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A REGIONAL R.L.P. MODEL OF TRADITIONAL AGRICULTURE

By

I. J .Singh
Assistant Professor

Department of Economics
Department of Agricultural Economics and Rural Sociology
The Ohio State University
2120 Fyffe Road
Columbus, Ohio 43210

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OUTLINE

*This paper is part of a dissertation to be submitted for a Ph.D. in Economics at the University of Wisconsin. A different version of this paper along with some empirical results will be presented at the Winter Meetings of the Econometric Society in a joint session with the A.A.E.A. on the session Economic Development of Agriculture.

1. INTRODUCTION

The purpose of this paper is to give an outline of a model that could be used for the analysis of agricultural transformation and is based upon an activity analysis of traditional agriculture.

Given the argument in favor of including the many strategic details of agricultural development in the analysis of the transition of traditional agriculture,* the question of methodology is mainly concerned with finding a tool of analysis that allows us to deal with such a vast variety of detail simultaneously. This task may be doomed if we were to attempt to include all the details discussed in the last paper , but it is possible to include for the analysis of a given region, those details that the research worker feels are crucial, keeping in mind those that he cannot include because he is unable to obtain the appropriate data, but which are important and also those he does not wish to include because they are not significant in the region he is analyzing. It is this approach of judicious exclusion that allows the construction of a model that will include most of the important and strategic details.

In the study of agricultural transition at the regional level the main concern has been with regional aggregates such as acreages sown to various crops, regional investments in various types of goods, and regional output. This is also the focus of our concern, but in addition we are concerned with the decisions and the constraints that become evident only at the farm level, in terms of choice between different opportunities available to the farmer, opportunities that are interdependent. It is the choice between these interdependent opportunities, that in the

*See Singh, "Strategic Details of Development in Traditional Agriculture," Occasional Paper No.

aggregate constitutes the regional variables , in terms of which we can trace the path of regional growth. The methodology must therefore be i) disaggregative enough in its examination of choice so that it discriminates between the various alternatives available to the farmers in a region; ii) able to account for the way farmers make decisions at the farm level; and iii) positive rather than normative, since its concern is with how decisions with regard to various opportunities were arrived at and not with how decisions ought to be made in terms of some optimum or normative decisions rules.

In view of this concern with decision making with regard to opportunities that are available to farmers and the constraints under which they operate in order to avail themselves of these opportunities, the appropriate starting point is the examination of various economic activities carried out by subsistence farmers.

2. THE ACTIVITIES OF THE SUBSISTENCE PRODUCTION FARM

The subsistence farming household is engaged in a number of activities throughout the year. Let us call all such possible activities the activity set in traditional agriculture. The activities in this set can generally be considered under eight general types: 1) purchasing activities, 2) production activities, 3) technological activities, 4) fertilizing activities, 5) consumption activities, 6) investment activities, 7) sales activities, and 8) financial activities. Most of these activities are interdependent and carried out by the same decision unit in traditional agriculture -- the farming household. These are briefly discussed below.

2.1 Purchasing Activities

Farming households to the extent that they are unable to supply all their input needs are obliged to purchase them. Purchasing activities are associated with the purchase of variable inputs. The purchase of variable inputs depends upon i) their relative marginal productivities in various uses (which in turn depend upon the production alternatives available to farmers), ii) their relative prices, iii) their regional availabilities and iv) the availability of close substitutes on the farm. The availability of close substitutes within the household is especially true for labor inputs, and subsistence farmers are unlikely to hire labor unless they have first exhausted the available family labor.

Purchased inputs require the availability of cash, and to the extent that this is limited in subsistence households they are unable to avail themselves of these inputs. The degree to which variable inputs are purchased and replace inputs provided by the household to the firm is a measure in part of the market orientation of the traditional farmer.

2.2 Production Activities

Production activities are those activities that transform inputs into final outputs. Since production is viewed as being carried out by a sequence of tasks, production activities define the set of the sequence of tasks required to produce some final output. The production activity need not consider all the tasks required to produce a final output, but may include only a subset of tasks, and use as inputs intermediate outputs from other activities (see technological activities below) to produce final outputs.

Production activities have associated with them i) a cost per unit level of the activity, which accounts for all the costs of the tasks included in the subset examined by the production activity, ii) an output usually in the form of a yield per unit level of the activity, and iii) input coefficients that define the use of various physical and financial resources per unit level of the activity.

Production activities may include production under i) different soil conditions, ii) different water availability conditions (irrigated vs. unirrigated acreage), iii) different tenure conditions (production on rented vs. owner-cultivated farms), iv) different farm sizes (large vs. small farms) and different technologies (animal draft vs. tractor operated farms) and include the production of most of the major final outputs in the region. The extent to which account is taken of the varying conditions under which production is carried out will depend upon the importance of these distinctions for any given region. Thus, the full set of production activities in a region would include the production of major farm outputs for each of the subsets of conditions one wishes to investigate. If different subsets are considered then attention has to be given to the conditions under which these subsets are aggregated to give us a regional description of the production possibilities. For a region that is relatively homogeneous with respect to farm types -- that is soils, tenure and farm size conditions -- it is fairly easy to view production activities as being determined by the technical conditions of production -- that is the availability of water and different technologies. It is possible to do this by selecting a relatively homogeneous region, and is the course followed in this study. However, there is no theoretical limitation to defining all the production possibilities in the region, though significant practical problems arise from the size of the set obtained and the subsequent analysis.

2.3 Technological Activities

Technological activities are similar to production activities in the sense that they have associated with them unit level costs and input coefficients defining resource use, but they distinctly differ in the sense that they can provide only intermediate outputs, and are used to analyze the technological choice available to farmers in the region with regard to the performance of a distinct agricultural task. The intermediate outputs from these activities are usually standardized, so that we can then consider the use of a standard output from various operations. To the extent that given agricultural tasks (either specific to a given crop output or basic to all crop outputs) are included in the set of technological activities, these tasks are not considered under production activities, nor is their cost included under them, since production activities use the output (in the form of a performed task -- the intermediate output) from the technological activities. The notion of technological activities can perhaps be illustrated by a concrete example.

Suppose we are concerned with the production of wheat on a given farm type. The production of wheat requires several tasks -- land preparation, planting, irrigation, cultivation, harvesting and transportation at different intensities. Now we could if we wish consider each task as being performed by a different operation, and proceed to consider each operation as a technological activity, and then combine these tasks in different ways to produce wheat. Alternatively, it is possible to analyze only selected tasks. Thus, for example, we could consider a production activity and label it "produce wheat" and include in its input use structure and costs all tasks associated with the production of wheat except irrigation since we wish to consider in detail the technological choice available to farmers in the performance of this task. Then it is possible to define say two technological activities -- "irrigate with bullocks and a persian wheel" and "irrigate with a diesel powered tubewell" -- which use variable and quasi-fixed resources, and produce a <u>standard</u> intermediate output we can call a "standard irrigation unit". Then we can consider the production activity using several units of this output -- say six irrigation -- to produce a final output of wheat.

The choice of one technological activity over another for the performance of a given task depends upon the relative costs of the operations, upon the relative availabilities of resource inputs needed by each operation, and any behavioral constraints that might be relevant to the adoption of a new operation. The actual choice between operations in traditional agriculture may also depend upon the relative cost of hired labor and the availability of family labor. There are two possibilities here: 1) a case where a given operation is so efficient in terms of reducing costs for performing a task over its competing operation that it reduces costs even when all labor used by it is evaluated at a market wage, and 2) a case where the operation reduces costs for performing a task over its competing operations only when some or all parts of the labor it uses is family labor evaluated at a zero marginal cost by the farmer. In the former case the choice is more likely to depend upon the availability of complementary resources and adoption behavior, but in the second case the choice will depend crucially upon the availability of family labor, for once family labor is exhausted in any period, the farmer has to hire labor at the market wage rate, and the opportunity cost of a unit of his own labor then rises to the market wage rate since it is assumed that his labor is a perfect substitute. Thus, when family labor becomes a constraint one can expect a switching of technology in cases of the second type, where operations that use family labor at a zero marginal cost become unprofitable.

2.4 Fertilizing Activities

The final output from production activities are in the form of some "base" yields expected. It is assumed that the choice to fertilize an acre of a planted crop is separated from the choice to plant, only in the sense that after planting the farmer may wish to improve his base yield by applying inorganic fertilizers, and obtaining incremental yields from various levels of fertilizer used. The only reason to separate this choice is to allow the farmer to decide at what level (including zero level) he wishes to apply inorganic fertilizers to any crop, and to do this on the basis of comparing comparative yields expected from all possible crops. Fertilizer use depends upon the increase in yields expected from various fertilizing activities, fertilizer prices, output prices, and the total regional availabilities of fertilizers. Fertilizer activities have associated with them the purchase costs of fertilizers used, labor and nutrient inputs per unit level of the activity, and an incremental yield expected per unit of the activity.

2.5 Consumption Activities

Consumption activities are those associated with the household's consumption of farm produced outputs. There are two types of consumptions associated with traditional agriculture: 1) the consumption of foodgrains and food items partly processed on the farm and 2) the consumption of fodder crops needed for the maintenance of livestock on the farm.

The main determinants of the amount of the subsistence food crop retained for consumption in a peasant household are similar to those relevant in the analysis of the supply of marketable surplus. There have been a number of attempts to estimate the marketable surplus function of a subsistence crop: KRISHNA (1962, 1965), NARAIN (1961), PARATHASARATHY and SUBBA RAO (1964). The determinants of the consumption of food crops by a peasant household are: 1) family size of the household, 2) the output of the subsistence food crop, 3) the total farm income lagged by one year, 4) the harvest price of the subsistence crop, 5) the output of the nearest competing crop in the consumption pattern of the household and 6) the harvest price of this competing crop.

The main determinants of the fodder requirements on the farm are the number of livestock animals maintained and the daily requirements per animal (a fixed cost) and also the amount of work required of draft animals (a variable in terms of extra fodder fed when animals are worked).

Both the requirements for food and fodder act as a constraint upon the sale of the subsistence crop and upon the production of alternative crops, thus reducing the amount of cash flows that can be generated by the farm. In the case of the need to set aside a large percentage of area for their production has an important bearing upon the cropping pattern as well as upon the replacement of animal power by machinery, since the latter is not only labor saving but also releases land for production.

2.6 Investment Activities

Investment activities are associated with investment in quasi-fixed inputs leading to the replacement of and additions to the capital stock. Investment purchases compete for cash with both consumption and production purchases. Those quasi-fixed factors that are not purchased but produced on the farm, compete with other production activities for the use of variable inputs, and provide a stream of services in the form of capacities for use on the farm.

All investments, whether purchased or produced on the farm, increase available capacities for use on the farm, while their physical depreciation reduces available capacities. Investment activities have associated with them a net cost calculated on a payback principle, a cash outlay for purchase, and an addition to quasi-fixed capacities that are in turn used by the other activities. Investment activities also have associated with then adoption constraint that describe the bahavior of farmers in the region with regard to their adoption. Investments become profitable if additions to capacity generate cost savings to justify the use of available cash over its internal rate of return in terms of its most profitable use for some other activity.

2.7 Sales Activities

The sales activities are associated with the sale of final crop outputs for cash. The cash incomes generated by these jointly meet the requirements for the purchase of variable inputs, of consumption goods and services and for the purchase of investment goods with a lag. The marketable surplus decision (decision to sell) is viewed as the residual of two decisions -- the decision to produce and the decision to retain for consumption, since it has been observed that the "critical magnitude that is first determined by the peasant is the quantity to be retained for consumption" (out of production) KRISHNA (1965), except in extraordinary circumstances where peasants have to resort to "distress selling" to meet fixed cash obligations. It is also assumed that outputs retained for consumption and sales exhaust the total output, and that no inventories are kept except in the form of outputs retained for consumption.

Sales activities have associated with them the labor and other inputs required to transport the goods to the market, and a cash flow from the sales that provides cash for the next cropping period.

2.8 Financial Activities

Financial activities are those associated with the cash flows in the current cropping period in the farming household, and as such can be considered as a monetary link that binds the other economic activities involving the use of cash inputs. (Sales activities are associated with a cash flow for the period following the current cropping period).

There are two types of financial activities: 1) banking (saving) activities associated with a net addition to the cash flow and ii) net borrowing activities associated with a net subtraction from the cash flow in the current period. The saving activity is associated with a minimum rate at which a farmer can bank his liquid funds as an alternative to using them and the borrowing activity is associated with different rates at which farmers can borrow working capital to augment their liquidity.

In addition to saving and borrowing, there are also two other activities associated with cash flow: i) cash expenditures on consumption by the farming household and ii) cash incomes from non-farm employment. The first uses cash available and the second adds to current liquidity. Both of these activities have been treated exogenously, however, and are not explicitly treated as activities. Thus, the cash available for use by all activities is derived from previous sales and past savings and current non-farm cash incomes and current net borrowings, while the cash is partly exhausted by current cash expenditures for consumption and debt debt repayments on past borrowings. It is possible, thus, to analyze the cash streams generated by all the activities rather than analyze only credit available for production activities.

3. ELEMENTS OF THE RECURSIVE PROGRAMMING MODEL

Most of the early concern with the microeconomic analysis of agricultural response was confined to the prediction of the production of agricultural commodities on the basis of statistically estimated relationships between outputs, inputs and prices on the basis of time series or cross-sectional data. This study of price elasticities has undergone continuous refinements through the introduction of lagged prices and outputs other important factors affecting output responses, NERIOVE (1958) but two major problems remained. Firstly, is the problem of the competition for inputs by several outputs in a multi-product household-firm unit and the inability to get at the details of technological change.

An alternative approach to the problem of supply response at the microeconomic level centered around production function analysis -- the response of outputs to input use. This approach has led to many important contributions, HEADY and DILLON (1961) YOTOPOLOUS (1966), but also suffered from the same setbacks as the supply elasticity approach in the sense that interdependence cannot be easily handled.

This concern with the interdependence of activities competing for a given set of inputs suggests that the best way perhaps to formulate the problem is in the format of activity analysis, and the methodology best suited for this purpose is that of linear programming, since it has the advantage of explicitly examining resource use alternatives in considerable detail. The large variety of problems that could be handled in the programming format has led to its increasing use in economic analysis in both agriculture HEADY and CANDLER (1958), industry KOOPMANS (1957), and other areas SAMUELSON, DORFMAN and SOLOW (1958). A major attempt to incorporate both the dynamics of supply response as well as the problems of interdependence and technological change in agriculture was made by DAY (1962). He suggested a methodology in which the problem of the estimation of supply would rest upon the technical facts of production and upon the basic economic fact of the allocation of scarce resources among alternative ends. This approach he called <u>recursive</u>

linear programming (abbreviated R.L.P.) and gave it an empirical basis in the study of production response and technological change of agriculture in the Mississippi delta DAY (1963). Since the methodology is based upon an analysis of activities it readily presents itself as a means for analyzing the activities of the subsistence production farm outlined in the last section.

A recursive linear programming model has three elements: a) the activity set, b) the objective function, and c) the constraint structure. (See DAY (1961), (1962), (1963), and (1965) for a detailed discussion of the theoretical framework of the methodology of R.L.P.). We have already discussed the possible activity set in the last section we wish to include in our model of subsistence farms and we now turn to the other two elements.

3.1 The Objective Function

The objective function describes the decision behavior of farming units and includes many elements of the details of decision making discussed in the last chapter. The objective function represents what farmers are attempting to maximize (minimize), and in quantitative terms is the sum of each activity level times its contribution to the maximizing (minimizing) objective.

What is included or excluded in the objective function depends upon the maximizing principle used. It is true that given the inadequate knowledge of the technical possibilities and the complex forces that change expectations it would be naive to suggest that farmers behave in some optimal manner; however, in order to explain any planning behavior the concept of optimization cannot be dispensed with. This is not to suggest that the optimizing principle is used as a tool for optimal choice or even that farmers are consciously pursuing this principle, but that it operates as a rough guide to choose among alternatives that are present in the farmer's environment. Some decision criteria has to be established or else choice would be impossible.

All programming models (including R.L.P.) use the optimizing principle explicitly through the objective function. The most commonly used specification of this principle in traditional agriculture is the maximization of short run profits or the maximization of short run returns to fixed resources JOHL and KAHLON (1969). We have already stated that in traditional agriculture objectives may be ordered -- this principle is now incorporated by stating that the farm has two objectives that are ordered: the first and foremost objective to to meet the requirements for food and fibre on the subsistence production farm and secondly, after these have been met to maximize short run profits (minimize short run cash costs). This decision rule differs from that of a firm minimizing cash costs in three instances: i) the consumption requirements act as a constraint upon the cost minimization, ii) the use of family labor is viewed as having a zero opportunity cost, and iii) the use of animal draft includes the cost of variables inputs only (mainly concentrates and fodder fed to animals when they work) but excludes the fixed fodder requirements for their maintenance.

The evaluation of the use of family labor at a zero marginal cost is not meant to imply that farm labor in traditional subsistence farms has a zero marginal value productivity as some have argued[1], but because the consumption constraints account explicitly for the costs of its use, and these costs are considered fixed by the farming household, in the sense that the amount of subsistence crops that he retains for consumption are invariant with the amount of family labor employed on the farm. Since the costs of maintaining are fixed in terms of outputs retained for consumption, their use has already been accounted for, and being fixed they do not enter in the decision criteria. The same argument holds for the fixed costs of maintaining the farm animal population in terms of fodder grown specially for their consumption.

3.2 The Structure of Constraints

All economic behavior is constrained and the objective function describing the optimizing principle is subject to a set of constraints. The constraint structure describes the set of conditions under which traditional farmers pursue their objectives and their activities. There are three broad categories of constraints considered: 1) resource constraints, 2) behavioral constraints, and 3) financial constraints.

Resource constraints include constraints upon the availability of i) variable inputs such as family labor, hired labor, fertilizers and animal draft, ii) quasi-fixed inputs in the form of limited capacities describing the flow of services from various investments in machinery, implements and other quasi-fixed assets, iii) fixed inputs in the form of regional resources of land and infrastructure of various types. One of the most important aspects of the availability of resources is their time dimension. If the various inputs are not available at a specific time, given the nature of agricultural production they may not be available at all. The strong seasonal pattern to resource use means that the same physical input available at different times in the cropping year has to be considered as several different inputs, since they have a different time subscript. The real resource constraints in agriculture cannot be accounted for unless this time dimension of resource use is considered explicitly.

Behavioral constraints describe the institutional and behavioral environment in which farmers pursue their objectives -- an environment that cannot be defined in explicit economic terms. Great ingenuity and the help of other disciplines may be needed to clarify the nature of these constraints, but the difficulty of this task should not blind us to its importance.

Three broad constraints are recognized in traditional agriculture: i) a set of flexibility constraints HENDERSON (1959) that place both an upper and lower limit on the extent to which farmers are willing to change their output of ary given crop in response to profitability, because they may expect such changes to be short lived, or their desire to diversify their portfolio of crop outputs due to risk or becuase they account for uncertainty in this manner. In this context flexibility constraints can be viewed as expressing farmers' response to risk and uncertainty DAY (1961) while their inclusion in a linear model can be viewed as an approximation of a non-linear form of the objective function DAY and AIGNER (1969), ii) a set of adoption constraints defining an upper limit on the rate of growth of certain activities to take account of the facts of adoption behavior with regard to new outputs and investments. Such adoption behavior is not peculiar to agriculture but is also observed in industry DAY (1969), TABB (1967) and ABE (1969), and iii) a set of consumption constraints which describe the limitation imposed, in the absence of trade and fully developed markets, by the need to produce family requirements on the farm in the form of food and fodder. These are the explicit formulation of the costs of maintaining family labor and animal draft as a first order objective on the subsistence production farm.

Financial constraints are of two types: i) an upper limit placed upon the amount of various types of credit available to the farmer from various sources and at various interest rates with varying terms of debt repayment, and ii) the constraint placed upon the activities that use cash by the total liquidity generated in any given cropping year.

3.3 The Concept of Planning Behavior

All economic decision making is encompassed by time. Production and consumption plans for a given period for example depend upon both the past and the future. The past affects production decisions because the current stock of resources -- physical and financial -- are a result of decisions made in the past with regard to investment and consumption. This current stock of resources then form the basis on which plans for the future are contingent. Current decisions on the other hand affect the future because partly they include anticipations about the future and partly because current decisions affect the stock of resources available for the future. Thus, economic decision making is both backward and future looking.

Though the importance of time in the economic decision-making process has been known, and though highly developed theoretical constructs have been devised to incorporate time, recursive programming makes use of a somewhat unique concept of planning behavior. The normally accepted concept, implied first by HICKS (1946) allows current plans to include expectations of economic variables in future periods, discounted appropriately over a fixed time horizon. Not only are current decisions made on the basis of expected future values, but decisions for future executions are also mapped out in the present. Thus, this concept -- dynamic programming -- determines both plans that are to be executed in the present period and anticipates plans that are currently designed for future execution. Now if all the expectations were exactly realized, then currently laid plans for future action would approximate any plans made in the future with regard to future action. But given the realities of economic life and the extreme uncertainty of the economic environment (especially in agriculture) even the best laid plans are bound to be unfulfilled.

In the actual decision-making process we observe that with the passage of time, future plans are revised in the light of current and temporary conditions. Thus, any analysis that tries to reflect the actual process of economic planning as it affects actual production and consumption decisions must provide for this continuous planning -- the continual regeneration of the planning process. Recursive programming provides for precisely this continuous planning and is distinguished from dynamic programming by this unique concept of planning behavior that is incorporates. Firstly, it describes optimization over a limited time period on the basis of knowledge gained from past experience and secondly it allows for the sequential regeneration of the planning problem. "A recursive programming problem is not solved by a single decision that claims to determine what action will be optimal in each planning period within the time horizon, as do current versions of dynamic programming." Instead, it recognizes that plans for the future must be changed during each succeeding planning period to account for the actual history of economic variables" (DAY, 1962). The decision process in this case is never terminal, just like in the real world, it is continually reformulated to account for newly acquired information, so that we get not an optimum solution, but sequential optimizing behavior. Traditional agriculture that is undergoing transformation is likely to conform very closely to this concept of planning behavior.

The model examined in the next section accounts for planning behavior by i) basing future price expectations on the prices received and paid by the farmers in the previous period (a simple lagged price expectation model) and ii) basing the resource, financial and behavioral constraints in the current cropping year upon the actual activity levels generated either by the model or available from exogenous regional data for the previous year.

Having examined the basic elements of a recursive programming model, we now turn to describing one such model for traditional agriculture.

4. A REGIONAL R.L.P. MODEL OF TRADITIONAL AGRICULTURE

This section develops a recursive programming model for analyzing integrated household-firm activities discussed in section 2. Consider a region homogeneous with regard to soil, climate, topology, tenure, farm size, and resource distribution. Let such a region consist of subsistence farmers in the main, organized into farming households and occupied mainly with the production of field crops. For such a homogeneous region, if carefully defined, it is possible to view a regional model with the following three elements: 1) the activity set, 2) the objective function, and 3) the constraint structure.

4.1 The Activity Set

The full activity set is as follows:

$$L_{11},\ldots,L_{1\tau},\ L_2,\ L_3,\ P_1,\ldots,P_g,\ F_1,\ldots,F_h, T_1,\ldots,T_m,\ C_1,\ldots,C_n,\ S_1,\ldots,S_q,\ I_1,\ldots,I_r,\ V_1,\ V_2,\ V_3 \qquad (1)$$

where:

$L_{11},\ldots,L_{1\tau}$ are activities associated with the purchase of hired labor during period τ in the cropping year, where t is a time subscript that allows a time dimension to resource use within a cropping year. (Thus, L_{11} is the purchase of hired labor during the month of July in the region.);

L_2 is an activity associated with the use of bullock day equivalents of animal draft power in the farms in the region;

L_3 is an activity associated with the use of tractor hours on the farms in the region;

$P_1,..,P_g$ are production activities, measured in acreage sown to important crops in the region, and include a) crops sown during two seasons in the cropping year -- rabi (winter) and Kharif (summer), b) crops sown on irrigated and unirrigated soil conditions, c) both traditional crops as well as new high yield varieties, d) crops grown for fodder and 3) production under two technologies for each final output -- traditional (bullock operated) and modern (tractor operated);

$F_1,..,F_h$ are fertilizing activities, measured in acreage fertilized of various crops with different combinations and different levels of inorganic fertilizer used in the region;

$T_1,..,T_m$ are technological activities associated with technological choices available to farmers in the region, for the performance of those agricultural tasks explicitly excluded for detailed analysis. These activities involve the "production" of intermediate outputs measured in standard units and used by the production activities;

$C_1,..,C_n$ are consumption activities involving the retention of farm outputs for consumption, measured in quintals, by the farming households in the region;

$S_1,..,S_q$ are sales activities involving the sale of final outputs for cash;

$I_1,..,I_r$ are investment activities associated with the purchase of quasi-fixed inputs to increase their capacities in the region;

V_1 is an activity associated with net savings at interest rates available for such a purpose to farmers in the region;

V_2 is an activity associated with the net borrowing of working capital at low interest rates for one cropping year by the farmers in the region;

V_3 is an activity associated with the net borrowing of working capital at high interest rates for one cropping year by the farmers in the region.

4.2 The Objective Function

The farmers in a relatively homogeneous farming region are said to maximize the following objective function:[2]

$$\text{Max}\ \Pi(t) = \sum_{j=1}^{q} Z^{s}_{jt-1}\ S^{*}_{jt} - \left[\sum_{j=1}^{n} Z^{1}_{t-1}\ L^{*}_{1it} + Z^{2}_{t-1}\ L^{*}_{2t} + Z^{3}_{t-1}\ L^{*}_{3t}\right.$$

$$\left. + \sum_{j=1}^{g} Z^{p}_{jt-1}\ P^{*}_{jt} + \sum_{j=1}^{m} Z^{t}_{jt-1}\ T^{*}_{jt} + \sum_{j=1}^{r} Z^{I}_{jt-1}\ I^{*}_{jt}\right]$$

$$+ b_{it}\ V^{*}_{it} - \left[b_{2t}\ V^{*}_{2t} + b_{3t}\ V^{*}_{3t}\right] \qquad (2)$$

$$t = 2, \ldots, \theta$$

where:

- Z^{s}_{jt} is the harvest price of the jth. final output in year t, (In Rs./Quintals)
- S^{*}_{jt} is the actual level of the jth. sales activity in year t, (In Quintals);
- Z^{1}_{t} is the cost of hiring a unit of labor during period i in year t, (In Rs./Man Day);
- $L^{*}_{1\ t}$ is the actual amount of hired labor used in period i in year t, (In Man Days);
- Z^{2}_{t} are the variable cash costs associated with the use of a bullock day equivalent of animal draft in year t, (In Rs./Bullock Day Equivalent);
- L^{*}_{2t} is the actual amount of animal draft power used in year t, (In Bullock Day Equivalents);
- Z^{3}_{t} are the variable cash costs associated with the use of tractor power in year t, (In Rs./hour);
- L^{*}_{3t} is the actual amount of tractor power used in year t, (In hours).
- Z^{p}_{jt} are the variable cash costs associated with the jth. production activity in year t, and include the costs for those tasks that have not been included in the technological activities, (In Rs./acre);
- P^{*}_{jt} is the actual acreage sown to the jth. production activity in year t, (In acres);
- Z^{t}_{jt} are the variable cash costs associated with the jth. technological activity in year t, and include those tasks explicitly excluded for analysis, (In Rs./Standard Unit);

T^*_{jt} is the actual level of use of the jth. technological activity in year t, (In Standard Intermediate Units);

Z^I_{jt} is the current cost of the jth. investment good computed on a payback principle; i.e., for the jth. investment good $Z^I_{jt} = (i + d + u)\ C_{jt}$ where i is the rate of interest on medium term loans, d is the financial depreciation rate and is taken to be 1/L where L is the use life of the good in years, u is an uncertainty factor reflecting uncertainty with regard to the expected returns from the investment and C_{jt} is the purchase price of the investment good in year t, (In Rs./Unit Investment);

I^*_{jt} is the actual level of investment in the jth. investment good in year t, (In Units);

b_{1t} is the rate of return expected from saving in year t, (A percentage);

V^*_{1t} is the actual level of savings in year t, (In Rupees);

b_{2t} is the interest rate on working capital loans available to the farmers in year t, (A percentage)

V^*_{2t} is the actual level of net borrowing in year t, (In Rupees);

b_{3t} is the interest rate at which the supply of loans for working capital becomes infinitely elastic in year t, (A percentage);

V^*_{3t} is the actual net borrowings on the second type of loan in year t, (In Rupees).

The objective function maximizes short run cash returns, explicitly excluding the costs associated with the use of family labor and the maintenance costs associated with the use of animal draft power.

4.3 The Constraint Structure

The objective function is maximized subject to the following set of constraints that hold for each year of the recursive programming model; i.e., for t = 2,..., θ :

4.3.1 Non-Negative Outputs

$$(P_1, \ldots, B) \geqq 0 \qquad (1)$$

This means that economic activities cannot be operated "backwards."

4.3.2 Variable Inputs

3.2.1 Labor

$$\sum_{j=1}^{q} \nu^{p}_{jt} P_{jt} + \sum_{j=1}^{h} \nu^{f}_{jt} F_{jt} + \sum_{j=1}^{m} \nu^{t}_{jt} T_{jt} + \sum_{j=1}^{q} \nu^{s}_{jt} S_{jt} \leq F_{\tau t} + L^{*}_{1\tau t} \qquad (2\ 1)$$

where ν^{k}_{j} is the labor input coefficient for period τ for a unit level of the jth. activity of the kth. type, $F_{\tau t}$ is the amount of family labor available and any hired labor such that

$$L^{*}_{1\tau t} \leq H_{\tau t} \qquad (2.1.a)$$

where $H_{\tau t}$ is the amount of hired labor available in the region in period τ in year t, in man days.

3.2.2 Animal Draft Power

$$\sum_{j=1}^{g} d^{p}_{j\tau} P_{jt} + \sum_{j=1}^{h} d^{t}_{j\tau} T_{jt} \leq B_{\tau t} \qquad (2\ 2)$$

$$\text{and} \qquad L_{2t} \leq TBL_{t} \qquad (2.2.a)$$

where $d^{k}_{j\tau}$ is the draft labor input coefficient for period τ for a unit level of the jth activity of the kth. type, $B_{\tau t}$ is the amount of animal draft available (in bullock days equivalents) in period τ in year t in the region, L_{2t} is defined above and TBL_{t} is the total amount of draft power available throughout the year.

3.2.3 Inorganic Fertilizers

$$\sum_{j=1}^{h} f^{*}_{ij} \; F_{jt} \leq A_{it} \qquad (2.3)$$

where f^{*}_{ij} is the amount of the ith fertilizer (i = 3, for all fertilizers expressed in nitrogen, phosphorus, and potash equivalents in kilograms of nutrient available) used for the jth fertilizing activity, and A_{it} is the total availability of the ith fertilizer in the region in year t in kilograms of nutrients available.

The input coefficients f^{*}_{ij} are derived from statistically fitted production functions of the type

$$\hat{y}_j = \hat{f}_j \; (N, P, K) \qquad (i)$$

where N, P, and K are amounts of nitrogen, phosphorus, and potash used on experimental plots and y_j is the recorded yield of the jth crop output. It is then possible to approximate non-linear production functions fitted to the data by several discrete linear segments representing each fertilizing activity.

4.3.3 Quasi-Fixed Input Capacities

The constraints on the capacities of quasi-fixed inputs take the general form

$$\sum_{j=1}^{g} a^{p}_{ij} \; P^{*}_{jt} + \sum_{j=1}^{m} a^{t}_{ij} \; T^{*}_{jt} + \sum_{j=1}^{q} a^{s}_{ij} S^{*}_{jt} \leq Q_{i\tau} \; (t) \qquad (3.3)$$

where $a^{k}_{ij\tau}$ is the amount of the capacity of the ith quasi-fixed input utilized by the jth. activity of the kth. type in period i, and Q is the amount of the ith. capacity availed in period τ in year t, where we can write

$$Q_{i\tau}(t) = Q_{i\tau}(t-1) + \theta_i \,.\; I^*_{i(t)} - \frac{2}{n_i} \sum_{j=\frac{n}{2}+1}^{h} \theta_{i\tau} I^*_i \;(t-j) \qquad (3.3.a)$$

where $Q_{i\tau}$ is the capacity of the ith. quasi-fixed input available in period τ at the end of the previous year, $\theta_{i\tau}$ is the amount of the addition to capacity in period τ as a result of a unit investment in the ith. quasi-fixed input, I^*_{it} is the actual investment in the ith input in the current period and where the physical depreciation of capacity is given by the last expression where n_i is the use life of the ith. investment good.

In addition for total tractor house we have

$$L^*_{3t} \leq TTH_t \qquad (3.3.b)$$

where $TTH_{(t)} = \sum_{\tau} Q_{\tau}(t)$ and are the total annual tractor hours of capacity available.

3.4.1 Land

$$\sum_{j=1}^{w} P^w_{jt} \leq N^w_t \qquad (4.1)$$

where P^w_{jt} is the jth, production activity that can be carried out suitably on land type w, where the land type is defined by soil type, season and the availability of artificial irrigation, and N^w_t is the amount of land type w available in the region in y year t. Thus, for example, for a given soil type we have four land categories -- winter irrigated and unirrigated and summer irrigated and unirrigated.

3.4.2 Canal Irrigation Capacity

$$T_{rt} + T_{kt} \leq CIR_t \qquad (4.2)$$

where T_{rt} is the technological activity "irrigate rabi crops by canal" and T_{kt} the activity "irrigate Kharif crops by canal," and CIR_t is the total amount of canal irrigation available in standard units in the region in year t.

4.3.5 Behavioral Constraints

Considering the retention of farm outputs for consumption as a first order objective we specify two sets of consumption constraints that take account of subsistence production in traditional agriculture.

3.5.1 Consumption Constraints (Food)

$$C_{jt} = \bar{C}_{jt} \qquad (5.1)$$

where C_{jt} is the consumption activity associated with . the consumption of the jth. subsistence crop and $\bar{C}_{jt}$ can be estimated from household data and have been found to depend mainly on the size of the farm household and the total output of the crop on the farm. SINGH (1969).

3.5.2 Consumption Constraint (Fodder)

The fodder requirements for maintaining draft and other farm animals are expressed by the constraint

$$P_{ft} \geq G_t + g_1 L^*_{2t} - g_{2j} I^*_{jt} \qquad (5.2)$$

where P_{ft} is the production activity "produce green fodder", G_t are the fixed fodder requirements for maintain the draft and livestock population in year t, g_1 is a coefficient giving the additional fodder requirements per day worked by the draft animals and g_{2j} is a coefficient giving the amount of fodder area released by replacing the use of animal draft power by the jth. investment good, all expressed in acre equivalents of fodder sown, and L^*_{2t} and I^*_{jt} have been defined.

3.5.3 Flexibility Constraints

The flexibility constraints are specified separately for irrigated and unirrigated production activities for each crop and take the general form

$$X_t \geq \min \begin{cases} (1 + \bar{\beta}) \; X_{t-1} \\ X_{t-1} + \bar{\gamma} \; (\Lambda_t - X_{t-1}) \end{cases} \qquad (5.3.a)$$

$$\text{and} \quad X_t \geq \max \begin{cases} (1 + \underline{\beta}) \; X_{t-1} \\ X_{t-1} - \underline{\gamma} \; (\Lambda_t - X_{t-1}) \end{cases} \qquad (5.3.b)$$

where X_t is the total acreage sown to a given crop in year t, Λ_t is the maximum desired acreage farmers are willing to plant to this crop in year t, and $\bar{\beta}$, $\bar{\gamma}$ and $\underline{\beta}$, $\underline{\gamma}$ are flexibility coefficients associated with the upper and lower bounds respectively such that $X_t = \sum_j P^*_{jt}$ where P^*_{jt} is the actual level of the production activity associated with the production of the jth. crop.

3.5.4 Adoption of New Crop Varieties

The adoption constraint on new crop varieties specifies an upper limit on acreage planted to new varieties under both irrigated and unirrigated conditions and takes the general form

$$X_t \leq \min \begin{cases} (1 + p_1)^n \; X_{t-n} \\ p_2 \, \bar{X}_t + (1 - p_2) \, X_{t-1} \end{cases} \tag{5.4}$$

where X_t is the total acreage planted to a new variety in year t, and p_1 and p_2 are adoption coefficients, and $\bar{X}_t$ is the maximum desired acreage that farmers are willing to plant to the new variety in year t. The tow sets of constrained taken together define an s-shaped path through time.

3.5.5 Adoption of New Investment Goods

The adoption of new investments are also characterized by an upper limit and the constraint on investments takes the general form

$$K_{jt} \leq \min \begin{cases} (1 + \alpha_1)^n \; K_{jt-n} \\ \alpha_2 \bar{K}_{jt} + (1 - \alpha_2) \, K^*_{jt-1} \end{cases} \tag{5.5}$$

where K_{jt} is the number of units of the jth. investment good in use in year 5, measured in units of capacity, α_1 and α_2 are adoption coefficients associated with an s-shaped adoption path, $\bar{K}_{jt}$ is the current maximum desired capacity in the jth. good, and K^*_{jt-1} is the actual capacity utilized in the previous year. Since net investment in capacity can be viewed as the difference in total capacity over two periods, we can write

$$I_{jt} = K_{jt} - K_{jt-1} \tag{5.5.a}$$

and then (5.5) can be expressed in terms of the investment activities as follows

$$I^*_{jt} \leq \min \begin{cases} \alpha_1 \; K^*_{jt-1} \\ \alpha_2 \; \bar{K}_{jt} - \alpha_2 \; K^*_{jt-1} \end{cases} \tag{5.5.b}$$

where I^*_{jt} is the investment in the jth. investment good in the current year and K^*_{jt-1} is the actual capacity utilized in the previous year.

4.3.6 Financial Constraints

There are two financial constraints, one on the cash available and the second on the credit available to farmers in the region.

3.6.1 Cash Constraint

$$\sum_{j=1}^{c} x_{jt}^{1} L_{1\,t}^{*} + x_{t}^{2} L_{2t}^{*} + x_{t}^{3} L_{3t}^{*} + \sum_{j=1}^{g} x_{jt}^{p} P_{jt}^{*} + \sum_{j=1}^{h} x_{jt}^{f} F_{jt}^{*} + \sum_{j=1}^{r} c_{jt} I_{jt}^{*} \leq \sum_{j=1}^{q} z_{jt-1}^{s} S_{jt-1}^{*} - \bar{E}_{t} + \bar{Y}_{t-1} + (1+b_1) V_{1t-1}^{*} - \left[(1+b_2) V_{2t-1}^{*} + (1+b_3) V_{3t-1}^{*} \right] \qquad (6.1)$$

where x_{jt}^{k} is the cash requirement per unit of the jth. activity of the kth. type in year t, c_{jt} is the purchase price of the jth. investment good, z_{jt-1}^{s} is the harvest price of the jth. final output in the previous year, $\bar{E}_t$ are cash purchases for consumption in the current year, and $\bar{Y}_{t-1}$ are any non-farm incomes earned by farming households in the previous year and both are exogenously estimated for the region, and other variables are defined. Thus, cash used cannot exceed the proceeds from last year's sales less any current cash consumption expenditures, plus any non-farm cash incomes and savings with accrued interest from the previous year, less the repayment in full of last year's net borrowings along with the accrued interest.

3.6.2 Credit Constraint

$$V_{2t} \leq \hat{s} \left(\sum_{j=1}^{q} z_{jt-1}^{s} S_{jt-1}^{*} \right) \qquad (6.2)$$

where the amount of net borrowings at the interest rate b_2 are restricted to some percentage $\hat{s}$ of the previous year's gross sales, providing an upper credit limit to the net borrowing at this rate.

4.3.7 Balance Equations

A set of equations defines transfers of final outputs, intermediate outputs and planted acreages for fertilizing and these are included under this section.

3.7.1 Final Output Balances

Total outputs are exhausted by sales and consumption and assuming no inventories (except in the form of outputs retained for consumption) we define the following output balances

$$S^*_{jt} + C^*_{jt} \leq \sum_{j}^{k} 1 \quad y^p_j \; P^*_{jt} \; + \sum_{j}^{l} 1 \quad y^f_j \; F^*_{jt} \qquad (7.1)$$

where S^*_{jt} and C^*_{jt} are the actual sales and consumption of the jth. final output in year t, y^p_j is the expected yield from k production activities associated with the jth. output, and are called "base yields", and y^f_j is the incremental yield expected (in addition to the base yield) from l fertilizing activities associated with the jth. output. The expected base yields are derived from available yield data and can be adjusted for weather variations by the use of a weather index to deflate the yields DOLL (1967), OURY (1965), SHAW (1964), STALLING (1961) and U.S.D.A. (1962). The expected incremental yields are derived from production functions fitted to field experimental data on fertilizer response.

3.7.2 Standard Intermediate Output Balances

The technological activities associated with any given task each produce the same intermediate output which is then used by the other activities. Thus, we have

$$\sum_{j=}^{g} 1 \quad r^p_{ij} \; P^*_{jt} + \sum_{j=}^{h} 1 \quad r_{ij} \; F^*_{jt} + \sum_{}^{l} 1 \; r^s_{ij} \; S^*_{jt} \leq \sum_{v} \; T^v_{it} \qquad (7.2)$$

where r^k_{ij} is the number of standard units of the ith. standard intermediate output used by the jth. activity of the kth. type, and T^v_{it} is the level of utilization of the vth. technological activity associated with the ith. task in year t. Thus $\sum_{v} T^v_i$ implies that there might be v possible operations by which the ith. agricultural task can be performed, and each is associated with a technological activity that produces a standard unit of the ith. task performed

3.7.2 <u>Fertilized Acreage Balances</u>

$$\sum_{j}\sum_{1}^{l} F^{*}_{jt} \leq \sum_{j}\sum_{1}^{k} P^{*}_{jt} \qquad (7.2)$$

where F^{*}_{jt} is the acreage fertilized of the jth. crop at l different levels, and P^{*}_{jt} is the acreage planted to the jth. crop under k different production activities. Thus, the fertilized acreage cannot exceed the planted acreages for any given crop in year t.

The model formulated above tries to incorporate many of the details of the household-firm interdependence, the details of technological change and the details of decision making in traditional agriculture. Its main problem as formulated is that it can be applied empirically to only a relatively homogeneous region in which development is already under way so that the traditional equilibrium is already undergoing transformation.

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