Evaluation of Pumping-Test Data
in Piketon, Ohio

A Senior Honors Thesis

Presented in Partial Fulfillment of the Requirements for Graduation with Distinction in Geological Sciences in the Undergraduaduate Colleges of The Ohio State University

by

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Abstract

Analytical methods of evaluating pumping-tests in water-table aquifers have historically made simplifying assumptions that resulted in imprecise values of hydraulic parameters. A new analytical method and computer program developed by Dr. Allen F. Moench was used to evaluate data from a pumping test conducted in a water-table aquifer near Piketon, Ohio, in 1963. The average anisotropy ratio was 8.1, lower than the previously calculated averages of 17 using the Neuman method and 71 using the Stallman method. The average value of specific yield was 0.3, which is higher than the previously calculated values of 0.09 using the Neuman method, 0.2 using the Stallman method.

Acknowledgments

I want to acknowledge several people who helped me with this project. First I want to thank my advisor Dr. E. Scott Bair for giving me the opportunity to work with a new analytical method and the freedom to learn on my own how to use it. Thank you to Drs. Gunter Faure and Gerald H. Newsom for sitting on my oral examination committee. Thanks to Terry Lahm for helping me find my way in and out of the computer, to Leslie McClenahen for running to program on the east-well data, and to Stanley Norris for making public his original pumping-test data. And I want to thank Dr. Halan Noltimier for having the faith in me to put me up to writing an honors thesis in the first place.

I also want to thank several people who kept me going through the process: the Friday lunch bunch (Heidi, Tim, and Tom) and the not-necessarily-Friday diner's club (Nancy, Karen, Diana, Gailee, and Mary). My mom's emotional support and critter feeding were invaluable, as were Kathy's chocolate infusions. And Mark, who kept me fed and sane so I could meet the deadline!

Figures

Figure 11: Effect on Type Curve With Varriation of Kz/Kr (Kv/Kh) Ratio.

Figure 12: Distance vs. Drawdown at t = 2000 minutes.

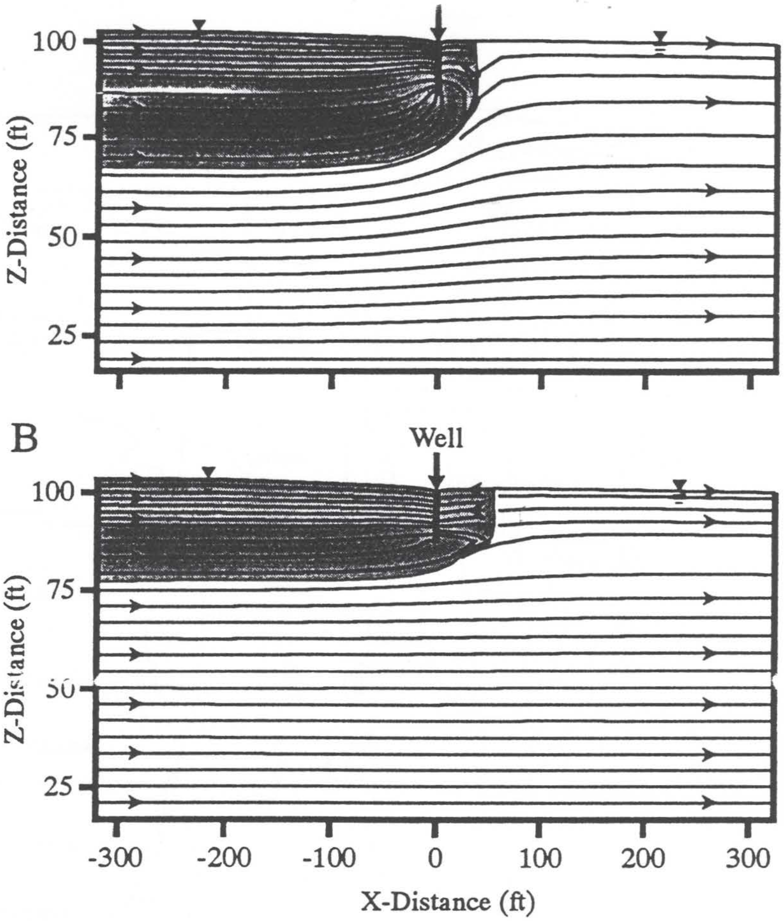
Figures 13-21: Grapghs of Observation Well Data with Matched Type Curves Generated Using WTAQ1. Match points are Indicated.

Introduction

Accurate analysis of the hydraulic properties of an aquifer is important not only in determining the amount of water available, such as for municipal or commercial use, but also in designing contaminant remediation systems. The rate of flow is controlled, in part, by the hydraulic conductivity, which is a vector property having both a magnitude and a direction. Commonly the rate of ground-water flow is considerably different in the horizontal direction than in the vertical direction. This is related to the anisotropy of an aquifer, in that the directional differences in the flow rates are proportional to the directional differences in hydraulic conductivity. Figures 1 and 2 show the variation in the way a contaminant moves through a hypothetical aquifer with varying anisotropy ratios. Underestimation of flow rates and misestimation of horizontal and/or vertical hydraulic conductivity can allow a contaminant to flow past wells designed to capture it for treatment, whereas overestimation of flow rates can lead to costly treatment of clean water.

Aquifer tests are one way to determine in-situ values of vertical and horizontal hydraulic conductivity and other hydraulic parameters of an aquifer. Proper interpretation of these tests helps minimize errors in the design of recovery wells for capturing contaminants. An aquifer test is a controlled field experiment, which consists of pumping a well at a constant rate for some amount of time and measuring the decline of water levels in nearby observation wells. The data are plotted and matched with mathematically-derived curves representing idealized behavior. From the equations describing the "type curves", the hydraulic parameters of the aquifer can be calculated.

Between 1953 and 1979, the U.S. Geological Survey and the Division of Water of the Ohio Department of Natural Resources conducted 13 controlled aquifer tests to determine the hydraulic properties of a sand and gravel aquifer near Piketon, Ohio (figure 3). The tests were



Vertical Exaggeration: 4X

Figure 1: Effect on Flow Depth of a Potential Contaminant due to Variation in Anisotropy Ratio (Bair and Lahm, 1996).

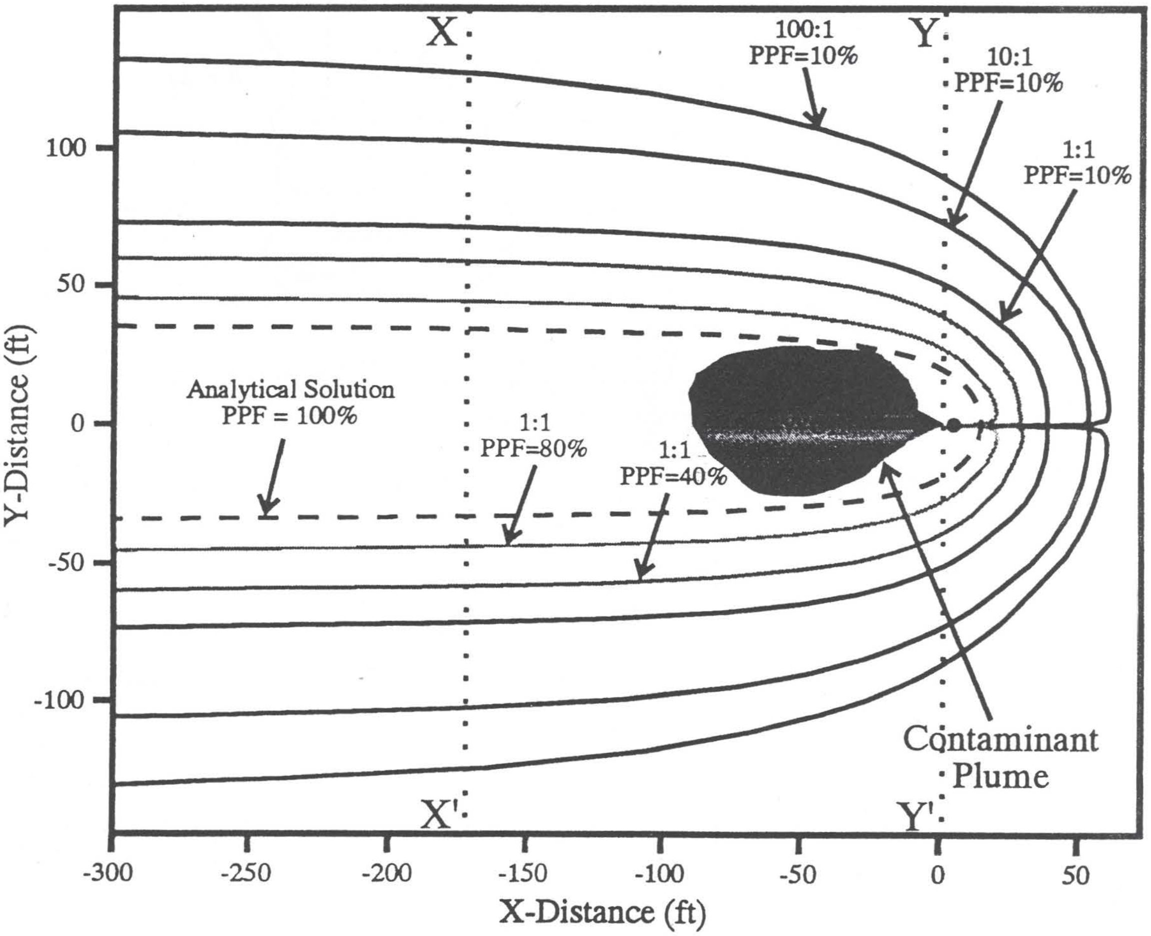


Figure 2: Effect on Radial Spread of a Potential Contaminant With Variation in Anisotropy Ratio (Bair and Lahm, 1995).

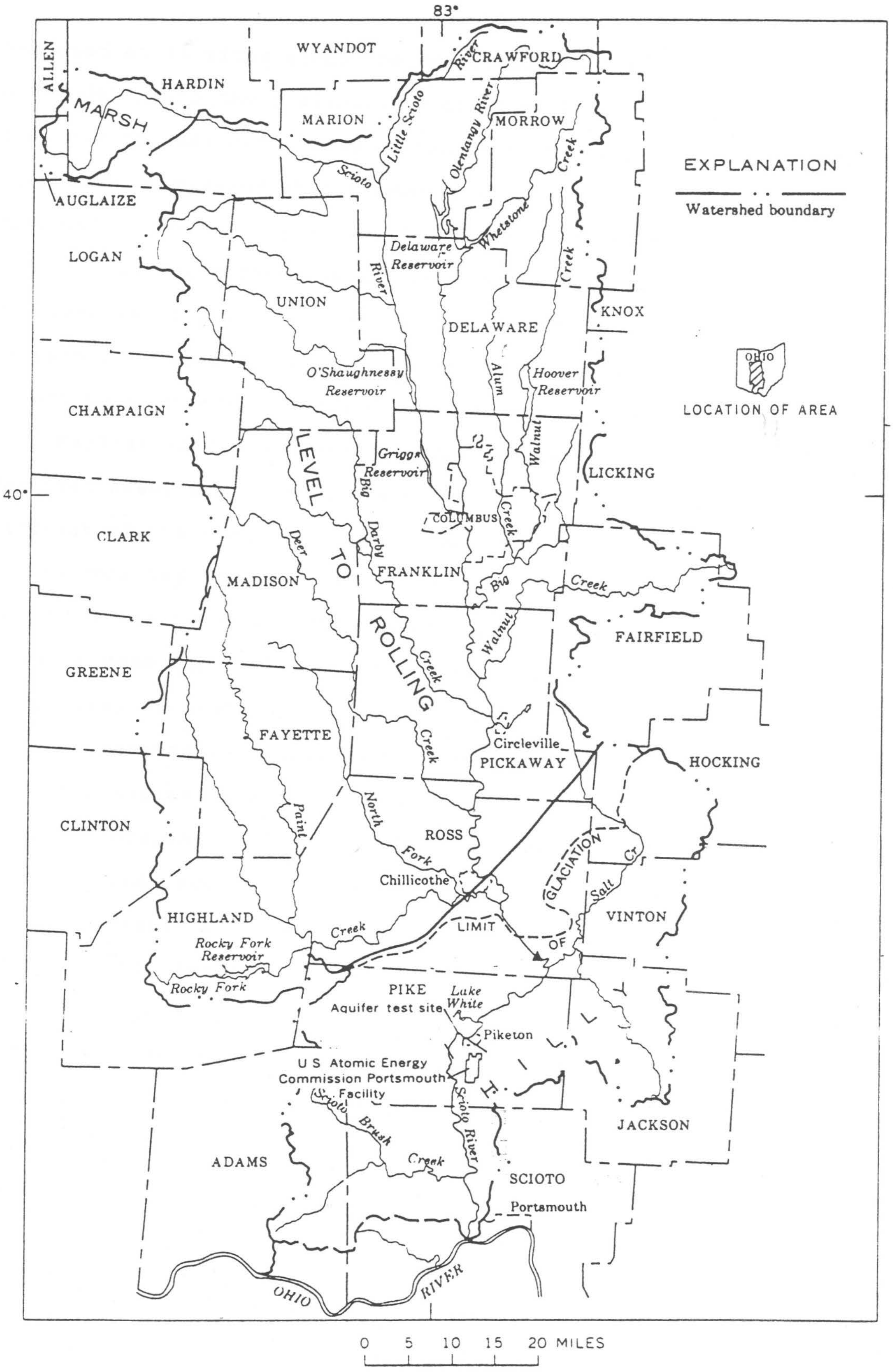


Figure 3: Map of Scioto River Basin Showing Aquifer Test Site (Norris and Fidler, 1969, p.2).

conducted at 11 sites along the Scioto River. The purpose of the tests was to determine the availability of 20 million gallons of water per day to supply the gaseous diffusion facility operated by the U. S. Nuclear Regulatory Commission at Portsmouth, Ohio, 25 miles to the south. Previously, the only water for industrial processing (mainly for cooling) was drawn from the Scioto River. Degradation of the quality of the river water over time due to increased population and industry upstream increased the cost of water treatment, causing plant engineers to look to ground water as a potential supply (Norris and Fidler, 1969).

Earlier analysis techniques for aquifer tests made certain idealized assumptions about the nature of the aquifer and the design and placement of the observation and pumping wells. For example, the pumping well had to fully penetrate the aquifer. Another assumption was that water was released instantaneously from storage in the partially saturated zone above the declining water table. By making these assumptions, accurate evaluation of vertical-flow components created by partially penetrating wells is not possible with analytical solutions.

In a series of articles published between 1993 and 1995, Dr. Allen F. Moench (Moench, 1993, 1994, 1995) proposed a new analytical method that takes into account the particular geometry of test wells and the slow and variable release of water from the unsaturated zone above the declining water table.

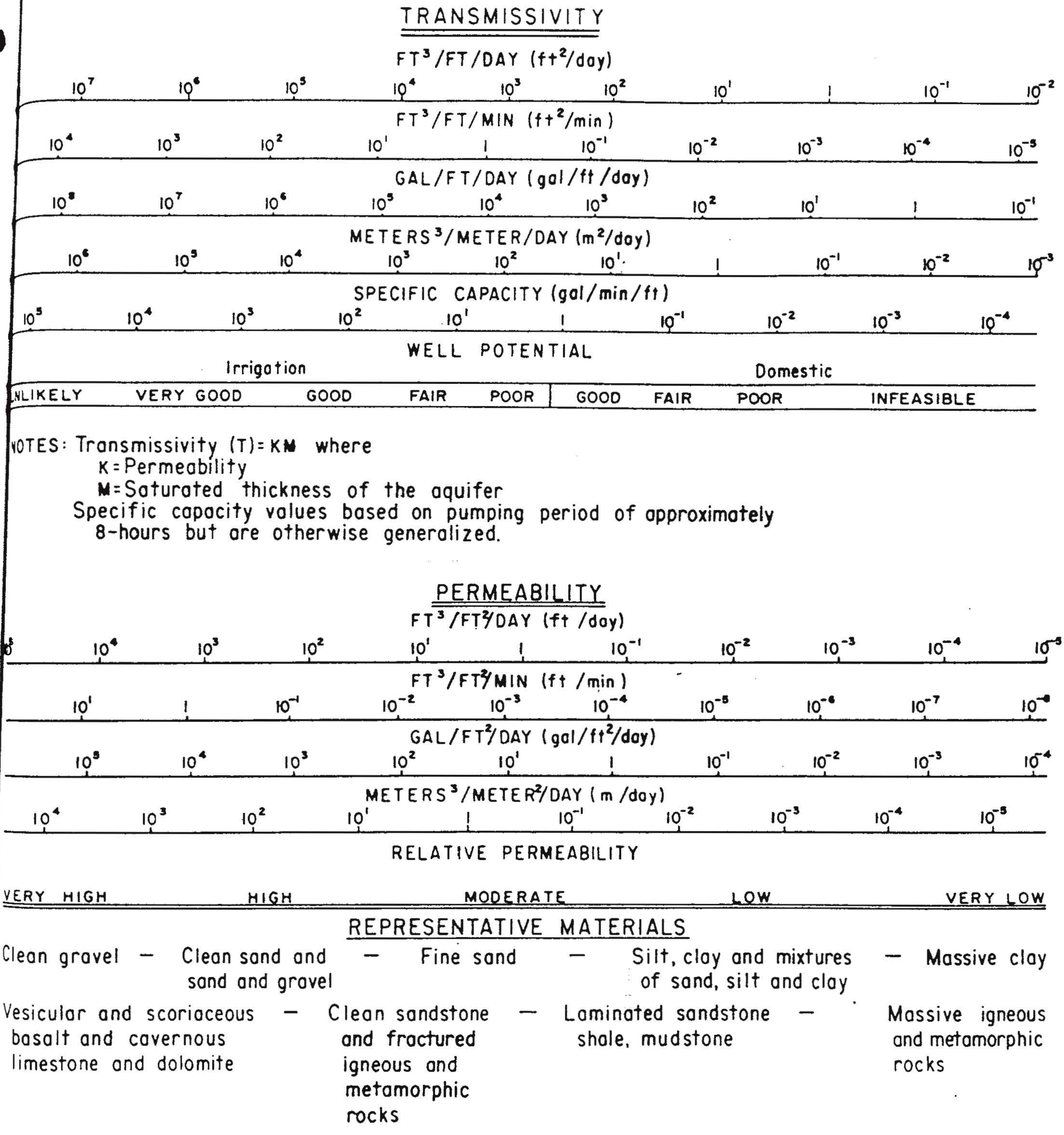
The purpose of this thesis is to reanalyze the data collected in the original 1963 nine-day pumping test at the Piketon site using the new method and computer program developed by Moench. I will compare the values of hydraulic properties computed using the Moench method with the values originally computed to determine whether accounting for specific well geometries and noninstantaneous release of water from storage gives more realistic results.

Definition of an Aquifer

An aquifer is defined as any part of a geologic unit that can hold or transmit enough water to supply wells. The porosity of the aquifer is the percentage of the rock that is void of material. The voids or pores can be openings between grains in the rock or fractures in the rock. The rate that ground-water flows through an aquifer is determined, in part, by grain shape and arrangement, the amount that the pores are connected, and the pattern of fractures in the rock. This is known as the effective porosity and is related to the permeability of the rock unit. Permeability is the ease with which ground-water flows through the aquifer (Domenico and Schwartz, 1990, p. 24-27). The technical term for permeability is hydraulic conductivity.

One parameter used to describe the flow of ground water through an aquifer is the hydraulic conductivity. Another is the transmissivity, which is the horizontal hydraulic conductivity multiplied by the thickness of the aquifer. Figure 4 shows the ranges that transmissivity and hydraulic conductivity can have in different geologic media. As shown, the permeability of geologic materials can vary over 11 orders of magnitude. For comparison, the range of temperatures on the surface of the earth is a little over two orders of magnitude. As a result of the great variability of permeability in geologic materials, it is important to measure permeability (hydraulic conductivity) as accurately as possible in the field. this is especially true when site-specific evaluations of ground-water resources or contaminant movement are being made.

The storage coefficient (S) is the amount of water that will come out of storage due to a decrease in the compression of the aquifer matrix and an increase in the expansion of the water molecule. Typical values of S are 0.001 to 0.00001. Specific yield (Sy) is the amount of water that will come out of storage that results from vertical drainage of the pores as the water table declines. Typical values of Sy range from 0.01 to 0.45. The latter parameter is the more important type of



NOTES: Transmissivity (T) = KM where
K = Permeability
M = Saturated thickness of the aquifer
Specific capacity values based on pumping period of approximately 8-hours but are otherwise generalized.

Figure 4: Ranges of Transmissivity and Hydraulic Conductivity for Various Materials (Ground Water Manual, 1981).

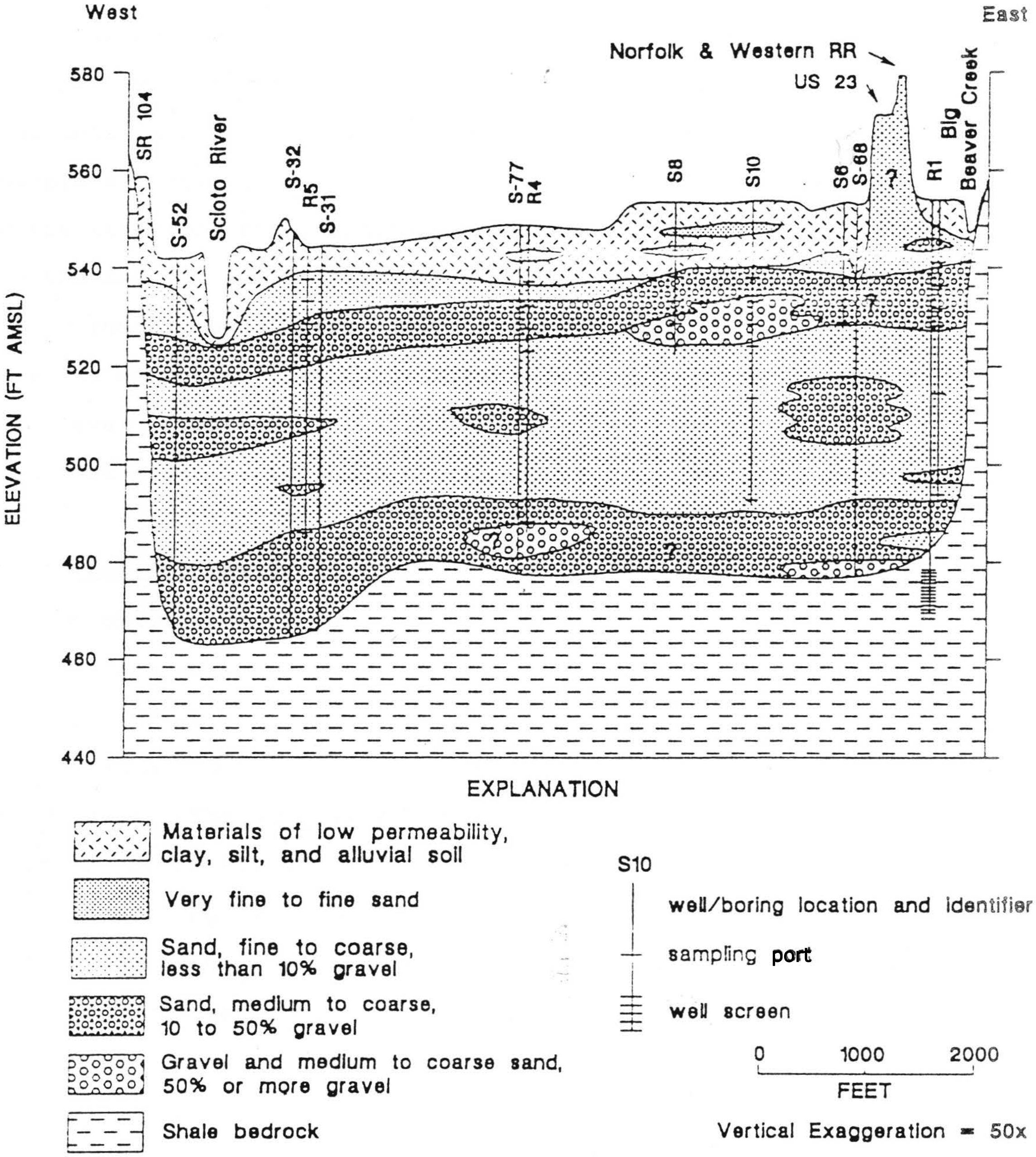


Figure 5: East-West Cross Section of Alluvial Valley and Aquifer near Test Site (Finton, 1994, p.53).

storage mechanism for an unconfined aquifer because in this type of aquifer water is mainly derived from pore-water drainage.

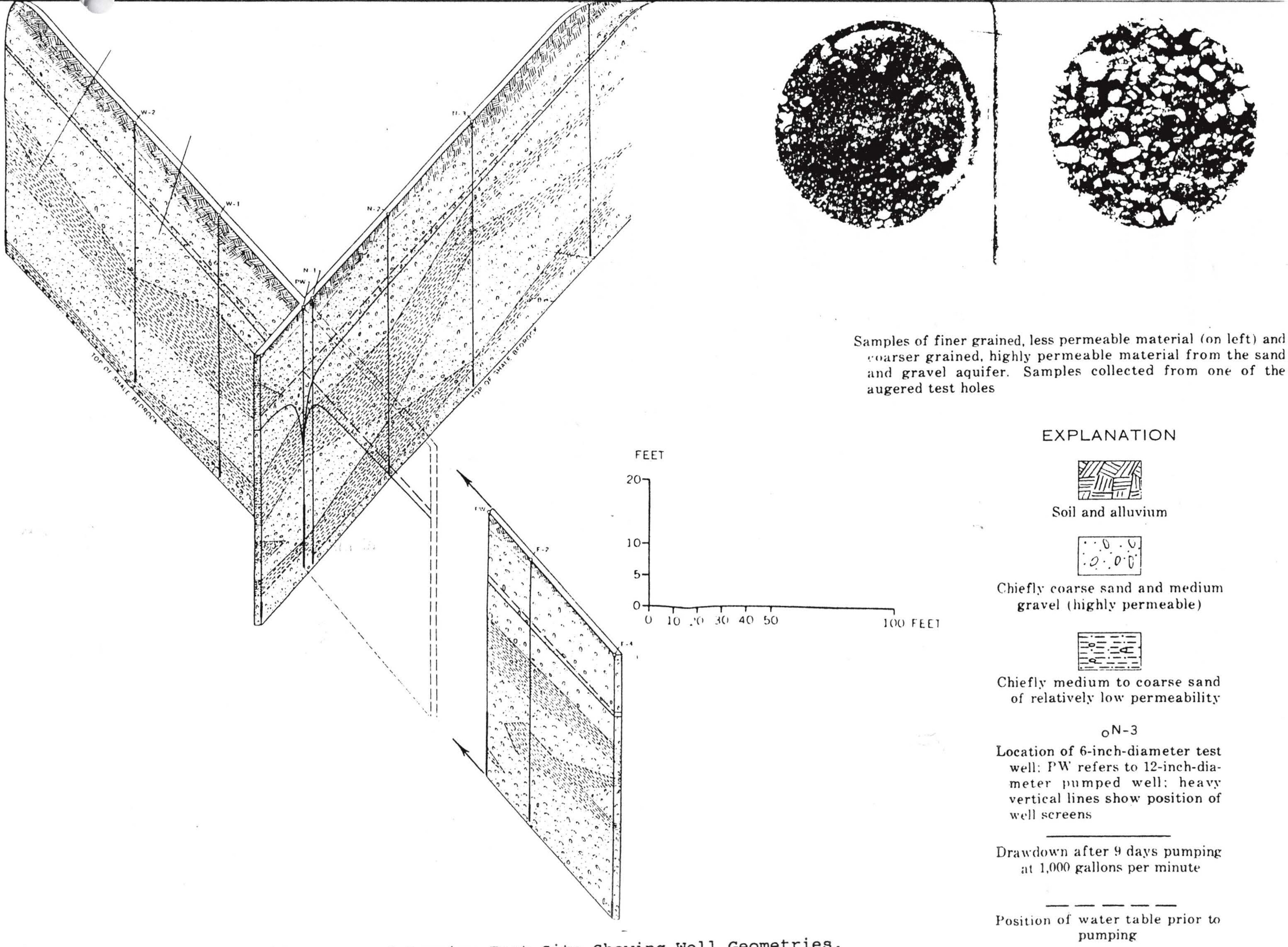
Description Of the Site

Piketon is located in the Scioto River Valley in southern Ohio, about 20 miles south of the limit of glaciation (Norris and Fidler, 1969). The aquifer at Piketon lies in a preglacial valley and is approximately 40 to 65 feet thick. The aquifer consists of sand and gravel outwash from the glaciers, overlain by fine-grained, poorly permeable alluvium from the Scioto River (figure 5). There are two discrete layers of finer-grained material, one thin zone a few feet above the bedrock and another 12 to 20 feet thick separating the aquifer into two roughly equal parts (Norris and Fidler, 1969, p.13). The aquifer is unconfined, with the water table lying 10 to 15 feet below ground level (figure 6). The Scioto River has downcut into the aquifer, and in most areas is in contact with the aquifer (Norris, 1983a).

Original Aquifer Tests and Analysis

The original aquifer tests were conducted in 1963 in October during the dry season when water levels in the river were stable and changes in infiltration would least affect results. Pumping lasted for nine days at a constant rate of 1000 gallons per minute (192,500 cubic feet per day). The pumping well was located 450 feet from the south bank of the river (figure 7). The observation wells were located at right angles along two lines, one parallel to the river extending through the pumping well (parallel line) and the other extending from the pumping well perpendicular to the river (river line) (Norris and Fidler, 1969).

The pumping well was 12 inches in diameter and 83 feet deep with a screen in the bottom 20 feet of the aquifer. The observation wells were six inches in diameter with screens in the bottom five feet of the aquifer (Norris and Fidler, 1969). Shallow drive-point wells were



Samples of finer grained, less permeable material (on left) and coarser grained, highly permeable material from the sand and gravel aquifer. Samples collected from one of the augered test holes

EXPLANATION

Soil and alluvium

Chiefly coarse sand and medium gravel (highly permeable)

Chiefly medium to coarse sand of relatively low permeability

oN-3

Location of 6-inch-diameter test well; PW refers to 12-inch-diameter pumped well; heavy vertical lines show position of well screens

Drawdown after 9 days pumping at 1,000 gallons per minute

Position of water table prior to pumping

Figure 6: Fence Diagram of Pumping Test Site Showing Well Geometries, With Grain-Size Scale (Norris and Fidler, 1969, Plate 1).

installed just below the water table at the same locations as the observation wells.

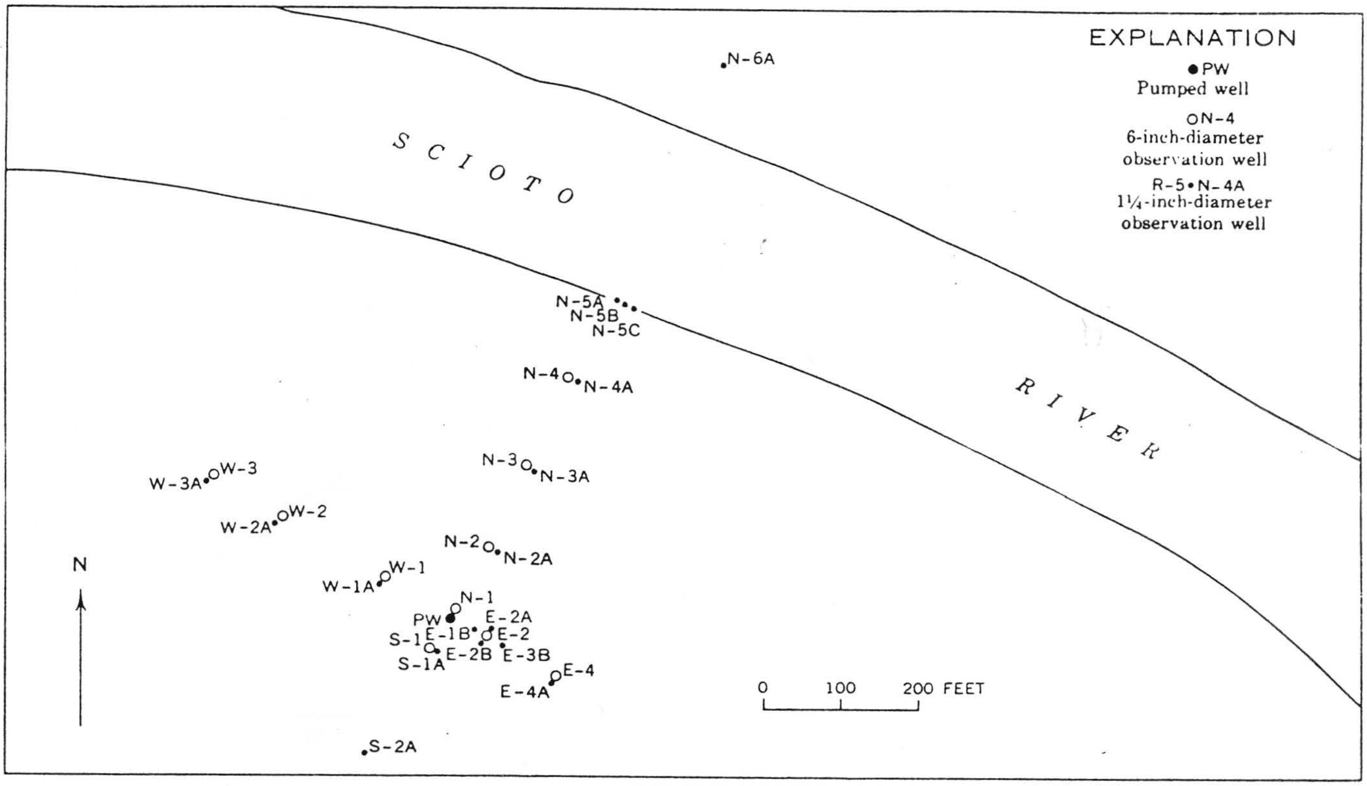


Figure 7: Location of Wells for Pumping Test (Norris and Fidler, 1969, p.15).

The drawdowns observed in the deep and shallow wells were averaged to "correct for errors resulting from partial penetration of the pumped well" (Norris and Fidler, 1969, p.26). Values of drawdown were adjusted for dewatering effects (Jacob, 1944).

Distance-drawdown formulas developed by Rorabaugh (1956) and converted to graphical form by Schafer and Kaser (1965) were used to determine an average transmissivity of 28,700 square feet per day (Norris and Fidler, 1969, p.24-28)(figure 8). The coefficient of storage was considered by the original investigators to be equivalent to the specific yield (Norris and Fidler, 1969, p.31). A distance-drawdown method developed in 1946 by Cooper and Jacob based on the Theis nonequilibrium formula for flow to a well in a confined aquifer (Theis, 1935) was used. The specific yield was determined to range from 0.1 to

0.85. Because of the induced infiltration from the river, specific yield values can be unrealistically large.

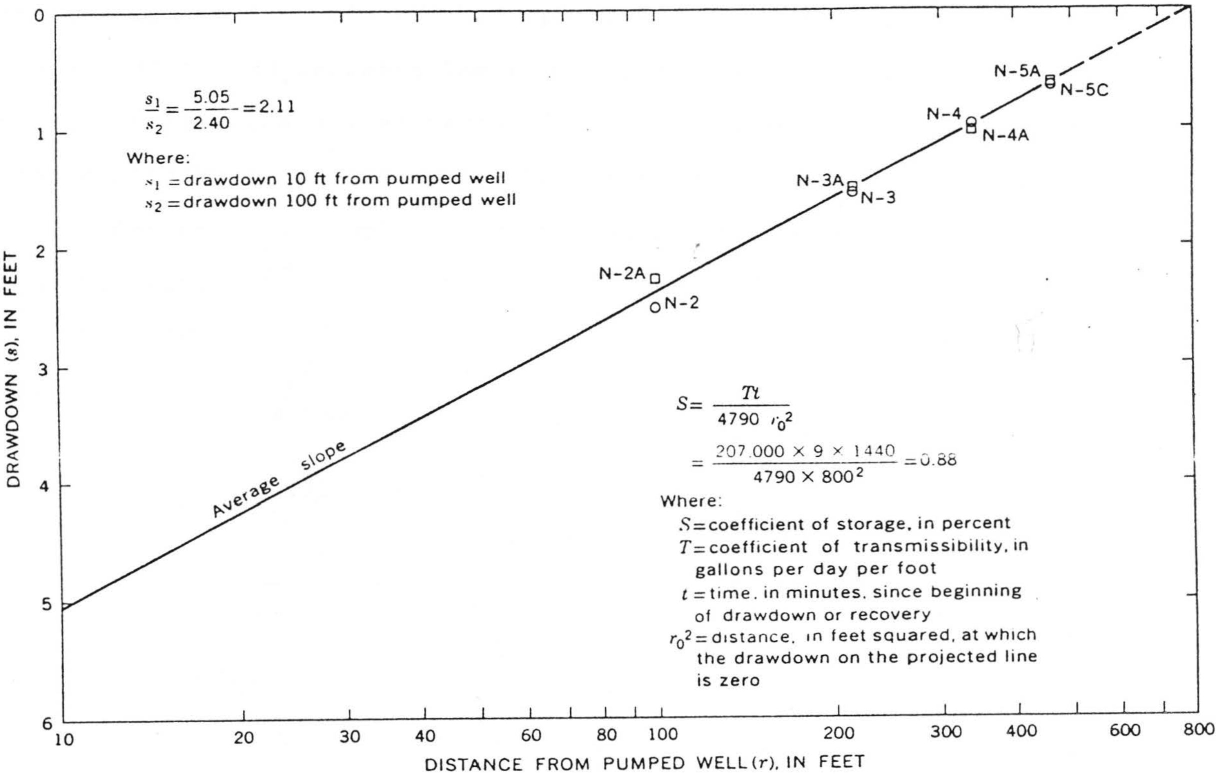


Figure 8: Graph of Drawdown vs. Radial Distance(Using Data From River Line Wells) Used to Determine Transmissivity and Storativity in Original Study (Norris and Fidler, 1969, p.27).

In 1966, the investigators used a method using special type curves developed by Stallman (1965) to determine a more accurate value of specific yield of 0.2, which is in the proper range for a water-table aquifer. Stallman used electric-analog simulations to study the effects on specific yield of vertical flow components, the difference between vertical and horizontal permeability, and the partial penetration of the pumped well.

The Moench and Neuman Method

Neuman (1973) addressed this same problem with a new set of universal type curves. To account for cases with partially penetrating wells, he developed an analytical solution and wrote a computer program to generate type curves for a particular geometry of pumping and observation wells. The computer program was called DELAY2. Using the Neuman method and assuming the pumping well was fully penetrating, Norris in 1991 reanalyzed three of the wells and calculated an average transmissivity of 36,368 square feet per day and a specific yield of .09. However, the complexity of the analytical solution required much computer time and greater precision than readily available on most computers (Moench, 1993).

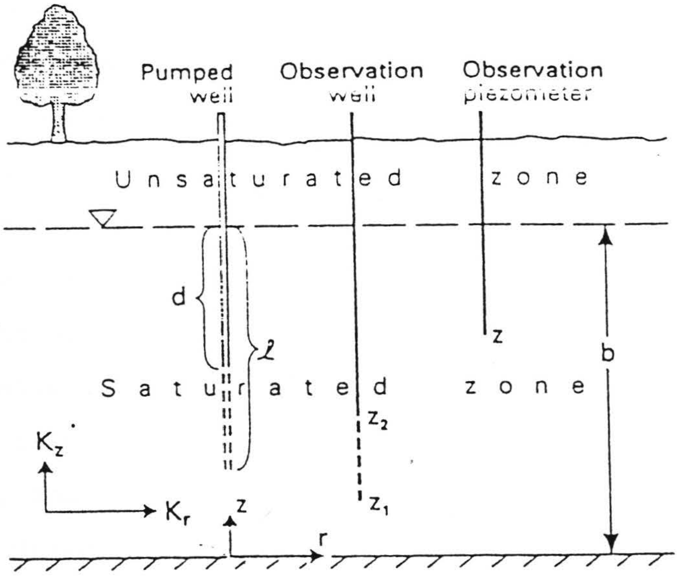


Figure 9: Schematic of Pumping and Observation Wells With Variables Used in Type Curve Computations (Moench, 1993, p.967).

In 1993, Moench proposed a different analytical solution for analyzing data from pumping tests in water-table aquifers, as he thought that calculated values for specific yield from aquifer tests were lower than values determined from water-balance calculations from field data or column drainage experiments conducted in the laboratory. He developed a new analytical solution and wrote a computer program called WATQ1 to generate type curves for pumping tests in water-table aquifers. This solution is specific to a particular aquifer and to a given geometry of wells. This new method particularly addresses the assumption that water drains instantly from the partially saturated zone above the water table. Figure 9 is a schematic of an idealized water-table aquifer with a partially penetrating pumping well, observation well, and piezometer showing how the wells are located within the aquifer (Moench, 1993).

The computer program written by Moench generates type curves for analyzing pumping tests in water-table aquifers and takes into account the effects of partial penetration and noninstantaneous pore drainage. The analytical solution is the sum of three drawdown components: the Theis (1935) solution for flow to a well in a confined aquifer, the deviation from Theis due to the effects of partial penetration developed by Hantush (1961), and the deviation due to effects of the free-surface drainage developed by Neuman (Moench, 1993).

The program was *relatively* straightforward to work with. It creates type curves by plotting dimensionless time against dimensionless drawdown. The curves are plotted on a log-log graph, at the same scale as the time and drawdown data measured during the field tests in the observation wells. The two graphs are placed on top of one another and a match point is chosen as in any other type-curve analysis method. Transmissivity and storativity are calculated from the equations for dimensionless drawdown (h_D) and time (t_D):

$$h_D = 4\pi T(h_i - h)/q_0$$
$$t_D = Tt/r^2 S$$

where

S = specific yield

T = transmissivity,

t = time, and

r = radial distance of the observation well from the pumped well,

hi = initial water level,

h = measured head, and

q_0 = pumping rate.

The user sets up a file of values, free format, that the program uses to compute the type curves. The user determines the length of time the curves will be computed for, number of log cycles on the time scale, and the number of points to be plotted per log cycle. The program computes type curves for up to six observation wells at one time.

A value for σ (the ratio S/Sy (storativity/specific yield)) and the ratio of vertical to horizontal conductivity (XKZKR) must be entered by the user, along with geometries of the pumped and observation wells. The user differentiates between a confined or unconfined aquifer; partially or fully penetrating pumped well; and a partially or fully penetrating observation well, or piezometer.

The values for σ and XKZKR determine the shape of the type curve. Figure 10 shows that an increase in σ increases the slope of the early-time part of the curve. This corresponds to increasing storativity, meaning more water is being released due to aquifer matrix compression. Figure 11 shows the effects of XKZKR on the shape of the curve. A decrease in the ratio lengthens the flat part of the curve and flattens the inflection of the tail during the late time. This can be related to a relatively higher vertical hydraulic conductivity, implying that a greater volume of water is released from the pores and that the water drains from the pores more quickly.

Data Analysis

I plotted measured drawdown against the known distance of each observation well from the pumping well after 2000 minutes, at which time the ground water was coming from pore-water drainage (figure 12). Graphed this way, the drawdown, at least in the wells closer to the pumped well, should fall on a straight line (Fetter, 1994, p.227). The three wells on the river line closest to the pumping well do fall on a straight line, with all of the wells falling near the straight line. From this graph I used the Jacob (1944) straight-line distance-drawdown

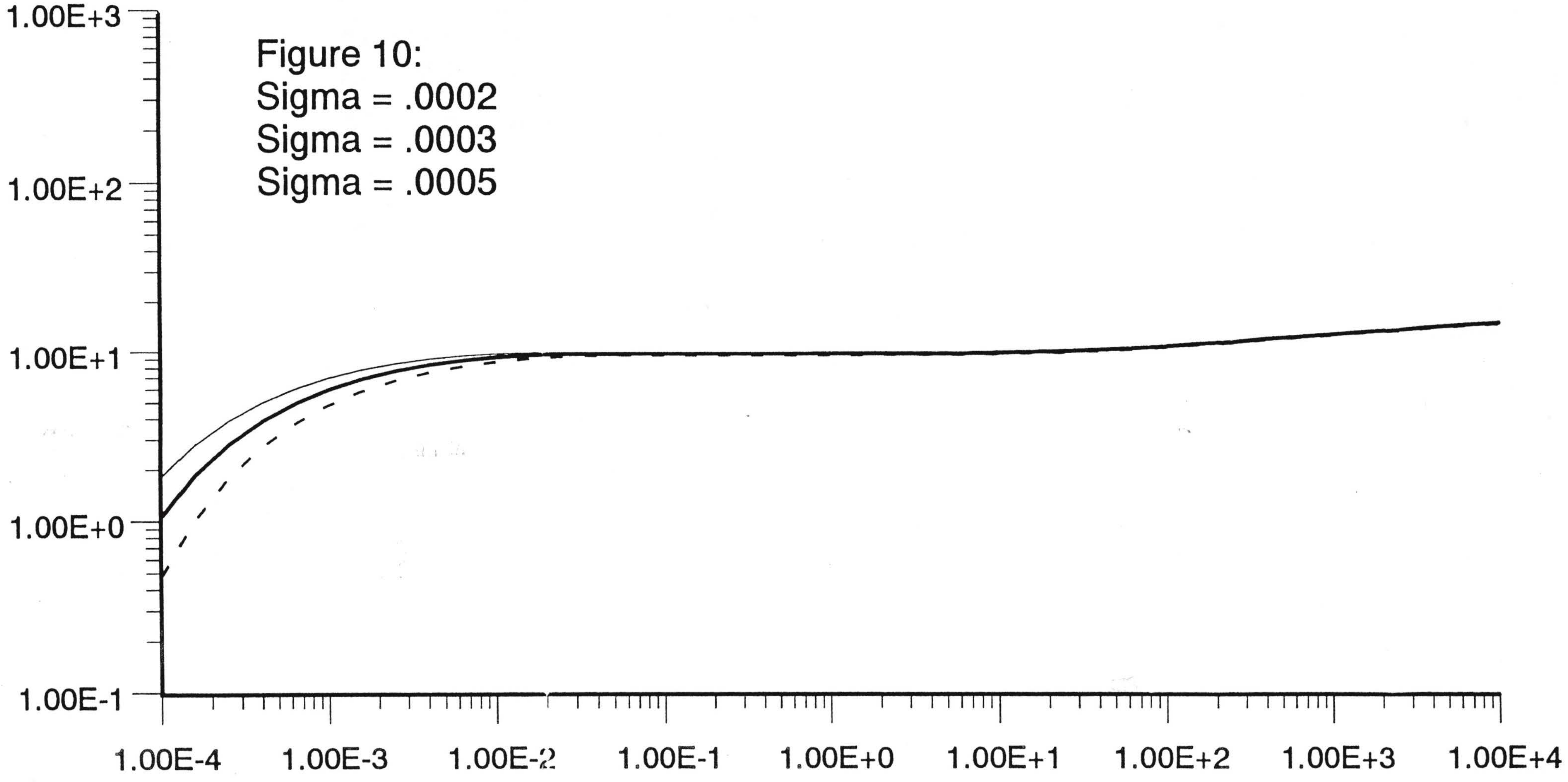


Figure 10:

Sigma = .0002
Sigma = .0003
Sigma = .0005
1.00E+3
1.00E+2
1.00E+1
1.00E+0
1.00E-1
1.00E-4
1.00E-3
1.00E-2
1.00E-1
1.00E+0
1.00E+1
1.00E+2
1.00E+3
1.00E+4

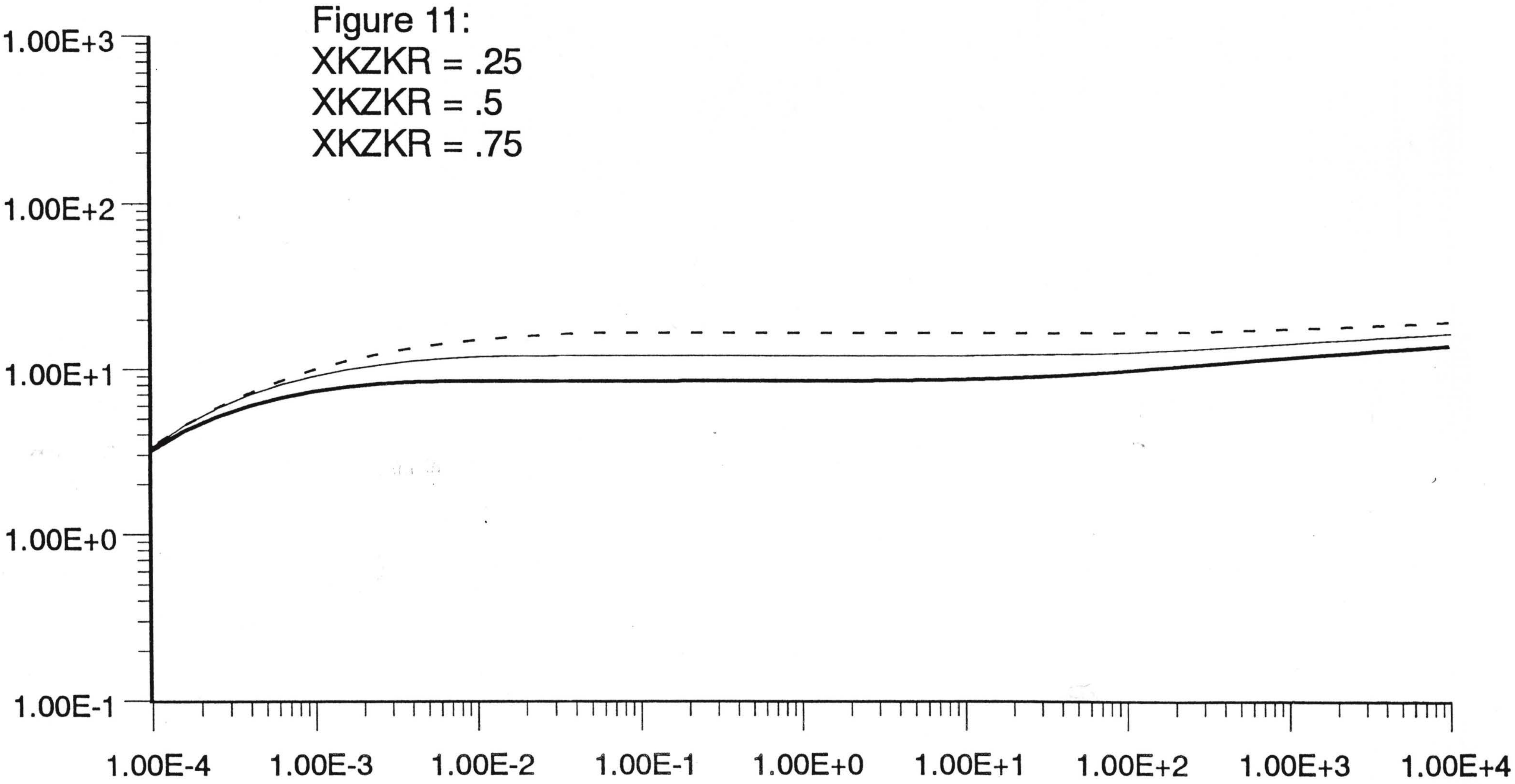


Figure 11:

XKZKR = .25
XKZKR = .5
XKZKR = .75
1.00E+3
1.00E+2
1.00E+1
1.00E+0
1.00E-1
1.00E-4
1.00E-3
1.00E-2
1.00E-1
1.00E+0
1.00E+1
1.00E+2
1.00E+3
1.00E+4

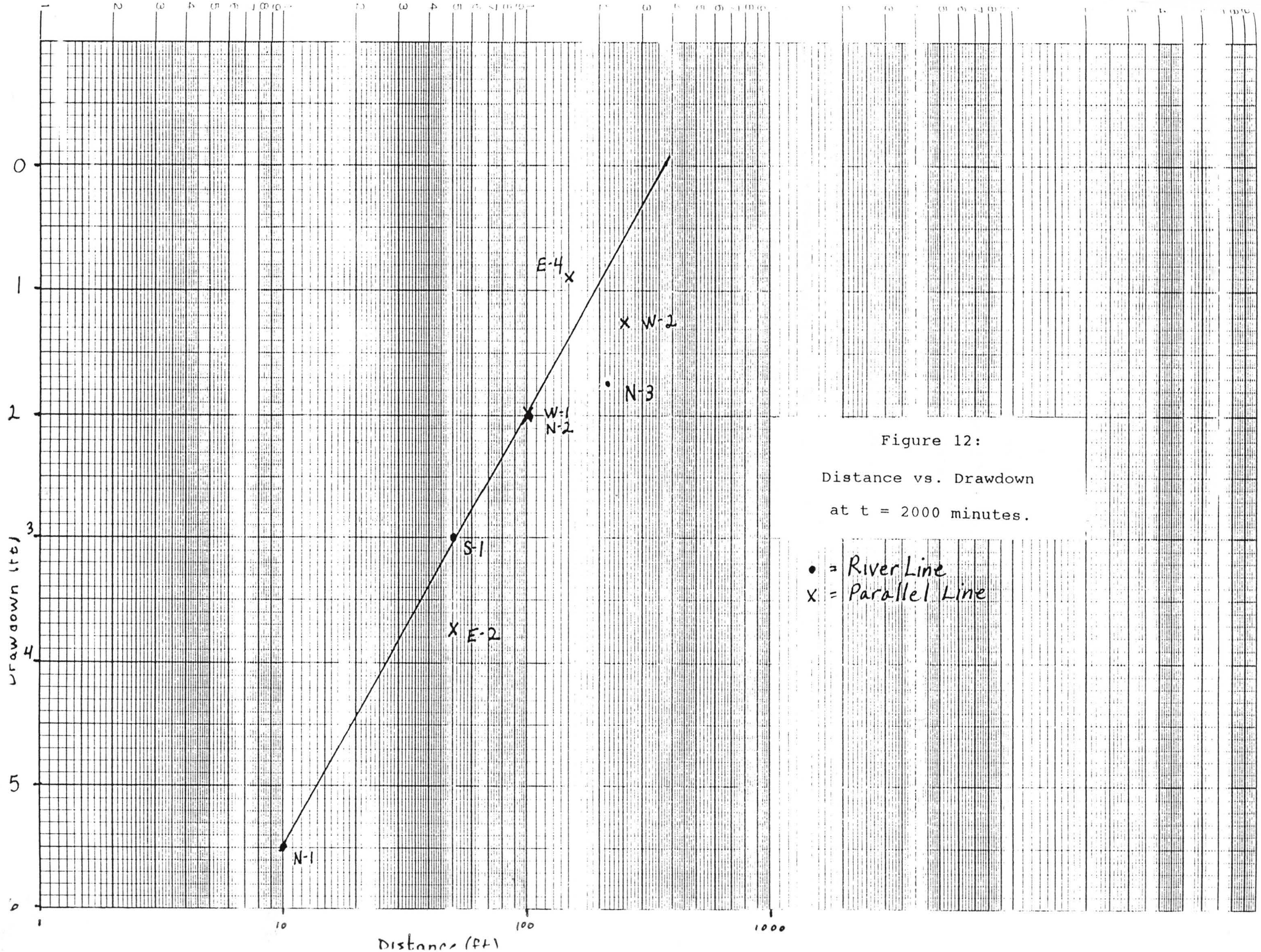


Figure 12:

Distance vs. Drawdown

at t = 2000 minutes.

analytical method to calculate an average transmissivity of 20,100 square feet per day and an average specific yield of 0.5.

To avoid accounting for the affects of infiltration from the river, I analyzed only data from wells more than 200 feet from the river. I plotted the original elapsed time and adjusted drawdown data provided by Norris (1991) in the GRAPHER program (figures 13-21, 19 omitted). I initially ran WTAQ1 using average water-table aquifer parameter values, and then adjusted the parameters until the theoretical type curves fit the actual field-data curves. It took approximately 15 tries to match the first curve, but once I found the range of parameters for the test site I was usually able to match each curve with three or four tries.

The results using the method developed by Moench are given in the tables at the end of this paper. I calculated the results in two ways. First I used results from all the observation wells; then I used the results from those wells with data curves that I was most precisely able to match to type curves. In the last tables I compare my results to those from the previous studies by Norris, who used different analysis techniques.

The average transmissivity of 31,052 square feet per day that I calculated is within the range of the previous averages. The standard deviation is 11,159 square feet per day, and two standard deviations about the mean value gives a range of 8468 to 53,104 square feet per day.

The average value of specific yield is higher than those previously calculated. This is what Moench predicted was happening in analysis of water-table aquifers. It is also what led him to develop his analytical method and computer program.

The average ratio of horizontal to vertical hydraulic conductivity of 9.3 that I calculated is less than that previously calculated. The standard deviation is 4.8. Only the previously calculated Kh/Kv ratio with the Stallman method falls within this range.

Figure 13:
Obs. Well N-1
r=10'

Curve N-1
S/Sy = .005
XKZKR = .1

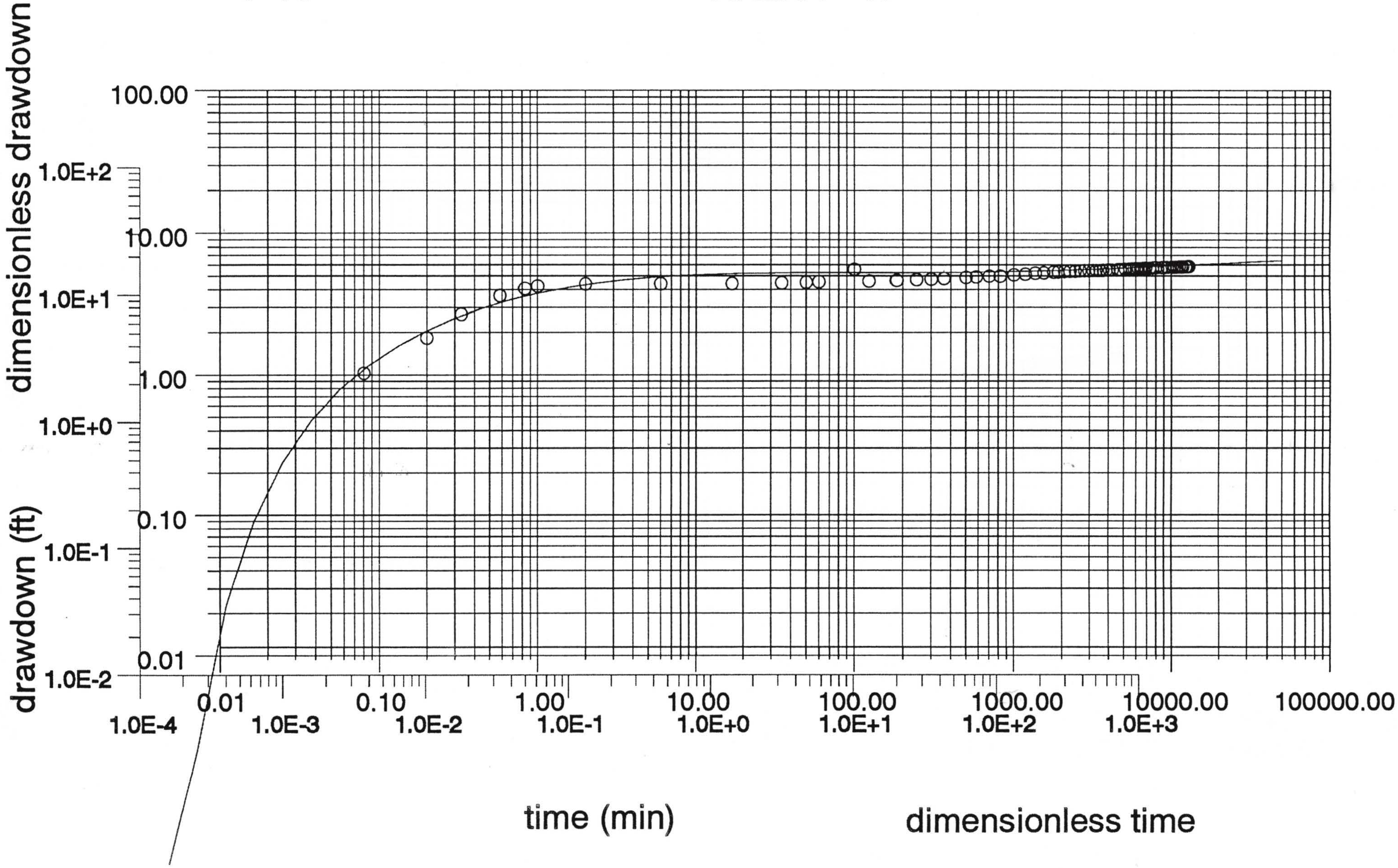


Figure 14:
Obs. Well N-2
r = 100'

S/Sy = .0007
XKZKR = .5

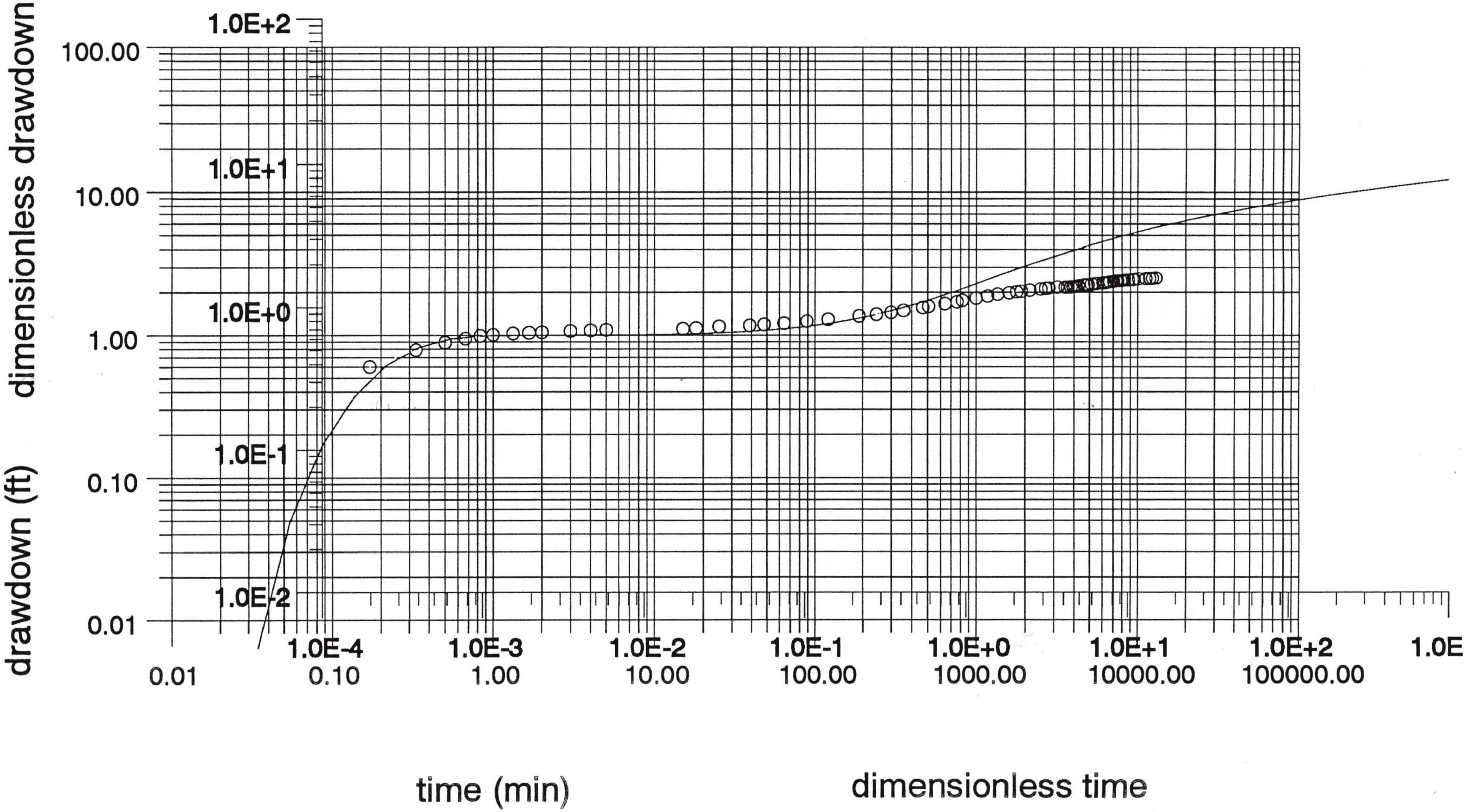


Figure 15:
Obs. Well N-3
r = 216'

Curve N-3
S/Sy = .8
XKZKR = .5

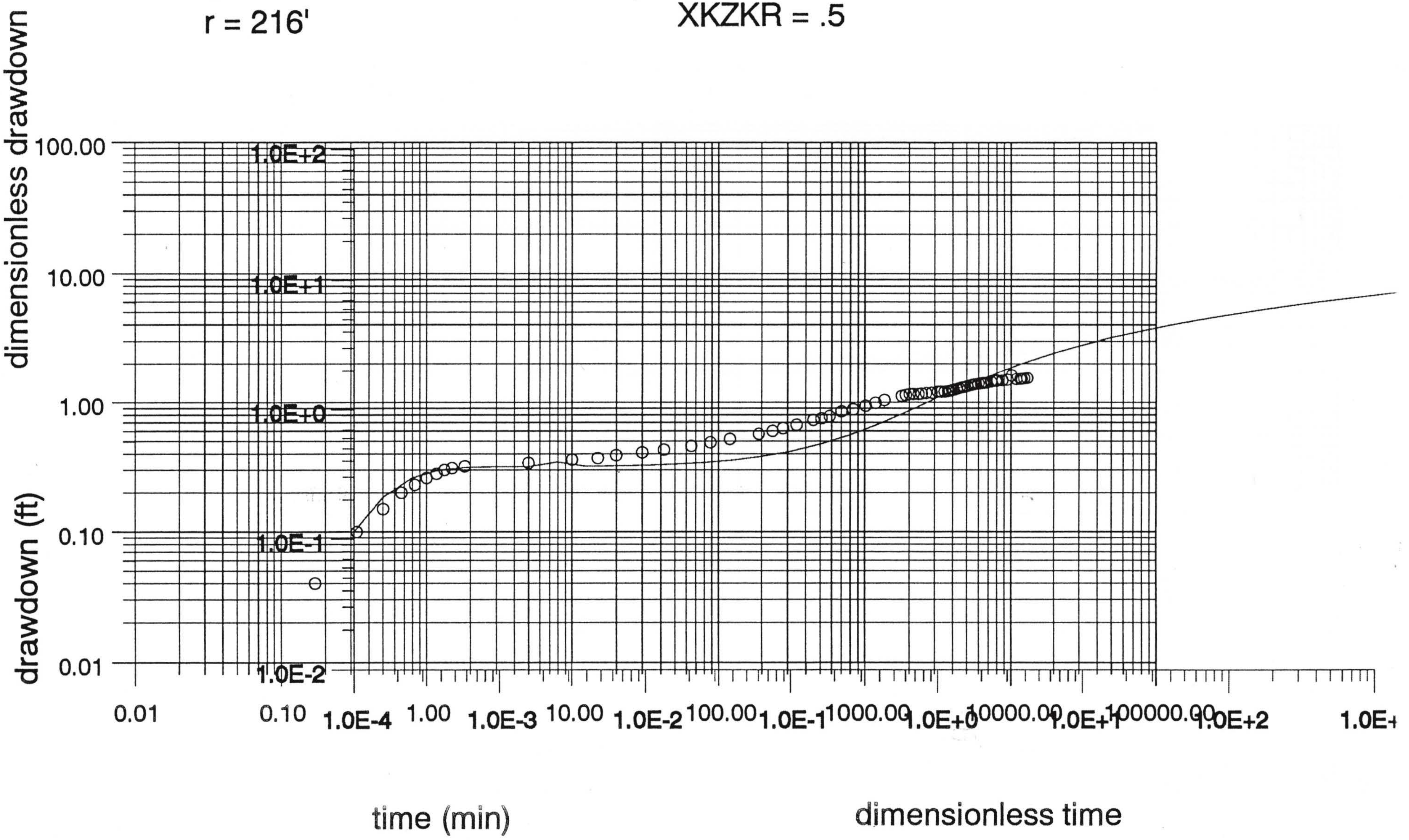


Figure 16:
Obs. Well S-1
r = 50'

Curve S-1
S/Sy = .005
XKZKR = .1

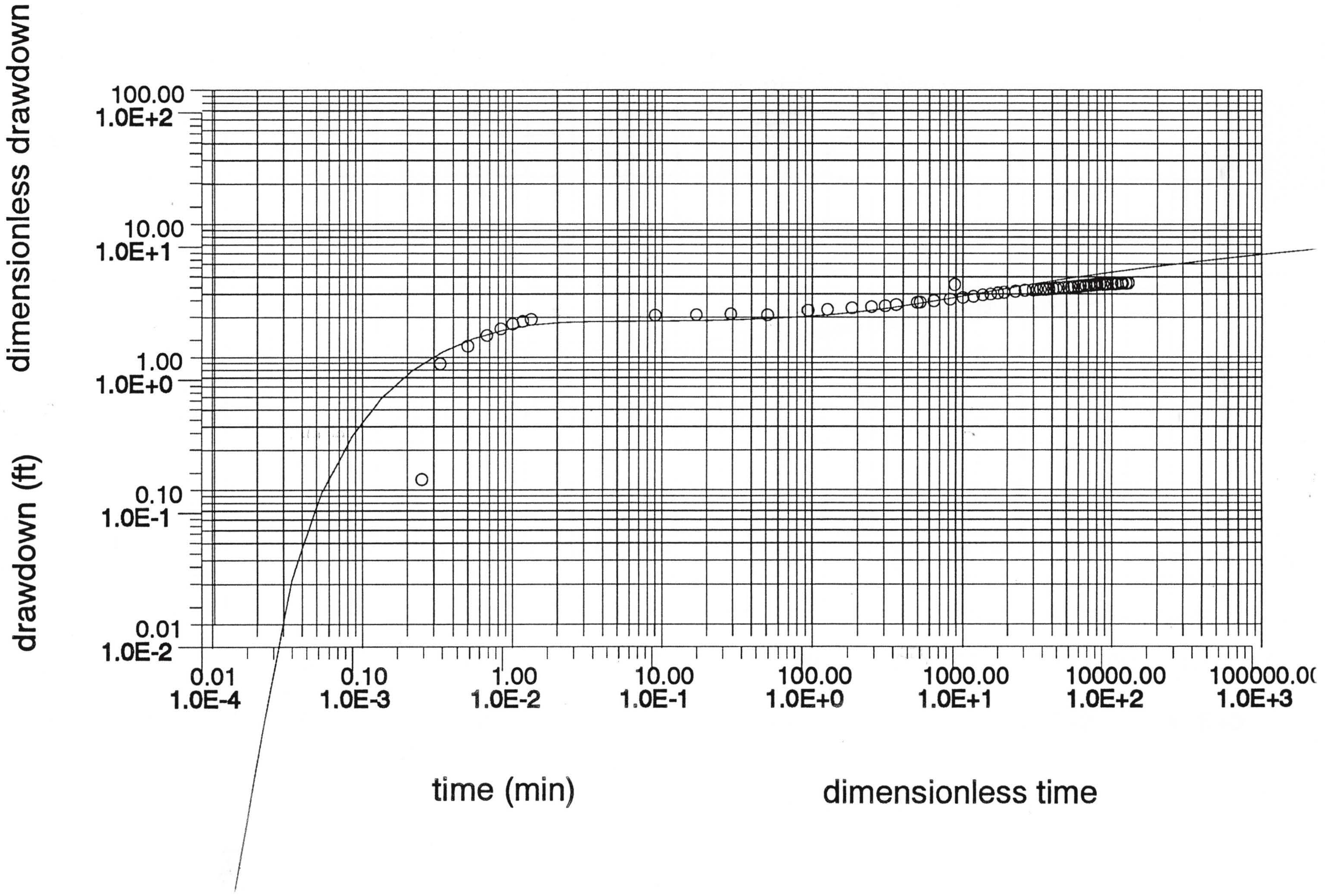


Figure 17:
Obs. Well W-1
r = 100'

Curve W-1
S/Sy = .003
XKZKER = .08

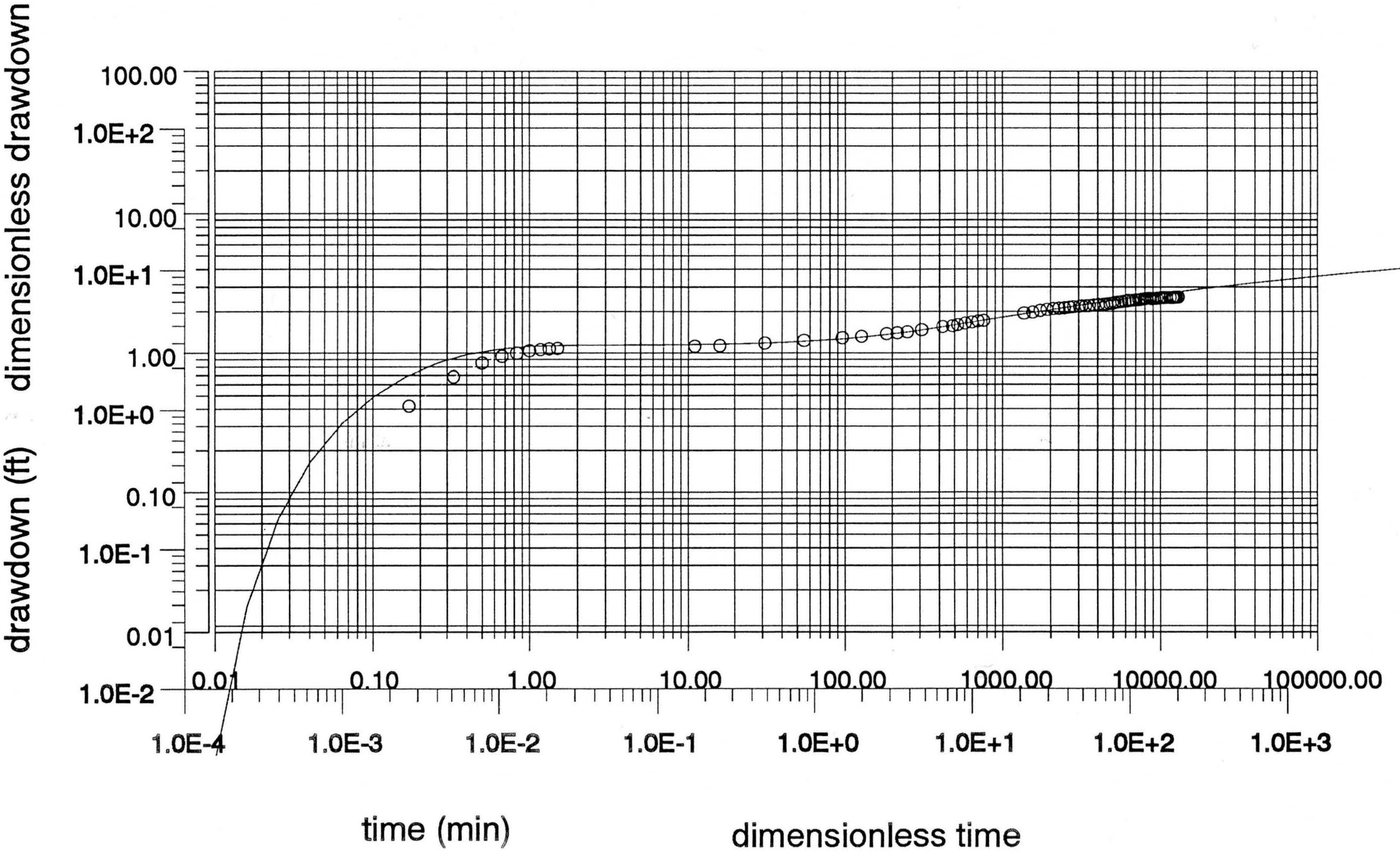


Figure 18:
Obs. Well W-2
r = 250

Curve W-2
S/Sy = .003
XKZKR = .5

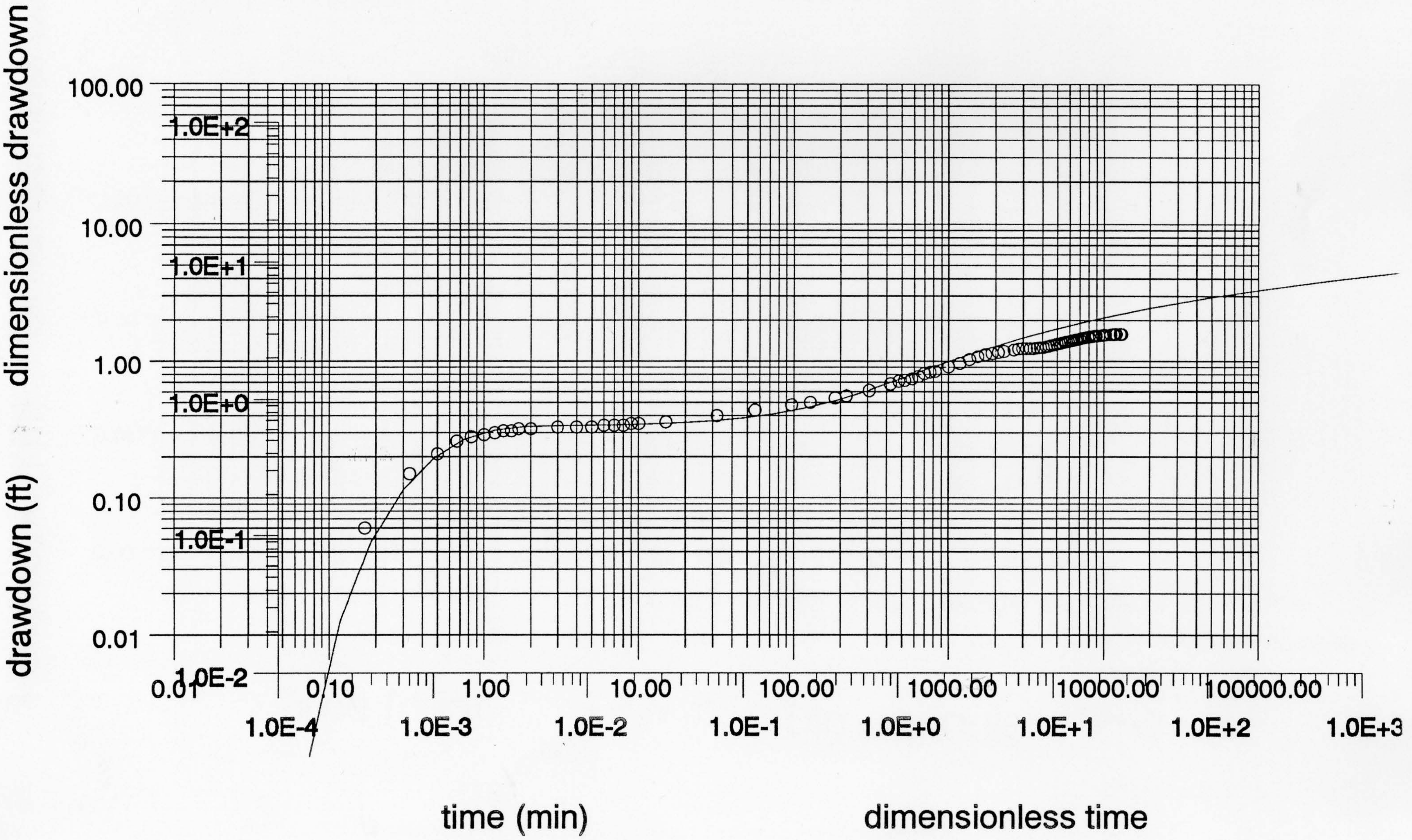
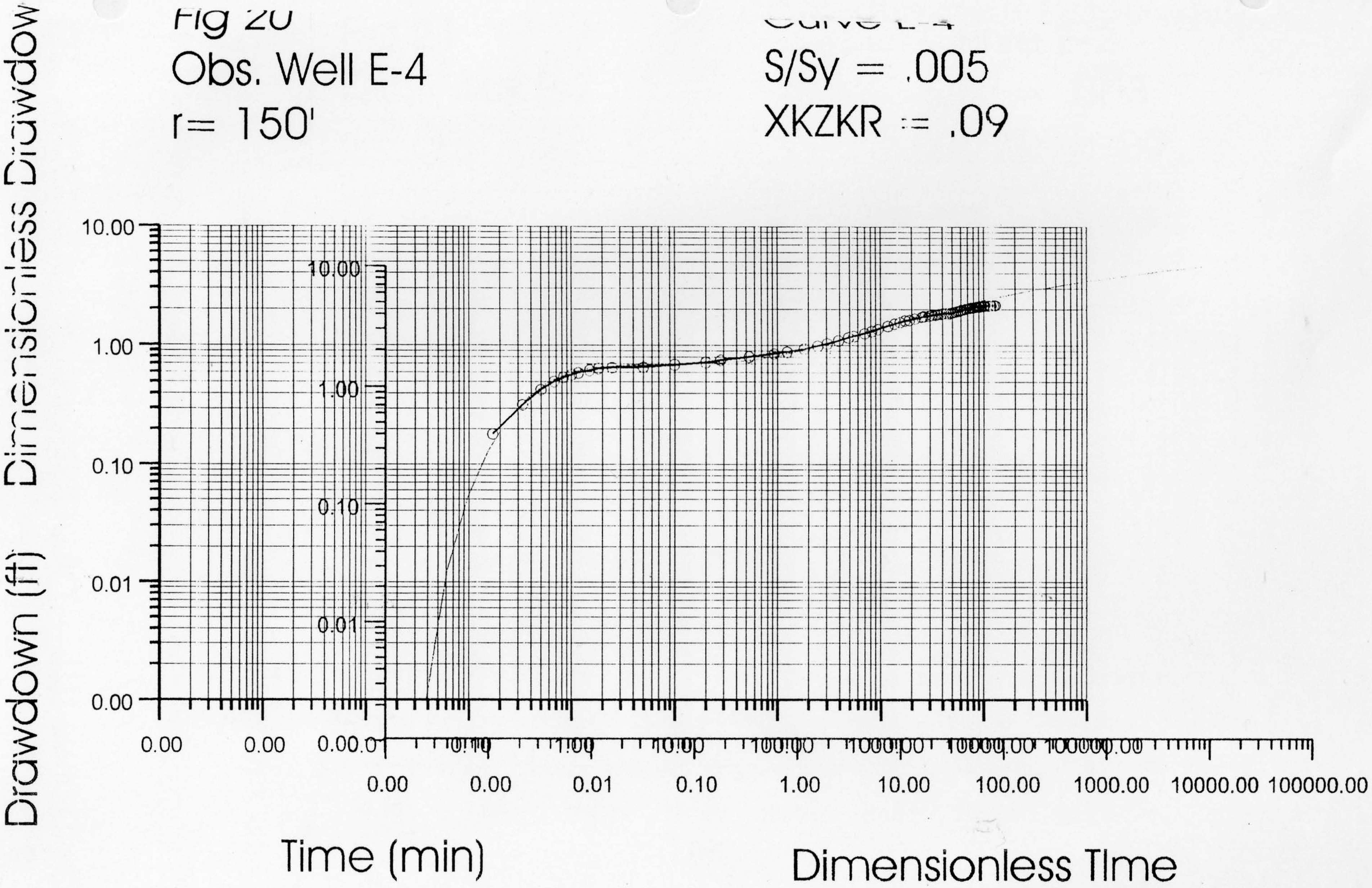


Figure 19 is missing from the original text.

Fig 20
Obs. Well E-4
r= 150'

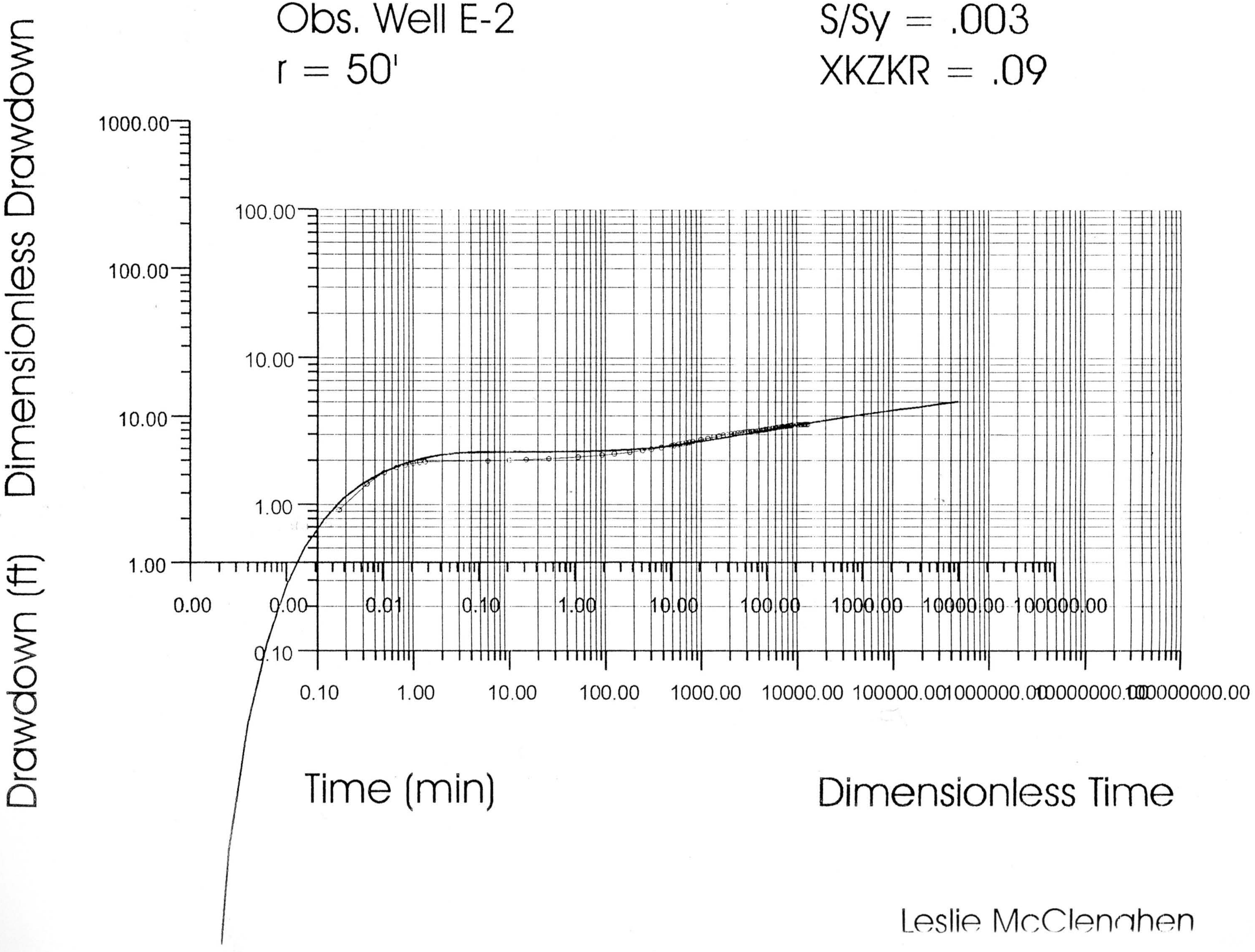
Curve E-4
S/Sy = .005
XKZKR = .09



Leslie McClenahen

Fig 21
Obs. Well E-2
r = 50'

Curve E-2
S/Sy = .003
XKZKR = .09



Leslie McClenahen

I was unable to fit the data from observation well W-3 to a type curve with any precision, so I omitted that well in calculating averages. I only analyzed data from the observation wells, as problems analyzing the data from the piezometers prevented me from using them for this paper. The inflection of the tail part of the theoretical curve in late time was less than in the generated type curves, meaning thad drawdown was greater than the theroetical behavior. I think it is a consequence of the layer of finer-grained material that divides the aquifer acting as a confining layer and causing the upper part of the aquifer to behave as a separate aquifer unit.

Conclusions

The values of transmissivity I calculated are not significantly different from those previously calculated. My horizontal to vertical hydraulic conductivity ratios are different from all of the ratios previously calculated. The Moench method is therefore important in analyzing pumping tests in water-table aquifers.

The ratio of horizontal to vertical hydraulic conductivity is an important aquifer parameter for designing contaminant remediation systems. The results obtained from the Piketon data using the program and analytical method developed by Moench are reasonable for this type of aquifer. As figures 1 and 2 show, the use of earlier methods in designing a recovery program would have led to overestimation of vertical contaminant travel and unnecessary expense. The new values would allow for a design using a smaller, more realistic safety factor.

I think that the lower values than previously obtained suggest that the layer of finer material affects the performance of the aquifer. As no earlier methods could account for this, earlier studies averaged the data from the bottom of the aquifer with the upper part of the aquifer. I think the result was that this effect was missed.

As methods for analyzing aquifer tests improve, the accuracy with which we can predict the flow of ground water and contaminants through an aquifer increases. Budget constraints seldom allow for the implementation of a "perfect" pumping-test design. Existing well designs are commonly used that are seldom perfect. As in the Piketon example shown above, certain aquifer characteristics are rarely ideal. The method for evaluating water-table aquifers developed by Moench accounts for much of the natural variability that might be present in a pumping test.

TABLE OF RESULTS

SELECTED DATA : Conform to Theory

OWell No.	r (ft)		T (ft2/day)	Kh (ft/day)	Kv (ft/day)	Kh/Kv	Sy	S/Sy
N-1	10		34000	524	52	10	2.400	0.005
N-2	100		9000	139	69	2	0.490	0.001
W-1	100		38300	589	47	13	0.150	0.003
E-2	50		38300	589	54	11	0.540	0.003
E-4	150		35600	548	49	11	0.180	0.005
average			31040	478	54	9	0.750	0.003
average w/o N-1							0.300	0.003

ALL DATA

OWell No.	r (ft)		T (ft2/day)	Kh (ft/day)	Kv (ft/day)	Kh/Kv	Sy	S/Sy
N-1	10		34000	524	52	10	2.400	0.005
N-2	100		9000	139	69	2	0.490	0.001
N-3	216		17000	262	131	2	0.840	0.800
S-1	50		30600	589	59	10	1.200	0.005
W-1	100		38300	589	47	13	0.150	0.003
W-2	250		29500	453	227	2	0.150	0.003
E-2	50		38300	589	53	11	0.540	0.003
E-4	150		35600	548	49	11	0.180	0.005
average			29000	447	86	8	0.740	0.017
(average w/o N-1)							0.390	0.136

COMPARISON WITH PREVIOUS ANALYSES

		T (ft2/day)	Kh (ft/day)	Kv (ft/day)	Kh/Kv	Sy
J.T.	river line	21500	330.7692308	61	6	0.32
Moench	parallel line	37400	575.3846154	59	10	0.24
Method	average	31,000	476.9230769	54	8	0.28
Norris	river line	38,200	588	22	27	0.1
Neuman	parallel line	34,500	531	14	37	0.09
Method	average	36,400	560	17	33	0.09
Norris	river line					
Stallman	parallel line					
Method	average	28,700	442	71	6	0.2
Norris	river line					.1-.85
original	parallel line					.1-.5
	average	28,700	441			

italicized = data from N-1 not used

blank cell = data not available

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