P H Y S I C A L   R E V I E W   L E T T E R S

Dipolar Bosons in a Planar Array of One-Dimensional Tubes

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We investigate bosonic atoms or molecules interacting via dipolar interactions in a planar array of one-dimensional tubes. We consider the situation in which the dipoles are oriented perpendicular to the tubes by an external field. We find various quantum phases reaching from a ''sliding Luttinger liquid'' phase to a two-dimensional charge density wave ordered phase. Two different kinds of charge density wave order occur: a stripe phase in which the bosons in different tubes are aligned and a checkerboard phase. We further point out how to distinguish the occurring phases experimentally.

Recent experiments in ultracold atoms have achieved the realization of various quantum phases. These phases reach from the superfluid and Mott insulator of bosons in optical lattices to a BCS phase and a Bose-Einstein condensate of molecules in fermionic gases [1]. In addition, it has been possible to engineer different trapping geometries, which enabled the study of the remarkable physics of one-dimensional quantum systems [2–4]. In such systems, known as Luttinger liquids [5], the interactions play a major role and lead to properties quite different from their higher-dimensional counterparts. This has been nicely demonstrated investigating the crossover between an array of decoupled one-dimensional tubes and a quasi-three-dimensional system by lowering the potential barrier between the tubes [2]. Since the interactions in most cold atomic gases have a short range of the order of a few nm and the distance between the tubes is typically 500 nm, the only coupling between the tubes is generally provided by the hopping of particles between tubes. For bosons this leads to a dimensional crossover between the peculiar one-dimensional phases and a quasi-three-dimensional super-fluid [6].

However, other fascinating phases may occur when one-dimensional tubes are directly coupled by interactions. These include, in particular, the so called ''sliding Luttinger liquid'' (SLL) [7–9] that was studied in connection with high-T_c superconductors and stripe physics [10] for fermionic systems. In the SLL phase the typical properties of one-dimensional systems, namely, algebraically decaying correlations, survive despite the coupling. Thus the Fermi liquid phase that usually occurs in more than one dimension is suppressed. Experimentally the SLL has not been observed yet.

In cold atomic gases these phases could not be explored so far due to the lack of interactions extending over the range of the intertube distances. However, the experimental progress in the realization of quantum degenerate atomic and molecular gases with dominating dipole-dipole interactions [11] have shown the potential to bridge that gap. Particularly promising are polar molecules with large

electric dipole moments [12–16], since the strength of the dipolar interaction is considerable over the range of tube spacings [17].

In this Letter we investigate the possible quantum phases of a dipolar bosonic gas in a planar array of one-dimensional homogeneous tubes (Fig. 1). This situation can be achieved experimentally by using a strong two-dimensional optical lattice. If sufficiently strong, this lattice suppresses the hopping of particles between different tubes. Even in this situation the dipole-dipole interaction couples the tubes. The dipole-dipole interaction is attractive or repulsive depending on the relative orientation of the dipoles. This anisotropy causes interesting phenomena such as an instability towards collapse [18,19]. Here we focus on the situation where the dipoles are aligned by an additional external field. The orientation is chosen perpendicular to the direction of the tubes, since this configuration is the most stable [20]. The orientation with respect to the plane of the array is varied.

Assuming vanishing hopping between the tubes, we find four different regimes: (i) a SLL with dominating superfluid correlations, (ii) a SLL with dominating charge density wave (CDW) correlations, (iii) a checkerboard or stripe CDW ordered phase (cf. Fig. 3 below), and
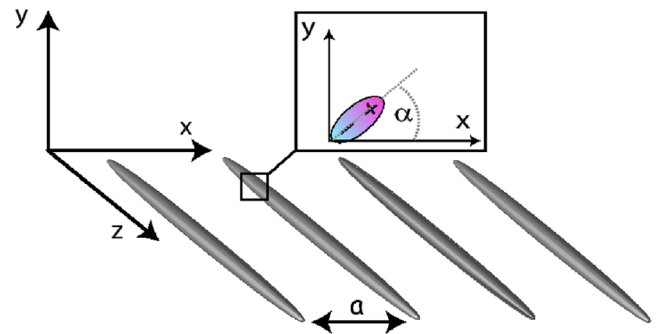

FIG. 1 (color online). A planar array of one-dimensional tubes. The dipoles point perpendicular to the tube direction. In the inset the angle α of the orientation of the dipoles with respect to the plane of the array is shown.

(iv) an instability towards collapse [22]. The finding of a SLL phase is novel in a bosonic system. The SLL has power-law correlation functions along the tubes that can be dominating superfluid correlations (i) or dominating CDW correlations (ii). The two regimes are connected by a crossover. We discuss how cold quantum gases could provide the opportunity to observe the SLL phase for the first time experimentally.

Interacting bosons in an array of one-dimensional tubes (cf. Fig. 1) can be described by the Hamiltonian

$$H = \sum_j \int dz \left(\frac{1}{2M} |\partial_z \hat{\Psi}_j(z)|^2 + \frac{g}{2} \hat{\rho}_j(z) \hat{\rho}_j(z) \right)$$
$$+ \sum_{j,j'} \int dz dz' V_d((j-j')a, 0, z-z') \hat{\rho}_j(z) \hat{\rho}_{j'}(z').$$
$$(1)$$

Here $\hat{\Psi}_j$ is the bosonic annihilation operator in tube j and $\hat{\rho}_j = \hat{\Psi}_j^\dagger \hat{\Psi}_j$ is the density operator. We use $\hbar = 1$. The tube distance is a, M is the mass of the particles, and g is the strength of the δ interaction between the particles.

The last term in Hamiltonian (1) stems from the dipole-dipole interaction. Its amplitude is $V_d(\mathbf{r}) = V_0[\hat{\mathbf{d}}^2 - 3(\hat{\mathbf{d}} \hat{\mathbf{r}})^2]/r^3$, where $\hat{\mathbf{d}} = (\cos\alpha, \sin\alpha, 0)$ denotes the direction of the dipole moment and $\hat{\mathbf{r}}$ is the unit vector. The interaction strength V_0 is given by $V_0 = d^2 \mu_0/(4\pi)$ for magnetic and $V_0 = d^2/(4\pi\epsilon_0)$ for electric dipoles. Here d is the strength of the dipole moment, and μ_0 (ϵ_0) is the vacuum permeability (permittivity).

The low-energy properties of the system can be described using the bosonization approach [5,7]. Two bosonic fields, $\hat{\phi}_j$ and $\hat{\theta}_j$, are introduced to describe the modes of the system, where $\hat{\phi}_j$ and $\hat{\theta}_j$ are related to the amplitude and phase of the operator $\hat{\Psi}_j$, respectively. The Hamiltonian in the absence of the dipolar interaction becomes

$$H_0 = \frac{u}{2\pi} \sum_j \int dz \left(K(\nabla \hat{\theta}_j(z))^2 + \frac{1}{K}(\nabla \hat{\phi}_j(z))^2 \right), \quad (2)$$

where the parameters u, the velocity, and K, the Luttinger parameter, depend on the microscopic details of the underlying system [5]. For weak interaction the relations $K \approx \pi\sqrt{\rho_0/Mg}$ and $u \approx \sqrt{g\rho_0/M}$ hold. Here ρ_0 is the average density of bosons.

The dipolar interaction can be expressed in the bosonization language using $\hat{\rho}_j(z) = \rho_0 - (1/\pi)\nabla \hat{\phi}_j(z) + \rho_0 \sum_{p\neq 0} e^{i2p[\pi\rho_0 z - \hat{\phi}_j(z)]}$, where p is an integer. The smooth part of the density $\sim \nabla \hat{\phi}_j$ yields a nonlocal quadratic term in the Hamiltonian, whereas the first harmonic $|p| = 1$ introduces backscattering terms. Neglecting higher harmonics, $|p| > 1$, the contribution to the Hamiltonian of the dipolar interaction thus becomes

$$H_d = \frac{1}{\pi^2} \sum_{j,n} \int dz dz' V_d(na, 0, z-z')$$
$$\times [\nabla \hat{\phi}_j(z) \nabla \hat{\phi}_{j+n}(z') + \hat{O}_{j,n}^b(z, z')], \quad (3)$$

where $\hat{O}_{j,n}^b(z, z') \propto \cos[2\hat{\phi}_j(z) - 2\hat{\phi}_{j+n}(z')]$ are the backward scattering operators.

Assuming that the average particle spacing is large compared to the size of the dipoles, the detailed structure of the interaction at short distances can be neglected [23,24], and any possible further contribution to the interaction on a length scale below a cutoff $1/\rho_0$ can be absorbed into the values of u and K. By this definition K can now range from very large values for a weakly interacting gas without dipolar interactions to values smaller than 1 in the presence of finite dipolar interactions as shown in Ref. [25] for a purely dipolar one-dimensional system. Reaching values of K smaller than 1 in a bosonic system is a characteristic of nonlocal interactions [5]. In the following we use the dimensionless ratio of the dipolar interactions and the s-wave scattering given by $\gamma = 4V_0 K/(a^2 u)$.

In order to determine the quantum phases occurring in the system, we investigate the relevance of the superfluid and CDW operators along and perpendicular to the tubes. To find the dominant correlations, we first consider the effects of the quadratic part of $H_0 + H_d$ by comparing the relevance of the density-density correlations $\langle \hat{\rho}_j(z) \hat{\rho}_j(0) \rangle \propto (1/z)^{2KG_{\phi,\parallel}}$ and the single particle correlations $\langle \hat{\Psi}_j^\dagger(z) \hat{\Psi}_j(0) \rangle \propto (1/z)^{G_{\theta,\parallel}/(2K)}$. The algebraic decay of both correlations is typical for a Luttinger liquid [5]. We find that the exponents depend on the strength of the dipolar interaction and the orientation of the dipoles through $G_{\phi,\parallel}(\gamma, \alpha) = 1/(2\pi)\sum_{q_\perp}[1 + \gamma\tilde{V}(q_\perp, 0)]^{-1/2}$ and $G_{\theta,\parallel}(\gamma, \alpha) = 1/(2\pi)\sum_{q_\perp}[1 + \gamma\tilde{V}(q_\perp, 0)]^{1/2}$. Here $\tilde{V}(q_\perp, q_\parallel = 0) = 2(a\rho_0)^2 - 4\cos(2\alpha)\text{Re}[\text{Li}_2(e^{iaq_\perp})]$, where $\tilde{V}(q_\perp, q_\parallel) = V_d(q_\perp, q_\parallel)/(4\pi V_0)$ denotes the dimensionless interaction strength, Li_2 the polylogarithm, and Re the real part. $q_\perp$ and $q_\parallel$ are the components of the momentum perpendicular and parallel to the tubes, respectively. To perform the calculations, a discrete Fourier transform is taken in the x direction perpendicular to the tubes, and a continuous Fourier transform along the tubes. We further approximate the momentum dependence of the dipolar interaction along the tubes by the low-momentum value at $q_\parallel = 0$, which yields the main contribution to the long-distance behavior of the correlation functions. This corresponds to using an effective interaction that is local along the tube and whose strength is given by the dipolar interaction integrated over the tube [26].

Comparing the obtained exponents, the CDW correlations in the tube dominate at long distances if

$$2KG_{\phi,\parallel} - \frac{1}{2K}G_{\theta,\parallel} < 0.$$

For decoupled tubes the functions reduce to $G_{\phi,\parallel} = 1$ and

$G_{\theta,\parallel} = 1$. In this case one recovers the crossover for a single tube [5,25] between a region in which the superfluid correlations are dominant for $K > 1/2$ and a region in which the CDW correlations are dominant for $K < 1/2$ (cf. $\gamma = 0$ line in Fig. 2).

For coupled tubes the intertube operators can become relevant and change the nature of the occurring quantum phases. If the prefactors of the operators $\hat{O}_{j,n}^{b}$ are small, their relevance can be determined by looking at their scaling dimension. The scaling dimension of the operators $\hat{O}_{j,n}^{b}$ is $2KG_{\phi,\perp,n}$ using $G_{\phi,\perp,n} = 1/(2\pi)\sum_{q_\perp}[1 + \gamma\tilde{V}(q_\perp,0)]^{-1/2}[1 - \cos(naq_\perp)]$. Therefore a backscattering operator between tubes at distance na becomes relevant and can induce charge ordering, if

$$2KG_{\phi,\perp,n} - 2 < 0.$$

In Fig. 2 the phase diagram depending on γ and K is shown for two different orientations of the dipoles [27]. Experimentally K can be varied by adjusting the short-range interaction between the particles. Varying the dipolar interaction strength changes both γ and K. In Fig. 2(a) the dipoles are pointing along the direction of the array ($\alpha = 0$) leading to an attractive intertube interaction at short distances. By contrast in Fig. 2(b) the dipoles are oriented perpendicular to the plane of the array ($\alpha = \pi/2$) resulting in a purely repulsive intertube interaction. The phase boundaries are shown for the case where the spacing between tubes equals the average particle distance, i.e., $a\rho_0 = 1$ [28].

For $\gamma \ll 1$ the behavior for the different orientations of the dipoles is very similar: For $K \gtrsim 1$ a SLL with dominant superfluid correlations [regime (i)] occurs. In contrast to an array of independent tubes, the presence of the dipolar interaction leads to a coupling of the densities in different tubes on long length scales. In the bosonization language the coupling is given by the forward scattering terms, i.e., in Hamiltonian (3) the terms that contain $\nabla\hat{\phi}_j(z)\nabla\hat{\phi}_{j'}(z')$ with $j \neq j'$.

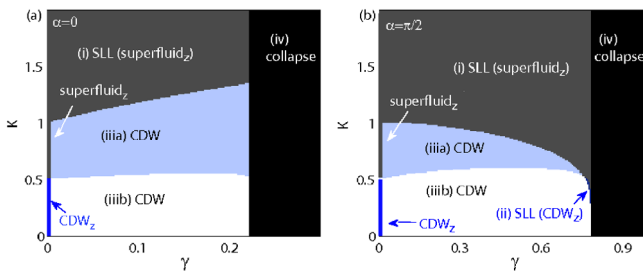The CDW correlations along the tubes become relevant for values $K \lesssim 1/2$. However, the CDW correlations perpendicular to the tubes become relevant already for larger values $K \lesssim 1$. In the regime $1/2 \lesssim K \lesssim 1$ [regime (iiia)], a recalculation of the intratube correlations taking into account the ordering in the perpendicular direction then leads to a CDW ordered phase in all directions. In this case, the order in the tube will be weaker than the order in the direction perpendicular to the tubes. In contrast, for $K \lesssim 1/2$, the order along the tubes is approximately as strong as the order perpendicular to the tubes, since the CDW ordering both along and perpendicular to the tubes is relevant [regime (iiib)].

Above a critical value of γ an instability towards collapse [regime (iv)] occurs. The instability occurs if there exists a $q_\perp$ for which $1 + \gamma\tilde{V}(q_\perp) < 0$. The physics of the instability is different for the shown cases $\alpha = 0$ and $\alpha = \pi/2$. For $\alpha = 0$ the attractive intertube interaction overcomes the repulsive intratube interation and causes the collapse of the particles inside the tubes and an alignment of these perpendicular to the tubes. However, the instability for $\alpha = \pi/2$ stems from a strong repulsive intertube interaction that dominates over the intratube interaction. This can occur, if the tubes are very close and the contact interaction is attractive. Here the particles collapse inside a tube in order to avoid the interaction with the particles in the neighboring tubes.

For intermediate values of γ the behavior depends on the orientation of the dipoles. For $\alpha = 0$ the CDW order can already be reached for values of $K > 1$; i.e., the dipolar interaction enhances the tendency of the system to order. By contrast for $\alpha = \pi/2$, a larger value of γ can destabilize the CDW ordering. Thereby a transition between a SLL and a CDW ordered phase seems to be possible simply by varying the orientation of the dipoles. Further for $\alpha = \pi/2$ a small region of SLL with dominating CDW order [regime (ii)] can be seen in Fig. 2(b) at large γ. Whether this survives for realistic experimental parameters is an open question.

In phase (iii), the form of the CDW order that occurs depends on the direction of the dipoles (Fig. 3). In particular, if the dipoles lie in the plane of the tubes, the interaction between tubes is attractive at short distances and therefore the CDW order of different tubes is aligned [29]. By contrast, if the dipoles are perpendicular to the plane, the interaction between tubes is repulsive and a
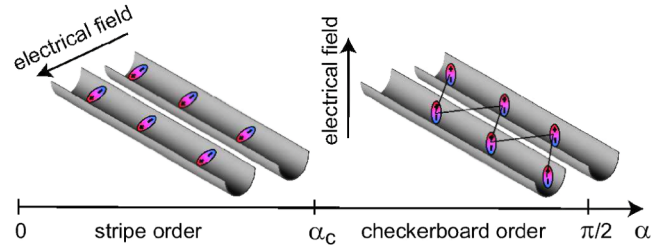

FIG. 2 (color online). Different quantum phases occurring for the directions (a) $\alpha = 0$ and (b) $\alpha = \pi/2$. Experimentally K can be changed varying the s-wave scattering length of the particles. Further tuning the dipolar interaction would correspond to changing both γ and K. The subscript "z" denotes the dominant correlations along the tube.

FIG. 3 (color online). A sketch of the transition between a stripe and checkerboard CDW order depending on the orientation of the dipoles is shown.

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checkerboard ordering is expected. To determine which form of CDW order the system takes for a given angle α, we perform a minimization of the energy of the system assuming CDW order along the tubes; i.e., we take the density in tube j to be of the form $\hat{\rho}_j(z) = \rho_0[1 + \cos(2\pi\rho_0 z - \bar{\phi}_j)]$. Here $\bar{\phi}_j$ describes the average phase in tube j. With this ansatz (and using $\delta z = z - z'$) the expression to be minimized reduces to

$$\sum_{j \neq j'} \int d\delta z V_d((j - j')a, 0, \delta z) \cos[2\pi\rho_0\delta z - 2(\bar{\phi}_j - \bar{\phi}_{j'})].$$

As expected, we find a transition between the stripe order for small α and the checkerboard order for α close to $\pi/2$ (see Fig. 3). Assuming only coupling between nearest-neighbor tubes, the transition takes place at $\cos^2(\alpha_c) = K_1(2\pi a \rho_0)/(2\pi a \rho_0 K_2(2\pi a \rho_0))$, where K_i are modified Bessel functions. At the transition point the tubes experience only a weak coupling to other tubes, and the correlations in the tube are superfluid or CDW dominated depending on the parameter regime. Taking the full dipolar interaction into account, we determine the transition point numerically [30]. The result can hardly be distinguished from the transition point found for nearest-neighbor coupling only.

The precise setup to observe the quantum phases experimentally depends on the realization of the dipolar particles. However, here we describe some of the basic characteristics of the phases that could be detected. The coupling of the tubes in the SLL phase can distinguish it from the Luttinger phase of decoupled tubes. This coupling could, e.g., be detected by exciting the dipole mode for part of the tubes and detecting the induced center of mass momentum of the remaining tubes. Further, the frequency of the dipolar mode can give information on the state inside the tubes [31]. The stripe and checkerboard orders show characteristic density-density correlations that could be detected measuring the noise-correlation spectrum of time-of-flight images [32]. Before performing the time-of-flight measurement, molecules could be dissociated while freezing their position by an additional strong optical lattice.

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