A DYNAMIC PROGRAMMING ANALYSIS OF A VARIABLE AMORTIZATION LOAN PLAN

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Returns on agricultural assets are risky. Although returns vary, obligations on debt instruments traditionally have remained constant. Because loan obligations are not positively correlated with returns to assets, constant debt obligations tend to destabilize the farm's financial position. Furthermore, a firm's financial position may be enhanced when loan terms allow for a positive correlation between returns to assets and debt obligations. Various "innovative" loan instruments which allow for this positive correlation have been advanced (see Lee and Baker for a review).

The purpose of this paper is to analyze the performance of an innovative prototype variable amortization loan repayment plan. This plan has been proposed by Baker (1976, 1986) and is unique in that it allows for a debt reserve, which provides liquidity in periods of adverse incomes. The variable amortization loan plan's performance is compared to a straight amortization loan and a flexible amortization loan. Each of these alternative loan plans is delineated in the following section.

To compare the three plans, dynamic programming models are applied. These models examine an investment in a hog finishing operation under differing loan repayment plans and levels, and consider conflicting objectives of borrowers and lenders. The models establish two rules: one for withdrawing funds from the operation, the second fixing the yearly principal repayments. Criteria for evaluating alternative loan plans include the future value of expected withdrawals from the hog finishing operation, the probability of suffering a cash shortfall, and the expected amount of debt outstanding at the end of the loan term.

Alternative Loan Repayment Plans

The traditional amortization loan is the most common loan repayment plan. Under this program, the total periodic loan obligation remains constant over the loan's life. Thus, over time, the principal and interest portions of the total payment vary, with the interest payment constituting a larger portion of the total payment in earlier years. The advantage of this plan is simplicity. A disadvantage is that the loan obligation does not vary with returns to assets.

Another loan repayment plan is a flexible amortization plan. This plan is offered by the Farm Credit System, and is the fundamental equivalent to a plan proposed by Lee. Total debt obligations are calculated in the same way as the amortization loan. The principal must be reduced annually to align with the amortization schedule. In any given year, farmers can make payments to further reduce the principal portion of the loan, thus reducing future interest payments. Prepayments of this nature made during times of financial well-being allow for a skipped payment in the future if need be. Prepayments are beneficial even if farmers choose not to take advantage of "skipping" a future payment, for the reduction of the principal portion results in a shorter loan life, and fewer interest payments. This flexibility is a primary advantage of the flexible amortization plan over the traditional amortization plan. Thus, the plan provides some flexibility in matching returns from assets to loan repayments. However, if principal has not been prepaid, the plan does not allow for lower debt repayments in adverse income years.

An alternative to the flexible amortization plan is Baker's variable amortization plan (VAP). This plan includes two key elements. First, a debt reserve is added to the total loan obligation. Equaling a certain percentage of the total loan, this debt reserve can be drawn on in adverse income years. For example, a 10 percent debt reserve could be added to the loan, which means that a $4,000 reserve exists on a $40,000 loan.

The second component is scheduling of loan repayments. According to Baker, loan debt obligations are determined by an amortization schedule based on the loan plus the debt reserve. The loan obligation would be allowed to "flex" based on returns to assets; thus repayments could be regulated based on an index of revenues and costs (see Baker [1986] for an example and further description). An advantage of the VAP is that it allows loan repayments to be positively correlated to return on assets. Furthermore, it provides greater flexibility over the flexible amortization plan.

Evaluating Alternative Loan Repayment Plans

Evaluation of the alternative repayment plans considers the borrower's (i.e., farmer's) and lender's perspectives. Borrowers prefer plans with lower interest costs, allowing faster withdrawals from a debt-financed investment. In the case of a farm, withdrawals are applied toward firm growth, investment in off-farm assets, and family living expenses. An objective which incorporates this perspective is the maximization of the present values of withdrawals.

Lenders, on the other hand, opt for plans with the highest probability of meeting loan obligations. In addition, lenders prefer that borrowers maintain liquid funds, such as in a savings account, which can be drawn on to meet loan obligations in adverse income years. An objective which minimizes the discounted present value of cash shortfalls can fulfill this perspective[1].

The Stochastic Dynamic Programming Models

Stochastic dynamic programming (DP) models are solved to evaluate the three alternative loan repayment plans. Results from the models are generated from three distinct perspectives: the borrower's, the lender's, and one of a borrower/lender. The DP models are of a hog facility investment which costs $80,000, and finishes 2000 hogs per year. This investment is financed with a debt of either $20,000 or $40,000. In either case, the loan spans ten years, with obligations due at year-end.

The DP models test the self-liquidating abilities of the alternative loan plan. Because of the models' evaluative nature, funds from other enterprises are not considered. Such a focus ignores possible cash flows from other farm enterprises which may provide funds to repay loan obligations. However, since the flexibility of loan plans is being evaluated, this perspective is appropriate.

Four DP models representing alternative loan repayment plans are solved. The first, which is entitled "BASIC DP," examines the firm with no debt financing. Results are applied as benchmarks for the other loan plans. The remaining three DP models are modifications of the BASIC DP model and incorporate the traditional amortization, flexible amortization, and variable

[1]An objective of minimizing the probability of cash shortfalls also is appropriate. The dynamic programming models were solved using this objective. Similar results were obtained.

amortization loans. Each of the four models are described in the following sub-sections.

The Basic Dynamic Programming Model

Stages of the BASIC DP model are monthly periods. The BASIC DP model contains three state variables: per hog direct returns, taxable income, and debt/saving balance. Per hog direct returns are stochastic while the remaining state variables are deterministic. Per hog direct returns equal gross revenue (from the sale of a 220 pound hog) less variable costs of raising a hog (from 40 pounds to 220 pounds). Taxable income accumulates throughout a year and accounts for annual income tax obligations. The debt/saving balance incorporates liquidity and additional debt requirements into the model. A positive balance results when funds generated from the hog operation have not been withdrawn. Positive balances can be used to counter cash shortfalls in future years. Negative amounts indicate that additional debt has been required to cover adverse hog return outcomes. The decision variable is the amount of funds to withdraw at the end of the year. This withdrawal can be used in a variety of ways, such as to increase the size of the farming operation, make off-farm investments, or cover family living expenses.

The BASIC DP model can be written as follows:

$$V_t(HR_t,\ TI_t,\ B_t) = \max_{W_t} E\{R_t(HR_t,\ TI_t,\ B_t) + \beta * V_{t+1}(HR_{t+1},\ TI_{t+1},\ B_{t+1})\} \qquad \text{(1-a)}$$

subject to:

(1-b) $HR_{t+1} = f_1(HR_t)$

(1-c) $TI_{t+1} = f_2(TI_t, HR_t)$

(1-d) $B_{t+1} = f_3(B_t, TI_t, HR_t)$

where:

$V_t(\cdot)$ is the recursive objective function;

HR_t is the per hog direct return state variable;

TI_t is the taxable income state variable;

B_t is the debt/saving balance state variable;

$E\{\cdot\}$ is an expectations operator;

$R_t(\cdot)$ is the current returns function;

β is the discount factor;

W_t is the amount of withdrawals;

$f_1(\cdot)$ is the stochastic state transition equation for per hog direct returns;

$f_2(\cdot)$ is the taxable income state transition equation, and

$f_3(\cdot)$ is the debt/saving balance state transition equation.

The Current Returns Function. The current returns function within the recursive objective function given in (1-a) equals:

(2) $R_t(HR_t, TI_t, B_t) = \lambda * W_t - (1 - \lambda) * AD_t$

where λ is a parameter which ranges between 0 and 1 and AD_t equals the amount of additional debt required at the end of the year. Specifically AD_t equals:

$AD_t =$

$$
\begin{cases}
-B_{t+1} + B_t & \text{when } B_{t+1} \text{ and } B_t \text{ are less than zero,} \quad (3\text{-a}) \\
-B_{t+1} & \text{when } B_{t+1} \text{ is less than zero and } B_t \text{ is greater than or equal to zero,} \quad (3\text{-b}) \\
0 & \text{when } B_{t+1} \text{ is greater than or equal to zero.} \quad (3\text{-c})
\end{cases}
$$

Equation (3-a) gives additional debt capital requirements when debt capital has been needed in previous years, as indicated by a negative debt/saving balance (i.e., B_t is less than zero). Equation (3-b) gives additional debt capital requirements when debt capital has not been accumulated previously (positive B_t). Equation (3-c) indicates that additional debt is not required when the debt/saving balance is positive.

The parameter λ in the current returns function (Equation 2) encompasses differences in the objectives of farmers (i.e., borrowers) and lenders. A λ value of 1 represents the farmer's objective and indicates that all weight is placed on the farmer's perspective, where the present value of withdrawals is maximized (presuming risk neutrality). When, on the other hand, λ equals 0 all weight is placed on the lender's perspective, and no additional debt is incurred. A λ between 0 and 1 provides for varying weightings between the farmer's and lender's perspective.

Withdrawals at the end of the year (W_t) cannot occur if the resulting debt/saving balance (B_{t+1}) is negative. This restriction prevents withdrawals from the asset base by using debt capital.

The discount rate equals 13 percent, or the average percentage return from the hog investment facility. Since the discount rate is higher than the

5 percent return on saving, an incentive exists to withdraw funds from the operation. However, these funds are presumed to be unavailable for use in meeting future loan obligations. Not withdrawing funds produces a return, as is described in the subsection on debt/saving balance state transition equation.

Hog Returns State Transition Equation. The stochastic, per hog direct return state transition is estimated using monthly data from the Livestock and Meat Situation and Outlook (U.S.D.A.). These returns are adjusted to reflect mid-west conditions.

Evaluation of various time-series models suggest that an AR(2) model adequately captures the series' time dependent nature. Resulting parameter estimates and t-statistics (in parentheses) are:

$$\text{(5-a)} \quad HR_{t+1} = \underset{(3.60)}{2.430} + \underset{(16.54)}{1.177}*HR_t - \underset{(-6.29)}{.4393}*HR_{t-1}$$

This equation has an adjusted R-square of .7360 and a standard error estimate of 7.239. Residuals show no sign of auto-correlation and the hypothesis of non-normally distributed residuals is rejected using the Jarque-Bera test statistic.

To reduce the dimensions of the DP models, a single direct return variable is included. Burt and Taylor's method of reducing an auto-regressive process results in:

$$\text{(5-b)} \quad HR_{t+1} = 1.6888 + .8177*HR_t$$

The reduced form has a standard error estimate of 6.6279.

Taxable Income State Transition Equation. The deterministic, taxable income state transition equation accumulates taxable income during a year. This transition equation can be expressed as:

$$TI_{t+1} =$$

$$\begin{cases} 0 & \text{for month twelve} \quad (6\text{-a}) \\ TI_t + HR_t*167 - FC/12 + i_1(TI_t)*TI_t & \text{otherwise} \quad (6\text{-b}) \end{cases}$$

where FC equals fixed costs, and $i_1(\cdot)$ is a function producing an interest rate. Equation (6-a) indicates that taxable income will equal zero at the beginning of each year. Ending year taxable income is withdrawn from the hog operation or flows into the debt/saving balance. Equation (6-b) gives taxable income changes between months during a year.

During a year, taxable income increases due to recognition of revenues and costs from hog sales, HR_t*167. The 2,000 hogs marketed during a year are presumed to move evenly through the hog facility. Thus, per hog direct returns are multiplied by 167, resulting in monthly revenues and variable costs. Fixed costs (FC) are presumed to occur evenly throughout the year, and are divided by 12 to arrive at monthly fixed costs. This amount (FC/12) reduces taxable income each month. Total fixed costs are adapted from Ohio Livestock Enterprise Budgets, 1987 and equal $4,000 per year.

The final term of (6-b), $i_1(TI_t)*TI_t$, gives returns on financial holdings or costs on operating debt. Positive taxable incomes from previous months are presumed to be placed in a saving account yielding a 5 percent return. Thus, $i_1(\cdot)$ equals .05 when TI_t is positive. Negative taxable incomes are presumed to be covered by an operating loan. The interest rate on the operating loan equals 11 percent.

__Debt/Saving Balance State Transition Equation.__ The deterministic, debt/saving balance state transition equation given in (1-d) can be rewritten as:

$$B_{t+1} =$$

$$\begin{cases} B_t & \text{for all months other than month twelve} \quad (7\text{-a}) \\ B_t + TAX\{TI_t + HR_t*167 - FC/12 + i_1(TI_t)*TI_t + i_2(B_t)*B_t\} - W_t & \text{for month twelve} \quad (7\text{-b}) \end{cases}$$

where $TAX\{\cdot\}$ is a function giving after-tax income and $i_2(\cdot)$ is a function giving the interest rate on the debt/saving balance. Equation (7-a) indicates that the debt/saving balance is static throughout the year. Equation (7-b) gives the debt/saving balance change at the end of the year.

At the end of the year, the debt/saving balance equals the previous debt/saving balance plus taxable income during the year, $TAX\{\cdot\}$, less withdrawals, W_t. Terms within $TAX\{\cdot\}$ give before-tax income, which includes:

1) accumulated taxable income from the previous eleven months, TI_t,
2) taxable income during month twelve, $HR_t*167 - FC/12 + i_1(TI_t)*TI_t$,
3) returns or costs on the debt/saving balance, $i_2(B_t)*B_t$. The function $i_2(\cdot)$ gives the yearly interest rate on the debt/saving balance. For positive balances, a 5 percent rate of return is applied because this is the average savings rate obtainable at commercial banks. For negative balances, a 10.5 percent interest rate to determines interest costs.

This percent is the average interest rate on intermediate term debt over the last 10 years.

Taxes are calculated based on the 1988 tax code. These taxes include two federal brackets, a 12 percent social security tax rate, and 5 Ohio tax brackets.

Note that the final term in (7-b) is withdrawals (W_t). As it is the decision variable, it reduces the debt/saving balance. The implicitly assumed return on withdrawals is the discount rate of 13 percent. In the case of positive balances, the decision variable's tradeoff is between 13 and 5 percent return on debt/saving balance. However, if withdrawals occur, these funds will not be available to repay debt. On the other hand, a positive debt/saving balance can be drawn on to repay debt. Thus, the debt/saving balance represents a liquid asset while withdrawals represent a non-liquid asset.

The Amortization Dynamic Programming Model (AMOR DP) Model

The AMOR DP model includes a traditional amortization loan which is used to finance the acquisition of the hog facility. The loan has a ten year period and has constant, yearly principal and interest payments. Two loan sizes, \$20,000 and \$40,000, are used. Yearly principal ($PRIN_t$) and interest payment (INT_t) for the two loan sizes are shown respectively in Panels A and B of Table 1.

Inclusion of the amortization loan requires no additional state variables. The debt/saving balance state transition equation, however, has to be modified to account for the principal and interest payments. Specifically, the debt/saving balance state transition equation is:

$$B_{t+1} =$$

$$\begin{cases} B_t & \text{for all months other than month twelve} \quad (8\text{-a}) \\ B_t + TAX\{TI_t + HR_t*167 - FC/12 + i_1(TI_t)*TI_t + i_2(B_t)*B_t - INT_t\} - PRIN_t - W_t & \text{for month twelve} \quad (8\text{-b}) \end{cases}$$

where INT_t equals the yearly interest payment and $PRIN_t$ equals the yearly principal payment. The loan is presumed to be paid during the first ten years. During these years, the debt/saving state transition equation varies due to the differing principal and interest payments.

The Flexible Amortization Dynamic Programing (FLEX DP) Model

The FLEX DP model modifies the terms of the previously described amortization loan. Under the flexible amortization loan terms, the principal portion of the payment can be prepaid. The specification of the FLEX DP model requires an additional state variable which gives the principal outstanding on the amortization loan (P_t). Also, an additional decision variable is required. This decision variable is the amount of principal paid each year (PP_t). The recursive equation for the model then becomes:

$$V_t(HR_t,\ TI_t,\ B_t,\ P_t) = \max_{W_t, PP_t} E\{R_t(HR_t,\ TI_t,\ B_t,\ P_t) + \beta * V_{t+1}(HR_{t+1},\ TI_{t+1},\ B_{t+1},\ P_{t+1})\} \quad (9)$$

The decision variables are restricted such that unscheduled principal payments and withdrawals do not result in a negative debt/saving balance.

The debt/saving balance state transition equation also has to be modified:

B_{t+1} =

(10-a) $\begin{cases} B_t & \text{for all months other than month twelve} \end{cases}$

(10-b) B_t + $TAX\{TI_t + HR_t*167 - FC/12 + i_1(TI_t)*TI_t + i_2(B_t)*B_t - .105*P_t\} - PP_t - W_t$ for month twelve

Furthermore, an additional state transition equation must be added to the model to give principal balance changes on the amortization loan. This state transition equation equals:

(11) $P_{t+1} = P_t - PP_t$

Note that the principal outstanding depends on the PP_t decision variable.

The Variable Amortization Dynamic Programming (VAP DP) Model

The VAP DP model incorporates a debt reserve within the amortization loan. The debt reserve is presumed to equal 10 percent of the outstanding principal balance. This addition does not require modification of the FLEX AMOR DP model's state transition equations, but the size of the amortization loan and the range on the debt repayment variable (PP_t) does have to be modified. Under the flexible amortization loan plan, the largest amortization principal balance is \$20,000 or \$40,000. Under the variable amortization loan, these sizes are increased by 10 percent to match the presumed debt reserve. The range on the decision variables allows use of the debt reserve. Therefore, at the end of a given year, it is possible to borrow an additional 10 percent of the outstanding balance to cover adverse income outcomes.

Solving the Dynamic Programming Models

To numerically solve the programming models, all state variables ranges must be divided into discrete states. Per hog direct returns have six states ranging in equal increments from -$20 to $40. Taxable incomes range from -$60,000 to $60,000 in $15,000 increments, resulting in nine states. The debt/saving balances range from -$400,000 to $400,000 in $10,000 increments, resulting in 81 states. The total yield from the BASIC and AMOR DP models is 4,374 states. In addition to these, the FLEX AMOR and VAR AMOR DP models have 11 additional states associated with the principal outstanding state variable. These states are divided to represent a principal payment under the amortization plan. The second column of panels A and B respectively show the states for the $20,000 and $40,000 loan for the FLEX DP model. These additional states result in a total of 48,114 states for the FLEX and VAP DP models.

All models are recursively solved beginning at the final period. Linear interpolation of the objective function is implemented for both the taxable income and debt/saving balance state variables in order to reduce biases resulting from discrete states and to increase the convergence rate. The BASIC DP program is recursively solved for five years, at which point the optimal decision rules converge. During the loan period, the remaining models do not converge because of the variance of the debt/saving balance state transition equation over the years of the amortization loans. To account for this non-convergence, the remaining models are solved for a fifteen year period. During the first five years (the final five years of the time frame) decision rules are generated. This results in a converged decision rule similar to that from the BASIC DP model. Finally, decision rules for the ten year loan repayment period are generated.

When evaluating the alternative loan repayment plans, state variable distributions that result from following the optimal decision rule are more useful than the decision rules. Therefore, optimal decision rules and the state transition equations are used to construct future state variable probability distributions following conditional probability methods (see Howard for a discussion). By using these methods, discrete joint probability density functions of the per hog direct returns, taxable income, and debt/saving balance can be ex ante forecasted, conditional on initial state variable levels and presuming that the optimal decision rules are followed. From the joint probability density function, the discrete marginal distributions of a single state variable can be found. These marginal distributions then can be applied to calculate expected values (see Schnitkey, or Novak and Schnitkey for a more detailed discussion of these methods).

Yearly conditional probability distributions have been computed using the same state variable discretation as those in the dynamic programming models. Unless noted otherwise, all conditional probabilities are calculated using initial state levels of a per hog direct return value of $10, a $0 taxable income, and a $0 debt/saving balance. The alternative loan repayment plans are evaluated using these criteria: (1) the yearly expected values of withdrawals, (2) the yearly expected values of the debt/saving balance, (3) the probabilities of having additional debt (i.e., having a negative debt/saving balances), and (4) the marginal debt/saving balance distributions at the end of the amortization loan (in year ten).

Results

The dynamic programming analysis results are delineated in the following four sub-sections. The first sub-section presents results for the BASIC DP model with λ values of 1, .33, and 0. The remaining three sub-sections give results for λ values of 1, .33, and 0 across the differing debt levels and loan repayment plans.

Results from the BASIC DP Model

Results from the BASIC DP model are of interest for two reasons. First, they provide an understanding of the dynamic factors at work. Second, given similar λ values, each of the remaining models' decision rules are the same as the BASIC DP models' decision rules after the ten year amortization loans have expired.

Converged optimal decision rules from the BASIC DP models can be described based on the debt/saving balance that is maintained. These balances vary depending on the level of λ. For a λ value of 1, which maximizes the present value of withdrawals (i.e, the farmer's perspective), withdrawals occur whenever positive debt/saving balances exist, and equal the amount of the positive debt/saving balances. For a λ value of .33, withdrawals occur on a schedule that builds a $30,000 saving balance, then maintains it at that level. Any funds which would result in debt/saving balances above $30,000 are withdrawn. For a λ of 0 (the lender's perspective) a positive debt/saving balance of $350,000 is built. Once debt/saving balances reach this level, withdrawals are possible.

Expected yearly withdrawals and debt/savings balances respectively are shown in Panels A and B of Figure 1 for the alternative λ values. In early years, withdrawals are higher for higher λ values because smaller savings balances are augmented to counter adverse outcomes. In later years, however, expected withdrawals are higher for lower λ values. This occurs because lower λ values have built saving balances that generate 5 percent returns. This produces greater taxable income, allowing larger withdrawals.

Note that consumption withdrawals trend downward from year two onward for λ =1 value and from year five onward for a λ =.33 value. This occurs because an increasing probability is in the debt region of the debt/saving balance. Transition matrices resulting from the BASIC DP model are not ergodic (see Howard for a discussion). A trapping state exists at approximately a -\$350,000 debt/saving balance. At this debt level, interest costs exceed taxable income for any level of per hog direct returns. Thus, debt is continually accumulated, creating a state equivalent to bankruptcy. This explains the downward trends in withdrawals (Panel A of Figure 1) and debt/saving balance (Panel B) for the two models.

The rate at which withdrawals and debt levels decline depends on the initial debt level. This is illustrated in Figure 2 which shows expected withdrawals and debt/saving levels for a λ value of .33 and beginning debt/saving balances of \$0 and \$40,000. The initial per hog direct return is \$0, with the initial taxable income at \$0.

The BASIC DP model with a λ =0 has two trapping states: one at the -\$350,000 debt/saving balance (i.e., bankruptcy) and the other at a \$350,000 debt/saving balance. Once a \$350,000 debt/saving balance has been reached, no probability exists of falling into debt. Thus, the firm never goes bankrupt. For beginning state intervals of a \$10 per hog direct return, a \$0 taxable income, and a \$0 debt/saving balance, the convergent probability of bankruptcy equals approximately 3 percent.

Results for a 1.0 λ Value

Since the decision rules from each model vary over the years of the loan, the optimal decision rules are not presented. Instead conditional probabilities for beginning state levels of a \$10 per hog direct return, a \$0 taxable income, and a \$0 debt/saving balance are shown. Given that the problems are calculated using the optimal decision rules, it assumes a basic understanding of the rules. In addition, conditional probabilities allow the performance of each loan repayment plan to be analyzed.

Figure 3 shows the yearly expected withdrawals from the AMOR and VAP DP models (results from the FLEX DP model are not presented because they are the same as the AMOR model results). This indicates λ levels of 1.0 do not result in any prepayment of principal. Panel A of Figure 3 shows expected withdrawals for a \$20,000 beginning amortization debt level while Panel B gives expected withdrawals for \$40,000.

In all years up to year ten, the VAP model results in higher expected withdrawals due to its flexibility. At the same time, expected debt/saving balances indicate that lower debt results from the variable amortization loan (Figure 4). In year seven, for example, approximately \$20,000 of debt is required for the amortization loan (given a beginning debt level of \$20,000). Additional debt of \$14,000 is needed for the variable amortization loan.

Higher debt levels under the amortization loan are primarily due to the higher probability of having additional debt, as shown in Figure 5. The yearly probabilities in Figure 5 give the probabilities of having negative debt/saving balances. Negative debt/saving balances result from cash shortfalls. The terms of the variable amortization loan allow for more of the cash shortfalls to be covered by the amortization loan, thus lowering probabilities.

Although the variable amortization loan results in lower probability of debt in years one through nine, it does not necessarily lead to lower probabilities in year ten, the final year of the amortization loan. This is illustrated in Table 2 which contains two panels giving results for the $20,000 and the $40,000 loans. For each loan size and the three loan repayment plans, the expected ending debt/saving balance, expected future value of withdrawals, and expected debt/saving balance plus withdrawals is given. Expected future value of withdrawals equal the withdrawals from years one through ten compounded to year ten. The expected debt/saving plus withdrawals can be used to evaluate the profitability of the loan repayment plans to the borrower. In addition, the debt/saving distribution in year ten and the probability of having debt is given in each panel.

The VAP results in significantly higher expected debt/saving balances plus withdrawals than do the other two loans. As stated before, however, the probability of having additional debt is higher under the variable amortization loan. For the $40,000 beginning amortization loan the probability of having debt is .1266 higher. At the same time, the VAP results in lower probability for debt/saving balances less than -$105,000. Approximately a .02 probability difference exists for both loan sizes.

<u>Results for a .33 λ Value</u>

Yearly expected withdrawals from the AMOR, FLEX, and VAP models are show in Figure 6. Similar to results from the DP models having a λ value of one, the VAP results in higher withdrawals in all years except in year ten. The

VAP's withdrawals are above the flexible amortization loan plan until year seven.

Debt/saving balances are lower under the VAP than the other two loan plans until at least year seven (Figure 7) because of the ability to retire debt instead of holding savings. Thus, the debt/saving distribution is partially truncated in the saving range. Note that the expected debt/saving balance is distinctly different between the flexible amortization and variable amortization loan plans, even though the flexible plan allows the amortization loan's principal to be prepaid. Differences result from the debt reserve under the VAP, which allows cash shortfalls to be covered.

As a result, the probability of having additional debt is lower under the VAP (Figure 8). Note also the probability of additional debt differs little between the amortization and flexible amortization loan plans, which suggests that the debt reserve under the VAP is key to providing flexibility in meeting cash shortfalls.

Table 3 presents summary conditional probability results in year ten. As with results when λ equals 1, the expected debt/saving balance plus withdrawals is higher for the VAP than it is for either of the other loans. Also, the probability of having a debt/saving balance less than -$55,000 is lower under the VAP. Unlike the case when λ equals 1, however, the probability of being in debt is lower.

Results for a 0.0 λ Value

Yearly expected withdrawals, debt/saving balances, and probability of additional debt are similar between the three loan repayment plans whether the λ value is 0 or .33. Thus, they are not shown. Only results from year ten are shown in Table 4. These results, too, are similar to those shown for a λ value of .33: the VAP produces higher expected debt/saving balance plus withdrawals, lower probability of having debt, and lower probability of having debt/saving balances below -$55,000.

Summary of Results

Results from the DP models can be summarized as follows:

1. For all λ values and initial amortization debt levels, the variable amortization loan results in higher total of expected withdrawals plus ending debt/saving balances than the other two repayment plans. Therefore, the variable amortization loan is more profitable from a borrowers standpoint, regardless of the level of emphasis given to the lender's objectives.
2. For λ values of .33 and 0, the probability of having debt is lower and the expected debt/saving balance is higher under the variable amortization loan plan than under the other two loan plans. This suggests that the variable amortization loan plan has advantages for lenders.
3. At a λ value of 1, the variable amortization loan plan results in a higher probability of having debt and a smaller debt/saving balance than the other loan plans. Lenders are not likely to prefer such a loan plan for profit maximizing, risk neutral borrowers.
4. Loan size does not influence the direction of results; so, debt levels do not seem to be a factor in preferring one loan plan over another.

Summary and Conclusions

Stochastic dynamic programming models have been solved which analyze the performance of three loan repayment plans: a traditional amortization loan, a flexible amortization loan, and a variable amortization loan. Performance has been monitored using differing loan sizes and objectives representing a borrower's perspective, a lender's perspective, and one that combines the two. Results indicate that borrowers may prefer the variable amortization loan because it has a debt reserve that serves as a liquidity source during periods of adverse income. Results further indicate that lenders may prefer the variable amortization plan, given that some restrictions are placed on withdrawals by the borrower.

These results suggest that the variable amortization has potential as a viable loan instrument in the agriculture sector. From the lenders perspective, loan terms must be scheduled such that additional debt is not generated by borrowers, an area which requires further research. Moreover, terms of the loan should be simplified for easier implementation.

Table 1. Principal Outstanding, Interest Payment, and Principal Payment for Two Amortization, Ten Year Loans.

Panel A. Beginning Debt = $20,000

Year	Principal Outstanding	Interest Payment	Principal Payment
0	20000		
1	18775	2100	1225
2	17421	1971	1354
3	15925	1829	1496
4	14272	1672	1653
5	12446	1499	1827
6	10427	1307	2018
7	8197	1095	2230
8	5732	861	2464
9	3009	602	2723
10	0	316	3009

Panel B. Beginning Balance = $40,000

Year	Principal Outstanding	Interest Payment	Principal Payment
0	40000		
1	37550	4200	2450
2	34842	3943	2708
3	31850	3658	2992
4	28544	3344	3306
5	24891	2997	3653
6	20854	2614	4037
7	16394	2190	4461
8	11465	1721	4929
9	6018	1204	5446
10	0	632	6018

Table 2. Expected Debt, Future Value of Withdrawals and Debt/Saving Distribution in Year 10 for Differing Debt Instruments and Beginning Debt Levels, Lambda = 1.0.

	Debt Instrument[1]		
	Amor.	Flexible Amor.	Variable Amor.
PANEL A. BEGINNING DEBT = $20,000			
Expected Ending Debt(-)/Saving(+)	-28,831	-28,831	-27,803
Expected Future Value of Withdrawals	168,248	168,248	186,055
Expected Debt/Saving plus Withdrawals	140,000	140,000	158,252
Debt/Saving Distribution (year 10)	Probability		
less than -$105,000	.0740	.0740	.0581
-$105,000 to -$55,000	.1219	.1219	.1242
-$55,000 to -$5,000	.3318	.3318	.4099
-$5,000 to +$5,000	.4236	.4236	.3621
$5,000 to $25,000	.0485	.0485	.0457
$25,000 to $55,000	.0000	.0000	.0000
Prob. of Having Debt	.5277	.5277	.5922
PANEL B. BEGINNING DEBT = $40,000			
Expected Ending Debt(-)/Saving(+)	-44,361	-44,361	-45,755
Expected Future Value of Withdrawals	131,091	131,091	163,113
Expected Debt/Saving plus Withdrawals	86,730	86,730	117,369
Debt/Saving Distribution (year 10)	Probability		
less than -$105,000	.1371	.1371	.1182
-$105,000 to -$55,000	.1679	.1679	.1968
-$55,000 to -$5,000	.3578	.3578	.4744
-$5,000 to +$5,000	.3063	.3063	.1959
$5,000 to $25,000	.0309	.0309	.0147
$25,000 to $55,000	.0000	.0000	.0000
Prob. of Having Debt	.6628	.6628	.7894

[1]See text for definition of differing debt instruments.

Table 3. Expected Debt, Future Value of Withdrawals and Debt/Saving Distribution in Year 10 for Differing Debt Instruments and Beginning Debt Levels, Lambda = .33.

	Debt Instrument[1]		
	Amor.	Flexible Amor.	Variable Amor.
PANEL A. BEGINNING DEBT = $20,000			
Expected Ending Debt(-)/Saving(+)	-3,364	-3,601	-1,897
Expected Future Value of Withdrawals	120,200	120,070	138,485
Expected Debt/Saving plus Withdrawals	116,836	116,469	136,588
Debt/Saving Distribution (year 10)	Probability		
less than -$105,000	.0448	.0434	.0303
-$105,000 to -$55,000	.0854	.0812	.0766
-$55,000 to -$5,000	.1456	.1477	.1596
-$5,000 to +$5,000	.0747	.0813	.0949
$5,000 to $25,000	.4875	.5225	.5432
$25,000 to $55,000	.1610	.1232	.0950
Prob. of Having Debt	.2758	.2723	.2665
PANEL B. BEGINNING DEBT = $40,000			
Expected Ending Debt(-)/Saving(+)	-17,327	-16,373	-11,215
Expected Future Value of Withdrawals	80,040	80,950	97,390
Expected Debt/Saving plus Withdrawals	62,713	64,577	86,175
Debt/Saving Distribution (year 10)	Probability		
less than -$105,000	.0881	.0868	.0599
-$105,000 to -$55,000	.0947	.0913	.0882
-$55,000 to -$5,000	.2008	.1918	.2048
-$5,000 to +$5,000	.0789	.0774	.0928
$5,000 to $25,000	.4399	.4550	.4823
$25,000 to $55,000	.0976	.0997	.0720
Prob. of Having Debt	.3836	.3699	.3529

1See text for definition of differing debt instruments.

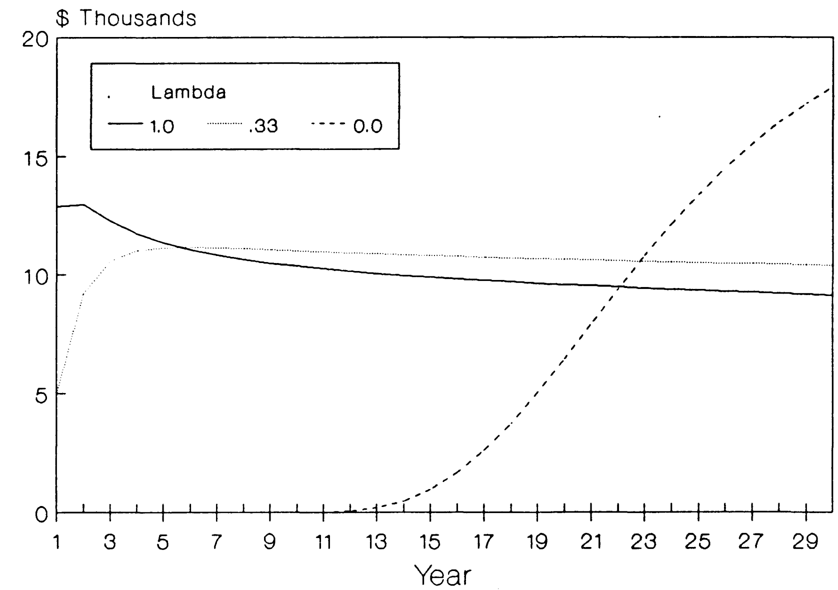
Table 4. Ending Debt, Future Value of Withdrawals and Debt/Saving Distribution in Year 10 for Differing Debt Instruments and Beginning Debt Levels, Lambda = 0.0.

	-------- Debt Instrument[1] -----		
	Amor.	Flexible Amor.	Variable Amor.
PANEL A. BEGINNING DEBT = $20,000			
Expected Ending Debt(-)/Saving(+)	77,381	78,654	89,315
Expected Future Value of Withdrawals	3,548	3,531	3,876
Expected Debt/Saving plus Withdrawals	80,929	82,185	93,191
Debt/Saving Distribution (year 10)	--------	Probability	---------
less than -$105,000	.0397	.0394	.0257
-$105,000 to -$55,000	.0463	.0459	.0357
-$55,000 to -$5,000	.0863	.0846	.0722
-$5,000 to +$5,000	.0246	.0248	.0213
$5,000 to $25,000	.0576	.0560	.0520
$25,000 to $55,000	.1074	.1055	.1014
greater than $55,000	.6381	.6438	.6917
Prob. of Having Debt	.1723	.1699	.1336
PANEL B. BEGINNING DEBT = $40,000			
Expected Ending Debt(-)/Saving(+)	39,777	41,545	56,734
Expected Future Value of Withdrawals	1,240	1,296	1,590
Expected Debt/Saving plus Withdrawals	41,017	42,841	58,324
Debt/Saving Distribution (year 10)	--------	Probability	---------
less than -$105,000	.0838	.0833	.0556
-$105,000 to -$55,000	.0797	.0788	.0638
-$55,000 to -$5,000	.1281	.1252	.1120
-$5,000 to +$5,000	.0327	.0323	.0300
$5,000 to $25,000	.0728	.0719	.0689
$25,000 to $55,000	.1231	.0997	.1213
greater than $55,000	.4798	.4877	.5484
Prob. of Having Debt	.2916	.2873	.2314

[1]See text for definition of differing debt instruments.

Figure 1. Expected Yearly Withdrawals and Debt Levels, No Debt.

Panel A. Expected Withdrawal Per Year.



Panel B. Expected Debt/Saving Balance Per Year.

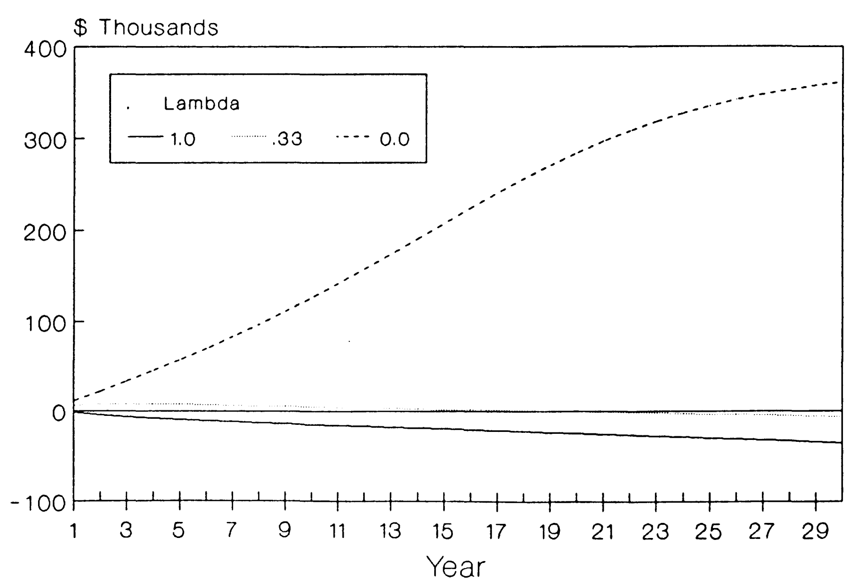
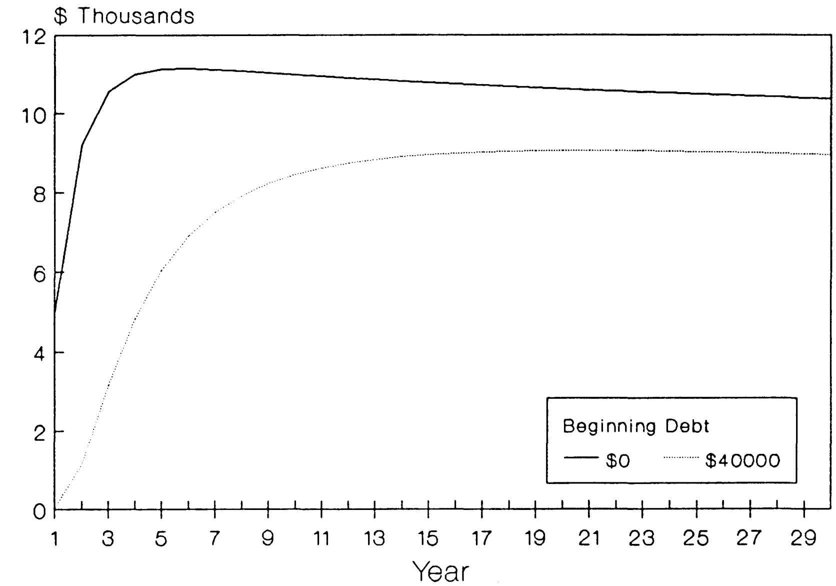


Figure 2. Expected Yearly Withdrawals and Debt Levels, Lambda = .33.

Panel A. Mean Withdrawal Per Year.



Panel B. Mean Debt/Saving Balance Per Year.

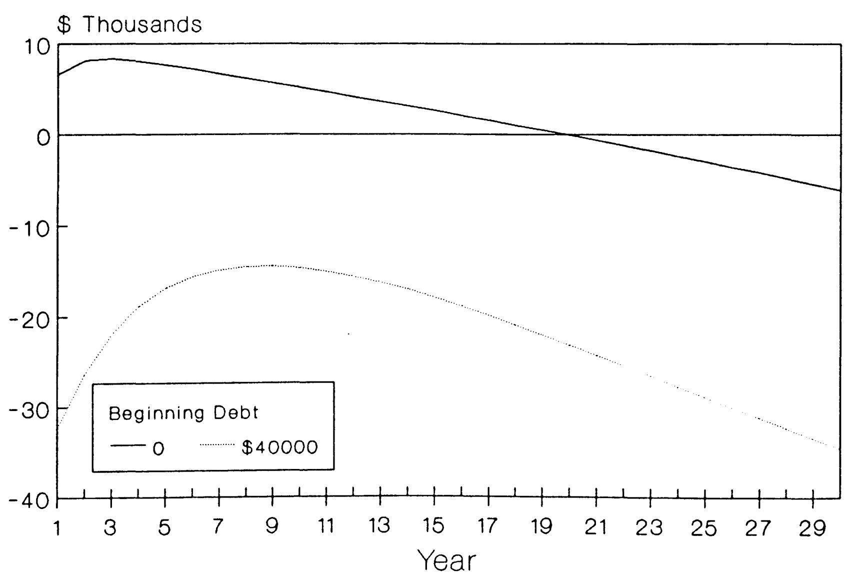
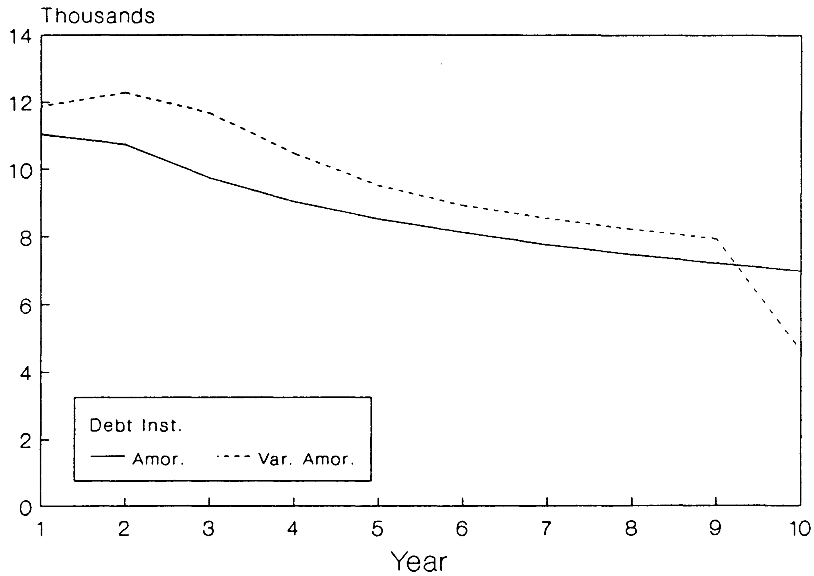


Figure 3. Expected Yearly Withdrawals, Lambda = 1.0.

Panel A. Beginning Debt = $20,000



Panel B. Beginning Debt = $40,000

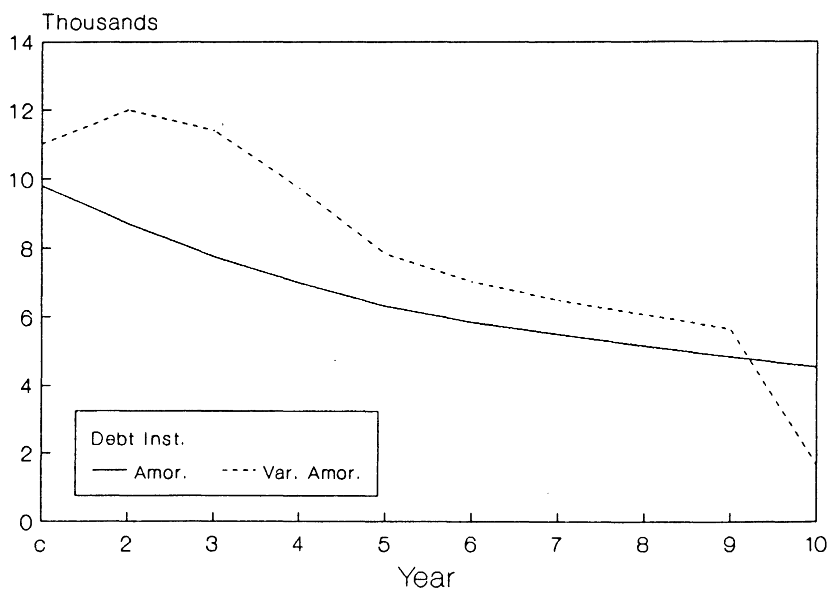
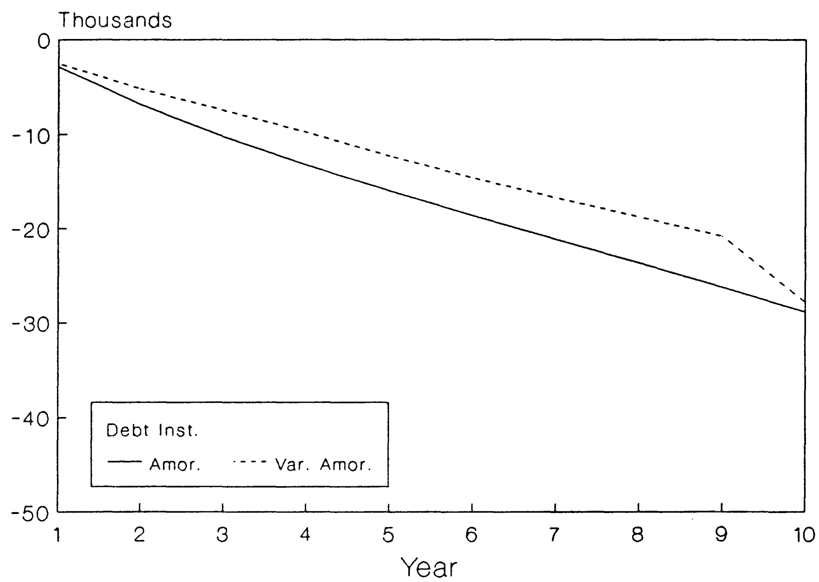


Figure 4. Expected Yearly Debt/Saving Balance, Lambda = 1.0.

Panel A. Beginning Debt = $20,000



Panel B. Beginning Debt = $40,000

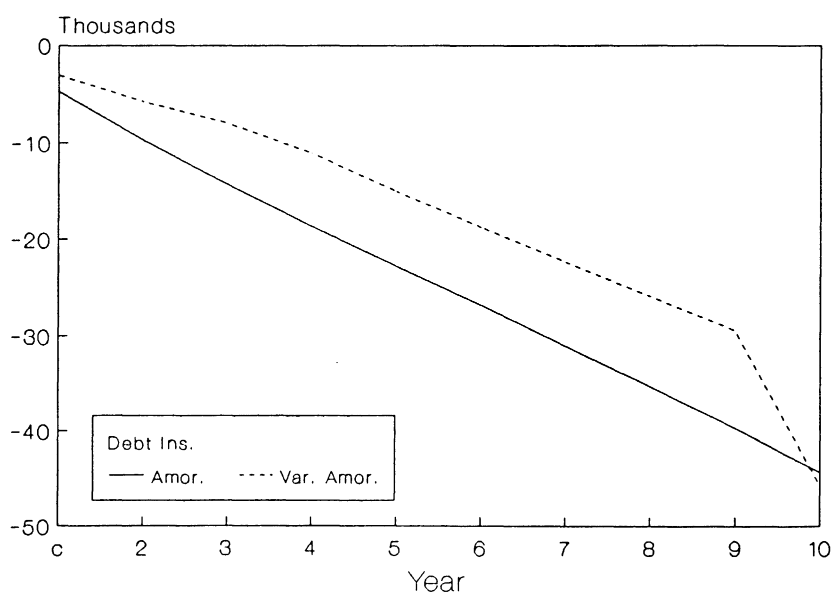
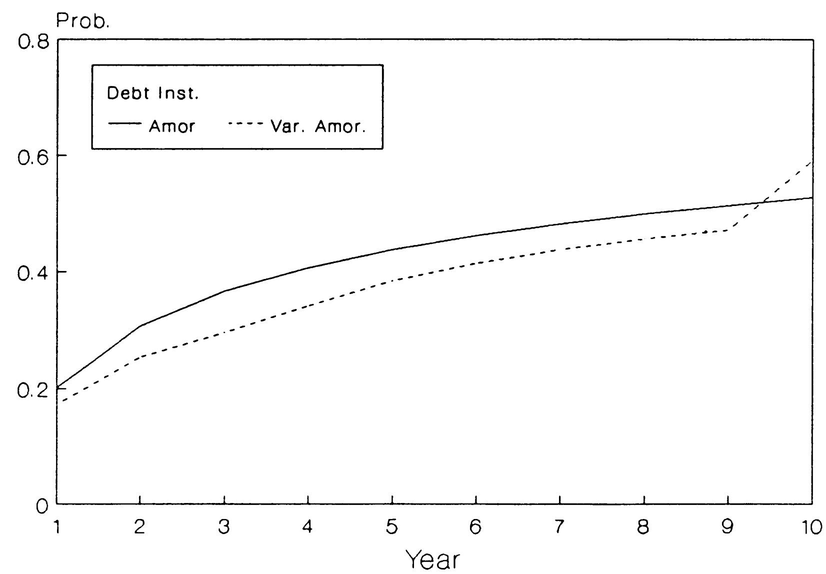


Figure 5. Probability of Having Additional Debt, Lambda = 1.0.

Panel A. Beginning Debt = $20,000



Panel B. Beginning Debt = $40,000

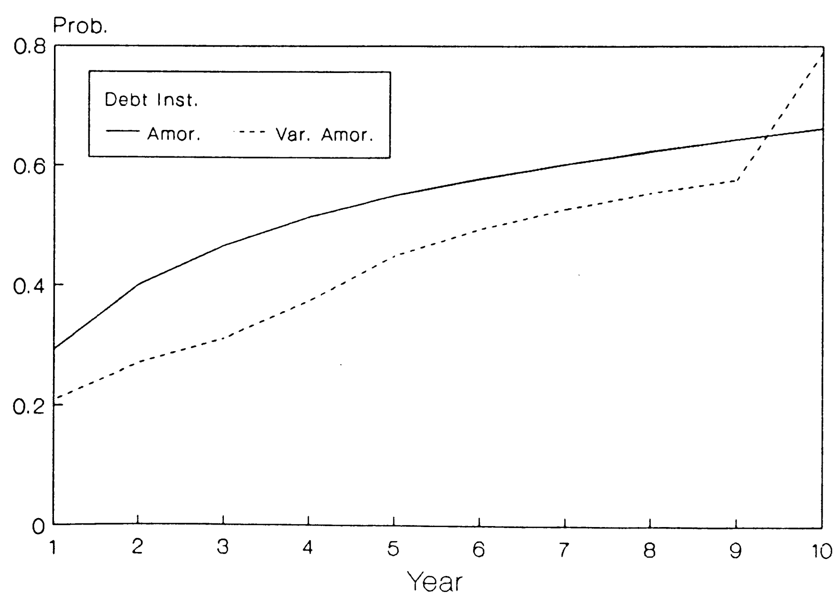
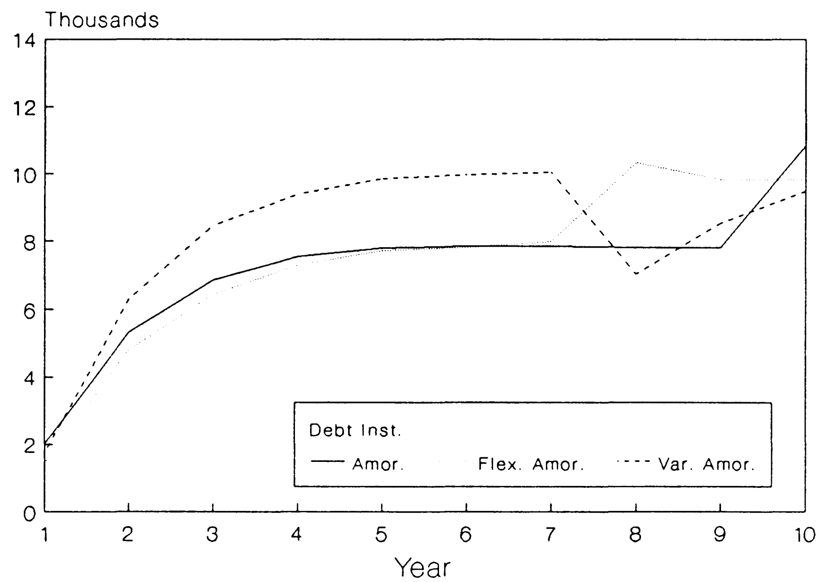


Figure 6. Expected Yearly Withdrawals, Lambda = .33.

Panel A. Beginning Debt = $20,000



Panel B. Beginning Debt = $40,000