Analyzing Dollar Cost Averaging

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Mathematics teachers are always looking for situations in which students can apply mathematical processes in meaningful real-world settings. We will present one such setting involving an investment strategy commonly called "dollar cost averaging."

The term "average" is commonly used in the classroom. Its usual meaning involves finding a "typical" value that generally represents all the data in a set. For instance if a student's scores on five tests are 68, 81, 72, 93 and 71, the mean score (one meaning of average) would be $\frac{68+81+72+93+71}{5} = \frac{385}{5} = 77$. This average score of 77 in some sense "smoothes out" the variation in the actual scores; if all five scores had been 77, the same sum would have resulted.

But the term "average" is used much more broadly than simply computing means. It is our goal in this article to display one of its many other roles (dollar cost averaging), this role being of special significance in discussing investment strategies. It is hoped that this discussion will enhance students' understanding of some of the varied ways in which this topic can be used.

To understand this strategy of dollar cost averaging, suppose that an investor, Drew, wishes to acquire stock shares in a particular company. His preference, of course, is to purchase these shares at their lowest price and to let their value appreciate. The problem, though, is that Drew is not able to predict with certainty the best time for this purchase. In spite of his best analysis, he might purchase the stock right before it decreases in value instead of preceding an increase.

One way to mitigate the negative effects of a bad timing choice is to buy shares of stock at several different times. In that way some shares are purchased at relatively higher prices, while others will be purchased at relatively lower prices. This staged purchase strategy, which enables Drew to "hedge his bets," is commonly called "dollar cost averaging" in that an 'average' purchase price results, a price that overall is between the highest and lowest possibilities.

To exemplify the use of this dollar cost averaging process, suppose that Drew decides to purchase $100 worth of ABC stock in each of six consecutive months. At the end of these six months, how many shares will Drew own, and what will be their value?

Suppose first that the prices of single shares of this ABC stock were as follows:

Month	Price per Share
1	$10
2	$ 8
3	$10
4	$12
5	$ 8

6	$10

In Month 1, Drew's $100 purchased 10 shares. In Month 2, his $100 purchased 12.5 shares $(\frac{100}{8})$; in months 3, 4, 5, and 6 he purchased respectively, 10, 8.33, 12.5 and 10 shares. In total, Drew has purchased 63.33 (to 2 decimal places) shares for a total cost of $600, or $9.47 per share on average. Since each share is worth $10 at the end of the cycle, Drew's portfolio is worth 63.33 (10) or $633.30.

If he had used all $600 in months 2 or 5, he would now have 6(12.5) = 75 shares worth $750 in total. But if his purchase had been entirely in month 4 (many people do enter the market at a high point, expecting it to go even higher), he would have $6(\frac{100}{12})$ =50 shares ultimately worth in total only $500. By using dollar cost averaging, Drew missed the best possible result but also avoided the least desirable outcome.

Let us now apply Drew's strategy to each of five possible additional scenarios:

Price Per Share

Month	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5
1	$10	$10	$10	$10	$10
2	$8	$12	$8	$12	$15
3	$6	$14	$6	$14	$5
4	$4	$16	$6	$14	$20
5	$2	$18	$8	$12	$5
6	$1	$20	$10	$10	$10

Number of Shares Purchased (All rounded to 2 decimal places):

Scenario 1: $\frac{100}{10}+\frac{100}{8}+\frac{100}{6}+\frac{100}{4}+\frac{100}{2}+\frac{100}{1}$

= 10 + 12.5 + 16.67 + 25 + 50 + 100 = 214.14

Scenario 2: $\frac{100}{10}+\frac{100}{12}+\frac{100}{14}+\frac{100}{16}+\frac{100}{18}+\frac{100}{20}=42.28$

Scenario 3: $\frac{100}{10}+\frac{100}{8}+\frac{100}{6}+\frac{100}{6}+\frac{100}{8}+\frac{100}{10}=78.33$

Scenario 4: $\frac{100}{10}+\frac{100}{12}+\frac{100}{14}+\frac{100}{14}+\frac{100}{12}+\frac{100}{10}=50.95$

Scenario 5: $\frac{100}{10}+\frac{100}{15}+\frac{100}{5}+\frac{100}{20}+\frac{100}{5}+\frac{100}{10}=71.67$

Two more meaningful statistics:

Average cost Per Share	Final Value of Portfolio
S1: $\frac{600}{214.4}=\$2.80$	214.14 (1) = $214.14
S2: $\frac{600}{42.28}=\$14.19$	42.28 (20) = $845.60
S3: $\frac{600}{78.33}=\$7.66$	78.33 (10)= $783.30

S4: $\frac{600}{50.95} = \$11.78$ 50.95 (10) = \$509.50

S5: $\frac{600}{71.67} = \$8.37$ 71.67 (10) = \$716.70

Since in each case \$600 was expended, the strategy yields the best results in S2 (a rising market), S3 (a market that first falls and then rises), and S5 (a wildly fluctuating market). It seems not to work well in S4 (a market that first rises and then falls) and S1 (a falling market).

But this is not the real point of the story. In each case Drew does better than if he had purchased all the shares at the highest price, and worse than if he had purchased all the shares at lowest price. His strategy avoids both the best and the worst possible outcomes. Looking back, he knows what he should have done to maximize his profits, but from his vantage point at the beginning of the process he would not have known the most advantageous purchase times. The dollar cost averaging process is designed not to maximize profits, but to avoid extremes which might be extraordinarily serendipitous but could also lead to disaster.

Another observation can be made. What now appears to be the worst outcome (S1) may turn out to be the best if the stock should rise significantly.

Questions for the Reader:

1. Apply the Dollar Cost Averaging process to other possible scenarios.
2. Discuss how this investment process applies when making periodic investments in interest-paying accounts when the available interest rate fluctuates over time.
3. Design and investigate other types of investment strategies.

Quote

"Mathematicians and physicists may manipulate abstruse symbols representing space, time, and quantity, but they first understood those entities as tiny children wanting a far-away toy, waiting for juice, or counting cookies. When we worked with children facing developmental problems who could nonetheless count, and even calculate, we found that numbers and computations lacked meaning for them unless we created an emotional experience of quantity by negotiating over pennies or candies."

Greenspan, S. I. & Shanker, S. G. (2004). *The first idea: How symbols, language, and intelligence evolved from our primate ancestors to modern humans*, 57. Da Capo Press. Cambridge, MA.