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## SUMMARY

Details of a theory of crosswind deflection of an air sprayer fan-shaped jet were developed. The effect of the radial divergence of the jet is taken into consideration as opposed to existing theories of plane-jet deflection. A system of coordinate transformations is derived to enable application of the theory to any radius on the jet midplane and to obtain three-dimensional deflected-midline trajectories for the jet. The deflecting crosswind is assumed essentially uniform over the region of interest in this version of the theory. Jet outlet velocity is assumed to be large in comparison with the crosswind velocity and jet integrity is presumed to be maintained in the deflection process.

# THEORY OF AIR-SPRAYER FAN JET DEFLECTION BY CROSSWIND FLOW*

In our earlier mathematical and experimental work [Brazee et al. (1981)], we established a base for a computer model for airflow in a fan (-shaped) jet produced typically by an orchard air sprayer. The model could be used to calculate velocity profiles for air sprayer jets (ASJ) issuing into either quiescent or moving ambient air. In a first attempt to study crossflow response, Fox et al. (1979) applied the plane-jet deflection theory of Abramovich (1963) to the jet model to account for crosswind and forward-travel effect. Since initial results were not completely satisfactory we carried out further theoretical developments and wind tunnel studies as reported in Fox et al. (1982). The purpose of the 1982 work was to measure in detail the effects of coflow, counterflow, and crossflow on an ASJ. A scale model of an orchard air sprayer system was placed in a wind tunnel that provided ambient crossflows.

The objective of the present study is to further develop the deflection theory for the ASJ by tying it directly to the fan jet theory developed earlier. A summary of the theory and a detailed experimental study of its applicability will be forthcoming later. The present report will outline the development of the deflection theory in detail, and also include details of the transformation formalism needed to carry the deflection problem into the general three-dimensional case.

The results developed in this study are demonstrated with a sample calculation in Appendix II, page 33. Figure II.1 shows a plot of the deflection of an air jet from a typical orchard air sprayer subjected to a crossflow velocity of 4.5 m/s. Growers often use sprayers when wind velocities are as great as 4.5 m/s. The large deflection of the air jet trajectory by travel and crossflow effects make it essential that growers consider wind effect when applying pest control agents with orchard air sprayers.

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## 1. A Theory of Fan Jet Deflection

A representative air mass element at some point C of a deflected fan jet is shown in Fig. 1. The constant ambient flow vector $\underset{\sim}{W}$ is parallel to the $q_1$-axis, and the jet centerline (midline) velocity $\underset{\sim}{U}$ at point C is at an angle $\alpha$ with respect to the $q_1$-axis. The unbalanced centrifugal force transmitted to the ambient air by the jet element is

$$F_c = -\frac{(\rho S_n ds)U^2}{R_c} , \tag{1-1}$$

while the unbalanced centripetal drag force acting on the jet element is

$$F_d = \tfrac{1}{2} c_d \rho (W \sin \alpha)^2 h_1 ds, \tag{1-2}$$

with

$$S_n = 2bh_1 . \tag{1-3}$$

The drag coefficient $c_d$ must be determined experimentally. It is assumed that the deflected jet's integrity is sufficiently maintained that the centerline velocity observed at distance s from the sprayer outlet along the curved centerline is equal to the velocity observed at distance s in the undeflected jet. The centerline-velocity equation for an undeflected fan jet was developed in our earlier work [Brazee et al.(1981)] as

$$U = \Delta U_m \cdot \Delta \overline{U}_m$$

$$= \frac{\Delta U_m}{2A_2(1-\mu)} \left\{ [\mu^2 A_1^2 + 4 \frac{F_o}{F_1} A_2(n_2 - \mu n_1)]^{1/2} - \mu A_1 \right\} , \tag{1-4a}$$

where

$$F_o/F) = \frac{b_o(r_o - x_{on})}{b(s+r_o)} , \tag{1-4b}$$

and

$$x_{on} = \frac{0.725 \ \delta(1+\mu) r_o}{0.725 \ \delta(1+\mu) + c_n(1-\mu) r_o} . \tag{1-4c}$$

Additional details on the application of Eqs. (1-4) can be found in the published article.

Since $F_c$ and $F_d$ act on different fluid masses, they can never be balanced, but they are nonetheless equal. Thus, equating $F_c$ and $F_d$ and using Eqs. (1-1) and (1-2) we get

$$\frac{1}{2}\, c_d \rho (W \sin\alpha)^2 h_1 ds = -\frac{U^2 \rho S_n\, ds}{R_c} \tag{1-5}$$

Application of (1-4a) and (1-3) followed by simplification of (1-5) yields the result

$$R_c \sin^2\alpha = -\frac{4b}{c_d W^2}\ \Delta\, \overline{U}_m^{-2}\,. \tag{1-6}$$

The radius of curvature of the jet midline is given by the familiar relationship

$$R_c = \frac{(1+q_2'^2)^{3/2}}{q_2''}\quad . \tag{1-7}$$

Since the first derivative of $q_2$ is

$$q_2' = \frac{dq_2}{dq_1} = \tan\alpha\quad , \tag{1-8}$$

we have

$$\sin\alpha = \frac{q_2'}{\sqrt{1+q_2'^2}}\quad . \tag{1-9}$$

The general relationships

$$(q_2''/q_2'^2) = \frac{d}{dq_1}\left(-\frac{1}{q_2'}\right) \tag{1-10a}$$

and

$$ds = (1+q_j'^2)^{1/2}\, dq_i\,,\quad i \neq j\quad , \tag{1-10b}$$

and the definitions

$$q_2' = \frac{1}{Z(s)} \tag{1-10c}$$

and

$$I(s) = \frac{[U(s)]^{-2}}{b(s)} \tag{1-10d}$$

together with Eqs. (1-7) and (1-9) lead to the differential equation for Z(s)

$$\frac{dZ}{ds} = \frac{c_d W^2}{4} \; I(s) \quad . \tag{1-11a}$$

Eq. (1-11a) can be solved numerically for Z(s) once the solution for $\Delta\bar{u}_m(s)$ and b(s) is started. The initial condition for starting the solution of Eq. (1-11a) is

$$Z(0) = 1/\tan|\alpha_0| \quad . \tag{1-11b}$$

The differential equations for the coordinates $(q_1, q_2)$ of the deflected jet midline path can be developed directly with the aid of Eq. (1-10b) as

$$\frac{dq_1}{ds} = \frac{Z}{\sqrt{1+Z^2}} \tag{1-12a}$$

and

$$\frac{dq_2}{ds} = \frac{1}{\sqrt{1+Z^2}} \quad . \tag{1-12b}$$

The trapezoidal method or Simpson's rule can be used conveniently to integrate Eqs. (1-11a), (1-12a), and (1-12b). It is evident that the integrations are performed along the curved midline path. Since Eq. (1-12b) always gives a positive result, it is required for numerical computation that

$$q_2 = \operatorname{sgn} \alpha_0 \int_0^s \frac{ds}{\sqrt{1+Z^2}} \quad . \tag{1-12c}$$

The computed midline paths can be compared to experimentally determined paths both for verifying applicability of the theory and to permit "inverse" evaluation of the drag coefficient $c_d$ by developing a set of paths corresponding to the set of $c_d$ values.

## 2. Coordinate Transformation System for Jet Deflection

The fan jet deflection theory developed in Section 1 is, of course, referred to an arbitrary "deflection plane"; i.e., a plane that contains the "crosswind" vector and also a map of the deflection path. In a physical sense, one may need to know both a fan jet midline path and its corresponding mean-velocity profile for a particular elevation angle (measured with respect to the sprayer centerline as vertex). To establish a rudimentary theory that helps answer these questions, we will assume that the horizontal effective wind vector is constant in magnitude and direction at all points.

Sprayer and Target Geometry. The first task is to construct a geometry of the ASJ and its target structure, and to define a set of coordinate frames upon which to base the deflection process. We presume that the sprayer travel is parallel to its centerline. It is important to note that the midplane of an ASJ is not necessarily perpendicular to the sprayer centerline. In fact, the "midplane" may not be a plane at all, but a cone with its apex and central axis coinciding with the sprayer centerline. The "midcone" may "open" either forward or backward with respect to sprayer travel depending on whether the ASJ tends to slant forward or backward. This type of jet structure can be represented as shown in Fig. 2.

In Fig. 2, the plane $\overline{OEG}$ is horizontal (parallel to level ground) and contains the sprayer centerline $\overline{IJ}$, which is also the apparent axial origin of the fan jet. The plane $\overline{EFGH}$ is vertical (perpendicular to $\overline{OEG}$), is parallel to $\overline{IJ}$, and is defined as the target plane. The plane $\overline{OFH}$ is the deflection plane containing the ASJ deflection path to be studied as required by the elevation h of the line $\overline{FH}$ (parallel to $\overline{OEG}$) above $\overline{OEG}$ and the distance $X_{oo}$, $\overline{AE}$, from the sprayer outlet. The cylindrical surface sector $\overline{ABCD}$ is an element of the cylindrical surface traveled by the sprayer outlet at the elevation angle of interest, and is contained in $\overline{OFH}$. Ultimately, the origins of the coordinate systems used to analyze jet deflection will be located at point D.

The slant of the jet will be specified by the angle $\gamma_s$ corresponding to the $\angle \overline{DOI}$ in Figure 2. The effect of slanting is to alter the geometrical parameters usually established by the sprayer structure. The corrected geometrical parameters for the slanted jet can be calculated as follows: The effective outlet radius $\overline{OD}$ is

$$r_o = \frac{r_{oo}}{\sin \gamma_s} \quad . \tag{2-1}$$

The elevation angle $\angle \overline{EOF}$ for an unslanted jet is

$$\psi_o = \tan^{-1} [h/(X_{oo} + r_{oo})] \quad . \tag{2-2}$$

Then, with reference to Fig. 2, the radial distance $\overline{OF}$ from the sprayer centerline $\overline{IJ}$ to the target line $\overline{FH}$ for an unslanted jet is

$$R_o = (X_{oo} + r_{oo})/\cos \psi_o \quad , \tag{2-3}$$

and the corresponding distance $\overline{OH}$ for a slanted jet is

$$R = R_o/\sin \gamma_s \quad . \tag{2-4}$$

The distance $\overline{DH}$ from the slanted jet outlet to the target line is

$$X_o = R - r_o \tag{2-5}$$

and the elevation angle is

$$\psi = \tan^{-1}\left(h/\sqrt{R^2 - h^2}\right) \quad . \tag{2-6}$$

The location of the jet analysis coordinate origins, D, can now be specified as ($l_o, h_o$), where

$$l_o = r_{oo} \cos \psi_o \tag{2-7a}$$

and

$$h_o = r_{oo} \sin \psi_o \quad . \tag{2-7b}$$

The Jet Coordinate Systems. Determination of the trajectory of the three-dimensional sprayer jet acted on by travel and crossflow effects requires introduction of vectors in several orthogonal coordinate systems. For example, the outlet centerline directions of air jets produced by real orchard sprayers often are at angles other than 90 deg to the longitudinal axis of the sprayer; wind velocity can occur at any azimuth and we must be able to determine jet deflection at any elevation angle in the vertical midplane of the outlet. The interaction of the vectors in several coordinate systems was solved by a series of matrix transformations involving either a rotation or a translation. A computer system carried out the necessary matrix transformations; data could be entered in any coordinate system and transformations made so results were referred to a base coordinate system.

The sequence of coordinate systems is as follows:

(1) The (x,y,z) coordinate system, CS[0], has its origin at point 0, which can be at any location on the "line of travel" of the sprayer centerline. The (x,y) plane is horizontal with the z-axis directed upward. The y-axis coincides with the sprayer centerline, positive in the direction of sprayer travel. The x-axis is directed toward the vertical target plane, $\overline{EFHG}$ in Fig. 2. Ultimately, all analysis results are referred to the (x,y,z) system. The point D in Fig. 2 is located at ($l_o$, 0, $h_o$) in CS[0].

(2) $CS[D_0](x_0, y_0, z_0)$ has its $x_0$-$y_0$ plane horizontal with its $x_0$-axis directed toward the $\overline{EFHG}$ target plane and parallel to the line $\overline{OG}$ of Fig. 2.

(3) $CS[D_1](x_1, y_1, z_1)$ can be generated from $CS[D_0]$ by rotation about the $z_0$-axis, through an angle $\frac{\pi}{2} - \gamma_s$, clockwise if $(\frac{\pi}{2} - \gamma_s) > 0$ or counterclockwise if $(\frac{\pi}{2} - \gamma_s) < 0$. This operation sets the $x_1$-axis parallel to $\overline{OE}$ (Fig. 2) and the $y_1$-axis parallel to $\overline{IJ}$ (Fig. 2). See Fig. 3(a).

(4) $CS[D_2](x_2, y_2, z_2)$ can be generated from $CS[D_1]$ by rotation about the $y_1$-axis through an angle $\psi_0$. This operation sets the $x_2$-axis parallel to $\overline{BF}$ (Fig. 2). See Fig. 3(b).

(5) $CS[D_3](x_3, y_3, z_3)$ can be generated from $CS[D_2]$ by rotation about the $z_2$-axis through an angle $(\frac{\pi}{2} - \gamma_s) > 0$ counterclockwise if $(\frac{\pi}{2} - \gamma_s) < 0$ or clockwise if $(\frac{\pi}{2} - \gamma_s) < 0$. This operation sets the $x_3$-axis parallel to $\overline{DH}$ (Fig. 2) See Fig. 3(c).

(6) The horizontal apparent airflow vector, $\underset{\sim}{w}$, which, in general, is the resultant of ambient mean airflow and sprayer travel will be defined in $CS[D_0]$. With the aid of the coordinate transformation sequence through $CS[D_0]$, $CS[D_1]$, $CS[D_2]$ and $CS[D_3]$, the resultant flow $\underset{\sim}{w}$ can be referenced to $CS[D_3]$ as $\underset{\sim}{w} = \{w_x, w_y, w_z\}_3$. The $x_3$-$y_3$ plane establishes a reference plane for analyzing the deflection process. Since $w_z$ is normal to the analysis plane, it is assumed to be inactive in the deflection process. Thus, the resultant of $w_x$ and $w_y$ components will be assumed to induce the jet deflection. A new coordinate system, $CS(x_4, y_4, z_4)[D_4]$ can then be defined with its $x_4$-axis in the same direction as the resultant vector $W$ in the $x_4$-$y_4$ plane, and with its $z_4$-axis coincident with the $z_3$-axis of $CS[D_3]$. The $x_3$-axis of $CS[D_3]$ defines at once the midline direction of an undeflected jet or the initial midline direction of a deflected jet in $CS[D_4]$. The angle between the $x_3$-axis of $CS[D_3]$ and the $x_4$-axis of $CS[D_4]$ is defined as $\alpha_0$. The deflection theory of Section 1 is to be applied in $CS[D_4]$ to generate the deflection field, which can then be referred back to $CS[D_1]$ or $CS[0]$. See Fig. 3(d).

<u>Ambient Wind and Travel Effects</u>. The mean wind vector $\underset{\sim}{U_W}$ is referred to the direction of the sprayer centerline, and therefore, is most readily defined in $CS[D_1]$ as shown in Fig. 4. Thus, the mean wind vector is

$$\underset{\sim}{U_W} = \{-U_W \sin\alpha_W,\ -U_W \cos\alpha_W,\ 0\}_1 \quad , \qquad (2\text{-}8a)$$

In the vector notation adopted in Eq. (2-8a), the three quantities inside the brackets are the $x_1$, $y_1$, and $z_1$ components, respectively, and the subscript one (1) following the second bracket indicates reference to $CS[D_1]$. The vector (2-8a) can be rewritten as

$$\underset{\sim}{U}_w = -U_w\{\sin \alpha_w, \cos \alpha_w, 0\}_1 \tag{2-8b}$$

or as

$$U_w = -U_w\{\cos(\gamma_s - \alpha_w), \sin(\gamma_s - \alpha_w), 0\}_0 \tag{2-8c}$$

in $CS[D_0]$.

The forward travel of the sprayer in effect produces an additional airflow imposed upon the jet. The apparent flow due to travel is introduced into the analysis as an apparent travel vector as shown in Fig. 4, and as written below in the form

$$\begin{aligned}\underset{\sim}{S} &= -S\{0,1,0\}_1 \\ &= -S\{\cos \gamma_s, \sin \gamma_s, 0\}_0 \quad .\end{aligned} \tag{2-9}$$

The definitions of $\underset{\sim}{U}_w$ and $\underset{\sim}{S}$ in either $CS[D_0]$ or $CS[D_1]$ allow flexibility in defining the ambient flow depending upon the input data for the analysis. The two vectors $\underset{\sim}{U}_w$ and $\underset{\sim}{S}$ can now be summed to obtain the resultant flow vector

$$\begin{aligned}\underset{\sim}{w} = \underset{\sim}{U}_w + \underset{\sim}{S} &= \{w_{xo}, w_{yo}, 0\}_0 \\ &= w\{\cos \Phi, \sin \Phi, 0\}_0 \\ &= \{[-U_w(\cos \gamma_s \cos \alpha_w + \sin \gamma_s \sin \alpha_w) - S \cos \gamma_s], \\ &\quad [-U_w(\sin \gamma_s \cos \alpha_w - \cos \gamma_s \sin \alpha_w) - S \sin \gamma_s], 0\}_0 \quad ,\end{aligned} \tag{2-10a}$$

which we now have chosen to define in $CS[D_0)$. in which

$$w = \sqrt{w_{xo}^2 + w_{yo}^2} \quad , \tag{2-10b}$$

and

$$\Phi = \tan^{-1}(w_{yo}/w_{xo}) \quad . \tag{2-10c}$$

This operation defines the apparent mean ambient flow vector with respect to the initial jet midline, and will help delineate effective wind azimuth change with respect to the jet as the elevation angles $\psi_o$ and $\psi$ change.

Construction of Transformation Matrices. The next task is to construct the orthonormal transformations based on the sequence of coordinate systems previously outlined. The transformation matrices for $CS[D_o]$ through to $CS[D_3]$ in order are:

$CS[D_o]$ to $CS[D_1]$ ,

$$P_1^{-1} = \begin{pmatrix} \sin \gamma_s & \cos \gamma_s & 0 \\ -\cos \gamma_s & \sin \gamma_s & 0 \\ 0 & 0 & 1 \end{pmatrix} ; \qquad (2\text{-}11)$$

$CS[D_1]$ to $CS[D_2]$,

$$P_2^{-1} = \begin{pmatrix} \cos \psi_o & 0 & -\sin \psi_o \\ 0 & 1 & 0 \\ \sin \psi_o & 0 & \cos \psi_o \end{pmatrix} ; \qquad (2\text{-}12)$$

and $CS[D_2]$ to $CS[D_3]$

$$P_3^{-1} = \begin{pmatrix} \sin \gamma_s & -\cos \gamma_s & 0 \\ \cos \gamma_s & \sin \gamma_s & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad (2\text{-}13)$$

Since these transformations are orthonormal, the inverse matrices $P_2^{-1}$ and $P_3^{-1}$ immediately yield their inverses by transposition, as

$$P_2 = (P_2^{-1})' , \qquad (2\text{-}14a)$$

and

$$P_3 = (P_3^{-1})' = P_1^{-1} . \qquad (2\text{-}14b)$$

The composite transformation from $CS[D_o]$ to $CS[D_3]$ is now available from

$$P^{-1} = P_1^{-1} P_2^{-1} P_3^{-1} . \qquad (2\text{-}15)$$

The apparent ambient flow vector is immediately available as

$$\underset{\sim}{w}_3 = \underset{\sim}{w}_o P^{-1} . \qquad (2\text{-}16)$$

In $CS[D_3]$, we define the resultant ambient flow direction in the $x_3$-$y_3$ plane by the relationships

$$\Phi' = \tan^{-1}(w_{3y}/w_{3x}) \tag{2-17a}$$

and

$$\alpha_0 = \pi - \Phi' , \tag{2-17b}$$

as indicated by Fig. 3(d). While Eq. (2-17a) is adequate in principle, computer implementation requires that $\Phi'$ be more specifically defined. The computation algorithm for $\Phi'$ is

$$\Phi' = \Phi', \quad w_{3x} < 0 \text{ and } w_{3y} < 0 \quad ; \tag{2-17c}$$

$$\Phi' = \pi + \Phi', \quad w_{3x} > 0, \quad \text{and } w_{3y} < 0 \text{ or } w_{3y} > 0 \quad ; \tag{2-17d}$$

$$\Phi' = 2\pi + \Phi', \quad w_{3x} < 0 \quad \text{and } w_{3y} > 0 \quad . \tag{2-17e}$$

With $\alpha_0$ now available, the final transformation $CS[D_4]$ can be completed. The transformation matrix is

$$P_4^{-1} = \begin{pmatrix} \cos\alpha_0 & \sin\alpha_0 & 0 \\ -\sin\alpha_0 & \cos\alpha_0 & 0 \\ 0 & 0 & 1 \end{pmatrix} , \tag{2-18a}$$

with

$$P_4 = (P_4^{-1})' \quad . \tag{2-18b}$$

This transformation automatically established the initial direction of the jet midline at an angle $\alpha_0$ with respect to the positive $x_4$-axis, along which the deflecting mean ambient flow component,

$$W = \sqrt{w_{3x}^2 + w_{3y}^2} = w_{4x} \tag{2-19}$$

is directed. If the matrix

$$P_h^{-1} = P^{-1}P_4^{-1} \tag{2-20}$$

is constructed, then the result

$$\underset{\sim}{w_4} = wP_h^{-1}$$

should be obtained with the form

$$\underset{\sim}{w}_4 = \{ W, 0, w_{4z} \}_4 \quad . \tag{2-21}$$

The inverse transformation

$$P_r = P_4 P_3 P_2 \tag{2-22}$$

will allow reference of any jet velocity or position vector to the CS[$D_1$].

Since the direction of the jet midline at the sprayer outlet coincides with the $x_3$-axis in CS[$D_3$], we can define a unit vector $\underset{\sim}{I}_3$ in the same direction by writing

$$\underset{\sim}{I}_3 = \{1,0,0\}_3 \quad . \tag{2-23}$$

Thus the direction of the jet at the outlet in CS[$D_4$] is indicated by the vector

$$\underset{\sim}{I}_4 = \underset{\sim}{I}_3 \underset{\sim}{P}_4^{-1} = \{\cos \alpha_0,\ \sin \alpha_0,\ 0\}_4 \ . \tag{2-24}$$

The local ambient flow condition of the sprayer outlet affects the configuration of the jet boundary layer, as embodied in our theory of the fan jet outlined in Section 1. It is necessary to know the magnitude of the coflow or counterflow velocity $U_h$. The value of $U_h$ can be determined with the aid of Eqs. (2-24) and (2-21) as

$$\begin{aligned} U_h &= \underset{\sim}{I}_4 \bullet \underset{\sim}{w}_4 \\ &= \{\cos \alpha_0,\ \sin \alpha_0,\ 0\}_4 \bullet \{W,\ 0,\ w_{z4}\}_4 \\ &= W \cos \alpha_0 \quad . \end{aligned} \tag{2-25}$$

The deflection theory of Section 1 is to be applied in CS[$D_4$] for the jet flow condition on that plane of elevation. Position-velocity vector pairs can be obtained in component form for any point of interest on the jet midline path with the aid of Eqs. (1-12a), (1-12b), and (1-4a). The position vector for the point of interest on the midline path is

$$\underset{\sim}{r}_4 = \{X,\ Y,\ 0\}_4 \tag{2-26}$$

where the components X and Y are obtained from Eqs. (1-12a) and (1-12b), respectively. With Eq. (1-4a) providing the midline velocity magnitude, the midline velocity vector corresponding to $\underset{\sim}{r_4}$ can be established with the aid of Eq. (1-10c) for the slope of the midline path; i.e., with

$$\tan \theta' = \frac{dy_4}{dx_4} = \frac{1}{Z(s)} \quad , \tag{2-27a}$$

The computation algorithm for $\theta'$ is

$$\theta' = -\frac{\pi}{2} \cdot \text{sgn } w_{3y} , \; Z(s) = 0 \quad ; \tag{2-27b}$$

$$\theta' = |\theta'| \cdot \text{sgn } w_{3y} , \; |w_{3x}| < \epsilon , \; \epsilon \sim 10^{-4} \quad ; \tag{2-27c}$$

$$\theta' = \pi + \theta' , \; \alpha_0 \geq 0 \text{ and } \theta' < 0 \quad ; \tag{2-27d}$$

$$\theta' = -(\pi + \theta') , \quad \alpha_0 < 0 \text{ and } \theta' < 0 ; \tag{2-27e}$$

$$\theta' = -\theta' , \quad \alpha_0 < 0 \text{ and } \theta' \geq 0 \quad , \tag{2-27f}$$

where $\theta'$ is evaluated via Eq. (2-27a). In cases excluded by the complete set of Eqs. (2-27b) through (2-27f), then $\theta = \theta'$.
$\theta = \theta'$.

Then, we have

$$\underset{\sim}{u_4} = \{ u_{4x} , u_{4y} , 0 \}_4 \quad , \tag{2-27g}$$

with

$$u_{4x} = \Delta\overline{U_m}(s) \cdot \cos \theta' \tag{2-27h}$$

and

$$u_{4y} = \Delta\overline{U_m}(s) \cdot \sin \theta' \cdot \text{sgn } \alpha_0 \quad . \tag{2-27i}$$

Then the total fluid velocity vector at $r_4$ is

$$\begin{aligned} \underset{\sim}{U_4} &= \underset{\sim}{u_4} + \underset{\sim}{w_4} \\ &= \{ u_{4x} + W , u_{4y} , w_{4z} \}_4 \quad . \end{aligned} \tag{2-28}$$

Finally, the position-velocity pair can be referred to CS[0] as

$$\underset{\sim}{r} = \underset{\sim}{r}_4 P_r + \{l_0, 0, h_0\}_0 \tag{2-29}$$

and

$$\underset{\sim}{U}_0 = \underset{\sim}{U}_4 P_r \quad . \tag{2-30}$$

## 3. Computer Implementation

The jet-deflection and coordinate transformation theory has been implemented in BASIC, and the program source listing is presented in Appendix I. The major sections of the program are executed in the following order:

(1) The input parameters of the problem are specified including the configuration of the air sprayer and its travel speed, location of the target plane and elevation of the target line; ambient mean wind magnitude and direction; basic parameters of the jet such as outlet velocity and boundary layer thickness.

(2) All internal variables and parameters that can be derived from the specifications entered in Step (1) are computed, in preparation for generating the coordinate transformation.

(3) The coordinate-transformation vectors and matrices are generated and the deflection CS[$D_4$] is established, based on the theory of Section 2.

(4) The configuration and midline velocity profile of the undeflected jet are computed. These computations are based on Eq. (1-4a) and more mathematical details presented in our earlier work, Brazee et al. (1981).

(5) The deflection theory of Section 1 is applied to the problems as posed in CS[$D_4$]. The midline-path position- velocity pairs are calculated for all desired points. Results are then subjected to transformation for reference to CS[0 ].

(6) Results are made available in numerical form or in graphical form on a CRT terminal.

The trapezoidal method is used to perform the numerical integrations of Eqs (1-11), (1-12a), and (1-12b). Preliminary results based on comparisons with experimental data indicate a value for $c_d$ of $c_d$ = 5. More detailed analysis will be available in a later publication.

A sample problem and computed results are presented in Appendix II.

## 4. Summary

Details of a theory of crosswind deflection of an air sprayer fan-shaped jet were developed. The effect of the radial divergence of the jet is taken into consideration as opposed to existing theories of plane-jet deflection. A system of coordinate transformations is derived to enable application of the theory to any radius on the jet midplane and to obtain three-dimensional deflected-midline trajectories for the jet. The deflecting crosswind is assumed essentially uniform over the region of interest in this version of the theory. Jet outlet velocity is assumed to be large in comparison with the crosswind velocity and jet integrity is presumed to be maintained in the deflection process.

## LIST OF SYMBOLS

$A_1, A_2$: Integral parameters of main region velocity profile [Brazee et al. (1981): $A_1 = 0.45$; $A_2 = 0.316$].

b: Local half-thickness of fan jet.

$b_o$: Jet outlet half-width.

$CS[D_i]$: Coordinate system $(x_i, y_i, z_i)$ with origin at point $D_i$, $i = 0,1,2,\ldots,4$

$c_d$: Empirical drag coefficient for crossflow jet deflection.

$c_n$: Dimensionless constant (0.27) for initial region of jet [Brazee et al. (1981)].

F: Jet area at distance s from outlet.

$F_o$: Jet outlet area.

$F_c$: Centrifugal force.

$F_d$: Centripetal force.

h: Elevation of target line above horizontal reference plane through sprayer centerline.

$h_o, l_o$: x,z coordinates in CS[O], of origins $D_i$ (i=0,1,...4) of the coordinate systems $CS[D_i]$.

$h_1$: Length of jet element measured normal to fan jet radius and parallel to jet midplane.

I(s): Defined function [See Eq. (1-10d)].

$\underset{\sim}{I_3}$: Special unit vector defined in $CS[D_3]$ [See Eq. (2-23)]; defines initial direction of jet in $CS[D_3]$.

$\underset{\sim}{I_4}$: Special unit vector in $CS[D_4]$ that defines initial direction of jet.

| | |
|---|---|
| $n_1, n_2$: | Integral parameters of jet outlet velocity profile [Brazee et al. (1981). Typical values: $n_1 = 0.835$, $n_2 = 0.743$]. |
| $P^{-1}$: | Transformation matrix $P^{-1} = P_1^{-1}P_2^{-1}P_3^{-1}$. |
| $P_h^{-1}$: | Transformation matrix for CS[$D_0$] to CS[$D_4$]. [See Eq. (2-20)]. |
| $P_i, P_i^{-1}$: | Forward, inverse orthonormal transformation matrices from CS[$D_i$] to CS[$D_{i-1}$], i=1,2,...4. |
| $P_i'$: | Transpose matrix of $P_i$, i=0,1,...4. |
| $P_r$: | Transformation matrix for CS[$D_4$] to CS[$D_1$]. [See Eq. (2-22)]. |
| $q_1, q_2$: | Jet midline coordinates in deflection plane. |
| $q_j'$: | Derivative $q_j' = dq_j/dq_i$. |
| $q_2''$: | Second derivative, $q_2'' = d^2q_2/dq_1^2$. |
| R : | Distance from sprayer centerline to target line, measured on fan jet radius, for slanted jet [See Eq. (2-4)]. |
| $R_0$: | Distance from sprayer centerline to target line, measured on fan jet radius, for unslanted jet [See Eq. (2-3).]. |
| $R_c$: | Local radius of curvature of jet midline. |
| $\underset{\sim}{r}$: | Position vector for jet midline path in CS[0]. |
| $r_{00}$: | Radius of sprayer outlet. |
| $r_0$: | Effective outlet radius for slanted fan jet. |
| $\underset{\sim}{r}_4$ : | Position vector for points on the jet midline path in CS[$D_4$]. |
| $-\underset{\sim}{S}$, S: | Apparent travel vector, and its magnitude for the air sprayer. |
| $S_n$: | Cross-sectional surface area of air sprayer fan jet. |
| s: | Distance measured along a curved jet midline. |
| U: | Air velocity. |
| $\underset{\sim}{U}_0$: | Total mean fluid velocity vector in CS[0]. |
| $U_{0m}$: | Maximum jet velocity at sprayer outlet. |

$\underset{\sim}{U}_4$: Total mean fluid velocity vector at $\underset{\sim}{r}_4$ in CS[$D_4$].

$U_h$: Ambient mean flow velocity at the sprayer outlet and parallel to the jet midplane.

$U_m$: Local midline (maximum) velocity in jet cross-section

$\underset{\sim}{U}_W, U_W$: Mean ambient wind vector and its magnitude.

$\underset{\sim}{u}_4$: Midline-velocity vector for deflected jet in the $x_4$-$y_4$ plane of CS[$D_4$] [See Eq. (2-27b)].

$u_{4x}, u_{4y}$: Midline-velocity vector components for deflected jet in $x_4$-$y_4$ plane of CS[$D_4$].

W: Deflecting mean ambeint flow component, $w_{4z}$, in $x_4$-$y_4$ plane in CS[$D_4$]; also deflecting mean ambient flow velocity in deflection theory.

$\underset{\sim}{w}$,w: Resultant ambient flow vector, and its magnitude [See Eq. (2-10a)].

$w_{xO}, w_{yO}$: Components in $x_O$,$y_O$ direction in CS[$D_O$] for resultant ambient flow vector $\underset{\sim}{w}$.

$\underset{\sim}{w}_4$: Resultant mean ambient flow vector in CS[$D_4$].

$X_{OO}$: Normal distance from sprayer outlet to vertical target plane measured in horizontal plane through sprayer centerline.

$X_O$: Distance from jet outlet to target line, measured on fan jet radius for slanted jet [See Eq. (2-5).].

$x_{On}$: Distance from apparent origin of jet boundary layer to the jet outlet.

Z(s): Defined function [See Eq. (1-10c)].

$\Delta U_m$: $U_m - U_h$.

$\Delta\bar{U}_m$: Normalized jet velocity $(U_m - U_h)/U_{Om} - U_h)$.

$\alpha$: Local angular inclination of deflected jet midline with respect to deflecting wind.

$\alpha_O$: Angle between the initial jet-midline direction and the positive $x_4$-axis in CS[$D_4$].

$\alpha_W$: Mean wind azimuth angle in CS[$D_1$] (See Fig. 4).

$\gamma_s$: Angle between jet midplane and sprayer centerline in direction of travel for slanted jet.

$\theta'$: Angle of slope of jet midline path in CS[$D_4$] [See Eq. (2-27a)].

$\mu$: Dimensionless parameter, $U_h/U_{Om}$.

$\rho$: Air mass density.

$\Phi$: Azimuth angle in CS[$D_O$] for the resultant ambient flow vector $\underset{\sim}{w}$ [See Eq. (2-10c)].

$\Phi'$: Azimuth angle in CS[$D_3$] for the resultant ambient flow vector $\underset{\sim}{w_3}$[See Eq. 2-17a)].

$\psi$: Elevation angle of target line relative to sprayer centerline for slanted fan jet.

$\psi_O$: Elevation angle of target line relative to sprayer centerline for unslanted fan jet.

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## APPENDIX I

## PROGRAM SOURCE FOR FAN JET DEFLECTION AND COORDINATE TRANSFORMATION

This appendix contains a source listing in BASIC for the program JET DEflection for a FAN Jet (JETDEFAN), which implements the jet deflection theory presented in Section 1 of this report. The sub-program Jet Deflection TRANsformation (JDTRAN), implements the coordinate transformation theory of Section 2. JDTRAN is designed to establish the coordinate transformation parameters, vectors, and matrices required, and enables reference of jet midline trajectories and mean velocities to a common three-dimensional coordinate system.

Position-velocity pairs of values are tabulated as program output for both the deflection coordinates, CS[$D_4$], and the sprayer base-reference coordinates, CS[0]. Up to 50 entries can be calculated as desired. The number of entries selected for computation may be in part limited by the output device, particularly if a CRT is used. The program version listed includes CRT output, but will copy the video display only on a printer. If greater printing capability is required, the program sections calling for output listing can be readily modified or duplicated to allow full output printing. We expect to prepare such a version later. A graphical display is available showing the jet midline trajectory in relation to the sprayer centerline, sprayer extreme edge, and the target plane. Note that the midline trajectory displayed is the two-dimensional projection of the three-dimensional trajectory on a horizontal plane through the sprayer centerline. An example of the graphical output appears in Fig. II.1 accompanying the sample problem presented in Appendix II.

Table A.I.1 contains a listing of the more important variables and the corresponding variable names in the program to assist in use or modification of JETDEFAN.

The program lists, in addition, the angle of deflection of the jet midline tangent from the original undeflected jet midline. The deflection angle DFLEK(I), is defined as

$$\text{DFLEK(I)} = \theta' - \alpha_0 \quad . \tag{A.I-1}$$

The yaw of the total velocity vector with respect to the midline tangent is also available, and is defined as

$$\text{SKEW(I)} = \theta' - \tan^{-1}(U_{4y}/U_{4x}) \quad , \tag{A.I-2}$$

with the total velocity vector $\underset{\sim}{U_4}$ as given by Eq. (2-28).

All angular inputs and outputs are in decimal degrees. All internal computations are carried out in radians. The factors DR and RD are the internal names for the degrees-to-radians and radians-to-degrees conversion factors, respectively.

```
1 REM * JETDTRAN SYSTEM *
2 CLS:PRINT@266,"JETDEFAN SYSTEM PREPARING FOR RUN"
3 DIMA(3,3),B(3,3),C(3,3),I1(3,3),I2(3,3),I3(3,3),I4(3,3)
4 DIMP2(3,3),P3(3,3),P4(3,3),IP(3,3),PR(3,3),IH(3,3),S(3)
5 DIMUW(3),W(3),W3(3),W4(3),AV(3),BV(3),WD(3)
6 DIMX2(3),V2(3),X4(3),V4(3),VT(3),U(51)
7 DIMXB(51),BT(51),YP(51),XJP(51),UM(51),SL(51),DFLEK(51)
8 DIMU1M(51,2),U2M(51,3),XM2(51,3),SKEW(51),RU2M(51),ZS(51)
10 READCN,CM,A1,A2,A3,FM,GRVTY,RHO,C1,C2,C3,N1,N2,CG$
11 READDR,RD,PI,IIMAX%,CD,GAS,THETA:RESTORE
12 DATA.27,.22,.45,.316,.251,1,1,1 21464,.45,.316,.251
13 DATA.875,.76," CHANGE(Y=YES OR (EN))"
14 DATA.174533E-01,57.29578,3.141593,50,5,1.5708,1.5708
15 CLS:PRINT"JET-DEFLECTION COORDINATE TRANSFORMATION";
20 PRINT" SYSTEM: JETDEFAN"
25 PRINT"JETDEFAN CONTROL":PRINT"OPERATION CALL CODES:"
30 PRINT" 1-ENTER BASE PARAMETERS":PRINT" 2-ENTER PROBLEM CON";
35 PRINT"DITIONS":PRINT" 3-GO TO JDTRAN SUBSYSTEM":PRINT" 4-";
40 PRINT"COMPUTE UNDEFLECTED JET":PRINT" 5-DEFLECT JET"
45 PRINT" 6-OUTPUT: JET CONFIGURATION IN DEFLECTION COORDINAT";
50 PRINT"ES":PRINT" 7-OUTPUT: JET DEFLECTION ANGLES":PRINT" 8";
55 PRINT"-OUTPUT: JET CONFIGURATION IN BASE COORDINATES"
57 PRINT" 9-PLOT JET-MIDLINE PROJECTION ON BASE X-Y PLANE"
58 PRINT"10-SET DISPLAY PRINT":PRINT"11-CANCEL DISPLAY PRINT"
59 PRINT"12-END"
60 INPUT"OPERATION DESIRED";NA%:GOTO70
63 IFNA%=4ORNA%=5THEN64ELSE65
64 CLS:PRINT@266,"EXECUTING JETDEFAN CALL CODE ";NA%;"."
65 ONNA%GOTO100,150,8000,640,4100,2300,2400,2500,2600
70 IFNA%>9THEN71ELSE63
71 NB%=NA%-9:ONNB%GOTO72,73,9415
72 KP%=1:KQ$="ON":GOTO74
73 KP%=0:KQ$="OFF"
74 CLS:PRINT@278,"DISPLAY PRINT ";KQ$:FORQQ%=1TO1000:NEXT:GOTO15
100 CLS:PRINT"BASE PARAMETER INPUT:":PRINT:C$=""
105 PRINT"U0M=";U0M;CG$;:INPUTC$
107 IFNOT(C$="")ANDNOT(C$="Y")THEN105ELSE110
110 IFC$=""THEN111ELSEINPUT"U0M=";U0M:C$="":GOTO105
111 PRINT"R0=";RO;CG$;:INPUTC$
112 IFNOT(C$="")ANDNOT(C$="Y")THEN111ELSE113
113 IFC$=""THEN115ELSEINPUT"R0=";RO:R00=RO
114 C$="":GOTO111
115 PRINT"B0=";B0;CG$;:INPUTC$
117 IFNOT(C$="")ANDNOT(C$="Y")THEN115ELSE120
120 IFC$=""THEN125ELSEINPUT"B0=";B0:C$="":GOTO115
125 PRINT"DELTA=";DELTA;CG$;:INPUTC$
127 IFNOT(C$="")ANDNOT(C$="Y")THEN125ELSE130
130 IFC$=""THEN135ELSEINPUT"DELTA=";DELTA:C$="":GOTO125
135 PRINT"THETA=";THETA*RD;" DEG.";CG$;:INPUTC$
137 IFNOT(C$="")ANDNOT(C$="Y")THEN135ELSE140
```

```
140 IFC$=""THEN146ELSEINPUT"THETA="; THETA:C$=""
145 THETA=THETA*DR:GOTO135
146 PRINT"CD="; CD; CG$; :INPUTC$
147 IFNOT(C$="")ANDNOT(C$="Y")THEN146ELSE148
148 IFC$=""THEN149ELSEINPUT"CD="; CD:C$="":GOTO146
149 CLS:GOTO25
150 CLS:C$="":PRINT"PROBLEM-CONDITIONS INPUT:":PRINT
151 PRINT"UH="; UH; CG$; :INPUTC$
152 IFNOT(C$="")ANDNOT(C$="Y")THEN151ELSE153
153 IFC$=""THEN155ELSEINPUT"UH="; UH:C$="":GOTO151
155 PRINT"XMIN="; X8MIN; CG$; :INPUTC$
157 IFNOT(C$="")ANDNOT(C$="Y")THEN155ELSE160
160 IFC$=""THEN165ELSEINPUT"XMIN=", X8MIN:C$="":GOTO155
165 PRINT"XMAX="; X9MAX; CG$; :INPUTC$
167 IFNOT(C$="")ANDNOT(C$="Y")THEN165ELSE170
170 IFC$=""THEN175ELSEINPUT"XMAX="; X9MAX:C$="":GOTO165
175 PRINT"IMAX="; IIMAX%; CG$; :INPUTC$
177 IFNOT(C$="")ANDNOT(C$="Y")THEN175ELSE180
180 IFC$=""THEN185ELSEINPUT"IMAX=", IIMAX%:C$="":GOTO175
185 DX=(X9MAX-X8MIN)/IIMAX%:DYP=DX
187 MAX%=IIMAX%+1:CLS:GOTO25
640 MU=UH/U0M
641 AALPH=A2/A1
642 FORK%=1TO51:XB(K%)=0:BT(K%)=0 NEXT IFMU<0THEN643ELSE644
643 MU=0:GOTO646
644 IFMU=0THEN646ELSEIFMU>1THEN645ELSE646
645 UH=U0M:PRINT"NEW UH":GOTO640
646 M1=(B0*RO)/CM
647 T2=MU*A1
650 S1=A1-A2
655 S2=1+MU
660 S3=1-MU
665 S4=N2-(MU*N1)
670 S5=1-(2*A1)+A2+(MU*S1)
675 S6=A1-A2+(MU*A2)
680 S7=S5-1+(2*A1):S8=1-S5:S9=1-(3*A1)+(3*A2)-A3
681 Q0=(.725*DELTA*S2)/(CN*S3):Q1=(S2*B0)/(CN*S3*S5)
682 Q2=S4/(S3*S7):Q3=(S4*B0)/(CM*A2*(S3[2)):SM1=MU/(S3*AALPH)
683 Q4=S4*B0/((S3[2)*A2):Q5=Q4*AALPH/CM IDUM=1.IALPH=1/AALPH
684 IALPH=IALPH+1:Q6=4*B0*A2*S4:F1=C1-C2 F5=1-(2*C1)+C2+(MU*F1)
685 F6=C1-C2+(MU*C2):F7=F5-1+(2*C1) F8=1-F5:F9=1-(3*C1)+(3*C2)
686 F9=F9-C3:P0=(2*C1)-C2:F2=1-P0 GOTO1050
704 IFINKEY$=""THEN704ELSE25
705 IFXP<XNTHEN706ELSE711
706 IFMU=0THEN707ELSE707
707 IFN2<1THEN708ELSE710
708 N2=N2+.01:IFIBLOK%=0THEN709ELSE640
709 PRINT"NOTE N2":IBLOK%=1:GOTO640
710 IBLOK%=0:PRINT"NOTE N2":GOTO740
711 PRINT"N2="; N2:IBLOK%=0
```

```
740 DU0M=U0M-UH
745 M3=DU0M/(2*A2*S3)
750 T1=((MU*A1)[2)
755 M4=4*M1*A2*S4
756 M7=4*A2*S4*B0*RPO
757 ASQ=T1/M7
758 BPFJ=(P5F1*B0)/(X6P1+RPO)
759 BMO=BPFJ:VXI=XP+RO:MF=CM:JSTRT%=0
760 P=XN
765 VMOM=2*RHO*THETA*B0*RO*N2*(DU0M[2)*FM
766 YB=B0*(((F8*(P+X50N))/(F5*(XN+X50N)))+1)
772 POWER=RHO*THETA*(DU0M[3)*(XP+RO)*BPFJ*A3*FP/GRVTY
810 BP=Q2*B0
815 FORI%=1TOMAX%
820 I=I%-1
824 X=X8MIN+(DX*I)
825 XARO=X+RO
826 IFX>XPTHEN828ELSE827
827 DDUM=DU0M:XB(I%)=X:GOTO1295
828 IFJSTRT%=0THEN829ELSE1165
829 HX=X-XP:H1XHF=HX/2:GOTO1165
995 U(I%)=DDUM+UH
1000 NEXTI%:CLS:GOTO25
1050 X50N=Q0/(1+(Q0/RO))
1055 RPO=RO-X50N
1060 X1N=((SQR((RPO[2)+(4*Q1*RPO)))-RPO)/2
1065 XN=X1N-X50N
1070 P5F1=Q2*RPO
1075 P6F2=((RPO/(S5*(X1N+RPO)))-1)/X1N
1080 BF=(1/P6F2)-RPO
1085 FCF=-(P5F1/P6F2)
1090 RADCL=SQR((BF[2)-(4*FCF))
1095 AROOT=((-BF)-RADCL)/2
1100 BROOT=((-BF)+RADCL)/2
1105 AROOT=AROOT-RPO
1110 BROOT=BROOT-RPO
1114 REM * ROOT DISCRIMINATOR AND SELECTOR SEGMENT *
1115 X6P1=0
1120 IFAROOT<=0THEN1125ELSE1130
1125 IFBROOT<=0THEN1150ELSE1145
1130 X6P1=AROOT
1135 IFBROOT<=0THEN1150ELSE1140
1140 IF(BROOT-AROOT)<0THEN1145ELSE1150
1145 X6P1=BROOT
1150 XP=X6P1-X50N
1155 X70=XP-((Q3*RPO)/(X6P1*RPO))
1160 GOTO705
1165 PA=MF
1170 PB=AALPH:PC=ASQ:UXI=VXI
1175 VA=UXI
```

```
1180 VB=BMO
1185 GOSUB2000:K1A=FCN
1190 VA=UXI+H1XHF
1195 VB=BMO+(K1A*H1XHF)
1200 GOSUB2000:K2A=FCN
1205 VB=BMO+(K2A*H1XHF)
1210 GOSUB2000:K3A=FCN
1215 VA=UXI+HX
1220 VB=BMO+(K3A*HX)
1225 GOSUB2000:K4A=FCN
1230 VXI=VXI+HX
1235 BMF=BMO+((HX/6)*(K1A+(2*K2A)+(2*K3A)+K4A))
1240 BMO=BMF
1241 BT(I%)=BMF
1242 XB(I%)=X
1245 IFJSTRT%=0THEN1250ELSE1265
1250 JSTRT%=1
1255 HX=DX
1260 H1XHF=HX/2
1265 DDUM=M3*((SQR(T1+(M7/(BMF*XARO)))-T2):GOTO995
1295 YIP=((BPFJ-B0)*((X+X50N)/X6P1))+B0
1296 BT(I%)=YIP GOTO995
2000 REM * SUBR ALGF1(PA,PB,PC,VA,VB) (=FCN) *
2005 PVAR=VA*VB*PC.RPVAR=SQR(PVAR):RVRP1=SQR(PVAR+1)
2010 FCN=PA/(1+(4*PB*RPVAR*(RVRP1+RPVAR))):RETURN
2100 KD=(CD*(WD(1)[2))/(2*B0*(U0M[2)*SA):COTA=1/TAN(ALFO)
2105 CSCA=1/SA:HSNIA=LOG(COTA+SQR((COTA[2)+1)) LO=(COTA*CSCA)
2110 LO=LO+HSNIA PY=0-DYP SMAX=XB(MAX%).JL%=0.JU%=0:LS=0:US=0
2115 FORL%=1TOMAX% PY=PY+DYP:YP(L%)=PY FYP=(KD*PY/2)+COTA
2120 GYP=FYP[2:PX=(PY*FYP)-((KD*(PY[2))/4):XP(L%)=PX
2125 HISNF=LOG(FYP+SQR((FYP[2)+1)):SPY=FYP*SQR((FYP[2)+1)
2130 SPY=(SPY+HISNF-LO)/KD:IFSPY>SMAXTHEN2180ELSE2135
2135 IFSPY>USTHEN2140ELSE2145
2140 JL%=JU%:LS=US JU%=JU%+1:US=XB(JU%):GOTO2135
2145 IFSPY<XPTHEN2150ELSE2155
2150 UM(L%)=DU0M GOTO2175
2155 IFLS<XPTHEN2160ELSE2165
2160 TL=XP:TU=US.LV=DU0M:UV=U(JU%):GOTO2170
2165 TL=LS:TU=US:LV=U(JL%):UV=U(JU%)
2170 UM(L%)=LV+((UV-LV)*((SPY-TL)/(TU-TL)))
2175 SL(L%)=SPY:TMAX%=L%
2180 U1M(L%,2)=UM(L%)/SQR(1+GYP):U1M(L%,1)=U1M(L%,2)*FYP
2181 IFFYP=0THEN2182ELSE2183
2182 TP8TA=PI/2:GOTO2185
2183 TP8TA=ATN(1/FYP):IFFYP<0THEN2184ELSE2185
2184 TP8TA=((PI/2)+TP8TA)+(PI/2)
2185 TP8TA=RD*TP8TA:DFLEK(L%)=(RD*ALFO)-TP8TA
2190 FORI%=1TO2 V4(I%)=U1M(L%,I%):NEXTI% V4(3)=0:X4(1)=XP(L%)
2195 X4(2)=YP(L%):X4(3)=0:KH%=1:GOSUB9170:FORI%=1TO3
2200 U2M(L%,I%)=V2(I%):XM(L%,I%)=X2(I%):NEXTI%
```

```
2201 IFNOT(V4(2)=0)ANDV4(1)=0THEN2203ELSE2202
2202 IFV4(2)=0ANDV4(1)=0THEN2204ELSE2205
2203 SKEW(L%)=TP8TA-(RD*(PI/2)) GOTO2210
2204 SKEW(L%)=0:GOTO2210
2205 TEMP=ATN(V4(2)/V4(1)):IFTEMP<0THEN2206ELSE2207
2206 TEMP=((PI/2)+TEMP)+(PI/2)
2207 SKEW(L%)=TP8TA-(RD*TEMP)
2210 SUM=0:FORI%=1TO3:SUM=SUM+(V2(I%)[2):NEXTI%
2215 RU2M(L%)=SQR(SUM):NEXTL% MAX%=TMAX%:CLS:GOTO25
2300 REM * DEFLECTION-COORDINATE CONFIGURATION *
2305 CLS:PRINT"FAN JET CONFIGURATION IN DEFLECTION COORDINATES"
2310 PRINT:PRINTTAB(1)"I";TAB(4)"SL".TAB(16)"X'";TAB(29)"Y'",
2315 PRINTTAB(41)"VX'";TAB(54)"VY'" FORI%=1TOMAX%
2320 PRINTTAB(0)I%;TAB(3)SL(I%);TAB(15)XJP(I%);TAB(27)YP(I%);
2325 PRINTTAB(40)U1M(I%,1);TAB(52)U1M(I%,2):NEXT
2328 IFKP%=0THEN2330ELSE2329
2329 GOSUB9411
2330 IFINKEY$=""THEN2330ELSE2335
2335 CLS.GOTO25
2400 REM * JET DEFLECTION ANGLES *
2405 CLS:PRINT"FAN JET DEFLECTION ANGLES(DEG)":PRINT
2410 PRINTTAB(1)"I";TAB(4)"SL";TAB(16)"X'";TAB(28)"Y'",
2415 PRINTTAB(38)"MIDLINE";TAB(50)"TOTAL VEL."
2420 PRINTTAB(39)"DEFL. ANGLE";TAB(52)"YAW ANGLE" FORI%=1TOMAX%
2425 PRINTTAB(0)I%;TAB(3)SL(I%);TAB(15)XJP(I%);TAB(27)YP(I%);
2430 PRINTTAB(39)DFLEK(I%);TAB(51)SKEW(I%):NEXT
2433 IFKP%=0THEN2435ELSE2434
2434 GOSUB9411
2435 IFINKEY$=""THEN2435ELSE2440
2440 CLS GOTO25
2500 REM * BASE-COORDINATE CONFIGURATION *
2505 CLS:PRINT@128,"FAN-JET CONFIGURATION IN BASE COORDINATES"
2507 PRINT
2510 PRINTTAB(1)"I";TAB(4)"SL";TAB(18)"X";TAB(31)"Y",
2512 PRINTTAB(47)"Z"
2515 FORI%=1TOMAX%:PRINTTAB(0)I%,TAB(3)SL(I%);TAB(15)XM2(I%,1);
2520 PRINTTAB(31)XM2(I%,2);TAB(45)XM2(I%,3):NEXT
2525 FORI%=1TO2:PRINT:NEXT.IFKP%=0THEN2526ELSE2528
2526 IFINKEY$=""THEN2526ELSE2529
2528 GOSUB9411:GOTO2526
2529 CLS
2530 PRINTTAB(1)"I";TAB(4)"VX";TAB(16)"VY";TAB(29)"VZ",
2535 PRINTTAB(41)"V(TOTAL)" FORI%=1TOMAX%:PRINTI%;
2540 PRINTTAB(3)U2M(I%,1),TAB(15)U2M(I%,2);TAB(28)U2M(I%,3);
2545 PRINTTAB(40)RU2M(I%) NEXT
2548 IFKP%=0THEN2550ELSE2549
2549 GOSUB9411
2550 IFINKEY$=""THEN2550ELSE2555
2555 CLS:GOTO25
2600 G0XMN=0:G1XMX=X00
```

```
2605 G2YMN=0:G3YMX=0:FORI%=1TOMAX%
2610 IFXM2(I%,1)>G1XMXTHEN2625ELSE2615
2615 IFXM2(I%,1)<G0XMNTHEN2620ELSE2630
2620 G0XMN=XM2(I%,1):GOTO2630
2625 G1XMX=XM2(I%,1)
2630 IFXM2(I%,2)>G3YMXTHEN2645ELSE2635
2635 IFXM2(I%,2)<G2YMNTHEN2640ELSE2650
2640 G2YMN=XM2(I%,2):GOTO2650
2645 G3YMX=XM2(I%,2)
2650 NEXT
2655 G4SPX=G1XMX-G0XMN:G5SPY=G3YMX-G2YMN
2660 IF(G4SPX/G5SPY>1.33333)THEN2665ELSE2670
2665 G6REF=62/G4SPX:GOTO2675
2670 G6REF=46/G5SPY
2675 CLS:G7H=-2*G0XMN*G6REF:FORI%=1TO48:G8V=I%-1:SET(G7H,G8V)
2685 NEXT:G7H=2*(X00-G0XMN)*G6REF:FORI%=1TO48:G8V=I%-1
2686 SET(G7H,G8V):NEXT
2687 G7H=2*(R00-G0XMN)*G6REF:FORI%=1TO48:G8V=I%-1:SET(G7H,G8V)
2688 NEXT
2690 FORI%=1TOMAX%:G7H=2*(XM2(I%,1)-G0XMN)*G6REF
2695 G8V=(G3YMX-XM2(I%,2))*G6REF:SET(G7H,G8V):NEXT
2700 IFINKEY$=""THEN2700ELSE2705
2705 CLS:GOTO25
3000 REM * SUBR TRAPZINT *
3005 NTGRL=((FA+FB)/2)*BNDF:RETURN
3010 REM * SUBR LOKGRATE *
3015 IFLASTX>=XPTHEN3047ELSE3020
3020 IFX<XPTHEN3047ELSE3025
3025 ONJC%GOTO3030,3065
3030 FB=KD/((DU0M[2)*BPFJ):BNDF=XP-LASTX:GOSUB3000
3035 ACCUM=ACCUM+NTGRL:ZPHLD=ACCUM:FA=FB:BNDF=X-XP
3040 FB=KD/((U(L%)[2)*BT(L%)):GOSUB3000:FA=FB:BNDF=DX
3045 GOTO3055
3047 ONJC%GOTO3050,3105
3050 FB=KD/((U(L%)[2)*BT(L%)):GOSUB3000:FA=FB
3055 ACCUM=ACCUM+NTGRL
3060 ZS(L%)=ACCUM:GOTO3125
3065 ZYHLD=1/SQR(1+(ZPHLD[2)):ZXHLD=ZPHLD*ZYHLD:BNDF=XP-LASTX
3070 FA=Z1A:FB=ZXHLD:GOSUB3000:ACCUM=ACCUM+NTGRL:Z1A=FB
3075 FA=Z3A:FB=ZYHLD:GOSUB3000:APILE=APILE+NTGRL:Z3A=FB
3080 BNDF=X-XP:Z4B=(1/SQR(1+(ZS(L%)[2)))
3082 Z2B=ZS(L%)*Z4B:CZ=Z2B:SZ=Z4B*SGN(ALFO)
3085 FA=Z1A:FB=Z2B:GOSUB3000:ACCUM=ACCUM+NTGRL
3090 FA=Z3A:FB=Z4B:GOSUB3000:APILE=APILE+NTGRL:BNDF=DX
3100 GOTO3120
3105 Z4B=(1/SQR(1+(ZS(L%)[2)))
3107 Z2B=ZS(L%)*Z4B:CZ=Z2B:SZ=Z4B*SGN(ALFO)
3110 FA=Z1A:FB=Z2B:GOSUB3000:ACCUM=ACCUM+NTGRL:Z1A=FB
3115 FA=Z3A:FB=Z4B:GOSUB3000:APILE=APILE+NTGRL:Z3A=FB
3120 XJP(L%)=ACCUM:YP(L%)=APILE*SGN(ALFO)
```

```
3125 LASTX=X:RETURN
4100 KD=(CD*(WD(1)[2))/4:XJP(1)=0:YP(1)=0
4102 ZS(1)=1/TAN(ABS(ALFO))
4105 UM(1)=U(1):SL(1)=0:U1M(1,1)=UM(1)*CA:U1M(1,2)=UM(1)*SA
4110 DFLEK(1)=0:FORI%=1TO2:V4(I%)=U1M(1,I%):NEXTI% V4(3)=0
4115 X4(1)=XJP(1):X4(2)=YP(1) X4(3)=0:KH%=1:GOSUB9170
4120 FORI%=1TO3:U2M(1,I%)=V2(I%) XM(1,I%)=X2(I%) NEXTI%
4125 L%=1:GOSUB4204
4127 IFDTEK%=0THEN4133ELSE4130
4130 DTEK%=0 GOTO4135
4133 SKEW(L%)=RD*(ALFO-TEMP)
4135 SUM=0 FORI%=1TO3
4140 SUM=SUM+(V2(I%)[2):NEXTI% RU2M(1)=SQR(SUM)
4145 LASTX=SL(1):FA=KD/((DU0M[2)*BT(1)):ACCUM=ZS(1) JC%=1
4150 BNDF=DX FORL%=2TOMAX% X=XB(L%) GOSUB3010:NEXTL%
4155 LASTX=SL(1):Z3A=1/SQR(1+(ZS(1)[2)):Z1A=ZS(1)*Z3A
4160 ACCUM=0:APILE=0:JC%=2 FORL%=2TOMAX%:X=XB(L%) GOSUB3010
4165 UM(L%)=U(L%):SL(L%)=X U1M(L%,1)=UM(L%)*CZ
4170 U1M(L%,2)=UM(L%)*SZ:IFZS(L%)=0THEN4175ELSE4176
4175 TP8TA=(-SGNW3(2)))*PI/2 GOTO4190
4176 TP8TA=ATN(1/ZS(L%)) IFABS(W3(1))<.0001THEN4177ELSE4178
4177 TP8TA=-SGN(W3(2))*ABS(TP8TA) GOTO4190
4178 IFALFO>=0THEN4179ELSE4180
4179 IFTP8TA<0THEN4181ELSE4190
4180 IFTP8TA<0THEN4182ELSE4183
4181 TP8TA=PI+TP8TA:GOTO4190
4182 TP8TA=-(PI+TP8TA):GOTO4190
4183 TP8TA=-TP8TA
4190 DFLEK(L%)=RD*(ALFO-TP8TA) FORI%=1TO2:V4(I%)=U1M(L%,I%)
4195 NEXTI%:V4(3)=0:X4(1)=XJP(L%) X4(2)=YP(L%):X4(3)=0 KH%=1
4200 GOSUB9170:FORI%=1TO3 U2M(L%,I%)=V2(I%):XM(L%,I%)=X2(I%)
4201 NEXTI%:GOSUB4204
4202 IFDTEK%=0THEN4235ELSE4203
4203 DTEK%=0:GOTO4240
4204 REM * SUBR SKEWER *
4205 IFVT(2)=0ANDVT(1)=0THEN4220ELSE4206
4206 IFVT(2)>0ANDVT(1)=0THEN4215ELSE4207
4207 IFVT(2)<0ANDVT(1)=0THEN4216ELSE4208
4208 TEMP=ATN(VT(2)/VT(1))
4209 IFVT(2)>=0ANDVT(1)>0THEN4233ELSE4210
4210 IFVT(2)>=0ANDVT(1)<0THEN4213ELSE4211
4211 IFVT(2)<0ANDVT(1)>0THEN4233ELSE4212
4212 IFVT(2)<0ANDVT(1)<0THEN4214ELSE4220
4213 TEMP=PI+TEMP:GOTO4233
4214 TEMP=-PI+TEMP:GOTO4233
4215 TEMP=PI/2:GOTO4233
4216 TEMP=-PI/2:GOTO4233
4220 SKEW(L%)=0:DTEK%=1
4233 RETURN
4235 SKEW(L%)=RD*(TP8TA-TEMP)
```

```
4240 SUM=0:FORI%=1TO3:SUM=SUM+(V2(I%)[2):NEXTI%
4245 RU2M(L%)=SQR(SUM):NEXTL%:CLS:GOTO25
8000 REM * JDTRAN: JET-DEFLECTION TRANSFORMATION SYSTEM *
8070 GOTO8330
8080 REM * SUBR MMULT *
8090 CT=0:FORI%=1TO3:FORJ%=1TO3:FORK%=1TO3
8100 CT=CT+(A(I%,K%)*B(K%,J%)) NEXTK%:C(I%,J%)=CT:CT=0
8110 NEXTJ%,I%:RETURN
8120 REM * SUBR VTRAN *
8130 CT=0:FORJ%=1TO3:FORK%=1TO3
8140 CT=CT+(AV(K%)*C(K%,J%)) NEXTK% BV(J%)=CT:CT=0 NEXTJ%
8150 RETURN
8160 REM * SUBR MOVCA *
8170 FORI%=1TO3:FORJ%=1TO3 A(I%,J%)=C(I%,J%):NEXTJ%,I%
8180 RETURN
8190 CLS:PRINT"JDTRAN PARAMETER INPUT:":PRINT
8200 PRINT"UNITS ARE TO BE IN KG-M-SEC SYSTEM. ANGLES ARE";
8210 PRINT" TO BE":PRINT"GIVEN IN DEGREES. ":PRINT
8220 PRINT"PARAMETER INPUT " PRINT KP%=0
8240 PRINT"DISTANCE TO VERTICAL TARGET PLANE, X00=";X00
8242 PRINTCG$;:INPUTC$
8243 IFNOT(C$="")ANDNOT(C$="Y")THEN8242ELSE8244
8244 IFC$=""THEN8250ELSEINPUT"X00=";X00:C$=""
8245 KP%=1:GOTO8240
8250 PRINT"ELEVATION OF TARGET LINE, H=";H
8252 PRINTCG$;:INPUTC$
8253 IFNOT(C$="")ANDNOT(C$="Y")THEN8252ELSE8254
8254 IFC$=""THEN8260ELSEINPUT"H=";H:C$=""
8255 KP%=1:GOTO8250
8260 PRINT"ANGLE, GAMMA SUB S,  BETWEEN JET MIDLINE AND ";
8270 PRINT"SPRAYER":PRINT"     CENTERLINE=";GAS*RD
8272 PRINTCG$;:INPUTC$
8273 IFNOT(C$="")ANDNOT(C$="Y")THEN8272ELSE8274
8274 IFC$=""THEN8280ELSEINPUT"GAMMA SUB S=";GAS
8275 GAS=GAS*DR:C$="":KP%=1 GOTO8260
8280 PRINT"ANGLE, ALPHA SUB W, BETWEEN WIND VECTOR AND UNIT ";:
8290 PRINT"TRAVEL":PRINT"     VECTOR=";AW*RD
8292 PRINTCG$;:INPUTC$
8293 IFNOT(C$="")ANDNOT(C$="Y")THEN8292ELSE8294
8294 IFC$=""THEN8300ELSEINPUT"ALPHA SUB W=";AW
8295 AW=AW*DR:C$="":KP%=1 GOTO8280
8300 PRINT"BASE WIND SPEED, UW=";VW
8302 PRINTCG$;:INPUTC$
8303 IFNOT(C$="")ANDNOT(C$="Y")THEN8302ELSE8304
8304 IFC$=""THEN8310ELSEINPUT"UW=";VW:C$=""
8305 KP%=1:GOTO8300
8310 PRINT"SPRAYER TRAVEL SPEED, S=";SV
8312 PRINTCG$;:INPUTC$
8313 IFNOT(C$="")ANDNOT(C$="Y")THEN8312ELSE8314
8314 IFC$=""THEN8320ELSEINPUT"S=";SV:C$=""
```

```
8315 KP%=1:GOTO8310
8320 IFNOT(KP%=1)THEN8330ELSE8324
8324 KP%=0:C%=2:GOTO8428
8330 CLS:PRINT"JDTRAN  JET-DEFLECTION TRANSFORM SUBSYSTEM"
8335 PRINT:PRINT"JDTRAN CALL CODES:"
8336 PRINT"1-DEFLECTION-TRANSFORMATION PARAMETER INPUT"
8337 PRINT"2-CONSTRUCT DEFLECTION TRANSFORMATION"
8350 PRINT"3-DISPLAY TRANSFORMATION-PARAMETER INPUT"
8355 PRINT"4-DISPLAY DERIVED TRANSFORMATION PARAMETERS"
8360 PRINT"5-DISPLAY VECTORS"
8370 PRINT"6-DISPLAY TRANSFORMATION MATRICES"
8375 PRINT"7-ENTER JET VECTORS FOR TRANSFORMATION OPERATION"
8380 PRINT"8-TRANSFORM JET VECTORS TO BASE COORDINATES"
8410 PRINT"9-RETURN TO JETDEFAN CONTROL"
8420 PRINT:INPUT"OPERATION DESIRED";C%  .
8428 IFC%=2ORC%=8THEN8429ELSE8430
8429 CLS:PRINT@266,"EXECUTING JDTRAN CALL CODE ";C%;" "
8430 ONC%GOTO8190,8440 ,8810 ,9310,8860 ,8920 ,8780,9170 ,9410
8432 FORI%=1TO3:CLS:FORJ%=1TO50:NEXTJ%
8433 PRINT@266,"ERROR TRAP ";ET%:FORJ%=1TO50
8434 NEXTJ%,I%:RESUME8330
8440 ET%=1
8442 ONERRORGOTO8432:RO=R00/SIN(GAS):P10=ATN(H/(X00+R00))
8450 S(1)=-SV*COS(GAS) S(2)=-SV*SIN(GAS):S(3)=0
8452 ET%=2
8460 R10=(X00+R00)/COS(P10) REL=R10/SIN(GAS) XO=REL-RO
8465 PSI=ATN(H/SQR((REL[2)-(H[2)))
8470 L0=R00*COS(P10):H0=R00*SIN(P10)
8472 ET%=3
8480 PHI=P10
8490 UW(1)=-VW*((COS(GAS)*COS(AW))+(SIN(GAS)*SIN(AW)))
8500 UW(2)=-VW*((SIN(GAS)*COS(AW))-(COS(GAS)*SIN(AW)))
8510 UW(3)=0:FORI%=1TO3 W(I%)=UW(I%)+S(I%):NEXT
8512 ET%=4
8520 W(3)=0:WA=SQR((W(1)[2)+(W(2)[2)):PUH=ATN(W(2)/W(1))
8530 SG=SIN(GAS):CG=COS(GAS):SF=SIN(PHI):CF=COS(PHI)
8540 NL=0:UN=1:I1(1,1)=SG I1(1,2)=CG:I1(1,3)=NL:I1(2,1)=-CG
8550 I1(2,2)=SG:I1(2,3)=NL:I1(3,1)=NL:I1(3,2)=NL I1(3,3)=UN
8560 I2(1,1)=CF:I2(1,2)=NL:I2(1,3)=-SF:I2(2,1)=NL I2(2,2)=UN
8570 I2(2,3)=NL:I2(3,1)=SF:I2(3,2)=NL:I2(3,3)=CF
8580 FORI%=1TO3:FORJ%=1TO3:I3(I%,J%)=I1(J%,I%)
8590 P2(I%,J%)=I2(J%,I%) P3(I%,J%)=I1(I%,J%) NEXTJ%,I%
8600 FORI%=1TO3:FORJ%=1TO3:A(I%,J%)=I3(I%,J%)
8605 B(I%,J%)=I2(I%,J%)
8610 NEXTJ%,I%:GOSUB8080 :GOSUB8160 :FORI%=1TO3:FORJ%=1TO3
8620 B(I%,J%)=I3(I%,J%) NEXTJ%,I%:GOSUB8080 FORI%=1TO3
8625 FORJ%=1TO3
8630 IP(I%,J%)=C(I%,J%):NEXTJ%,I%:FORI%=1TO3:AV(I%)=W(I%):NEXT
8640 GOSUB8120 :FORI%=1TO3:W3(I%)=BV(I%):NEXT
8642 ET%=5
```

```
8650 WYX3=W3(2)/W3(1):PPHI=ATN(WYX3)
8651 IFW3(1)<=0THEN8653ELSE8652
8652 PPHI=PI+PPHI:GOTO8654
8653 IFW3(2)>=0THENPPHI=(2*PI)+PPHIELSE8654
8654 ALFO=PI-PPHI:SA=SIN(ALFO)
8660 CA=COS(ALFO):I4(1,1)=CA:I4(1,2)=SA:I4(1,3)=NL:I4(2,1)=-SA
8670 I4(2,2)=CA:I4(2,3)=NL:I4(3,1)=NL:I4(3,2)=NL:I4(3,3)=UN
8680 FORI%=1TO3:FORJ%=1TO3:P4(I%,J%)=I4(J%,I%):NEXTJ%,I%
8690 FORI%=1TO3:FORJ%=1TO3:A(I%,J%)=P4(I%,J%)
8695 B(I%,J%)=P3(I%,J%)
8700 NEXTJ%,I%:GOSUB8080:GOSUB8160 :FORI%=1TO3:FORJ%=1TO3
8710 B(I%,J%)=P2(I%,J%):NEXTJ%,I%:GOSUB8080 :FORI%=1TO3
8715 FORJ%=1TO3
8720 PR(I%,J%)=C(I%,J%):NEXTJ%,I%:FORI%=1TO3:FORJ%=1TO3
8730 A(I%,J%)=IP(I%,J%):B(I%,J%)=I4(I%,J%):NEXTJ%,I%:GOSUB8080
8740 FORI%=1TO3:FORJ%=1TO3:IH(I%,J%)=C(I%,J%):NEXTJ%,I%
8750 FORI%=1TO3:AV(I%)=W(I%):NEXT:GOSUB8120 :FORI%=1TO3
8760 W4(I%)=BV(I%):NEXT:WD(1)=W4(1):WD(2)=0:WD(3)=0
8762 UH=CA*WD(1):ET%=0:ONERRORGOTO0
8770 GOTO8330
8780 CLS:INPUT"JET COORDINATE (X,Y,Z)";X4(1),X4(2),X4(3)
8790 INPUT"JET VELOCITY (VX,VY,VZ)";V4(1),V4(2),V4(3)
8800 GOTO8330
8810 CLS:PRINT"INPUT PARAMETERS ":PRINT
8820 PRINT"R00=";R00:PRINT"X00=";X00:PRINT"H=";H
8830 PRINT"GAMMA SUB S=";GAS*RD;" DEGREES"
8840 PRINT"ALPHA SUB W=";AW*RD;" DEGREES"
8850 PRINT"UW=";VW:PRINT"ABS(S)=";SV
8852 PRINT"U0M=";U0M;"   MU=";MU;"   N1=";N1;"   N2=";N2
8854 PRINT"B0=";B0;"   DELTA=";DELTA;"   CD=";CD:GOTO9298
8860 CLS:PRINT"SYSTEM VECTORS:":PRINT
8870 PRINT"S(";S(1);",  ";S(2);",  ";S(3);")"
8880 PRINT"UW(";UW(1);",  ";UW(2);",  ";UW(3);")"
8890 PRINT"W(";W(1);",  ";W(2);",  ";W(3);")"
8900 PRINT"W3(";W3(1);",  ";W3(2);",  ";W3(3);")"
8910 PRINT"W4(";W4(1);",  ";W4(2);",  ";W4(3);")":GOTO9298
8920 CLS:PRINT"MATRIX DISPLAY CONTROL:"
8930 A$="":INPUT"LIST MATRIX OR (EN)";A$:IFA$=""THEN8330
8940 FORI%=1TO3:FORJ%=1TO3
8950 IFA$="P1I"THEN9010 ELSE8960
8960 IFA$="P2I"THEN9020 ELSEIFA$="P3I"THEN9030 ELSE8970
8970 IFA$="P4I"THEN9040 ELSEIFA$="P2"THEN9050 ELSE8980
8980 IFA$="P3"THEN9060 ELSEIFA$="P4"THEN9070 ELSE8990
8990 IFA$="PI"THEN9080 ELSEIFA$="PR"THEN9090 ELSE9000
9000 IFA$="PHINV"THEN9100 ELSE8930
9010 A(I%,J%)=I1(I%,J%):GOTO9110
9020 A(I%,J%)=I2(I%,J%):GOTO9110
9030 A(I%,J%)=I3(I%,J%):GOTO9110
9040 A(I%,J%)=I4(I%,J%):GOTO9110
9050 A(I%,J%)=P2(I%,J%):GOTO9110
```

```
9060 A(I%,J%)=P3(I%,J%):GOTO9110
9070 A(I%,J%)=P4(I%,J%):GOTO9110
9080 A(I%,J%)=IP(I%,J%):GOTO9110
9090 A(I%,J%)=PR(I%,J%):GOTO9110
9100 A(I%,J%)=IH(I%,J%):GOTO9110
9110 NEXTJ%,I%
9120 CLS:PRINT"MATRIX ",A$;" =":PRINT:PRINT
9130 FORI%=1TO3
9140 PRINTTAB(4)A(I%,1);TAB(24)A(I%,2);TAB(44)A(I%,3)
9150 PRINT:NEXT
9158 IFKP%=0THEN9160ELSE9159
9159 GOSUB9411
9160 B$=INKEY$:IFB$=""THEN9160 ELSE8920
9170 FORI%=1TO3:VT(I%)=V4(I%)+W4(I%):NEXTI%
9180 FORI%=1TO3:AV(I%)=VT(I%):FORJ%=1TO3:C(I%,J%)=PR(I%,J%)
9190 NEXTJ%,I%:GOSUB8120 :FORI%=1TO3:V2(I%)=BV(I%):NEXTI%
9200 FORI%=1TO3:AV(I%)=X4(I%):NEXTI%:GOSUB8120 :FORI%=1TO3
9210 X2(I%)=BV(I%):NEXTI%:X2(1)=X2(1)+L0:X2(3)=X2(3)+H0
9212 IFKH%=1THEN9214ELSE9220
9214 KH%=0:RETURN
9220 CLS:PRINT"JET ";
9230 PRINT"POSITION-VELOCITY TRANSFORMATION RESULTS:":PRINT
9240 PRINT"X4(";X4(1);",  ";X4(2);",  ";X4(3);")"
9250 PRINT"V4(";V4(1);",  ";V4(2);",  ";V4(3);")"
9260 PRINT"W4(";W4(1);",  ";W4(2);",  ";W4(3);")"
9270 PRINT"VT(";VT(1);",  ";VT(2);",  ";VT(3),")"
9280 PRINT"X2(";X2(1);",  ";X2(2);",  ";X2(3);")"
9290 PRINT"V2(";V2(1);",  ";V2(2);",  ";V2(3);")"
9298 IFKP%=0THEN9300ELSE9299
9299 GOSUB9411
9300 B$=INKEY$:IFB$=""THEN9300 ELSE8330
9310 CLS:PRINT"DERIVED PARAMETERS:":PRINT
9320 PRINT"RO=";RO;"   XO=";XO
9330 PRINT"R0=";R10
9340 PRINT"PSI=";PSI*RD;"   PSI SUB 0=";P10*RD
9350 PRINT"PHI(LC)=";PHI*RD
9360 PRINT"L0=";L0;"   H0=";H0
9370 PRINT"ABS(W)=";WA
9380 PRINT"PHI(UC)=";PUH*RD;"   PHI'(UC)=";PPHI*RD
9390 PRINT"ALPHA SUB 0=";ALFO*RD
9392 PRINT"UH=";UH;"   X0N=";X50N:PRINT"XN=";XN;"   XP=";XP
9400 GOTO9298
9410 CLS:GOTO25
9411 QX%=15360:FORQY%=1TO16:FORQQ%=QX%TOQX%+63
9412 LPRINTCHR$(PEEK(QQ%));:NEXTQQ%:LPRINT" ":QX%=QX%+64
9413 NEXTQY%:RETURN
9415 CLS:END
```

## APPENDIX II

## SAMPLE JET-DEFLECTION PROBLEM SOLUTION WITH THE JETDEFAN SYSTEM

The sample problem presented in this section is to provide a performance test and to clarify application of the system. Table A.II.1 is a listing of the input variables and parameters in appropriate units, m-kg-sec. Printed input values and output results complete this section as produced by video display copy. Fig. II.1 is a graphical display of the jet midline path.

TABLE A.II.1 INPUT LIST FOR SAMPLE JET DEFLECTION COMPUTATION

1. Jet outlet velocity, $U_{Om}$ = 57.5 m/sec
2. Jet outlet radius, $r_{OO}$ = 0.635 m
3. Jet outlet half-width, $b_O$ = 0.162 m
4. Effective outlet boundary layer thickness, $\delta$ = 0.162 m
5. Sprayer travel speed, S = 1.8 m/sec
6. Distance, sprayer outlet to target plane, $X_{OO}$ = 2.44 m
7. Target line elevation, h = 3.66 m
8. Deflection drag coefficient, $c_d$ = 5
9. Jet slant angle, $\gamma_s$ = 80 (Forward slant)
10. Mean ambient wind, $U_W$ = 4.5 m/sec
11. Mean wind azimuth, $\alpha_W$ = 45
12. Nominal jet midline length for problem, XMAX = 8 m
13. Nominal number of analysis points, IMAX = 9

INPUT PARAMETERS

R00= .635
X00= 2.44
H= 3.66
GAMMA SUB S= 80.0001 DEGREES
ALPHA SUB W= 45 DEGREES
UW= 4.5
ABS(S)= 1.8
U0M= 57.5 MU= 0 N1= 875 N2= .76
B0= .162 DELTA= 162 CD= 5

DERIVED PARAMETERS

RO= .644796 XO= 4.20924
R0= 4.7803
PSI= 48.9391 PSI SUB 0= 49.9642
PHI(LC)= 49.9642
L0= .408474 H0= .486183
ABS(W)= 5.91144
PHI(UC)= 47.4337 PHI'(UC)= 39.9587
ALPHA SUB 0= 140.041
UH=-3.59759 X0N= .259759
XN= .317402 XP= 31792

SYSTEM VECTORS

S(-.312566 , -1 77265 , 0 )
UW(-3.68618 , -2.58109 , 0 )
W(-3.99875 , -4.35375 , 0 )
W3(-3.59758 , -3.01432 , 3.59394 )
W4( 4.69348 , 3.57628E-06 , 3.59394 )

MATRIX P1I =

| | | |
|---|---|---|
| .984808 | .173648 | 0 |
| -.173648 | .984808 | 0 |
| 0 | 0 | 1 |

MATRIX P2I =

| | | |
|---|---|---|
| .643266 | 0 | -.765643 |
| 0 | 1 | 0 |
| .765643 | 0 | .643266 |

MATRIX P3I =

| | | |
|---|---|---|
| .984808 | -.173648 | 0 |
| .173648 | .984808 | 0 |
| 0 | 0 | 1 |

MATRIX P41 =

| | | |
|---|---|---|
| -.766508 | .642235 | 0 |
| -.642235 | -.766508 | 0 |
| 0 | 0 | 1 |

MATRIX P2 =

| | | |
|---|---|---|
| .643266 | 0 | 765643 |
| 0 | 1 | 0 |
| -.765643 | 0 | .643266 |

MATRIX P3 =

| | | |
|---|---|---|
| .984808 | .173648 | 0 |
| -.173648 | .984808 | 0 |
| 0 | 0 | 1 |

MATRIX P4 =

| | | |
|---|---|---|
| -.766508 | -.642235 | 0 |
| .642235 | -.766508 | 0 |
| 0 | 0 | 1 |

MATRIX PI =

| | | |
|---|---|---|
| .593716 | -.281014 | -.754011 |
| .281014 | .95045 | -.132952 |
| .754011 | -.132952 | .643266 |

MATRIX PR =

| | | |
|---|---|---|
| -.413839 | - 76558 | -.492569 |
| .492471 | -.643341 | .586161 |
| -.765643 | 0 | .643266 |

MATRIX PHINV =

| | | |
|---|---|---|
| -.274611 | 596704 | -.754011 |
| - 825811 | - 54805 | -.132952 |
| -.492569 | 586161 | .643266 |

FAN JET CONFIGURATION IN DEFLECTION COORDINATES

| I | SL | X′ | Y′ | VX′ | VY′ |
|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | -44. 0742 | 36. 9285 |
| 2 | . 888889 | -. 677389 | . 575527 | -24. 5028 | 21. 2315 |
| 3 | 1. 77778 | -1. 33711 | 1. 17051 | -13. 2572 | 12. 7833 |
| 4 | 2. 66667 | -1. 94298 | 1. 81927 | -7. 88711 | 9. 38555 |
| 5 | 3. 55556 | -2. 43919 | 2. 55108 | -4. 16116 | 7. 74849 |
| 6 | 4. 44445 | -2. 68865 | 3. 38535 | - 579585 | 6. 54791 |
| 7 | 5. 33333 | -2. 51001 | 4. 21547 | 2 46401 | 4. 38227 |
| 8 | 6. 22222 | -1. 92217 | 4. 84908 | 3 23817 | 2. 15469 |
| 9 | 7. 11111 | -1. 1324 | 5. 24136 | 2 84929 | . 991513 |
| 10 | 8 | -. 277128 | 5. 47607 | 2. 2797 | . 463974 |

FAN JET DEFLECTION ANGLES(DEG)

| I | SL | X′ | Y′ | MIDLINE DEFL ANGLE | TOTAL VEL YAW ANGLE |
|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | 3. 2007 |
| 2 | . 888889 | -. 677389 | . 575527 | 949957 | 6. 07597 |
| 3 | 1. 77778 | -1. 33711 | 1. 17051 | 3 99889 | 12. 2239 |
| 4 | 2. 66667 | -1. 94298 | 1. 81927 | 9 9994 | 21. 2499 |
| 5 | 3. 55556 | -2. 43919 | 2. 55108 | 21 8043 | 32. 1671 |
| 6 | 4. 44445 | -2. 68865 | 3. 38535 | 44 983 | 37. 1985 |
| 7 | 5. 33333 | -2 51001 | 4. 21547 | 79 389 | 29. 1747 |
| 8 | 6. 22222 | -1. 92217 | 4. 84908 | 106 401 | 18. 4419 |
| 9 | 7. 11111 | -1. 1324 | 5. 24136 | 120. 854 | 11. 6984 |
| 10 | 8 | - 277128 | 5. 47607 | 128 537 | 7. 69725 |

FAN-JET CONFIGURATION IN BASE COORDINATES

| I | SL | X | Y | Z |
|---|---|---|---|---|
| 1 | 0 | .408474 | 0 | 486183 |
| 2 | .888889 | .972234 | 148336 | 1 1572 |
| 3 | 1.77778 | 1.53827 | .270628 | 1 83091 |
| 4 | 2.66667 | 2.10849 | .317097 | 2 50962 |
| 5 | 3.55556 | 2.67424 | .226181 | 3 183 |
| 6 | 4.44445 | 3.18833 | -.119554 | 3 79489 |
| 7 | 5.33333 | 3.52321 | -.790366 | 4 19348 |
| 8 | 6.22222 | 3.59198 | -1.64804 | 4 27533 |
| 9 | 7.11111 | 3.45832 | -2.50504 | 4 11625 |
| 10 | 8 | 3.21997 | -3.31081 | 3.83255 |

| I | VX | VY | VZ | V(TOTAL) |
|---|---|---|---|---|
| 1 | 31.7318 | 6.3915 | 43.3557 | 54.1062 |
| 2 | 15.9021 | 1.50655 | 24.5144 | 29.2592 |
| 3 | 7.08774 | -1.66786 | 14.0232 | 15.8009 |
| 4 | 3.19209 | -3.59312 | 9.38639 | 10.5454 |
| 5 | .843943 | -5.39246 | 6.59152 | 8.55798 |
| 6 | -1.22951 | -7.36205 | 4.12361 | 8.52735 |
| 7 | -3.55558 | -8.29892 | 1.35502 | 9.12964 |
| 8 | -4.97298 | -7.45851 | -.332029 | 8.97051 |
| 9 | -5.38487 | -6.41247 | -.822286 | 8.41385 |
| 10 | -5.40895 | -5.63702 | -.850946 | 7.85855 |

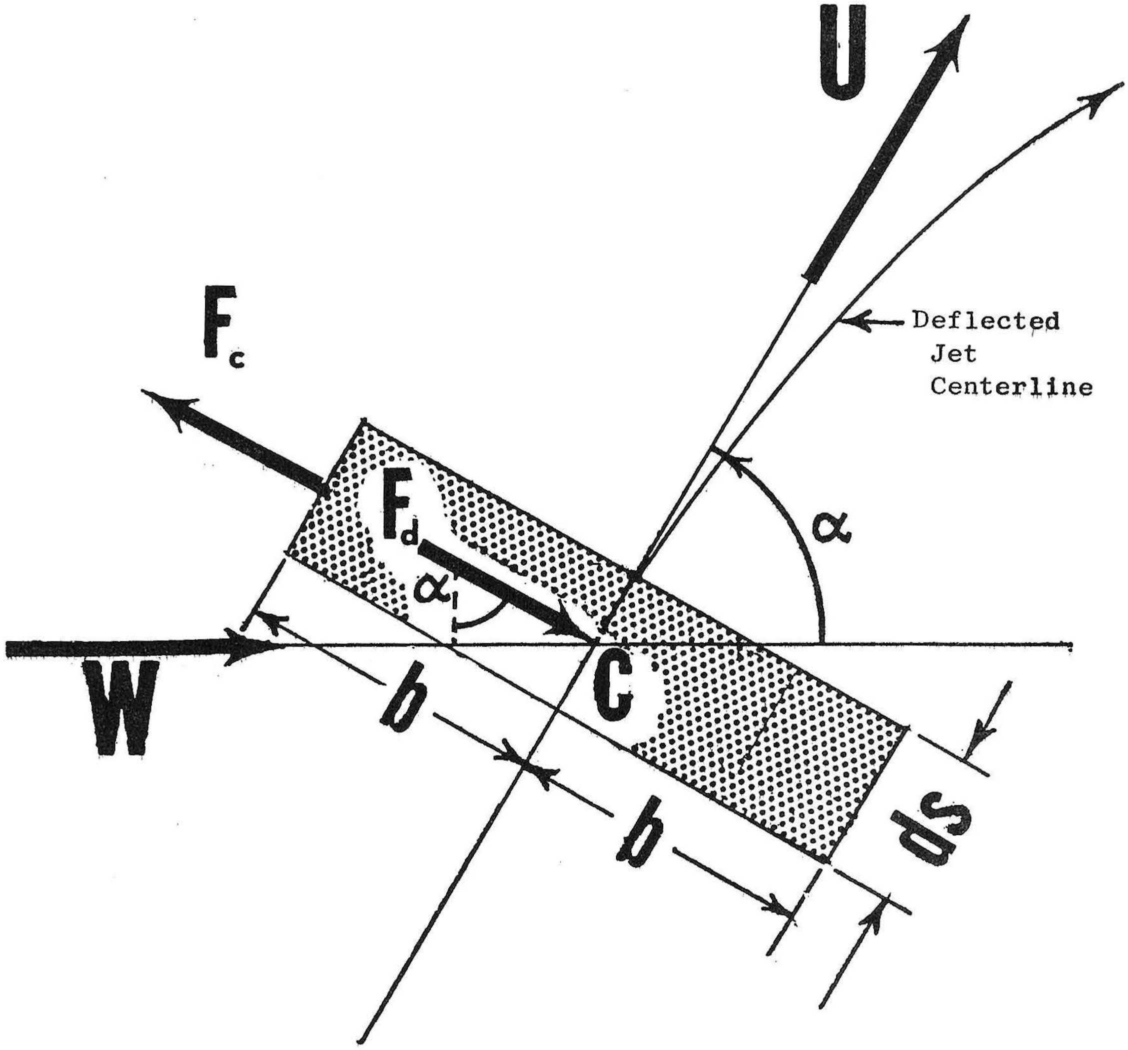


Figure 1. Deflection dynamics of an air-mass element of a fan jet.

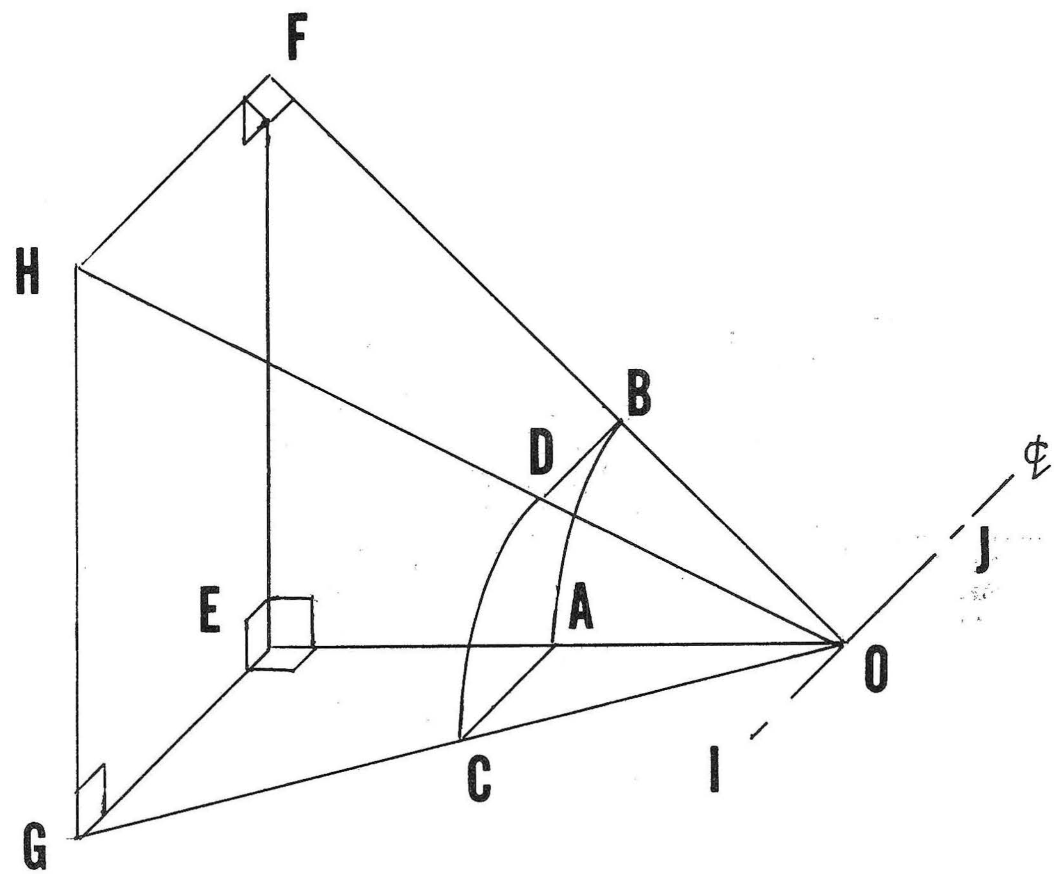


Figure 2. Geometric diagram for air sprayer configuration with slanted fan jet, target plane, EFHG, and target line FH. Line IJ is the sprayer centerline, and the curved element ABCD is on the surface "generated" by the sprayer outlet as it travels parallel to IJ. The plane OEG is horizontal.

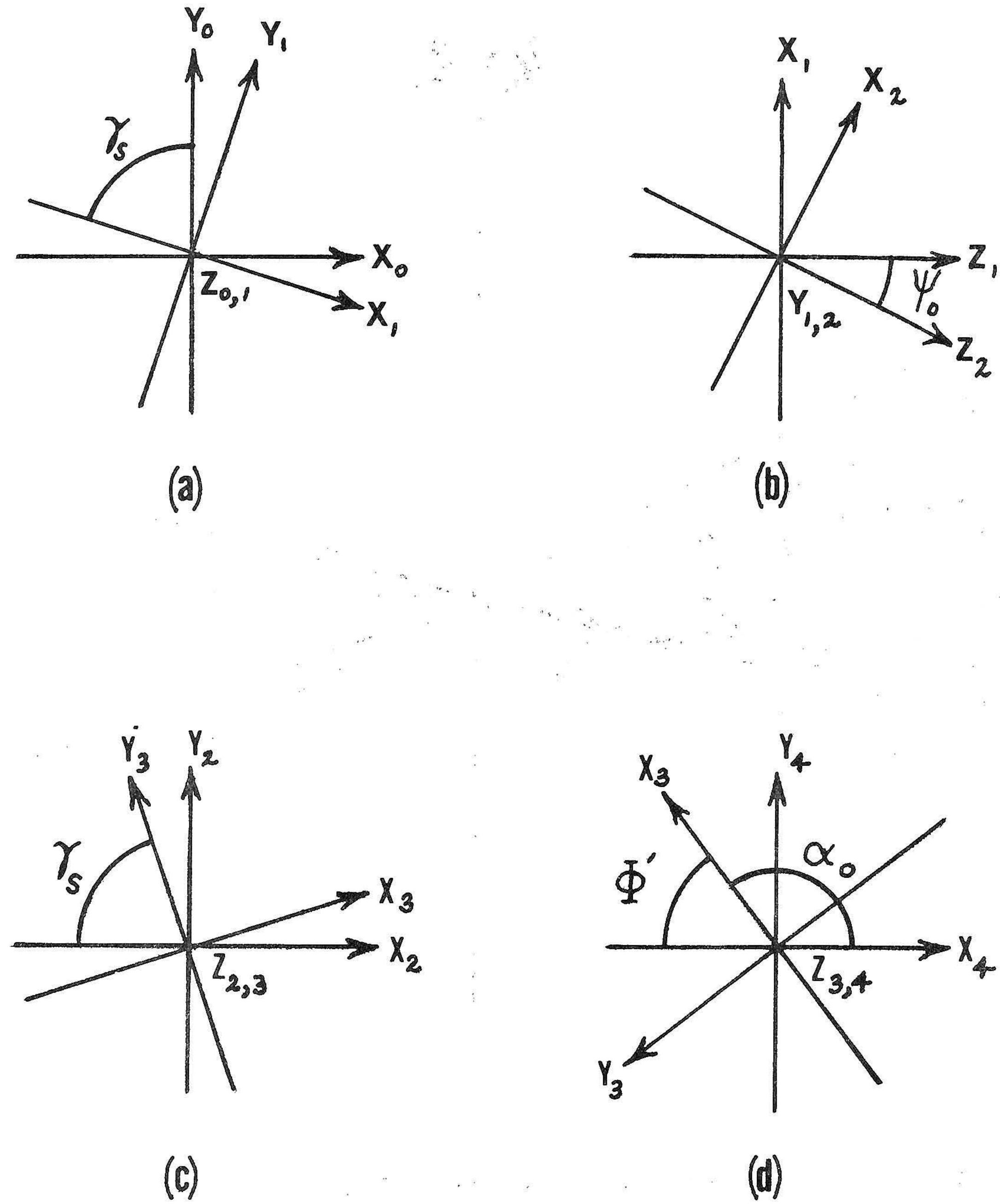


Figure 3. The coordinate transformation sequence (a) $CS[D_0]$ to $CS[D_1]$, (b) $CS[D_1]$ to $CS[D_2]$, (c) $CS[D_2]$ to $CS[D_3]$, and (d) $CS[D_3]$ to $CS[D_4]$.

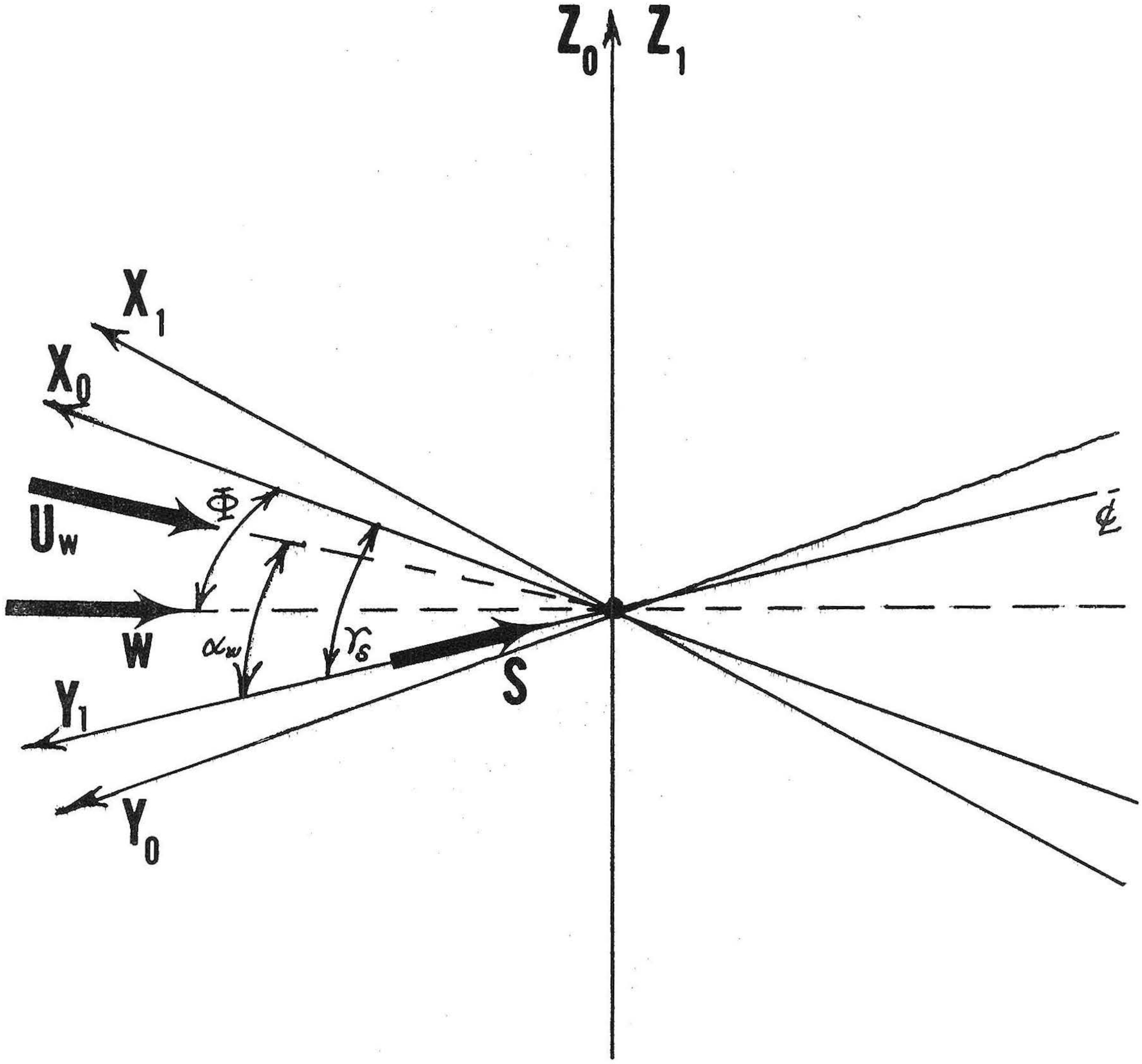


Figure 4. The mean ambient wind vector $\underset{\sim}{U}_w$, the vector of apparent flow due to travel $\underset{\sim}{S}$, and the resultant flow vector w, shown in relation to the coordinate systems CS[$D_0$] and CS[$D_1$].

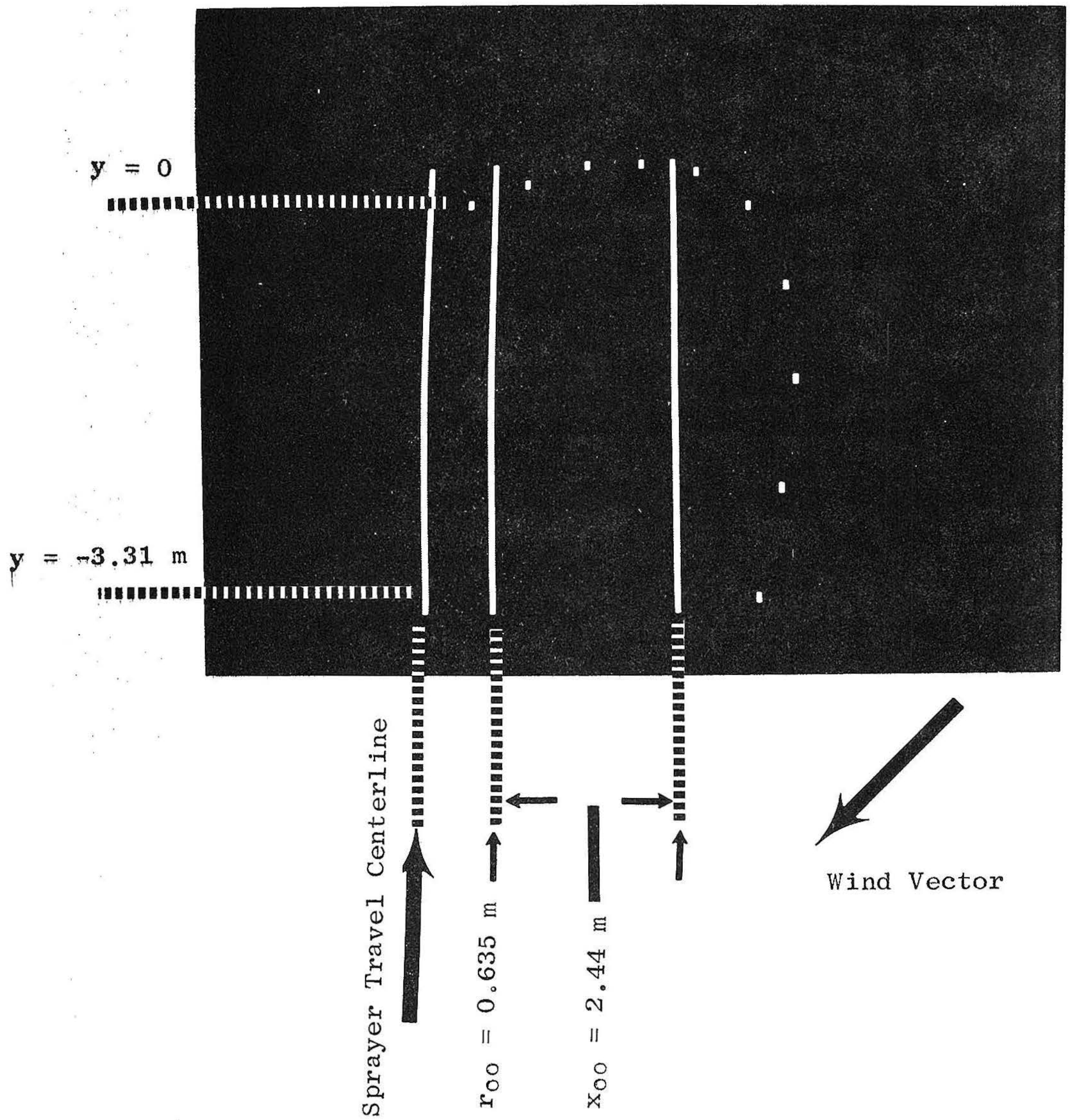


Figure II.1. Computed graphical display of deflected jet midline.

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