Mersenne Primes

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Marin Mersenne's mathematical contributions helped paved the way for advances in mathematics. In this article, his life is not only depicted but his mathematical intellect is highlighted through his achievements. Since his time, technology has helped further the finding of Mersenne primes. Prime numbers are large concepts for students to learn in school. Mersenne primes further examine the study of prime numbers.

Introduction

Prime numbers have intrigued the minds of mathematicians throughout history. Prime numbers are characterized by only having two factors: 1 and itself. The development of these numbers was a major advancement in number theory. Whether mathematicians studied these numbers in their free time or they studied them to make advances in number theory, many positive contributions have come from such investigations. One such mathematician who made a great advance in the study of prime numbers was Marin Mersenne. He is most widely known for a certain kind of prime number, Mersenne prime numbers.

Mersenne primes are a special type of prime number. They have distinct characteristics in comparison to "regular" prime numbers. The mathematical concept of Mersenne primes was developed in the search for perfect numbers. A perfect number is a number in which the sum of its proper factors equals the original number.

For example, 6 is a perfect number. The factors of 6 are: 1,2,3,6. When we add the factors together that are less than 6 we get: 1+2+3=6

History

Marin Mersenne was born in 1588 in France. He studied at the Jesuit Collége du Mains of La Fléche (Wanko, 2005) and became an ordained priest in 1614. Mersenne is described in many different ways by historians. Some believe he was "a shrewd thinker, a brilliant experimentalist, an inferior mathematician" while some even believe he needed psychiatric evaluation (Buske & Keith, 2000). Although, many people only know of Mersenne for his famous conjecture for which numbers are named for him, he was also an important influence on young mathematicians. He became a mentor to young researchers in the fields of mathematics and science. Mersenne was discouraged because these fields were in decline, unable to attract a sufficient number of scholars. He created a research center in Europe and used it to keep in contact with other mathematicians and scientists and notify them of any recent discoveries in their fields. He was a mentor to a young Pascal. Without such mentoring, it is hard to say if we would have the same theorems we do

(Image Courtesy of http://fr.academic.ru/dic.nsf/frwiki/1240555

Fig 1 Marin Mersenne (1588-1648)

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today. Mersenne was friends with other famous mathematicians such as Descartes and Fermat. Unlike Mersenne, they did not enjoy reading the works of other scholars because they feared they would be overly influenced by their discoveries. "Mersenne felt that much more could be accomplished through shared ideas and communication and thus took it upon himself to correspond with young mathematicians and scientists and provide them with detail of others' findings" (Wanko, 2005). After Mersenne had passed away in 1648, it was determined that he had seventy eight different correspondents all across Europe (Wanko, 2005). Mersenne's famous conjecture came in 1644, only four years prior to his death.

During his time, Mersenne had to compute all calculations by hand. He spent countless hours trying to find values of k.

Brief explanation of primes

Before we delve further into Mersenne primes, it is helpful to first provide a brief overview of prime numbers. Recall that a prime number only has two factors: one and the number itself. For example, five is a prime number because it only has 2 factors, 1 and 5. If a number has more than these two factors it is said to be composite. Primes are considered a building block in mathematics. As the Fundamental Theorem of Algebra indicates, all integers may be represented as a unique product of primes. Prime numbers are important in mathematics, and it is crucial for students to learn about them while in middle school.

Mersenne Primes

The discovery of Mersenne primes can be attributed to contributions made by Euclid, the Pythagoreans, and Marin Mersenne himself. The history begins with an increasing interest in finding perfect numbers.

During Mersenne's time, the Pythagoreans had only found four perfect numbers: 6, 28, 496, and 8128. The fact that only four perfect numbers had been found presented a challenge to find more perfect numbers. Euclid wrote a series of books entitled Elements. In Proposition 36 of book IX of the *Elements*, Euclid states a formula to find perfect numbers.

Euclid's formula stated:

If $2k-1$ is prime for $k>1$, then $n=2k-1(2k-1)$ is a perfect number.

For example if $k=2$ then $22-1=3$ (which is prime). Now we can plug 3 into Euclid's equation. $n=22-1(3)=6$, which is a perfect number.

How does Euclid's formula for perfect numbers relate to the primes that Mersenne discovered? "If Euclid's theorem were true, the problem of finding larger perfect numbers could be boiled down to finding values for k where the expression 2^k-1 is prime," (Wanko, 2005). This is when Marin Mersenne stepped in and formed his conjecture. He was also interested in finding more perfect numbers, so he took it on himself to find values of k to satisfy Euclid's formula. Mersenne was aware that in order for 2^k-1 to be prime, k itself must also be prime. In 1644, he claimed that he had found all the prime numbers that make 2^k-1 prime. His conjecture consisted of the numbers: 2, 3, 5, 7, 13, 17, 19, 31, 67, 127, and 257. These numbers then became known as the Mersenne Primes.

Mersenne's list of possible values for k was questioned by some in regards to his degree of accuracy. During his time, Mersenne had to compute all calculations by hand. He spent countless hours trying to find values of k. Since they were all worked out by hand, there were probably some computational errors. "It wasn't until 1947 that all of the possible Mersenne primes (p) were checked using computers," (Wanko, 2005). Much to the surprise of many mathematicians, his conjecture was correct (although not complete).

Mersenne's original conjecture:
$M = 2^k - 1$ is prime for $k = 2, 3, 5, 7, 13, 17, 19, 31, 67, 127,$ and 257.

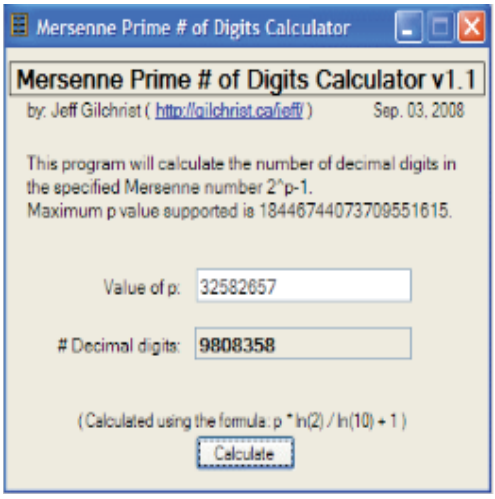
Actual Primes (Wanko, 2005): With $k \leq 257$, $M = 2^k - 1$ is prime for $k = 2, 3, 5, 7, 13, 17, 19, 31, 67, 127,$ and 257.

One may notice that this list of primes starts out by listing the prime numbers in order, but isn't 11 also a prime number? Mersenne noticed that not every prime number satisfied the equation. For instance, if we try to use 11 we get: $2^11 - 1 = 2048$. We know that 2048 can't be prime because it is divisible by 2 (2×1024).

Fig 2 Beware of false generalizations!

Mersenne Primes and Technology

In the seventeenth century, when Mersenne made his famous conjecture, the only method mathematicians had for finding large numbers was by hand. Now, with the aid of technology, people continue to search for larger and larger prime and Mersenne prime numbers. "In 1996, George Woltman, using networking software by Scott Kurowski, created the database GIMPS (Great Internet Mersenne Prime Search), which coordinates the efforts of more than 8,000 computer users internationally in a practice known as distributed internet data processing" (Buske & Keith, 2000). The main server then distributes and collects information from all the people participating. Anyone can help participate in the search if they have the right type of computer. Monetary rewards are even being offered to whoever finds the first ten-million-digit prime number. In 1999, the search found its then record high prime $2^{6972593} - 1$. It was found by Nayan Hajratwala of Michigan using the GIMPS software that he downloaded on his computer (Buske & Keith, 2000).This number is over two million digits long and could fill over seven miles of digits and commas (Buske & Keith, 2000). Then, in 2004, another prime became the largest known world prime, $2^{24036583} - 1$, consisting of over seven million digits (Wanko, 2005). At the time of this writing, the largest known Mersenne prime is $2^{43112609} - 1$. To date there are 47 known Mersenne primes and the search is still ongoing and it is surprising at how large these numbers can be.



(Image Courtesy of gilchrist.ca/jeff/MprimeDigits/index.html)

Fig 3 Mersenne digits calculator

Conclusion

In conclusion, Marin Mersenne made a significant contribution to mathematics. His intellect and mathematical curiosity about perfect numbers helped to pave the way for his conjecture. Today, with the help of computers, mathematicians have found the largest prime number to date through the

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use of Mersenne primes. Maybe in the near future the next largest prime number will be found. After all, there is an infinite number of them!

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Behind the scenes

"When an idea is served up from behind the scenes [unconsciousness], your neural circuitry has been working on it for hours or days or years, consolidating information and trying out new combinations."

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"Insanity is doing the same thing, over and over again, but expecting different results."

- Albert Einstein