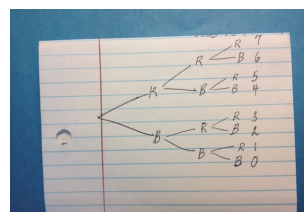


Figure 2. Tree diagram to predict of the *Liar's Bingo* number

Through repeated presentation of *Liar's Bingo* in our classrooms and at conferences, we collected some interesting patterns. We have been asked intriguing questions and developed some questions of our own, some of which we have been *able* to answer, others we are continuing to persevere in solving. For example, some have made the assertion that if you consider the black numbers as positive and the red numbers as negative, they sum to 77. After consultation with other tables, this was revised to if a strip has three black and three red numbers, then if the black numbers are positive and the red numbers are negative, they sum to 77. Some other patterns:

- If the last number on the strip is odd, then the first five numbers are even. And vice-versa, if the last number of the strip is even, then the first five numbers are odd.
- If you lie and say the last square is red, then that number is odd. And vice-versa, if you lie and say the last square is black, then that number is even.

A colleague who attended a conference where we presented the activity also continued to ponder the activity after the conference. He came back a few days later and wanted to know if we could not only say what the number in the square about which we lied *was*, but could we also say what color it was, or where it was in the strip. This led to a discussion of representing *Liar's Bingo* as a function and determining that it was not a one-to-one function.

As we presented *Liar's Bingo* to more of our classes, there were a few questions that we began to ask and then seek to answer:

- Are the strips that Steve Phelps created the only possible strips?
- How many possible strips are there?
- How could we create strips for three-digit numbers?
- How many possible strips are there for three-digit numbers? For four-digit numbers?
- Could we create strips using a different base?

As we grappled with some of these questions, we realized the rich mathematics into which we were delving and began to discuss extensions of this activity beyond the initial class time of ‘playing’ the game, in order to lure our students deeper into the patterns of *Liar’s Bingo*.

## Extensions

In order to create opportunities for students to persevere in uncovering more of the mathematics embedded in *Liar’s Bingo* after initially finding patterns, playing the game, and understanding why *Liar’s Bingo* works, Shelly Sheats Harkness created Extensions 1, 2, and 3. These tasks are included at the end of this article. By using these extensions, teachers can take a long-term view of planning around highly-connected opportunities of problematic practices that occur over longer periods and provide opportunities for students to step away, reflect, and return to *Liar’s Bingo*. Therefore, student engagement with *Liar’s Bingo* might span either contiguous or non-adjacent class sessions.

Interestingly, when we used these extensions with a group of conference attendees, some thought Extension 2 (create the strip that begins with number 43) was easier than Extension 3 (what numbers must be on the strip that is Black, Red, Red, Red, Red, Black) but some thought Extension 3 was easier than Extension 2. Teachers who use these extensions may want to focus on classroom conversations about which were easier and why. For Extension 2, after some initial confusion and grappling with how to begin, one group of attendees figured out the solution by thinking about the number 43 and asking themselves, “So, to get 43, what colors do you need when playing the game if lied ... using BBBRR, we played the game to figure the other numbers.” For another group that worked on Extension 3, they began by lying about the first number - Red, Red, Red, Red, Red, Black - which meant the first number had to be 76. They then “figured out the number for each spot by playing the game” (Extension 3).

Given the importance of providing students with opportunities to persevere and persist in their mathematical thinking, rich tasks such as *Liar’s Bingo* provide sound terrain for such explorations. Just as important as interesting tasks, though, is the instructional context in which such tasks are situated. The opportunities for conjecture, patterning, and reasoning afforded by *Liar’s Bingo* are predicated on pedagogies that allow for open-ended investigation and collaboration over a sustained duration – perhaps well-beyond that of a single class period. (In fact, we are still trying to make sense of why, if a strip has three black and three red numbers, then if the black numbers are positive and the red numbers are negative, they add up to 77.) It is, perhaps, tautologous to state that perseverance may only develop in contexts where perseverance is possible; nevertheless, our experiences with *Liar’s Bingo* remind us that commitment to broad, engaging, open-ended mathematical experiences provide remarkable dividends in terms of opportunities to develop mathematical practices. One participant remarked, “Ha! There’s nothing mathematical. It’s just a code.” Perhaps this thought is not unique as students determine how the facilitator knows what number has been lied about; however, delving into the deeper patterns of *Liar’s Bingo* and persistent investigation hopefully leads students to an understanding that their “code” can be expressed with mathematics and can serve as a launch point for further investigations.



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Extension 1

A color-coded table/chart for the strips that start with 0-7

0	60	50	44	42	41
1	61	51	45	43	40
2	62	52	46	40	43
3	63	53	47	41	42
4	64	54	40	46	45
5	65	55	41	47	44
6	66	56	42	44	47
7	67	57	43	45	46

0					
1					
2					
3					
4					
5					
6					
7					

A color-coded table/chart for the strips that start with 10-17

10	70	40	54	52	51
11	71	41	55	53	50
12	72	42	56	50	53
13	73	43	57	51	52
14	74	44	50	56	55
15	75	45	51	57	54
16	76	46	52	54	57
17	77	47	53	55	56

10					
11					
12					
13					
14					
15					
16					
17					

What color changes do you see between the matrices for 0-7 and 10-17? Why do these changes need to happen?

Color the chart/table below for the strips that begin with 50-57.

50					
51					
52					
53					
54					
55					
56					
57					

How did you determine the color pattern? Will some columns in all of the possible tables/charts always be the same color? Why or why not?

Extension 2

Create the strip that begins with the number 43 ... Show all work and explain your thinking.

43					
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Extension 3

What numbers must be on the strip below? ... Show all work and explain your thinking.

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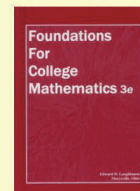
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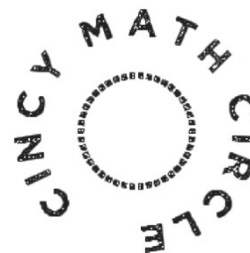
Math Teacher Circles are content-based professional development for K-12 teachers of mathematics. The purpose is to build a community of K-12 math teachers and mathematicians, by engaging in collaborative problem solving while working on rich mathematical tasks. And the best part – they're completely FREE!



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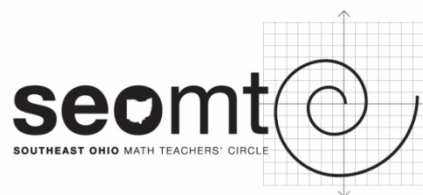
#### Cincinnati Math Teacher Circle

- Wiki: <http://cincymathcircle.wikispaces.com/>
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- Started in 2011 with over 100 participants each year
- Meetings monthly with the first two sessions on Aug 22, 2015 and Oct 10 @ Madeira HS. Visit their website for other meeting dates for the 2015-2016 school year.
- Supported by the University of Cincinnati Department of Mathematics MAT Program, Math for America seed grant, and Madeira High School (facilities)



#### Southeast Ohio Math Teacher Circle

- Website: [www.seomtc.weebly.com](http://www.seomtc.weebly.com)
- Email: [seomtcircle@gmail.com](mailto:seomtcircle@gmail.com)
- Meetings are typically held Saturdays 9 am- noon in the Athens area. Visit their website for meeting dates for the 2015-2016 School year.
- Funded by Ohio Board of Regents *Improving Teacher Quality* grant



#### Columbus Math Teacher Circle

- Website: <http://columbusmathcircle.wordpress.com>
- Visit their website for meeting dates and locations for the 2015-2016 School year.
- Funded by grants from American Institute of Mathematics (AIM) and OSUs Steam Factory.

