Curve fitting with the Bubble Board

Alexandrea N. Hammons, Alfinio Flores, John A. Pelesko, and L. Charles Biehl

The bubble board is a device to create simultaneously 56 identical soap bubbles. Students study the relation between time and the number of remaining bubbles for different concentrations of glycerin and use linear, exponential, and logistic decay models to fit the data.

Introduction

Soap bubbles are fascinating objects, and high school students are eager to study them. In the activities presented here, students experiment with different populations of soap bubbles, collect data on how the populations decrease over time, and use graphing calculators or spreadsheets to fit different mathematical models to the data. Students can generate a population of soap bubbles by using drinking straws and a bubble board, a simple apparatus that allows students to simultaneously form 56 almost identical soap bubbles. A similar device was proposed by Rämme (2001) as a tool for illustrating data collection and curve fitting. Using the bubble board, students explore the number of remaining bubbles over time and use different mathematical models to fit the data. Complete lab manuals for students and teachers are available on-line from the MEC Lab (Hammons 2009; Hammons & Biehl 2009a, 2009b). The board is made with a perforated slate of polycarbonate and clear drinking straws. One end of the straws is dipped in soap solution, then the board is inverted so that the other end of the straws is submerged in a water tank, and the bubbles are formed (see Figure 1).

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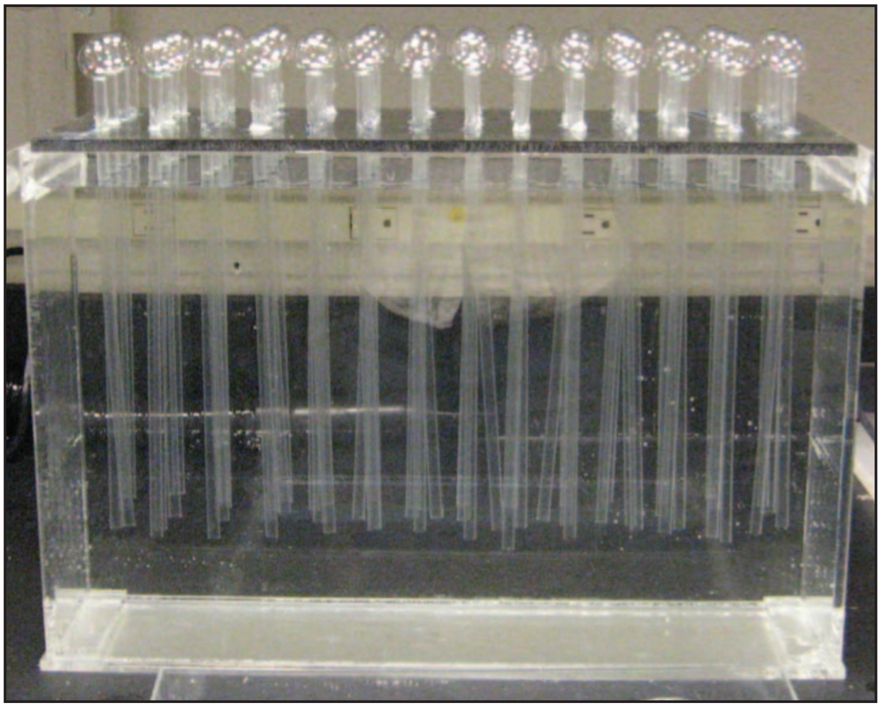

Fig 1 Bubble Board

Different factors affect the durability of soap bubbles, such as humidity of the air, temperature, volume of the bubble, temperature of water, water vapor inside the bubble, etc. (Behroozi and Olson 1994).

Over time, the bubbles will burst. Students count the number of bubbles remaining at given time intervals. To facilitate the counting, soap bubbles are made to last longer by adding small concentrations of glycerin to the bubble solution. Students can then study the relationship between time and the number of remaining bubbles for different concentrations of glycerin. With these activities, students can study scatter plots and curve fitting using different mathematical models.

In this article, we first describe how the straws and bubble board are assembled, then how bubbles are formed, and finally how data are collected. We also discuss activities for students to fit curves to sets of data using linear, quadratic, exponential, and logistic models. Lists of materials and resources needed, places where these can be acquired, and instructions to build the board are given in the appendices.

Assembling the straws and the Bubble Board

For instructions on how to build the bubble board itself see Appendix C. When the board is ready, the straws, approximately 23 cm long and 5.9 mm in diameter, are inserted into the holes so that segments of 3 cm are on the side that will be dipped in the soap solution. Approximately 19.5 cm of each straw is on the side that will be submerged in water. The air displaced from the straws by the incoming water forms the bubbles. The straws will not be exactly the same length, so the part that is below the board may vary by one or two mm. Some bubbles will be slightly larger in volume (about 1%) than others. However, the difference in diameter of the spheres is hardly noticeable. Straws that are markedly longer than the rest can be trimmed. For purposes of dipping the straws in the soap solution, it is better to have all straw segments on the top side the same length. A water tank about 26 cm deep, 38 cm long, and 11 cm wide is used with the board. Students can use cookie trays or another shallow tray big enough for dipping the tips of the straws. See complete list of materials in Appendix A.

Making the bubbles

Students make three mixes (400 g each) of water and dish soap, 1% soap by weight, with three different concentrations of glycerin (2%, 4%, 6%). For further instructions on how to make these mixes, see Hammons (2009). Next, students pour one mix into the tray. For more uniformity in the duration of the bubbles across trials, students should wet the straws before the first trial by placing the board over the tank, dipping both ends of the straws in the water. The students should shake excess water from board over a sink and pat ends of straws with a dry paper towel. Next, students dip the shorter side of the straws into the soap mix. Then they pull up the board so that each straw is coated with a soap film. Students invert the board and place it gently into the top of a water tank so that the longer side of the straws is submerged (Figure 1). The displaced air will inflate the bubbles. Sometimes not all 56 bubbles will form, so students should quickly count how many are there at the beginning. Students need to place the board very gently onto the tank to minimize having bubbles slide off the tops of straws. If any bubbles slide off, to make observation easier, students can simply pop those bubbles and then note the new starting number.

Percentage of glycerin and survival time of bubbles. Different factors affect the durability of soap bubbles, such as humidity of the air, temperature, volume of the bubble, temperature of water, water vapor inside the bubble, etc. (Behroozi and Olson 1994). In this activity, students keep most factors constant between experiments and vary only the concentration of glycerin. The length and diameter of straws is kept constant, so the amount of air displaced and volume of the bubbles is also constant. Other factors that affect the durability of the bubble, such as humidity and temperature, are assumed to be constant within the room during the experiment, although they can vary from one room to another and from one day to another. The concentration of dish soap is kept constant in the mixes. Students repeat the experiments for 2%, 4%, and 6% concentrations of glycerin (i.e., A, B, and C).

Collecting Data

There are several ways in which students can record when a bubble bursts. One way uses on-line split timers with a stopwatch option (see appendix B for one). Once the bubble board has been inverted over the water tank, students click "start" and press the "split" button each time a bubble bursts. Data collection may be challenging as occasionally multiple bubbles pop simultaneously. Students do their best to be accurate, but know that human error is part of any experiment and that reasonable results will likely be obtained despite this obstacle. To increase accuracy, at least two students record data for each trial with their eyes on the board and hand on the mouse to click a split as each bubble bursts. When all bubbles have popped, or after the predetermined time, the students stop the timer. Students copy and paste the splits into Notepad or Word for later analysis. For each concentration, with available time, students repeat the experiment at least two additional times (for a total of three) and up to nine total times. Students record how many bubbles reamain after specified intervals of time into the Excel template. For solution A, students may find that the bubbles pop too quickly to accurately record individual pops. In this case, they simply count the number of bubbles remaining at one minute intervals.

Displaying and analyzing data

Students use data averaged over several trials to construct a graph corresponding to each concentration of glycerin. Figure 2 provides an example of these plots.

Linear models. Students first visually find a straight line that fits the data fairly well and estimate its slope and points of intersection. Then they check their estimation by using Excel or a graphing calculator to fit a line to the data (Figure 3). Students discuss why a line with a negative slope is a good model for the number of remaining bubbles. They notice, perhaps with some teacher prompting, that in all three cases the linear model predicts zero bubbles earlier than in the actual data, and realize that they look for a curve that fits the pattern of points better.

Looking for alternative models. Given that linear models do not provide very good fit, students use other models available on a graphing calculator. They should fit equations and discuss the appropriateness of each resulting model. For example, quadratic functions give a good fit on the way down from the vertex, but then go to zero too quickly and do not fit the lingering tail. Students also use a spreadsheet to fit decay curves to the scatter plots. The graphs in Figure 4 were generated using the Trendline feature (exponential function) in Microsoft Excel. A template is available from the MEC Lab (Hammons 2009). Instructions for this template are given there in the Teacher's Manual (Hammons and Biehl 2009a).

Hopefully students see that the exponential decay model offers a good fit for the lingering tail, but not quite so good for the beginning of the experiment.

Figure 5 shows the graph of the number of bubbles that pop out in each interval of

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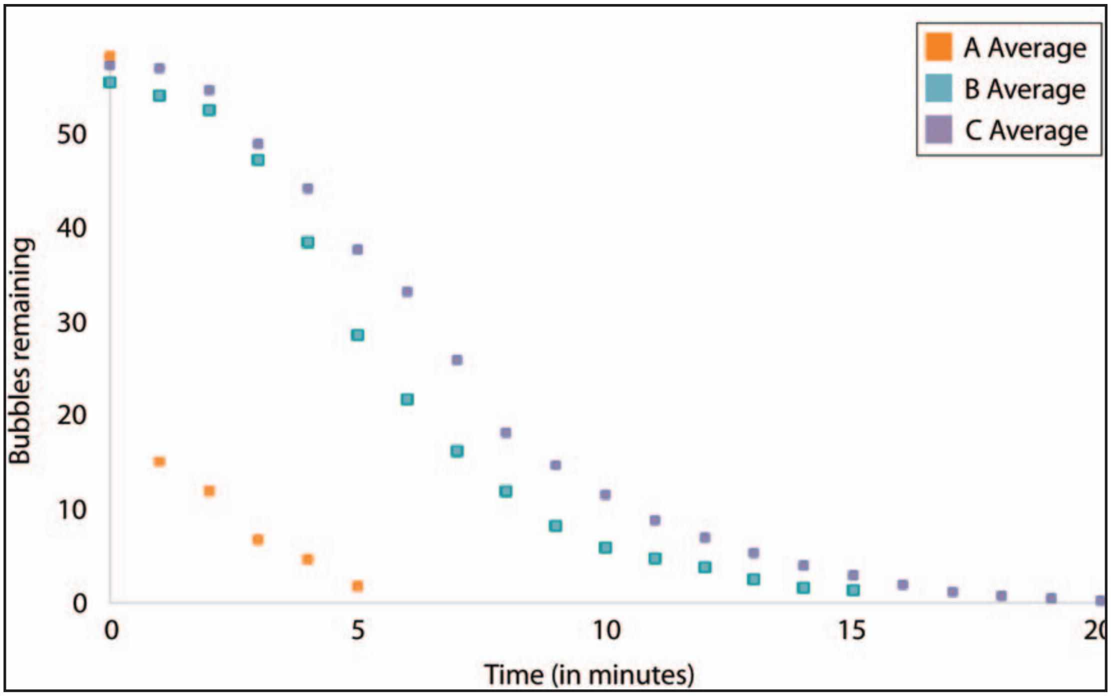

Fig 2 Remaining bubbles for different concentrations of glycerin

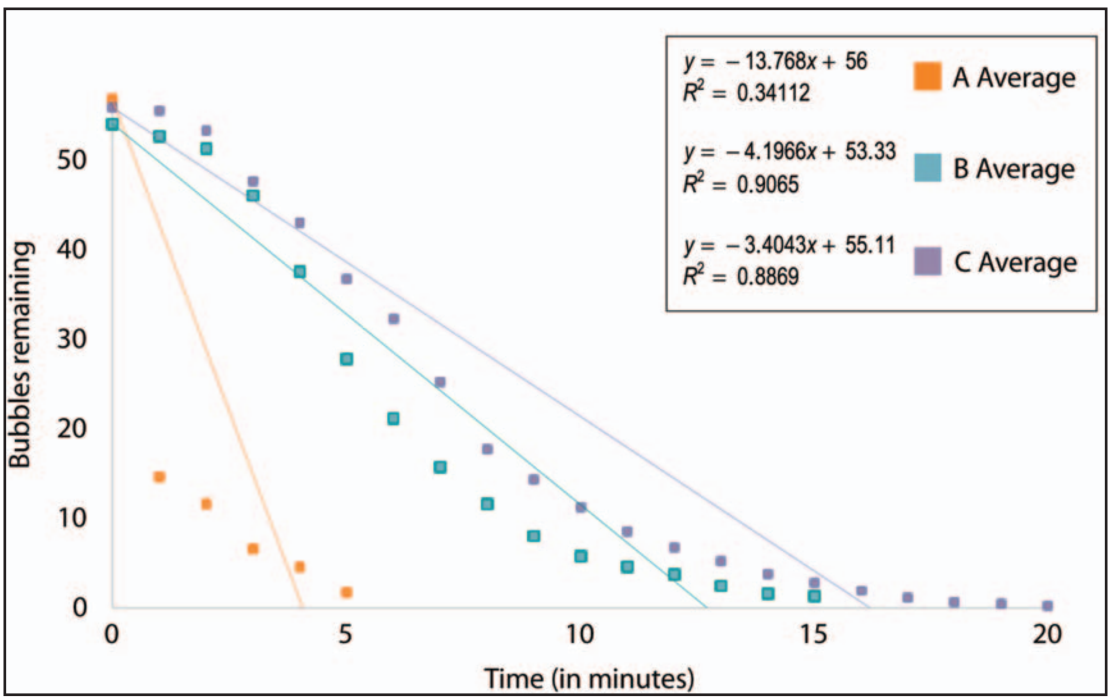

Fig 3 Linear models for different concentrations

time. The fastest decline of bubbles does not occur when there is the largest number of bubbles, which is what an exponential decay model would predict, but actually begins a little later. This may lead students to think that the popping of bubbles is not quite analogous to an exponential decay phenomenon.

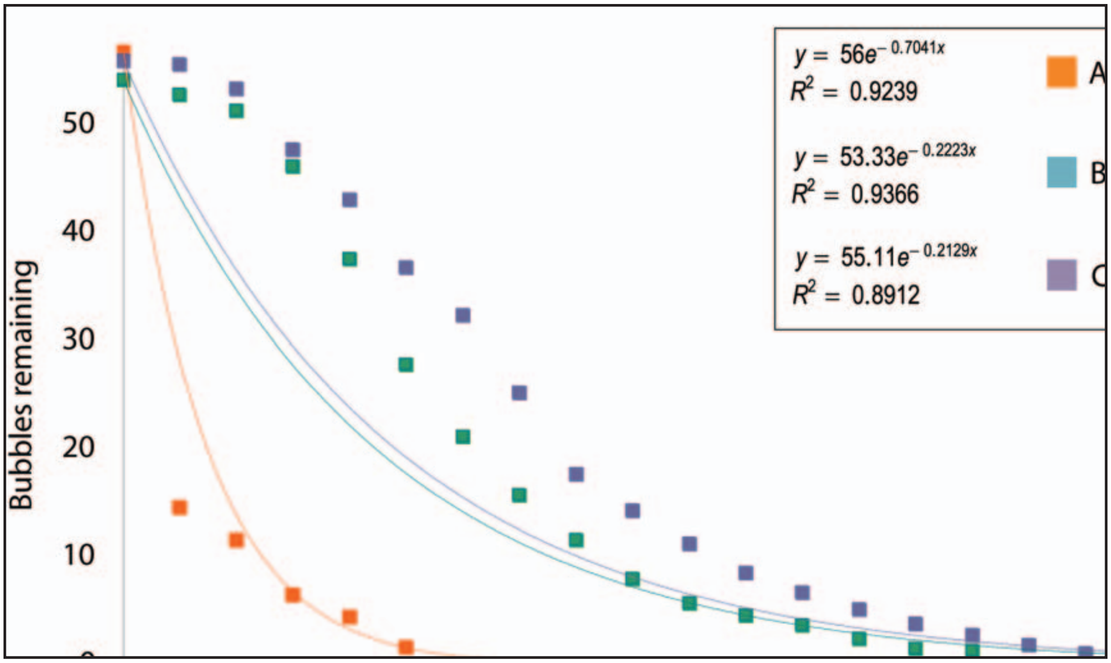

Fig 4 Exponential decay models for bubbles

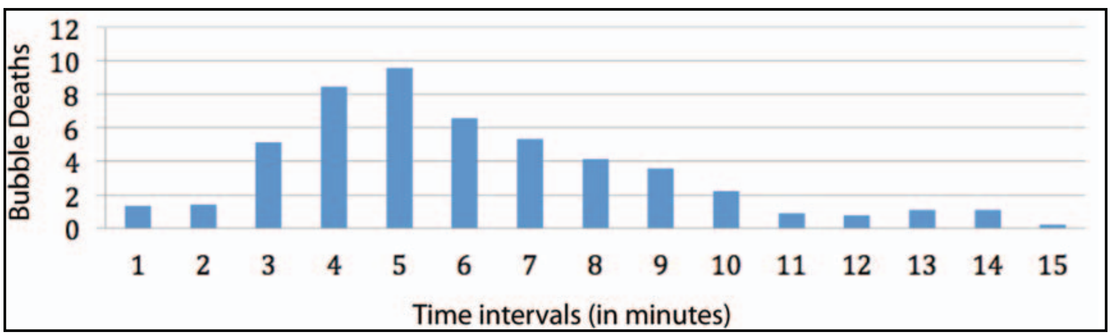

Fig 5 Number of bubbles burst in each time interval (4% mix)

Students try yet another model to better reflect the behavior of the bubbles. Using a graphing calculator, they can use logistic regression to obtain the equation $y = \frac{c}{1+ae^{-bx}}$ that best fits the data. Using the data obtained above for the averages for 6% concentrations, the corresponding values are a = .3249557511188; b = - 0.2950237437; and c = 75.581055453284.

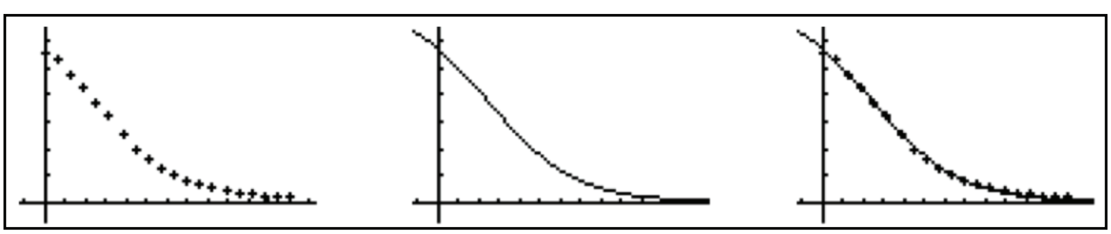

Fig 6 (Left) Original data; (Middle) Logistic curve; (Right) Curve fit to original data

As for the logistic fit, the teacher can guide a discussion with the class so that everyone understands what are some of the factors that make the logistic model a good fit for this experiment. First, because the parameter *b* in the logistic equation above is negative, the equation corresponds to a function describing decay, rather than growth. Second, in a logistic model two factors affect the rate of change of the population, not just one as in the exponential model, where the size of the population determines the rate of change. By having glycerin in the solution, bubbles are almost guaranteed to have a minimum lifespan of some duration, so at the beginning few bubbles will burst. Once they begin to pop, the rate would reasonably be somewhat exponential, and the hangers-on would then pop at a decreasing rate, proportional to the number of bubbles still extant, thus leveling the number of remaining bubbles.

Students can then analyze and discuss the graphs and different curves of fit as a group, noting the emergent patterns and considering ways to improve data modeling.

The bubble board in the classroom

There are several ways to use bubble boards in the classroom, ranging from teacher demonstrations to students working in teams and taking turns to collect data. For example, for a class with 24 students and two boards, 12 students are assigned to each board. They work in two teams of three groups of four and take turns to collect data. Two groups work at a bubble board at a time. The other groups are responsible for studying and discussing the handout on the regression models available on the calculator (*Calculator Regression Models*, 2011). Other configurations of students are suggested in the Teacher's Manual (Hammonds and Biehl, 2009a).

For the experimental part, students in each team need to know the necessary tasks to be done (measuring, recording data, entering data into computer or graphing calculator, etc.) and who is responsible for each task. They should also take turns in the different roles. The activity with the bubble board can be spread out over two or more class periods. In the first session, students at one board collect data until all bubbles have popped or after 5 minutes for solution A (2%), 10 minutes for B (4%), or 15 minutes for C (6%). In subsequent sessions, students bring their data together so that data for the averages of several trials are graphed, rather than individual trials. Students then analyze and discuss the graphs and different curves of fit as a group, noting the emergent patterns and considering ways to improve data modeling. Depending on students' familiarity using a spreadsheet or graphing calculator to fit regression models to sets of data, they will need more or less guidance from the teacher.

Concluding remarks

Based on the enthusiastic participation of our students, we are confident that your students will thoroughly enjoy the experiments described in this article. That said, students will need detailed instructions ahead of time so that they can work independently during the bubble formation and data collection. Once students have collected the data, the teacher needs to provide guidance on the use of different models to fit the data, and this may take more than one session. However, by using data that they generate, in an activity that they witnessed and enjoyed, students are more likely to participate in a discussion to make sense of the different mathematical models and why or why not they are a good fit for the data.

The bubble board activity can also serve as a teachable opportunity for the use of logistic models of growth or, as in the case of the soap bubbles, decay. Logistic models are useful in many fields, for example, in epidemiology, demography, and chemistry. Although the scope of the AP Statistics course (Legacy, 2008) limits itself to the linearization of data and does not address other models like the ones used in this article, the *Common Core State Standards for Mathematics* (Common Core State Standards Initiative 2010) recommends the use of exponential and logarithmic functions in modeling. One of the examples for modeling suggested by the *Common Core State Standards* is that of bacterial population growth (p. 72) which can be modeled using logistic curves. With the help of technology, high school students can successfully (and meaningfully) use logistic and other models to better describe and understand phenomena.

Acknowledgements

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Appendix A: Materials

- Tank, ≥ 22 cm deep, 10-14 cm wide, ≥ 38 long
- Tray big enough to hold tank (to prevent leakage and spills)
- Ruler, cm
- Straight drinking straws. We used 23 cm (9 inch) clear straws, with a diameter of 5.9 mm (0.23 inch). Other sizes are also appropriate.
- Permanent marker
- Bubble board with 56 holes that fit the straws' diameters
- Large beakers, ≥ 400 ml capacity
- 2.5 cm masking tape (if beakers do not have white label space)
- Pencil and paper
- Scale (grams)
- Small beaker, 100-250 ml capacity
- 12 g dish soap (Dawn)
- 48 g 86-88% Glycerin
- Pipettes
- 1140 ml tap water, plus enough to fill tank
- Stirring rod
- Computer with spreadsheet and internet
- Graphing calculators with different regression models (such as TI-84)
- 1 – Shallow cookie tray or 21.6 by 28 cm (8 1/2 by 11 in) Box Frame (sold to frame pictures), to hold soap solution
- Paper towels

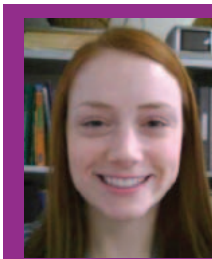
Appendix B: Sources for materials and teacher resources

- Glycerin can be bought in small amount in a pharmacy or in larger quantities, for

example at Acros Organics http://www.acros.com/
- Unwrapped straws are quite inexpensive.
- The water tank can be purchased from Educational Innovations Inc. http://www.teachersource.com/BiologyLifeScience/LifeScience/DemoTank.aspx
- An on-line stopwatch is available at http://www.online-stopwatch.com/split-timer/
- Excel worksheet for fitting curves is available from MEC-Lab http://meclab.pbworks.com/w/page/11422602/Bubble-Board (click the Data Analysis template link at the lower part of the page)

Appendix C: Constructing the bubble board

The board itself is made of a 16 cm by 38 cm rectangular piece of polycarbonate (Lexan), 0.5 cm thick. The board has 4 rows of 14 circular holes, at a distance of 2.5 cm apart from each other. We used straws with a length of 23 cm and diameters of 5.9 mm but other sizes of straws are available. The diameters of the holes need to match the diameter of the straws so that they can hold a straw with the same diameter tightly. If the fit is not quite tight, straws can be glued to the board. Teachers can drill the holes themselves or have the holes drilled at a local store. It is best to use clear, non-bending straws.



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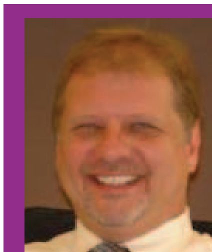

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