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BIASED ESTIMATES OF Ω_m FROM COMPARING SMOOTHED PREDICTED VELOCITY FIELDS TO UNSMOOTHED PECULIAR-VELOCITY MEASUREMENTS

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ABSTRACT

We show that a regression of unsmoothed peculiar velocity measurements against peculiar velocities predicted from a smoothed galaxy density field leads to a biased estimate of the cosmological density parameter Ω_m, even when galaxies trace the underlying mass distribution, and galaxy positions and velocities are known perfectly. The bias arises because the errors in the predicted velocities are correlated with the predicted velocities themselves. We investigate this bias using cosmological N-body simulations and analytic arguments. In linear perturbation theory, for cold dark matter power spectra and Gaussian or top-hat smoothing filters, the bias in Ω_m is always positive, and its magnitude increases with increasing smoothing scale. This linear calculation reproduces the N-body results for Gaussian smoothing radii $R_s \gtrsim 10\ h^{-1}$ Mpc, while nonlinear effects lower the bias on smaller smoothing scales, and for $R_s \lesssim 3\ h^{-1}$ Mpc, Ω_m is underestimated rather than overestimated. The net bias in Ω_m for a given smoothing filter depends on the underlying cosmological model. The effect on current estimates of Ω_m from velocity-velocity comparisons is probably small relative to other uncertainties, but taking full advantage of the statistical precision of future peculiar-velocity data sets will require either equal smoothing of the predicted and measured velocity fields or careful accounting for the biases discussed here.

Subject headings: cosmology: theory — galaxies: distances and redshifts — methods: numerical

1. INTRODUCTION

One of the most popular approaches to constraining the mass density parameter, Ω_m, the ratio of the average matter density to the critical density, is based on comparisons between the galaxy density field mapped by redshift surveys and the galaxy peculiar velocity field inferred from distance-indicator surveys (see the review by Strauss & Willick 1995). While the numerous implementations of this approach differ in many details, they are all motivated by the linear-theory formula for the peculiar-velocity field,

$$\boldsymbol{v}(\boldsymbol{x}) = \frac{H_0 f(\Omega_m)}{4\pi} \int \delta(\boldsymbol{x}') \frac{(\boldsymbol{x}' - \boldsymbol{x})}{|\boldsymbol{x}' - \boldsymbol{x}|^3} d^3x' , \qquad (1)$$

or its divergence,

$$\nabla \cdot \boldsymbol{v}(\boldsymbol{x}) = -a_0 H_0 f(\Omega_m)\delta(\boldsymbol{x}) , \qquad (2)$$

where $\delta(\boldsymbol{x}) \equiv \rho(\boldsymbol{x})/\bar{\rho} - 1$ is the mass density contrast, $f(\Omega_m) \approx \Omega_m^{0.6}$, H_0 is the Hubble parameter, and a_0 is the present value of the expansion factor (Peebles 1980).[2] "Velocity-velocity" comparisons start from the observed galaxy density field, predict peculiar velocities via equation (1) or some nonlinear generalization of it, and compare to estimated peculiar velocities (e.g., Kaiser et al. 1991; Strauss & Willick 1995; Davis, Nusser, & Willick 1996; Willick et al. 1997; Willick & Strauss 1998; Blakeslee et al. 1999). "Density-density" comparisons start from the observed radial peculiar velocity field, infer the three-dimensional velocity field using the POTENT method of Bertschinger & Dekel (1989), and compare the velocity divergence to the observed galaxy density field using equation (2) or a nonlinear generalization of it (e.g., Dekel et al. 1993; Hudson et al. 1995; Sigad et al. 1998; Dekel et al. 1999). Because the radial velocity field must be smoothed before computing the three-dimensional velocity field via POTENT, density-density comparisons in practice always compare the smoothed galaxy density field to predictions derived from the smoothed peculiar-velocity field. Velocity-velocity comparisons, on the other hand, usually smooth the galaxy density field to suppress nonlinear effects and shot noise, but compare the velocity predictions from these smoothed density fields directly to the estimated peculiar velocities of individual galaxies or groups. (The spherical-harmonic analysis of Davis et al. 1996 is an important exception in this regard.)

The avoidance of smoothing the data is often seen as an advantage of the velocity-velocity approach, since smoothing a noisy estimated velocity field can introduce statistical biases that are difficult to remove. However, in this paper we show that comparing smoothed velocity predictions to unsmoothed velocity measurements generally leads to biased estimates of $f(\Omega_m)$, even when the galaxy positions and velocities are known perfectly. The reason for this bias is fairly simple: the errors in the predicted velocities are correlated with the predicted velocities themselves, violating the conventional assumption that an individual galaxy's velocity can be modeled as a "large-scale" contribution predicted from the smoothed density field plus an *uncorrelated* "small-scale" contribution.

Galaxy redshift surveys map the galaxy density field, $\delta_g(\boldsymbol{x})$, rather than the mass density field, $\delta(\boldsymbol{x})$, so inferences from velocity-velocity and density-density comparisons often assume a linear relation between galaxy and mass density contrasts, $\delta_g(\boldsymbol{x}) = b\delta(\boldsymbol{x})$, and therefore constrain the quantity $\beta \equiv f(\Omega_m)/b$ rather than $f(\Omega_m)$ itself. The results reported in this paper emerged from a more general investigation of the effects of complex galaxy formation models on

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[2] Because galaxy distances are inferred from their redshifts via Hubble's law, uncertainties in H_0 and a_0 do not introduce any uncertainty in peculiar velocity predictions; if one adopts km s^{-1} distance units in place of Mpc, then H_0 and a_0 do not appear in equations (1) or (2).

estimates of β (Berlind, Narayanan, & Weinberg 1999; A. A. Berlind, V. K. Narayanan, & D. H. Weinberg, in preparation). However, the statistical bias in $f(\Omega_m)$ that we find applies even when galaxies trace mass exactly, so here we focus on this simpler case. (Throughout this paper we use the term "bias" to refer to systematic statistical errors rather than the relation between the distributions of galaxies and mass.) We further restrict our investigation to the case in which galaxy positions and velocities are known perfectly, ignoring the additional complications that arise in analyses of observational data.

2. RESULTS

We have carried out N-body simulations of three different cosmological models, all based on inflation and cold dark matter (CDM). The first is an $\Omega_m = 1$, $h = 0.5$ model ($h \equiv H_0/100$ km s^{-1} Mpc^{-1}), with a tilted power spectrum of density fluctuations designed to satisfy both *COBE* and cluster normalization constraints. The cluster constraint requires $\sigma_8 \approx 0.55$ (White, Efstathiou, & Frenk 1993), where σ_8 is the rms linear density fluctuation in spheres of radius 8 h^{-1} Mpc. Matching the *COBE* DMR constraint and $\sigma_8 = 0.55$ with $h = 0.5$ requires an inflationary spectral index of $n = 0.803$ if one incorporates the standard inflationary prediction for gravitational wave contributions to the *COBE* anisotropies (see Cole et al. 1997 and references therein). The other two models have $\Omega_m = 0.2$ and 0.4, with a power-spectrum shape parameter $\Gamma = 0.25$ (in the parameterization of Efstathiou, Bond, & White 1992) and cluster-normalized fluctuation amplitude $\sigma_8 = 0.55\Omega_m^{-0.6}$. These two models are open models with no cosmological constant, Λ. Since Λ has little or no effect on peculiar velocities at fixed Ω_m, our results for the two open models should also hold for flat-Λ cosmologies having the same values of Ω_m and the same matter power spectrum. We ran four independent simulations for each of the three cosmological models, and the results we show below are averaged over these four simulations. All simulations were run with a particle-mesh (PM) N-body code written by C. Park, which is described and tested in Park (1990). Each simulation uses a 400^3 force mesh to follow the gravitational evolution of 200^3 particles in a periodic cube 400 h^{-1} Mpc on a side, starting at $z = 23$ and advancing to $z = 0$ in 46 steps of equal expansion factor a. We form the mass density field by clouds-in-cells (CIC) binning the evolved mass distribution onto a 200^3 grid. We smooth this density field with a Gaussian filter of radius R_s and derive the linear-theory–predicted velocity field using equation (1). Finally, we linearly interpolate this velocity field to the galaxy positions to derive predicted galaxy peculiar velocities, $\boldsymbol{v}_{\rm pred}$.

Figure 1 compares the true velocities of particles ($\boldsymbol{v}_{\rm true}$) from one of the $\Omega_m = 1$ simulations to the velocities predicted ($\boldsymbol{v}_{\rm pred}$) by equation (1) from the mass density field smoothed with Gaussian filters of radius $R_s = 3$, 5, 10, and 15 h^{-1} Mpc (Figs. 1*a*–1*d*, respectively). The points in Figure 1 show one Cartesian component of the particles' velocities. If we make the assumption, common to most velocity-velocity comparison schemes, that each galaxy's velocity consists of a large-scale contribution predicted from the density field plus an uncorrelated small-scale contribution, then the best-fit slope of the $\boldsymbol{v}_{\rm true} - \boldsymbol{v}_{\rm pred}$ relation should yield the parameter $f(\Omega_m)$, in this case $f(\Omega_m) = 1$, with the scatter about this line yielding the dispersion of the small-scale contribution. However, it is clear from Figure 1 that this slope increases systematically with increasing R_s. (We note that the best-fit line, which minimizes $\sum |\boldsymbol{v}_{\rm true} - \boldsymbol{v}_{\rm pred}|^2$, is shallower than the line one would naively draw through these data points by eye, since it is vertical scatter rather than perpendicular scatter that must be minimized.)

The filled points in Figure 2 show the estimated $f(\Omega_m)$ as a function of R_s for the $\Omega_m = 1$ (*circles*) and $\Omega_m = 0.2$ (*squares*) cosmological models. The solid lines show the true value of $f(\Omega_m)$. In both cases, the estimated value of $f(\Omega_m)$ is quite sensitive to the smoothing scale: it is slightly underestimated at small scales, but increasingly overestimated at large scales. The $\Omega_m = 0.4$ model yields similar results, so we do not plot it separately. We also investigated $\Omega_m = 1$ simulations with a factor of 2 lower force resolution (200^3 force mesh instead of 400^3) and found identical results, so even at small smoothing scales our results are not affected by the simulations' limited gravitational resolution. The breakdown of linear theory at small scales is not surprising; however, the systematic failure of this method at large smoothing scales has not, to our knowledge, been previously discussed. The dependence of the estimated $f(\Omega_m)$ on the smoothing scale used for velocity predictions is our principal result.

We can understand the origin of the large-scale bias in $f(\Omega_m)$ by considering the case in which galaxy peculiar velocities are given exactly by linear theory. In this case,

$$\boldsymbol{v}_{\rm true}(\boldsymbol{x}) = (2\pi)^{3/2} H_0\, f(\Omega_m) \int e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \frac{i\delta_{\boldsymbol{k}}\boldsymbol{k}}{|\boldsymbol{k}|^2}\, d\boldsymbol{k}\,, \quad (3)$$

where $\delta_{\boldsymbol{k}}$ are the Fourier modes of the density field, and the integral extends over all of $\boldsymbol{k}$-space. Predicted velocities, however, are estimated from the density field smoothed with a window function, $W(r)$, of characteristic scale R_s. Therefore,

$$\boldsymbol{v}_{\rm pred}(\boldsymbol{x}) = (2\pi)^{3/2} H_0\, f(\Omega_m) \int \tilde{W}(kR_s) e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \frac{i\delta_{\boldsymbol{k}}\boldsymbol{k}}{|\boldsymbol{k}|^2}\, d\boldsymbol{k}\,, \quad (4)$$

where $\tilde{W}(kR_s)$ is the Fourier transform of the window function. The error in the predicted velocity of a galaxy at position $\boldsymbol{x}$ is therefore

$$\Delta\boldsymbol{v}(\boldsymbol{x}) = \boldsymbol{v}_{\rm true} - \boldsymbol{v}_{\rm pred} = (2\pi)^{3/2} H_0\, f(\Omega_m) \int [1 - \tilde{W}(kR_s)] \times e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \frac{i\delta_{\boldsymbol{k}}\boldsymbol{k}}{|\boldsymbol{k}|^2}\, d\boldsymbol{k}\,. \quad (5)$$

Note that in equation (4) we have defined $\boldsymbol{v}_{\rm pred}$ to be the velocity that would be predicted assuming the correct value of Ω_m. In practice, since we do not know the value of $f(\Omega_m)$ beforehand, we derive its value from the slope of the $\boldsymbol{v}_{\rm true}$ versus $f^{-1}\boldsymbol{v}_{\rm pred}$ relation (this is equivalent to assuming $\Omega_m = 1$ when computing $\boldsymbol{v}_{\rm pred}$).

If $\Delta\boldsymbol{v}$ were uncorrelated with $\boldsymbol{v}_{\rm pred}$, then the slope of the $\boldsymbol{v}_{\rm true}$ versus $f^{-1}\boldsymbol{v}_{\rm pred}$ relation would be an unbiased estimator of $f(\Omega_m)$. However, if $\Delta\boldsymbol{v}$ is positively correlated with $\boldsymbol{v}_{\rm pred}$, then the slope of the relation is no longer $f(\Omega_m)$, since points preferentially scatter above the line for positive $\boldsymbol{v}_{\rm pred}$ and below the line for negative $\boldsymbol{v}_{\rm pred}$. This steepening of the $\boldsymbol{v}_{\rm true} - \boldsymbol{v}_{\rm pred}$ relation is just the behavior seen in Figure 1. Equations (4) and (5) show that $\Delta\boldsymbol{v}$ and $\boldsymbol{v}_{\rm pred}$ will be correlated as long as some Fourier modes contribute to both

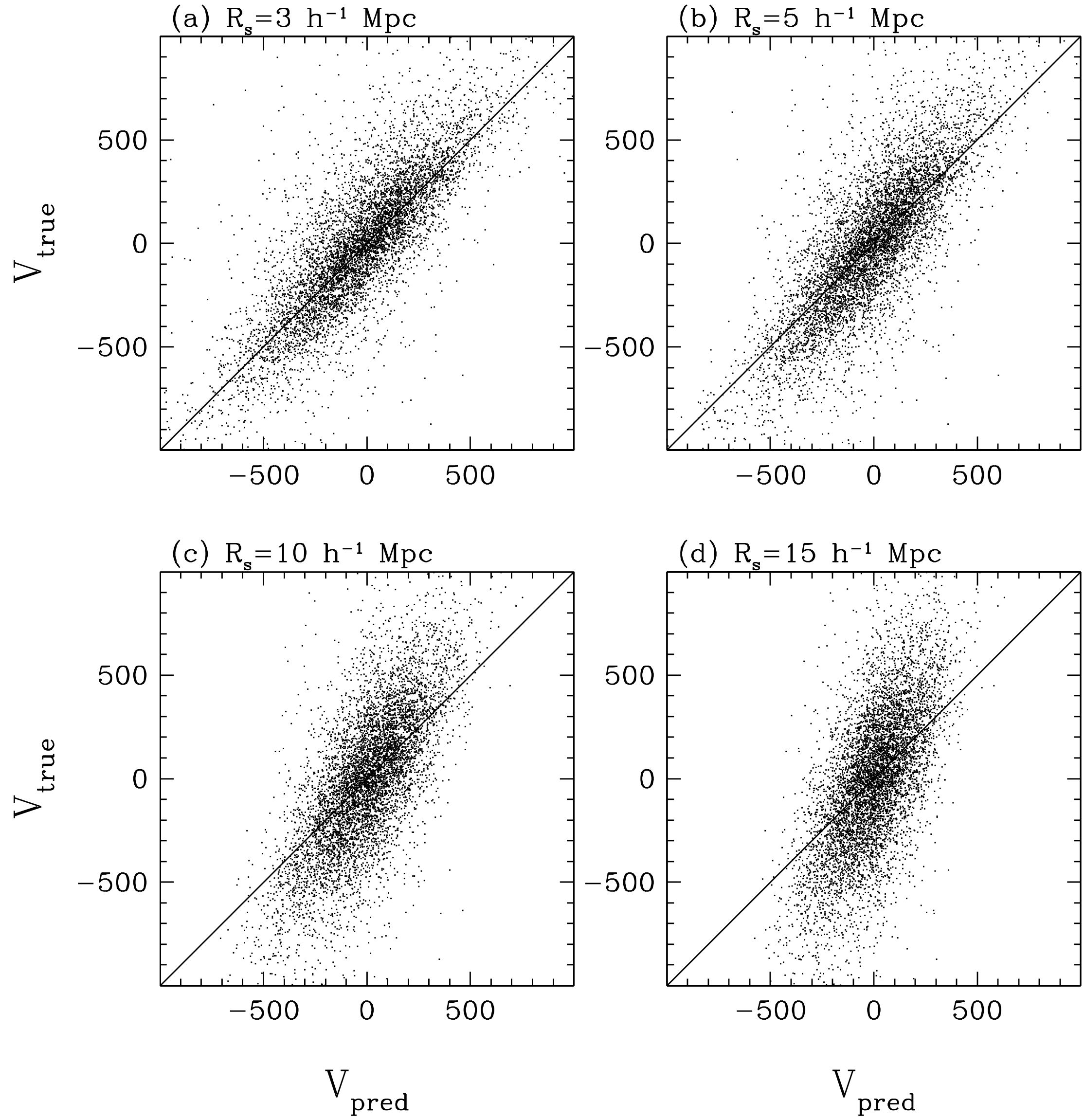

FIG. 1.—True velocities of particles from an $\Omega_m = 1$, CDM, cosmological N-body simulation, compared to the velocities predicted by linear theory from the mass-density field after it is smoothed with Gaussian filters of radius $R_s = 3$, 5, 10, and 15 h^{-1} Mpc (panels *a*, *b*, *c*, and *d*, respectively). The points show one Cartesian component of the particles' velocities. Solid lines show the relation $\boldsymbol{v}_{\rm true} = \boldsymbol{v}_{\rm pred}$.

integrals, which happens for any smoothing function other than a step function in $\boldsymbol{k}$-space.

We can quantitatively understand this bias by considering how $f(\Omega_m)$ is measured. For an ensemble of N points $(\boldsymbol{v}_{{\rm true},i}, f^{-1}\boldsymbol{v}_{{\rm pred},i})$, the slope of the best-fit line (assuming $\langle \boldsymbol{v}_{\rm true}\rangle = \langle \boldsymbol{v}_{\rm pred}\rangle = 0$) is

$$\begin{aligned} {\rm slope} &= \frac{\sum (f^{-1}\boldsymbol{v}_{{\rm true},i} \cdot \boldsymbol{v}_{{\rm pred},i})}{\sum (f^{-2}\boldsymbol{v}_{{\rm pred},i} \cdot \boldsymbol{v}_{{\rm pred},i})} \\ &= f(\Omega_m)\,\frac{(1/N)\sum [(\boldsymbol{v}_{{\rm true},i} - \boldsymbol{v}_{pred_i}) \times \boldsymbol{v}_{pred_i} + (\boldsymbol{v}_{pred_i} \cdot \boldsymbol{v}_{pred_i})]}{(1/N)\sum (\boldsymbol{v}_{{\rm pred},i} \cdot \boldsymbol{v}_{{\rm pred},i})} \\ &= f(\Omega_m)\left(1 + \frac{\langle \Delta\boldsymbol{v} \cdot \boldsymbol{v}_{\rm pred}\rangle}{\langle \boldsymbol{v}_{\rm pred} \cdot \boldsymbol{v}_{\rm pred}\rangle}\right). \end{aligned} \tag{6}$$

Equation (6) shows how a nonzero cross-correlation between $\Delta\boldsymbol{v}$ and $\boldsymbol{v}_{\rm pred}$ changes the measured slope of the velocity-velocity relation. We can compute this effect in the linear regime for a given power spectrum of density fluctuations, $P(k)$, and window function, $\tilde{W}(kR_s)$. Using equations (4) and (5), we have

$$\frac{\langle \Delta\boldsymbol{v} \cdot \boldsymbol{v}_{\rm pred}\rangle}{\langle \boldsymbol{v}_{\rm pred} \cdot \boldsymbol{v}_{\rm pred}\rangle} = \frac{\int_0^\infty \tilde{W}(kR_s)[1 - \tilde{W}(kR_s)]P(k)dk}{\int_0^\infty \tilde{W}^2(kR_s)P(k)dk}\,. \tag{7}$$

For Gaussian and top-hat window functions and a range of CDM power spectra, we find that the bias given by equation (7) is always positive and is always an increasing function of R_s. The dashed lines in Figure 2 show the slope computed (from eqs. [6] and [7]) using the linear mass power spectra of the simulations and the same Gaussian window functions that were used to measure $f(\Omega_m)$ (*filled symbols*). The striking similarity on large smoothing scales between the N-body data and this linear-theory calculation supports our conclusion that the large-scale bias is indeed caused by the cross-correlation between $\Delta\boldsymbol{v}$ and $\boldsymbol{v}_{\rm pred}$, which, in turn, is caused by the comparison of a smoothed prediction to unsmoothed data.

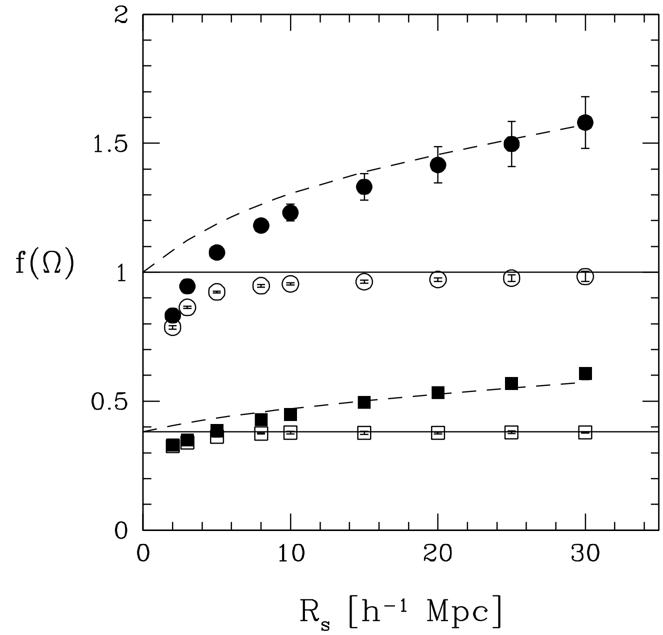

FIG. 2.—Estimates of $f(\Omega_m)$ from the slope of the relation between true galaxy velocities and velocities predicted by linear theory from the smoothed density field, as a function of the smoothing radius, R_s, for CDM models with $\Omega_m = 1$ (*circles*) and 0.2 (*squares*). Points represent the mean result of four simulations of each model, and error bars show the uncertainty in the mean derived from the dispersion among the simulations. Filled symbols show the estimated $f(\Omega_m)$ when the density field is smoothed with a Gaussian filter of radius R_s. Open symbols show the estimated $f(\Omega_m)$ when the density field is smoothed with a sharp low-pass $\boldsymbol{k}$-space filter (with a cut at $k_{\rm cut}$), where R_s is the radius of a Gaussian filter that falls to half its peak value at $k = k_{\rm cut}$. Dashed lines show the linear-theory prediction of the bias in the estimates of $f(\Omega_m)$ (eqs. [6] and [7]) from comparing smoothed velocity predictions to unsmoothed velocity measurements. Solid lines show the true values of $f(\Omega_m)$.

From equation (7), it is evident that the linear-theory cross-correlation between $\Delta \boldsymbol{v}$ and $\boldsymbol{v}_{\rm pred}$ will be zero if there is no smoothing at all, or if the smoothing function is a step function in $\boldsymbol{k}$-space, in which case the product $\tilde{W}(kR_s)[1 - \tilde{W}(kR_s)]$ is always zero. The open symbols in Figure 2 show the results of a velocity-velocity analysis of the same simulations, with the linear-theory velocities now predicted from a density field smoothed with a sharp, low-pass $\boldsymbol{k}$-space filter. Specifically, we set to zero all Fourier modes with $k > k_{\rm cut}$, and plot the new estimates of $f(\Omega_m)$ at the values of R_s for which a Gaussian filter falls to half its peak value at $k = k_{\rm cut}$ (i.e., $e^{-k_{\rm cut}^2 R_s^2/2} = 0.5$). Using the sharp $\boldsymbol{k}$-space filter causes the bias to vanish completely on large scales, yielding estimates of $f(\Omega_m)$ that are correct and independent of smoothing length. This result further supports our interpretation of the cause of the large-scale velocity-velocity bias.

Figure 2 shows that $f(\Omega_m)$ is underestimated at small scales in the N-body simulations. The linear-theory bias discussed above and shown by the dashed line in Figure 2 is always positive. Therefore, there must be a countervailing effect that biases $f(\Omega_m)$ estimates in the opposite direction on small scales. In highly nonlinear regions of the density field, such as the cores of galaxy clusters, linear-theory velocity predictions have large errors. However, errors caused by virial motions are uncorrelated with the predicted velocities because these virial motions have random directions. Such errors add random scatter to the velocity-velocity relation, but they do not change its slope. In mildly nonlinear regions of the density field, on the other hand, galaxy velocities still follow coherent flows, but these flows may no longer be accurately predicted by linear theory. In the case of a galaxy falling toward a large overdensity, linear theory will correctly predict the direction of motion, but it will overestimate the infall speed because it incorrectly assumes that the overdensity has grown at the linear-theory rate over the history of the universe, while in reality the overdensity grows to large amplitude only at late times, when it becomes nonlinear. In such regions, $\Delta \boldsymbol{v}$ will be opposite in sign to $\boldsymbol{v}_{\rm pred}$, causing an anticorrelation between the two quantities. The opposite happens in underdense regions, but since fewer galaxies reside in these regions and the velocity errors are smaller in magnitude, the net effect is still an anticorrelation between $\Delta \boldsymbol{v}$ and $\boldsymbol{v}_{\rm pred}$.

In order to show how these different effects come into play, we adopt a fluid-dynamics description and divide an individual galaxy's velocity into a mean flow, $\bar{\boldsymbol{v}}$, and a random "thermal" velocity, $\boldsymbol{\sigma}$, so that $\boldsymbol{v}_{\rm true} = \bar{\boldsymbol{v}} + \boldsymbol{\sigma}$. Here $\bar{\boldsymbol{v}}(\boldsymbol{x})$ is the average velocity of galaxies at spatial position $\boldsymbol{x}$, and therefore $\langle \boldsymbol{\sigma} \cdot \bar{\boldsymbol{v}} \rangle = 0$ by definition. Let $\boldsymbol{v}_{\rm lin}$ denote the velocity predicted in linear theory from the *unsmoothed* density field (eq. [1]). Equation (5) applies to the case where the velocity field is exactly linear, $\boldsymbol{v}_{\rm true} = \boldsymbol{v}_{\rm lin}$, but more generally,

$$\boldsymbol{v}_{\rm true} = \bar{\boldsymbol{v}} + \boldsymbol{\sigma} = \boldsymbol{v}_{\rm pred} + (\boldsymbol{v}_{\rm lin} - \boldsymbol{v}_{\rm pred}) + (\bar{\boldsymbol{v}} - \boldsymbol{v}_{\rm lin}) + \boldsymbol{\sigma} \ , \quad (8)$$

and therefore,

$$\Delta \boldsymbol{v} = \boldsymbol{v}_{\rm true} - \boldsymbol{v}_{\rm pred} = (\boldsymbol{v}_{\rm lin} - \boldsymbol{v}_{\rm pred}) + (\bar{\boldsymbol{v}} - \boldsymbol{v}_{\rm lin}) + \boldsymbol{\sigma} \ . \quad (9)$$

This equation shows the three possible sources of error in the smoothed linear theory prediction of galaxy velocities. The first term represents the effect caused by comparing a smoothed quantity with an unsmoothed quantity in linear theory, and is given by equation (5). The second term represents the inadequacy of using a linear-theory velocity estimator in regions where nonlinear effects are important. The third term represents errors caused by galaxies' random thermal motions.

As shown in equation (6), the bias in $f(\Omega_m)$ depends on the cross-correlation of these errors with $\boldsymbol{v}_{\rm pred}$,

$$\langle \Delta \boldsymbol{v} \cdot \boldsymbol{v}_{\rm pred} \rangle = \langle (\boldsymbol{v}_{\rm lin} - \boldsymbol{v}_{\rm pred}) \cdot \boldsymbol{v}_{\rm pred} \rangle + \langle (\bar{\boldsymbol{v}} - \boldsymbol{v}_{\rm lin}) \cdot \boldsymbol{v}_{\rm pred} \rangle + \langle \boldsymbol{\sigma} \cdot \boldsymbol{v}_{\rm pred} \rangle \ . \quad (10)$$

The first term is positive and causes an overestimate of $f(\Omega_m)$ for nearly all smoothing functions. Our calculation of this effect via equation (7) shows that it is zero for no smoothing and increases monotonically with smoothing scale. We have argued above that the second term is generally negative and causes an underestimate of $f(\Omega_m)$. Since this effect arises from nonlinearity in the density field, it should dominate on small scales and vanish with increased smoothing of the density field. Finally, the third term is equal to zero, because the thermal velocities have random directions. A combination of the first two terms of equation (10) explains the scale dependence of $f(\Omega_m)$ estimates in Figure 2. For large smoothing of the density field, the first term dominates and we overestimate $f(\Omega_m)$, whereas for small smoothing the second term dominates and we underestimate $f(\Omega_m)$. The estimate of $f(\Omega_m)$ is unbiased at the smoothing scale where these two effects cancel, but this scale should itself depend on the specifics of the underlying cosmological model. The numerical results in Figure 2 confirm this prediction: the $f(\Omega_m)$ estimate is unbiased at $R_s = 5\ h^{-1}$ Mpc in the $\Omega_m = 0.2$ model (with $\sigma_8 = 1.44$; *squares*) and at $R_s = 4\ h^{-1}$ Mpc in the $\Omega_m = 1$ model (with $\sigma_8 = 0.55$; *circles*). The smoothing scale for unbiased estimates could also depend on the assumed relation between galaxies and mass, a point we will investigate in future work. It is therefore not possible to remove this bias simply by choosing the right smoothing scale in a model-independent way.

If we had adopted a higher order perturbative expansion for predicting velocities from the smoothed density field, then equation (10) would still hold, with $\boldsymbol{v}_{\rm lin}$ replaced by $\boldsymbol{v}_{\rm per}$, the perturbative prediction in the absence of smoothing. The first term on the right-hand side would still be positive, since some Fourier modes would contribute to both $(\boldsymbol{v}_{\rm per} - \boldsymbol{v}_{\rm pred})$ and $\boldsymbol{v}_{\rm pred}$. The second term could be positive or negative, depending on the approximation and the smoothing scale. However, while a higher order approximation might reduce the magnitude of the second term relative to the linear approximation, it would not necessarily reduce the net bias in $f(\Omega_m)$, since this depends on the relative magnitude and sign of the first two terms.

3. DISCUSSION

The implications of our results for existing estimates of $f(\Omega_m)$ (or, more generally, of β) are probably limited. As already mentioned, density-density comparisons via POTENT are not influenced by the effects discussed here, because they compare density and velocity divergence fields smoothed at the same scale. The analysis of Davis et al. (1996), a mode-by-mode comparison of density and velocity fields, is also not affected, since the two fields are again compared at the same effective "smoothing". If the observed velocities are unsmoothed, a comparison in which velocities are predicted using a truncated spherical harmonic expansion of the density field (e.g., Blakeslee et al. 1999) may behave rather like our sharp $\boldsymbol{k}$-space filter analysis (Fig. 2, *open symbols*), since for a Gaussian field the different spherical harmonic components are statistically uncorrelated (A. Nusser 1999, private communication; Fisher et al. 1995). Among recent velocity-velocity studies, our procedure here is closest to the VELMOD analyses of Willick et al. (1997) and Willick & Strauss (1998), who used a 3 h^{-1} Mpc Gaussian filter to compute the predicted velocity field. These authors chose their smoothing scale partly on the basis of tests on N-body mock catalogs, and our results in Figure 2 suggest that biases in $f(\Omega_m)$ should indeed be small for this smoothing. It therefore appears unlikely that the effects discussed here can resolve the discrepancy between recent estimates of $f(\Omega_m)$ (or β) from velocity-velocity and density-density comparisons (e.g., Willick et al. 1997 versus Sigad et al. 1998). However, we have shown that the disappearance of the bias in $f(\Omega_m)$ at the 3 h^{-1} Mpc smoothing scale occurs because of a cancellation between positive and negative biases, and that the scale at which this cancellation occurs depends at least to some degree on the underlying cosmological model. As improvements in observational data reduce the statistical uncertainties in peculiar-velocity data, control of the systematic uncertainties that arise from comparing smoothed velocity predictions to unsmoothed data will become essential to obtaining robust estimates of the density parameter.

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