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NONNUCLEOSYNTHETIC CONSTRAINTS ON THE BARYON DENSITY AND OTHER COSMOLOGICAL PARAMETERS

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ABSTRACT

Because the baryon-to-photon ratio η_{10} is in some doubt, we drop nucleosynthetic constraints on η_{10} and fit the three cosmological parameters (h, Ω_M, η_{10}) to four observational constraints: Hubble parameter $h_o = 0.70 \pm 0.15$, age of the universe $t_o = 14^{+7}_{-2}$ Gyr, cluster gas fraction $f_o \equiv f_G h^{3/2} = 0.060 \pm 0.006$, and effective shape parameter $\Gamma_o = 0.255 \pm 0.017$. Errors quoted are 1 σ, and we assume Gaussian statistics. We experiment with a fifth constraint $\Omega_o = 0.2 \pm 0.1$ from clusters. We set the tilt parameter $n = 1$ and the gas enhancement factor $\Upsilon = 0.9$. We consider cold dark matter models (open and $\Omega_M = 1$) and flat ΛCDM models. We omit HCDM models (to which the Γ_o constraint does not apply). We test goodness of fit and draw confidence regions by the $\Delta\chi^2$ method. CDM models with $\Omega_M = 1$ (SCDM models) are accepted only because the large error on h_o allows $h < 0.5$. Baryonic matter plays a significant role in Γ_o when $\Omega_M \sim 1$. Open CDM models are accepted only for $\Omega_M \gtrsim 0.4$. The combination of the four other constraints with $\Omega_o \approx 0.2$ is rejected in CDM models with 98% confidence, suggesting that light may not trace mass. ΛCDM models give similar results. In all of these models, $\eta_{10} \gtrsim 6$ is favored strongly over $\eta_{10} \lesssim 2$. This suggests that reports of low deuterium abundances on QSO lines of sight may be correct and that observational determinations of primordial ^{4}He may have systematic errors. Plausible variations on n and Υ in our models do not change the results much. If we drop or change the crucial Γ_o constraint, lower values of Ω_M and η_{10} are permitted. The constraint $\Gamma_o = 0.15 \pm 0.04$, derived recently from the *IRAS* redshift survey, favors $\Omega_M \approx 0.3$ and $\eta_{10} \approx 5$ but does not exclude $\eta_{10} \approx 2$.

Subject headings: cosmology: theory — elementary particles — nuclear reactions, nucleosynthesis, abundances

1. INTRODUCTION

In a Friedmann-Lemaître big bang cosmology, the universal baryonic mass-density parameter $\Omega_B(\equiv 8\pi G\rho_B/3H_0^2)$ may be calculated from

$$\Omega_B h^2 = 3.675 \times 10^{-3}(T/2.73\ \mathrm{K})^3\eta_{10} = 3.667 \times 10^{-3}\eta_{10}\ , \tag{1}$$

where h is defined by the present Hubble parameter $H_0[h \equiv H_0/(100\ \mathrm{km\ s^{-1}\ Mpc^{-1}})]$, T is the present microwave background temperature, and η_{10} is the baryon-to-photon number ratio in units of 10^{-10}. The last member of equation (1) is obtained by setting $T = 2.728$ K (Fixsen et al. 1996). In principle, η_{10} is well determined (in fact, overdetermined) by the observed or inferred primordial abundances of the four light nuclides D, ^{3}He, ^{4}He, and ^{7}Li, if the number of light-neutrino species has its standard value $N_\nu = 3$. For some years it has been argued that η_{10} is known to be 3.4 ± 0.3 (Walker et al. 1991; these error bars are about "1 σ"; cf. Smith, Kawano, & Malaney 1993) or at worst 4.3 ± 0.9 (Copi, Schramm, & Turner 1995a; cf. Yang et al. 1984) and that equation (1) is a powerful constraint on the cosmological parameters Ω_B and h.

In practice, it seems recently that η_{10} may not be so well determined and even that the standard theory of big bang nucleosynthesis (BBN) may not give a good fit. With improved abundance data, it appears that the joint fit of the theory to the four nuclide abundances is no longer good for any choice of η_{10} (Hata et al. 1995). These authors offer several options for resolving the apparent conflict between theory and observation. Although they suggest that some change in standard physics may be required (e.g., a reduction in the effective value of N_ν during BBN below its standard value 3), they note that large systematic errors may compromise the abundance data (cf. Copi, Schramm, & Turner 1995b). The nature of such errors is unclear, and this remains controversial. Other authors have reacted to the impending crisis in self-consistency by simply omitting one or more of the four nuclides in making the fit (Dar 1995; Olive & Thomas 1997; Hata et al. 1996, 1997; Fields et al. 1996).

This controversy has been sharpened by new observations giving the deuterium abundances on various lines of sight to high-redshift QSOs. These data should yield the primordial D abundance, but current results span an order of magnitude. The low values, D/H by number $\approx 2 \times 10^{-5}$ (Tytler, Fan, & Burles 1996; Burles & Tytler 1996), corresponding to $\eta_{10} \approx 7$ in the standard model, have been revised slightly upward [D/H $\approx (3–4) \times 10^{-5}$ (Burles & Tytler 1997, 1998a, 1998b); $\eta_{10} \approx 5$], but it still seems impossible to reconcile the inferred abundance of ^{4}He ($Y_P \approx 0.234$; Olive & Steigman 1995, hereafter OS) with standard BBN for this large value of η_{10} (which implies $Y_{\mathrm{BBN}} \approx 0.247$) unless there are large systematic errors in the ^{4}He data (cf. Izotov, Thuan, & Lipovetsky 1994, 1997). Such low D/H values have also been challenged on observational grounds by Wampler (1996) and by Songaila, Wampler, & Cowie (1997), and deuterium abundances nearly an order of

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magnitude higher, D/H $\approx 2 \times 10^{-4}$, have been claimed by Carswell et al. (1994), Songaila et al. (1994), and Rugers & Hogan (1996) for other high-redshift systems with metal abundances equally close to primordial. Although some of these claims of high deuterium have been called into question (Tytler, Burles, & Kirkman 1997), Hogan (1997) and Songaila (1997) argue that the spectra of other absorbing systems require high D/H (e.g., Webb et al. 1997). If these higher abundances are correct, then D and ^{4}He are consistent with $\eta_{10} \approx 2$, but modelers of Galactic chemical evolution have a major puzzle: How has the Galaxy reduced D from its high primordial value to its present (local) low value without producing too much ^{3}He (Steigman & Tosi 1995), without using up too much interstellar gas (Edmunds 1994; Prantzos 1996), and without overproducing heavy elements (cf. Tosi 1996 and references therein)? It appears that η_{10}, although known to order of magnitude, may be among the less well known cosmological parameters at present. Despite this, large modern simulations that explore other cosmological parameters are often limited to a single value of $\eta_{10} = 3.4$ (e.g., Borgani et al. 1997).

In this situation it may be instructive, as a thought experiment, to abandon nucleosynthetic constraints on η_{10} entirely and ask the question, If we put η_{10} onto the same footing as the other cosmological free parameters and apply joint constraints on all these parameters based on other astronomical observations and on theory and simulation, what values of η_{10} and the other parameters are favored? This may indicate the most promising avenue to a resolution of the controversy over η_{10}.

We discuss the following popular CDM models: (1) Open or closed cold dark matter model with cosmological constant $\Lambda = 0$ (CDM model). The "standard" (flat) CDM model (SCDM), which is an Einstein-de Sitter model, is covered as a special case of this. (2) The flat CDM model with nonzero Λ (ΛCDM model). In a flat model with both hot and cold dark matter, with $\Lambda = 0$ (HCDM model), the constraints will be different; we defer these HCDM models to a later paper. Nonflat models with nonzero Λ are not necessarily ruled out by "fine-tuning" arguments and may be of interest (Steigman & Felten 1995), but at the moment we are not compelled to resort to these.

Our approach will be to let three parameters range freely, fit the constraints (observables) other than nucleosynthetic constraints, test goodness of fit by χ^2, and draw formal confidence regions for the parameters by the usual $\Delta\chi^2$ method. Because statistical results of this kind are sometimes controversial, we intend to keep the work conceptually simple, review the constraints in a helpful way, and discuss our method carefully. Error bars are $\pm 1\ \sigma$ unless stated otherwise. The $\Delta\chi^2$ approach is revealing because, in the linear approximation, the confidence regions obtained are rigorous as probability statements and require no "a priori" probability assumptions about the unknown parameters. Most of our results are not surprising, and related work has been done before (Ostriker & Steinhardt 1995; White et al. 1996; Lineweaver et al. 1997; White & Silk 1996; Bludman 1997), but not with these three free variables and the full χ^2 formalism. It is well known that recent cosmological observations and simulations, particularly related to the "shape parameter" Γ and the cluster baryon fraction (CBF), pose a challenge to popular models, and that there is some doubt whether any simple model presently fits all data well. Our work, which begins by discarding nucleosynthetic constraints, provides a new way of looking at these problems. The CBF and Γ constraints have not been applied jointly in earlier work. We find that, given our conservative (generous) choice of error bars on h, the SCDM model is disfavored somewhat, but by no means excluded, if we are willing to accept $\eta_{10} \gtrsim 9$. But even with the generous error on h, and allowing Ω_M to range freely, large values ($\gtrsim 5$) of η_{10} are favored over small values ($\lesssim 2$). This suggests that the low D abundances measured by Burles and Tytler may be correct and that the observed (extrapolated) primordial helium-4 mass fraction ($Y_P \approx 0.23$; cf. OS and Olive, Skillman, & Steigman 1997, hereafter OSS), thought to be well determined, may be systematically too low for unknown reasons.

2. CDM MODELS: PARAMETERS AND OBSERVABLES

2.1. *Parameters*

We will take the CDM models to be defined by three free parameters: Hubble parameter h; mass-density parameter $\Omega_M = 8\pi G\rho_M/3H_0^2$; and baryon-to-photon ratio η_{10}, related to Ω_B by equation (1). Here Ω_M by definition includes all "dynamical mass": mass that acts dynamically like ordinary matter in the universal expansion. It is not limited to clustered mass only. Other free parameters having to do with structure formation, such as the tilt parameter n, could be added (White et al. 1996; Kolatt & Dekel 1997; White & Silk 1996), but we will try in general to avoid introducing many free parameters, so as to avoid generating confidence regions in more than three dimensions. We will, however, show results for two values of n (1 and 0.8), and for a few alternative choices of other parameters affecting some of the observables.

2.2. *Observables*

We will consider five observables (constraints) that have measured values and errors that we assume to be normal (Gaussian). The five observables are (1) measured Hubble parameter h_o; (2) age of the universe t_o; (3) dynamical mass-density parameter Ω_o from cluster measurements or from large-scale flows; (4) gas-mass fraction $f_o \equiv f_G h^{3/2}$ in rich clusters; and (5) "shape parameter" Γ_o from structure studies. In much of our work we will drop one or another of these constraints.

An observable w_o has the central value $\langle w \rangle$ and the standard deviation σ_w. The theoretical expression for this observable is given by a known function, w, of the three free parameters. The χ^2 contribution of this observable is written as $\chi^2 = (\langle w \rangle - w)^2/\sigma_w^2$. This sets up the usual conditions for the total χ^2 (which, assuming the errors are uncorrelated, is a sum of χ^2 contributions from different observables) to find the confidence regions for the free parameters (Cramer 1946; Bevington & Robinson 1992; Press et al. 1992; Barnett et al. 1996). We state below the theoretical expression w and the observational constraint $w_o = \langle w \rangle \pm \sigma_w$ which we assume.

There are other constraints that could be applied, including cluster abundance, the height of the "acoustic peak" in the angular fluctuation spectrum of the cosmic background radiation, the Sunyaev-Zeldovich effect in clusters, the Lyα forest, and theoretical constraints on Λ (White et al. 1996; White & Silk 1996; Lineweaver et al. 1997; Myers et al. 1997; Rauch, Haehnelt, & Steinmetz 1997; Weinberg et al. 1997; Bi & Davidsen 1997; Fan, Bahcall, & Cen 1997;

Martel, Shapiro, & Weinberg 1998). We omit these here but intend to pursue them in subsequent work.

2.3. *Observed Hubble Parameter* h_o

For the Hubble parameter the observable h_o is simply fit with the parameter h. Measurements of h still show scatter that is large compared with their formal error estimates (Bureau, Mould, & Staveley-Smith 1996; Tonry et al. 1997; Kundić et al. 1997; Tammann & Federspiel 1997). This indicates systematic errors. We do not presume to review this subject. To be conservative (permissive), we take $h_o = 0.70 \pm 0.15$. Some may think that a smaller error could be justified. In § 3.2 we will experiment with shrinking the error bar.

2.4. *Observed Age of the Universe* t_o

Pre-*Hipparcos* observations gave the ages of the oldest globular clusters as $t_{\rm GC} \approx 14 \pm 2$ Gyr (Bolte & Hogan 1995; Jimenez 1997; D'Antona, Caloi, & Mazzitelli 1997; cf. Cowan et al. 1997; Nittler & Cowsik 1997). Some of the analyses incorporating the new *Hipparcos* data derive younger ages, $t_{\rm GC} \approx 12$ Gyr (Chaboyer et al. 1998; Reid 1997; Gratton et al. 1997). However, as Pont et al. (1998) and M. H. Pinsonneault (1998, private communication) emphasize, there are systematic uncertainties in the main-sequence fitting technique at the 2 Gyr level, and Pont et al. (1998) use *Hipparcos* data to derive an age of 14 Gyr for M92.

The theoretical age for the $\Lambda = 0$ models is given by $t = 9.78h^{-1}f(\Omega_M; \Lambda = 0)$ Gyr (Weinberg 1972, eqs. [15.3.11] and [15.3.20]). The "observed" age of the universe, t_o, should exceed $t_{\rm GC}$ by some amount Δt. The best guess for Δt might be 0.5–1 Gyr. Most theorists believe that Δt must be quite small (2 Gyr at most), but we know of no conclusive argument to prove this, and we do not want long-lived models to suffer an excessive χ^2 penalty. We could treat Δt as another free parameter, but to avoid this and keep things simple, we introduce asymmetric error bars. We believe that $t_o = 14^{+7}_{-2}$ Gyr is a fair representation of t_o derived from present data on $t_{\rm GC}$. This allows enough extra parameter space at large ages to accommodate a conservative range of Δt; extremely large ages will be eliminated by the h_o constraint in any case. The χ^2 analysis will still be valid with the unequal error bars if we assume the χ^2 contribution as $(14 - t)^2/\sigma_t^2$, where σ_t takes the value 2 Gyr when $t < 14$ Gyr and the value 7 Gyr when $t > 14$ Gyr.

2.5. *Observed Mass Density* Ω_o

The observed Ω at zero redshift, determined from clusters, has recently been reported as

$$\Omega_{\rm CL} = 0.19 \pm 0.06({\rm stat}) \pm 0.04({\rm sys}) \qquad (2)$$

(Carlberg, Yee, & Ellingson 1997), where the respective errors are statistical and systematic. This is based on the M/L ratio in clusters and the luminosity density of the universe. If light traces dynamical mass, then we expect that $\Omega_{\rm CL}$ can be directly fit with Ω_M.

There is a difficulty in using equation (2) as a constraint on the underlying parameter Ω_M. Many consumers of equation (2) and earlier results (cf. Carlberg et al. 1996) have failed to notice that the result is model dependent, because the clusters in the sample have substantial redshifts ($0.17 < z < 0.55$). In their analysis Carlberg et al. (1997) assumed $q_0 = 0.1$ (e.g., $\Omega_M = 0.2$ and $\Lambda = 0$). When the result is $\langle\Omega_{\rm CL}\rangle = 0.19$, clearly the analysis is approximately self-consistent, but this does not give us sufficient guidance in exploring other values of Ω_M. For example, effects of nonzero Λ could be substantial. We believe that if the analysis of Carlberg et al. (1997) were repeated for a ΛCDM model, the resulting $\langle\Omega_{\rm CL}\rangle$ might be smaller, perhaps 0.12 rather than 0.19. The parameters Ω_M and Λ need to be incorporated more fully into the analysis.

Looking at these and earlier data, we choose, somewhat arbitrarily, to use instead of equation (2) a somewhat more permissive Ω constraint from clusters:

$$\Omega_o = 0.2 \pm 0.1 \qquad (3)$$

(R. G. Carlberg 1997, private communication). If the critical ("closure") M/L ratio in B_T magnitude is 1500 h $(M/L)_\odot$ (Efstathiou, Ellis, & Peterson 1988), then equation (3) requires that the mean M/L in B_T for galaxies in the local universe be about $(300 \pm 150)h$ (cf. Smail et al. 1996). This agrees well with modern reviews (Bahcall, Lubin, & Dorman 1995; Trimble 1987).

We will assume in some of our examples that Ω_o can be directly fit with Ω_M (light traces mass; "unbiased"), with Ω_o given by equation (3). Obviously, under this assumption, the χ^2 contribution from the observed Ω_o will rule out the SCDM model ($\Omega_M = 1$) with high confidence.

Bias is possible, with the most likely bias being $\Omega_o < \Omega_M$. This would be the case of additional unclustered or weakly clustered dynamical mass. Because such weakly clustered mass is quite possible, we will also do other cases with an alternative to the cluster constraint, as follows. Dekel & Rees (1994; cf. Dekel 1997) studied large-scale flows around voids and concluded that Ω_M must be quite large: $\Omega_M >$ (0.4, 0.3, 0.2) at confidence levels (1.6 σ, 2.4 σ, 2.9 σ). All values $\Omega_M \geq 0.6$ were permitted. To use this one-way constraint as the third observable in a χ^2 fit, we need a substitute function $\chi^2(\Omega_M)$ having the properties $\chi^2(0.4) \approx (1.6)^2$, $\chi^2(0.3) \approx (2.4)^2$, $\chi^2(0.2) \approx (2.9)^2$. The function

$$\chi^2(\Omega_M) = \begin{cases} (0.6 - \Omega_M)^2/(0.125)^2 & (\Omega_M < 0.6) \\ 0 & (\Omega_M \geq 0.6) \end{cases} \qquad (4)$$

is a good approximation. For $\Omega_M \geq 0.6$, this χ^2 implies a "perfect fit" to the Ω observable. This leaves parameter space open to large Ω_M. We apply the Dekel-Rees constraint instead of the cluster constraint if we just substitute equation (4) for the usual χ^2 term arising from Ω_o.

2.6. *Observed Cluster Gas Mass Fraction* f_o

Theorists and observers (White & Frenk 1991; Fabian 1991; Briel, Henry, & Böhringer 1992; Mushotzky 1993) have long argued that the large observed gas mass fraction in clusters, f_G, is a valuable cosmological datum and poses a serious threat to the SCDM model. This argument was raised to high visibility by the quantitative work of White et al. (1993), and now the problem is sometimes called the "baryon catastrophe" (Carr 1993) or "baryon crisis" (Steigman & Felten 1995).

At the risk of boring the experts, we must emphasize that the following argument does not assume that most of the mass in the universe, or any specific fraction of it, is in rich clusters. Rather, we will use f_G, not as a constraint on Ω_M, but as a constraint on the universal baryon fraction, the ratio Ω_B/Ω_M. The idea is that the content of a rich cluster is a fairly unbiased sample of baryonic and dark matter. This is suggested by simulations, which are discussed below.

The measurement of f_G poses problems, but this is not the place for a lengthy review. Magnetic pressure (Loeb & Mao 1994) may cause systematic errors, but these are probably not large and do not provide a promising escape hatch for the SCDM model (Felten 1996). Reported values of f_G derived by various methods show quite a wide range from cluster to cluster and also from groups through poor and rich clusters (Steigman & Felten 1995; White & Fabian 1995; Lubin et al. 1996; Mohr, Geller, & Wegner 1996). Loewenstein & Mushotzky (1996) emphasize that the range in f_G is wider than expected from simulations, and they suggest that some significant physics may be missing from the simulations. Cen (1997) argues that the spread may be caused by projection effects in the measurements of f_G, arising because of large-scale pancakes and filaments. Evrard, Metzler, & Navarro (1996), using gasdynamical simulations to model observations, find that the largest error in f_G arises from measurement of the cluster's *total* mass and that this error can be reduced by using an improved estimator and by restricting the measurement to regions of fairly high overdensity. Evrard (1997) applies these methods to data for real clusters and finds $f_G h^{3/2} = 0.060 \pm 0.003$. This subject is still controversial so, to be conservative, we will double his error bars and take as our constraint

$$f_o \equiv f_G h^{3/2} = 0.060 \pm 0.006 . \tag{5}$$

Note that this is quite a large gas fraction ($f_G \approx 17\%$ for $h \approx 0.5$), in general agreement with earlier results.

The functional dependence of f_G on the cosmological parameters also poses problems. The *universal* baryonic mass fraction is Ω_B/Ω_M, but not all baryons are in the form of gas, and furthermore, selection factors may operate in bringing baryons and dark matter into clusters. White et al. (1993) introduced a "baryon enhancement factor" Υ to describe these effects as they operate in simulations. Υ may be defined by

$$f_{G0} = \Upsilon \Omega_G/\Omega_M , \tag{6}$$

where Ω_G is the initial contribution of *gas* to Ω_M (note that $\Omega_G \leq \Omega_B$) and f_{G0} is the gas mass fraction in the cluster immediately after formation. We will shortly set Υ equal to some constant. Υ is really the *gas* enhancement factor, because the simulations do not distinguish between baryonic condensed objects, if any (galaxies, stars, machos), and nonbaryonic dark matter particles. All of these are lumped together in the term $(\Omega_M - \Omega_G)$ and interact only by gravitation.

If all the baryons start out as gas ($\Omega_G = \Omega_B$) and if gas turns into condensed objects only *after* cluster formation, then equation (6) may be rewritten

$$f_G + f_{\rm GAL} = \Upsilon \Omega_B/\Omega_M , \tag{7}$$

where f_G is the present cluster gas mass fraction and $f_{\rm GAL}$ is the present cluster mass fraction in baryonic condensed objects of all kinds (galaxies, stars, machos). We wish to carry along an estimate of $f_{\rm GAL}$ to show its effects. White et al. (1993) took some pains to estimate the ratio $f_G/f_{\rm GAL}$ within the Abell radius of the Coma cluster, counting only galaxies (no stars or machos) in $f_{\rm GAL}$. They obtained

$$f_G/f_{\rm GAL} = 5.5h^{-3/2} . \tag{8}$$

This is large, so unless systematic errors in this estimate are very large, the baryonic content of this cluster (at least) is dominated by the hot gas. Carrying $f_{\rm GAL}$ along as an indication of the size of the mean correction for all clusters, and solving equations (7) and (8) for $f_G h^{3/2}$, we find

$$f_G h^{3/2} = [1 + (h^{3/2}/5.5)]^{-1}(\Upsilon \Omega_B/\Omega_M)h^{3/2} , \tag{9}$$

where Ω_B is given from η_{10} and h by equation (1). This is the appropriate theoretical function of the free parameters to fit to the observation.

The second term in brackets in equation (9) is the small correction term due to $f_{\rm GAL}$. In deriving this $f_{\rm GAL}$, given by equation (8), White et al. (1993), using observations in the inner parts of bright ellipticals by van der Marel (1991), assumed that within cluster galaxies the mean ratio of baryonic mass to blue light is $6.4\ h\ (M/L)_\odot$. We note that this correction term would be larger if cluster galaxies or the cluster as a whole contained baryonic machos amounting to $\sim 20\ (M/L)_\odot$, as suggested for the halo of our Galaxy by theories of observed microlensing events (Chabrier, Segretain, & Méra 1996; Fields, Mathews, & Schramm 1997; Natarajan et al. 1997). Indeed, Gould (1995) has even suggested that the mass in machos could be comparable to that in the gas component, in which case $f_{\rm GAL} \approx f_G$.

What value of the gas enhancement factor Υ should be used in equation (9)? A value $\Upsilon = 3$–5 would do away with the "baryon catastrophe" for the SCDM model. There is no plausible way to obtain an Υ this large. White et al. (1993), when they assumed zero-pressure gas to explore maximizing Υ, always found $\Upsilon \leq 1.5$ in simulations. More realistic simulations with gas pressure give $\Upsilon \approx 0.9$ (Evrard 1997) or values even as small as $\frac{2}{3}$ (Cen & Ostriker 1993). The gas preferentially stays out of the clusters to some extent rather than concentrating itself there. Gas can support itself through pressure and shocks, while CDM cannot. We will set $\Upsilon = 0.9$ in most of our examples. This is representative of results from simulations, and it is close to unity, so these cases will also illustrate the approximate consequences if gas is neither enhanced nor excluded in clusters.

Cen (1997) finds in simulations that the determination of f_G from X-ray observations is biased toward high f_G by large-scale projection effects; i.e., the calculated f_G exceeds the true f_G present in a cluster. This bias factor can be as large as 1.4. Evrard et al. (1996) and Evrard (1997) have not observed such a bias in their simulations. If Cen is correct, we could explore the effect of such a bias in our statistical tests by using for Υ, instead of 0.9, an "effective value" $\Upsilon \approx 0.9 \times 1.4 \approx 1.3$. Since this would also demonstrate the impact of any effect that may cause Υ to exceed unity moderately, we will show results for $\Upsilon = 1.3$ as well as for $\Upsilon = 0.9$.

Equations (7)–(9) above were derived under the assumption that all baryonic condensed objects (galaxies, stars, machos) form from the gas *after* the collapse of clusters occurs. If, instead, all such objects were formed *before* collapse, equation (7) should be replaced by

$$f_G + \left(\frac{\Upsilon - f_G}{1 - f_G}\right) f_{\rm GAL} = \Upsilon \Omega_B/\Omega_M , \tag{10}$$

reflecting the fact that the baryons in condensed objects now escape the gas enhancement occurring during cluster formation. Since these effects are not large for $\Upsilon \approx 1$ and equation (7) is likely to be closer to the true situation than equation (10), we will make no further use of equation (10).

2.7. *Shape Parameter* Γ_o *from Large-Scale Structure*

The last observable we use is the "shape parameter" Γ, which describes the transfer function relating the initial perturbation spectrum $P_I(k) \propto k^n$ to the present spectrum $P(k)$ of large-scale power fluctuations, as observed, e.g., in the galaxy correlation function. When the spectral index n of $P_I(k)$ has been chosen, Γ is determined by fitting the observed $P(k)$. There are some notational differences among papers on this subject. Sometimes Γ is used to mean simply the combination $\Omega_M h$. We will avoid this usage here.

Results of observations may be cast in terms of an "effective shape parameter" Γ (White et al. 1996), which we will take as our observable. Studies show that for the usual range of CDM models, with or without Λ, the expression for Γ is

$$\Gamma \approx \Omega_M h \exp[-\Omega_B - (h/0.5)^{1/2}(\Omega_B/\Omega_M)] - 0.32(n^{-1} - 1) \tag{11}$$

(Peacock & Dodds 1994; Sugiyama 1995; Liddle et al. 1996a, 1996b; White et al. 1996; Liddle & Viana 1997; Peacock 1997). For $n \approx 1$, if Ω_B and Ω_B/Ω_M are small, we have $\Gamma \approx \Omega_M h$. The Harrison-Zeldovich (scale-invariant, untilted) case is $n = 1$. We will adopt $n = 1$ for our standard case and experiment with different n in § 3.3. The approximation of equation (11) has been tested (and, we believe, is valid) only for models in which both n and the exponential term are fairly near unity.

This Γ is the parameter of the familiar BBKS approximation to the transfer function (Bardeen et al. 1986; Peacock 1997). The BBKS transfer function does not fit the data continuously from long to short wavelengths, and Hu & Sugiyama (1996) have developed a more elaborate approximation for short wavelengths ($\lesssim 3h^{-1}$ Mpc), useful especially in cases with large Ω_B/Ω_M. Recent comparisons with data (Webster et al. 1998) still use Sugiyama's (1995) form for Γ as in equation (11) above, and this BBKS approximation is adequate in the regime (3–100)h^{-1} Mpc, where Γ is determined.

For the observed value of Γ, we take

$$\Gamma_o = 0.255 \pm 0.017 \tag{12}$$

(Peacock & Dodds 1994; cf. Maddox, Efstathiou, & Sutherland 1996). This is based on the galaxy correlation function, and it assumes that light traces mass. The very small errors, from Peacock & Dodds (1994), result from averaging several data sets and may not be realistic. Later, we will explore the consequences of inflating these errors and/or moving the central value. Equations (11) and (12) imply, very roughly, that $\Omega_M h \approx 0.25$.

The shape parameter can be derived from the galaxy peculiar velocity field instead of the density field. The result from that technique, analogous to $\Omega_M h = 0.25$, is, very roughly and for $n = 1$,

$$\Omega_M h^{1.2} = 0.350 \pm 0.087(90\%\mathrm{CL}) \tag{13}$$

(Zaroubi et al. 1997a), where $\Omega_B h^2 = 0.024$ has been assumed and CL stands for confidence level. Equation (13) ostensibly includes an estimate of cosmic variance (cf. Kolatt & Dekel 1997; Zaroubi et al. 1997b). Equation (13) may be used to yield an estimate of Γ_o as follows: Adjust the error bar in equation (13) from 1.65 σ to 1 σ. Evaluate Ω_M from equation (13) at the midpoint of the "interesting" range of h, viz. $h = 0.7$. Substitute the resulting parameters, including Ω_B, into the right-hand side of equation (11), and evaluate. The result is

$$\Gamma_o = 0.32 \pm 0.05 . \tag{14}$$

The independent estimates in equations (12) and (14) agree tolerably within the stated errors. Any difference, if real, could be caused by galaxy bias.

The shape-parameter constraint is in a sense the least robust of the constraints we have discussed since it is not part of the basic Friedmann model. Rather, it depends on a theory for the primordial fluctuations and how they evolve. If the Friedmann cosmology were threatened by this constraint, we believe that those who model large-scale structure would find a way to discard it. Therefore, we will also explore some consequences of removing this constraint.

3. CDM MODELS: RESULTS

3.1. *CDM with Standard Constraints*

We begin the presentation of our results by adopting a standard case with only four constraints, dropping the Ω_o constraint. For this standard case we assume $n = 1$ and $\Upsilon = 0.9$, and we apply the following "standard constraints": $h_o = 0.70 \pm 0.15$, $t_o = 14^{+7}_{-2}$ Gyr, $f_o \equiv f_G h^{3/2} = 0.060 \pm 0.006$, and $\Gamma_o = 0.255 \pm 0.017$. Then χ^2 is the sum of four terms.

Results for our standard case are displayed in Figures 1, 2, and 3. Figures 1 and 2 are a pair that can be understood geometrically. The function $\chi^2(h, \Omega_M, \eta_{10})$ is computed on the three-dimensional parameter space. It has a minimum $\chi^2_{\min}$ in this space, which in this case is 1.2 for one degree of freedom (dof) and is located at (0.57, 0.61, 8.7). This value of $\chi^2_{\min}$ is acceptable; it is the 73% point of the distribution. We may draw a closed surface that encloses this point, defined

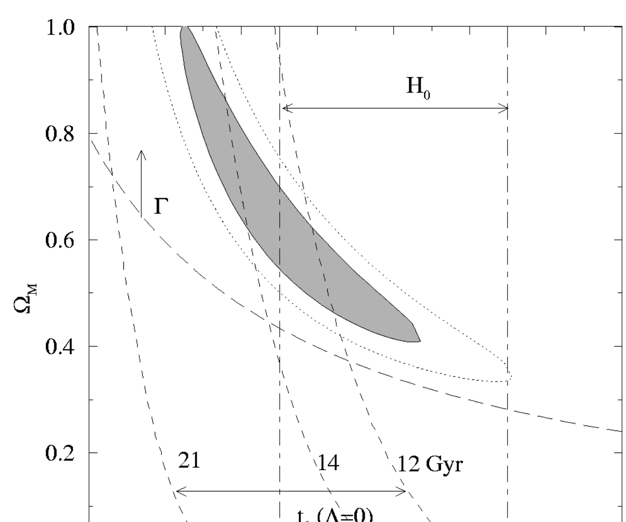

FIG. 1.—The 68% (*shaded region*) and 95% (*dotted line*) confidence regions (CRs) in the (H_0, Ω_M) plane for CDM models ($\Lambda = 0$) with our four standard constraints. The CRs are closed curves. Individual constraints in this plane are also shown schematically, as explained in the text.

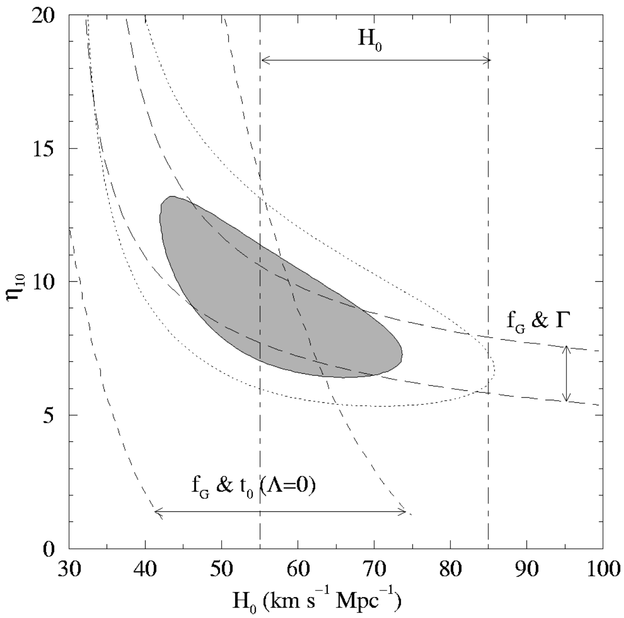

FIG. 2.—Same as Fig. 1, but in the (H_0, η_{10}) plane. Individual and paired constraints are also shown schematically. See text.

by setting

$$\Delta\chi^2 \equiv \chi^2(h, \Omega_M, \eta_{10}) - \chi^2_{\min} = 2.3 \ . \quad (15)$$

The quantity $\Delta\chi^2$ is distributed like a χ^2 variable with 3 dof (Press et al. 1992; Barnett et al. 1996). Our surface $\Delta\chi^2 = 2.3$ is at the 49% point, so it is a 49% confidence region ("CR") for the three parameters jointly. Furthermore, its projections on the orthogonal planes (h, Ω_M; Fig. 1) and (h, η_{10}; Fig. 2) give the 68% CRs for the parameters pairwise. These 68% CRs are shown as closed curves in Figures 1 and 2. Similarly, we construct 95% CRs in these planes by replacing 2.3 by 6.0 in equation (15).

We also show in Figures 1 and 2 projected CRs obtained by computing χ^2 for single observables alone, or for pairs of observables. They are drawn by setting $\Delta\chi^2 = 1$ and projecting. These regions are not closed. They are merely intended to guide the reader in understanding how the various constraints influence the closed contours, which show our quantitative results.

One-dimensional confidence intervals (CIs) may similarly be constructed for any parameter by projecting closed surfaces in three-space onto a single axis. These CIs may be described by a likelihood function. Figure 3 shows the likelihood function $\mathscr{L}(\eta_{10})$ for the parameter η_{10}. Table 1 shows the one-parameter CIs for the CDM models.

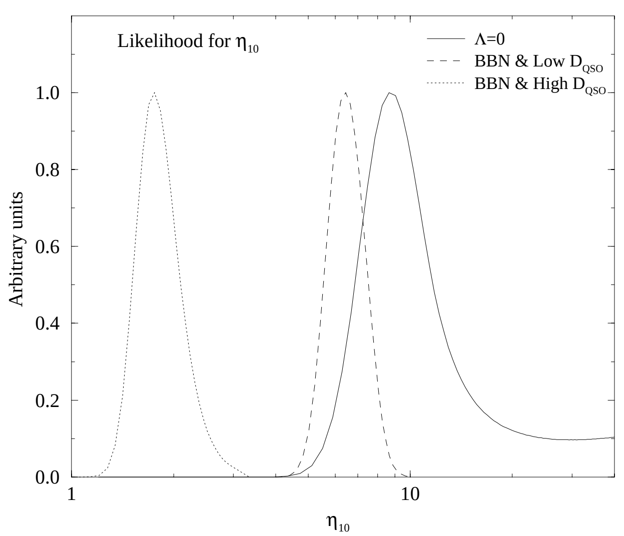

FIG. 3.—Likelihood $\mathscr{L}(\eta_{10})$ as a function of η_{10} for CDM models ($\Lambda = 0$) with our four standard constraints (*solid line*). Also shown are the corresponding BBN-predicted likelihoods for the high D abundance (*dotted line*) and for the low D abundance (*dashed line*) inferred from QSO absorbers (cf. Hata et al. 1997).

3.2. *CDM: Discussion*

It is well known that the condition $\Omega_M h \approx 0.25$ poses some threat to the SCDM ($\Omega_M = 1$) model. Figure 1 shows that this threat is far from acute, with our more accurate form of the Γ constraint given in equation (11), as long as the error on h_o is large (0.15) and BBN constraints are discarded. (Note again that we have not applied the constraint $\Omega_o = 0.2 \pm 0.1$.) Even our 68% contour extends to $\Omega_M = 1$. With the large error bar, the corresponding value of H_0, $h = 0.43$, is accepted. The exponential term in equation (11) becomes significant because the f_G constraint forces Ω_B to increase with Ω_M, allowing the product $\Omega_M h$ to exceed 0.25. This has been noted before (White et al. 1996; Lineweaver et al. 1997).

We have tested the SCDM model by fixing Ω_M at unity and fitting the four standard constraints with the remaining two parameters. The CRs for $\eta_{10} - H_0$ are shown in Figure 4. We find $\chi^2_{\min} = 3.4$ for 2 dof (82% CL), which is acceptable. However, this case encounters severe problems since only $h < 0.48$ and $\eta_{10} > 8$ are accepted at the 95% contour. Indeed, $\eta_{10} \gtrsim 8$ if $h \approx 0.4$ and $\eta_{10} \approx 15$ if $h \approx 0.48$. Such large η-values pose a serious threat to the consistency between the predictions of BBN and the primordial abundances of the light elements inferred from observations (e.g., Hata et al. 1996). Indeed, this "solution" is only acceptable because of our very generous error bar for h and because we have discarded BBN constraints.

When h is better known, the situation for SCDM will change. As an illustration, in Figure 5 we return to our three standard variables but replace our standard constraint on H_0 with $h_o = 0.70 \pm 0.07$, assuming, arbitrarily, a 10% error. The $\chi^2_{\min}$ is now 2.2 for 1 dof (86% CL), so we can still accept the basic Friedmann model, but SCDM is now excluded strongly. In this case the corresponding allowed range of η, shown in Figure 6, is not in strong conflict with BBN although the predicted ^{4}He abundance is larger than that inferred from observations of extragalactic II regions (OS; OSS).

Returning to our CDM case with standard constraints, it is less well known that the Γ constraint also poses a threat to *low*-density models (Liddle et al. 1996a; Kolatt & Dekel

TABLE 1

BEST-FIT PARAMETERS AND ERRORS FOR FOUR STANDARD CONSTRAINTS

Parameter	$\Lambda = 0$, CDM Model	$k = 0$, ΛCDM Model
η_{10}	$8.7^{+2.3}_{-1.6}$ (>6.1)	$8.4^{+2.1}_{-1.5}$ (>5.8)
Ω_B	$0.10^{+0.08}_{-0.04}$ (>0.04)	$0.08^{+0.06}_{-0.03}$ (>0.03)
Ω_M	$0.61^{+0.20}_{-0.14}$ (>0.39)	$0.53^{+0.19}_{-0.11}$ (>0.35)
H_0 (km s^{-1} Mpc^{-1})......	57^{+11}_{-10} (36–80)	62^{+13}_{-11} (39–87)
t_0 (Gyr)	$12.6^{+1.9}_{-1.6}$ (9.7–16.6)	$12.9^{+1.5}_{-1.4}$ (10.5–16.1)

NOTE.—Error bars are for 68% CI; range in parentheses is for 95%.

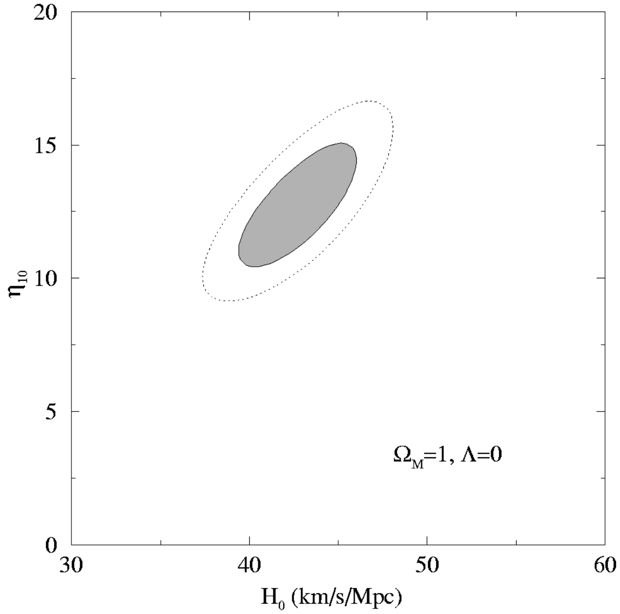

FIG. 4.—The 68% (*shaded region*) and 95% (*dotted line*) CRs in the (H_0, η_{10}) plane for CDM models ($\Lambda = 0$) with our four standard constraints, but with Ω_M now fixed at unity (SCDM models).

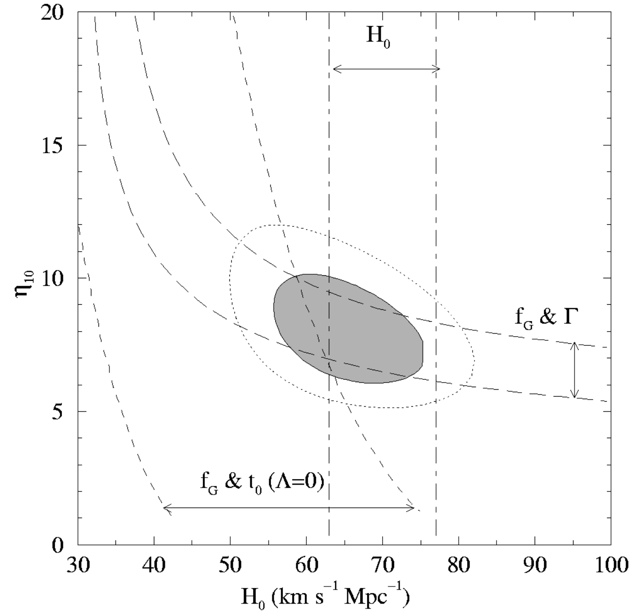

FIG. 6.—Same as Fig. 2, but with the constraint $h_o = 0.70 \pm 0.15$ replaced by $h_o = 0.70 \pm 0.07$.

1997; White & Silk 1996). From Figure 1 it is apparent that there is tension between our four standard constraints and $\Omega_o = 0.2 \pm 0.1$. Since the 95% contour does not even extend downward to $\Omega_M = 0.3$, we refrained from using this cluster constraint. The t_o and Γ_o constraints, combined, force the parameters upward out of the lower part of the figure and favor $\Omega_M \gtrsim 0.4$. We could nevertheless force a fit to all five constraints and draw CRs. We have done this, and we find that $\chi^2_{\rm min}$ is 7.8 for 2 dof (98% CL); i.e., we reject the combined fit with 98% confidence.

Among our principal results is that our standard CDM case favors large values of the baryon-to-photon ratio,

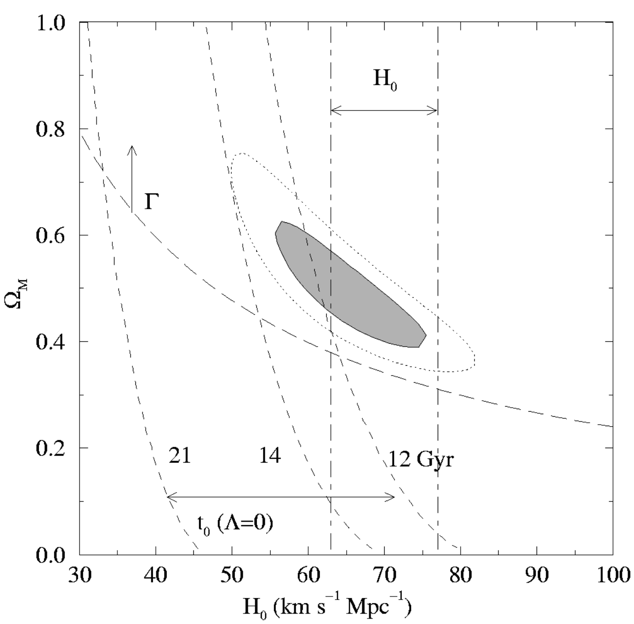

FIG. 5.—Same as Fig. 1, but with the constraint $h_o = 0.70 \pm 0.15$ replaced by $h_o = 0.70 \pm 0.07$.

$\eta_{10} = 8.7^{+2.3}_{-1.6}$ (see Fig. 3 and Table 1). It is the f_o and Γ_o constraints that, together, force us to large η_{10}. Also shown in Figure 3 are the likelihoods for η derived in Hata et al. (1997) for the high deuterium abundance inferred for some QSO absorbers (Songaila et al. 1994; Carswell et al. 1994; Rugers & Hogan 1996) and for the lower D abundance inferred for others (Tytler et al. 1996; Burles & Tytler 1996). It is clear from Figure 3 that our results here favor the high-η, low-D choice, which is consistent with local deuterium observations (Linsky et al. 1993) and Galactic chemical evolution (Steigman & Tosi 1992, 1995; Edmunds 1994; Tosi 1996). BBN consistency with the observed lithium abundances in very metal-poor halo stars requires that these stars have reduced their surface lithium abundance by a modest factor, $\lesssim$2–3. However, for consistency with standard BBN predictions for helium, our high value for η requires that the abundances inferred from the low-metallicity, extragalactic H II regions are systematically biased low. This high-η range is consistent with estimates of the baryon density derived from observations of the Lyα forest (Hernquist et al. 1996; Miralda-Escudé et al. 1996; Rauch et al. 1997; Croft et al. 1997; Weinberg et al. 1997; Bi & Davidsen 1997).

3.3. *CDM: More Variations*

Because we dropped the cluster-determined constraint $\Omega_o = 0.2 \pm 0.1$, it is of interest to see how the CRs in Figure 1 are affected if we apply instead an alternative constraint to Ω_M. If, for example, we adopt the Dekel-Rees constraint, $\Omega_{\rm DR}$ (eq. [4]), which implies a substantial contribution to Ω_M arising from mass not traced by light, this does not change Figure 1 by much because $\Omega_M > 0.4$ was favored already by our four standard constraints. Small h and large Ω_M are now favored slightly more. Because this makes little difference, we will proceed in most cases without any constraint Ω_M. The consequences for η are found in Table 2.

Figure 7, the analog of Figure 1, shows the effects of some other variations, taken one at a time. Here we consider only

TABLE 2

VARIATIONS: BEST-FIT VALUES AND ERRORS FOR η_{10}

Variation	η_{10}
With Ω_{DR}	$9.2^{+2.2}_{-1.5}$ (>6.5)
"Red" tilt $n = 0.8$	$10.8^{+3.5}_{-2.0}$ (>7.3)
Positive gas bias $\Upsilon = 1.3$......	$5.7^{+1.2}_{-0.9}$ (>4.0)
Without Γ; with Ω_{CL}	3.1 ± 1.6 (<6.5)
$\Gamma = 0.25 \pm 0.05$	$8.2^{+3.2}_{-2.2}$ (>4.2)
$\Gamma = 0.15 \pm 0.04$	$4.6^{+1.9}_{-1.2}$ (2.1–9.2)

NOTE.—Error bars are for 68% CI and range in parentheses is for 95%.

$\Lambda = 0$ models and show only the 95% CRs. The corresponding likelihoods for η are shown in Figure 8. Tilt in the primordial spectrum, for example, has been investigated in many papers (Liddle et al. 1996a, 1996b; White et al. 1996; Kolatt & Dekel 1997; White & Silk 1996; Liddle & Viana 1997). We show the effect of a moderate "red tilt" ($n = 0.8$ instead of $n = 1$). The χ^2_{min}-value is 1.5 for 1 dof (78% CL). The favored likelihood range for η_{10} is now higher, although $\eta_{10} \approx 7$ is still allowed (see Fig. 8). With this tilt the Γ constraint favors higher Ω_M, so that the SCDM model is allowed for h up to nearly 0.5. However, as can be seen in Table 2, the higher allowed range for η threatens the consistency of BBN. Conversely, a "blue" tilt, $n > 1$ (Hancock et al. 1994), would move the CR downward and allow models with $\Omega_M \leq 0.3$ at high h.

Changing to a gas enhancement factor $\Upsilon = 1.3$ (a modest positive enhancement of gas in clusters) instead of 0.9 does not change the contours in Figure 1 by much, particularly at the low-Ω_M end, where the exponential term in Γ is close to unity.

The χ^2_{min}-value for this case is 1.1 for 1 dof (71% CL). Although the acceptable range for η_{10} moves downward (see Fig. 8), $\eta_{10} \leq 4$ is still excluded, disfavoring the high D

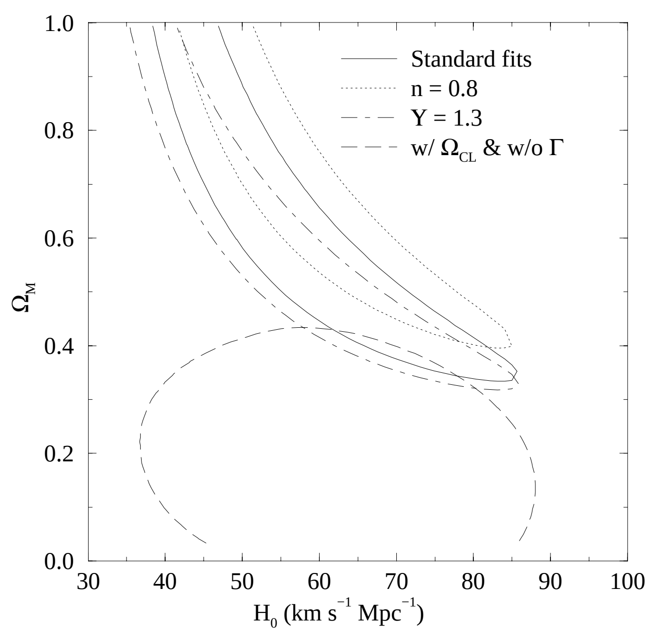

FIG. 7.—Same as Fig. 1 for variations on our CDM results ($\Lambda = 0$) for four standard constraints. The variations are taken one at a time. Only 95% CRs are shown. The result from Fig. 1 (no variation; *solid curve*); "red" tilt $n = 0.8$ (instead of $n = 1$; *dotted curve*); gas enhancement factor $\Upsilon = 1.3$ (instead of 0.9; *dot-dashed curve*); without the Γ constraint *and* with the cluster constraint $\Omega_0 = 0.2 \pm 0.1$ (*dashed curve*).

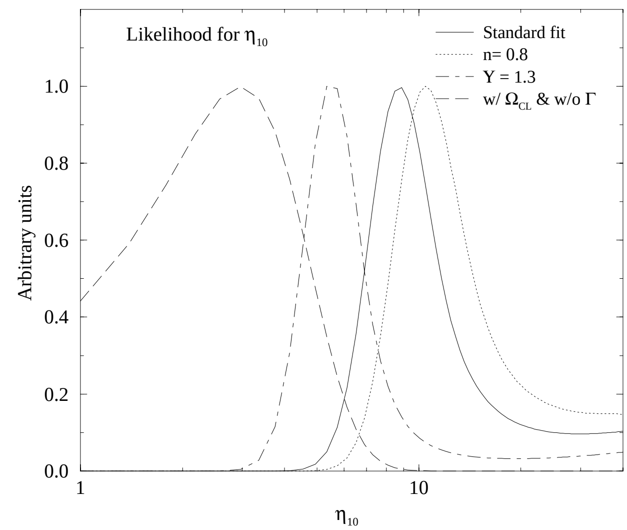

FIG. 8.—Likelihoods $\mathscr{L}(\eta_{10})$ as a function of η_{10} for the CDM models ($\Lambda = 0$) with the same variations as in Fig. 7.

abundance inferred from some QSO absorbers and favoring a higher helium abundance than is revealed by the H II region data. In § 2.6 we mentioned the possibility that the fraction of cluster mass in baryons in galaxies, isolated stars, and machos (f_{GAL}) might be larger—even much larger—than is implied by equation (8). Equations (8) and (9) show that a large f_{GAL} would affect the CRs in much the same way as a *small* Υ, favoring even higher values of Ω_M and η_{10}.

The Γ constraint is crucial for our standard results favoring high Ω_M and high η. If, for example, we drop the Γ constraint and in its place use the cluster estimate $\Omega_o = 0.2 \pm 0.1$, low Ω_M and low η are now favored (see Figs. 7 and 8).

Earlier we mentioned that the Peacock & Dodds (1994) estimate of Γ_o may have unrealistically small error bars. Given that the shape parameter plays such an important role in our analysis, we have considered the effects of relaxing the uncertainty in Γ_o. In Figures 9 and 10, the analogs of Figures 1 and 2, we show our results for $\Gamma_o = 0.25 \pm 0.05$. As expected our CRs have expanded and the best-fit values of Ω_M, h, and η_{10} have shifted: $\Omega_M = 0.48^{+0.22}_{-0.15}$, $h = 0.58 \pm 0.22$, and $\eta_{10} = 8.2^{+3.2}_{-2.2}$. Now the SCDM model with $\Omega_M = 1$, $h = 0.45$, and $\eta_{10} = 13$ is acceptable (80%). Although the uncertainties are larger, low η_{10} is still disfavored. If we add the Dekel-Rees estimate of Ω_M, the five-constraint fit favors somewhat higher values of Ω_M and η_{10} and slightly lower values of h. In contrast, if instead we include the cluster estimate, we find a barely acceptable fit ($\chi^2_{min} = 5.0$ for 2 dof, 92% CL), which favors lower values of Ω_M and η_{10} and slightly higher values of h.

3.4. *CDM: A Smaller Shape Parameter*

The results above show that the Peacock-Dodds shape parameter $\Gamma_o \approx 0.255$, which we have used, clashes with $\Omega_M \approx 0.2$ (the estimate from clusters). The agreement does not become good even if error bars ± 0.05 on Γ_o are assumed.

A new determination of Γ_o from the *IRAS* redshift survey (Webster et al. 1998) gives $\Gamma_o = 0.15 \pm 0.08$ at 95% confidence. Assuming that the statistics are roughly Gaussian, we can represent this at $\pm 1\ \sigma$ as

$$\Gamma_o = 0.15 \pm 0.04 . \quad (16)$$

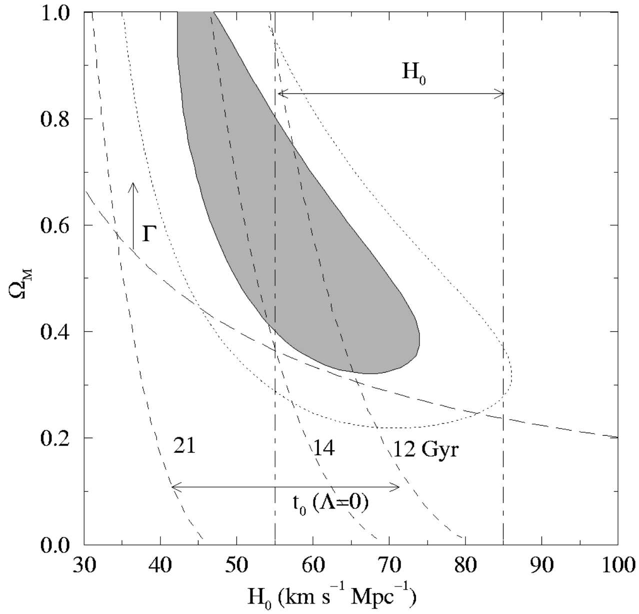

FIG. 9.—Same as Fig. 1, but with the shape-parameter constraint $\Gamma_o = 0.25 \pm 0.05$ (instead of 0.255 ± 0.017).

Combining equation (16) and equation (12) (used earlier) in quadrature would give $\Gamma_o = 0.239 \pm 0.016$, very close to equation (12). Throwing in equation (14) would bring us even closer to equation (12), which dominates because of its small error. Combining the estimates would be unwise, because they do not agree well.

This small value from the *IRAS* survey is not entirely new (cf. Fisher, Scharf, & Lahav 1994). It has received little attention because the larger value, $\Gamma_o \approx 0.25$, was already seen as a major challenge to the popular SCDM model ($\Omega_M = 1$). The smaller value poses an even more severe challenge to the SCDM model. But it gives more scope to low-density models, which are popular now. Until the

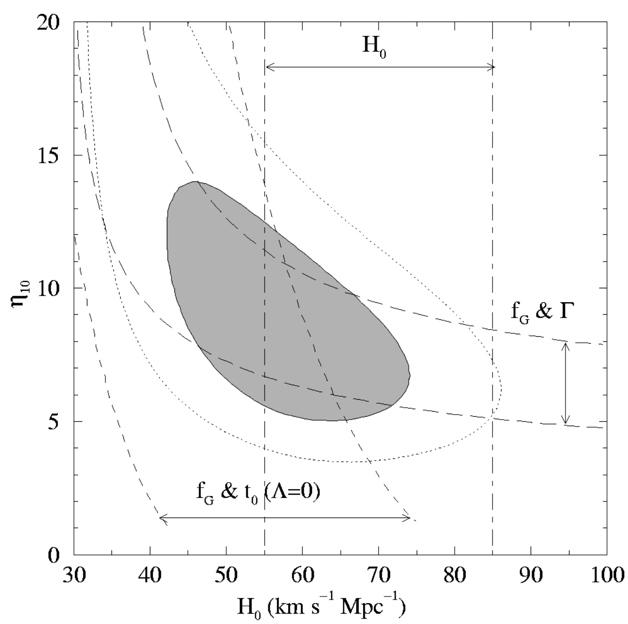

FIG. 10.—Same as Fig. 2, but with the shape-parameter constraint $\Gamma_o = 0.25 \pm 0.05$ (instead of 0.255 ± 0.017).

discrepant values of Γ_o are understood, we think it wise to show joint CRs using separately the larger and the smaller values of Γ_o.

Figures 11 and 12, analogs of Figures 1 and 2, show CRs for our four standard constraints, but with $\Gamma_o = 0.255 \pm 0.017$ replaced by $\Gamma_o = 0.15 \pm 0.04$. The $\chi^2_{\rm min}$ is 0.63 for 1 dof (good) and is located at $(h, \Omega_M, \eta_{10}) = (0.60, 0.30, 4.6)$.

The CRs now exclude the SCDM model strongly and favor low density. The value $\eta_{10} \approx 5$, favored by the Burles & Tytler (1997, 1998a, 1998b) deuterium abundance determination (see § 1), is now near the point of optimum fit. The CRs clearly would accept the added cluster constraint $\Omega_M \approx 0.2$ if we were to apply it. But note that in our three-

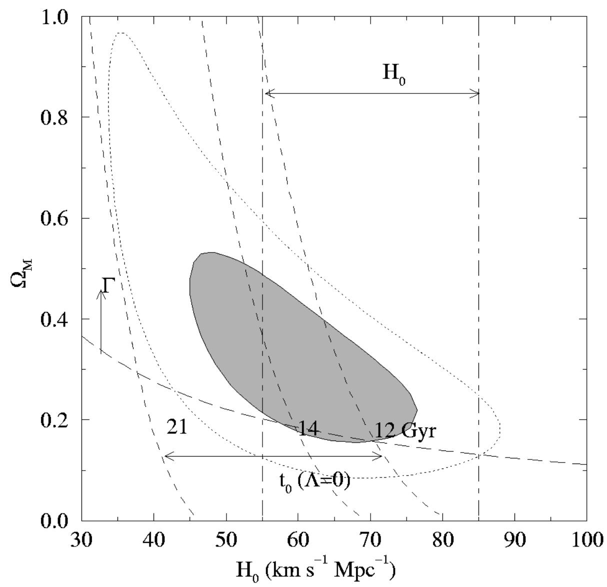

FIG. 11.—Same as Fig. 1, but with the shape-parameter constraint $\Gamma_o = 0.15 \pm 0.04$ (instead of 0.255 ± 0.017).

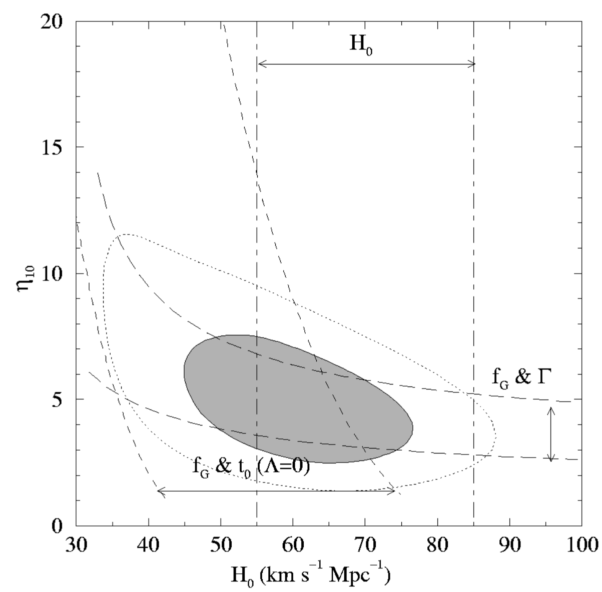

FIG. 12.—Same as Fig. 2, but with the shape-parameter constraint $\Gamma_o = 0.15 \pm 0.04$ (instead of 0.255 ± 0.017).

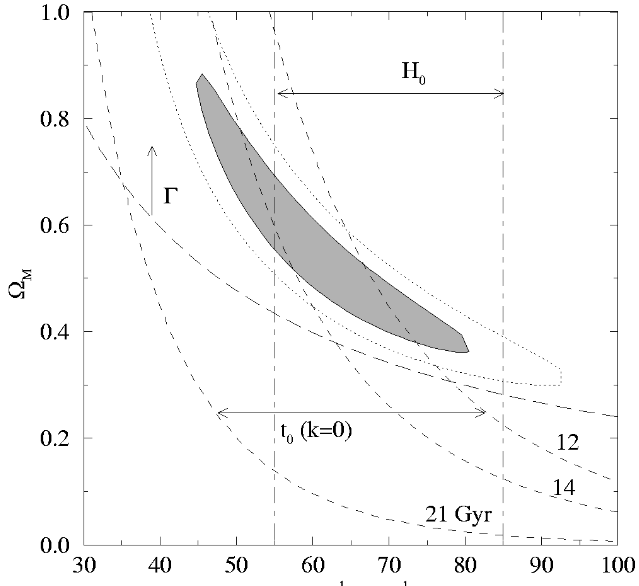

FIG. 13.—Same as Fig. 1, but for ΛCDM (flat) models

dimensional CRs, low Ω_M goes with low η_{10}, because of the f_G constraint. For example, the combinations (0.7, 0.2, 3) and (0.7, 0.3, 5) give good fits, while (0.7, 0.2, 5) and (0.7, 0.3, 3) give poor fits. In general, the CRs give some preference to $\eta_{10} \approx 5$. But even a value as small as $\eta_{10} \approx 2$ now lies within the 95% CI and cannot be excluded without BBN evidence (see Table 2).

4. ΛCDM MODELS: RESULTS

Turning to models with nonzero Λ, we consider here only the popular flat ($k = 0$) "ΛCDM" models with $\Omega_\Lambda = 1 - \Omega_M$, where $\Omega_\Lambda \equiv \Lambda/(3H_0^2)$. This means that there are still only three free parameters. The five constraints discussed in § 2 are still in force, except that the product of the age and the Hubble parameter is a different function of $\Omega_M = 1 - \Omega_\Lambda$: $t = 9.78h^{-1}f(\Omega_M; k = 0)$ Gyr (Carroll, Press, & Turner 1992, eq. [17]). For a given $\Omega_M < 1$, the age is longer for the flat ($k = 0$) model than for the $\Lambda = 0$ model. Figures 13 and 14 show the results for our four standard constraints, with no direct Ω_M constraint. Figures 13 and 14 differ very little from Figures 1 and 2, of which they are the analogs. The longer ages do allow the CRs to slide farther down toward large h and small Ω_M. The $\chi^2_{\rm min}$ is 0.8 for 1 dof (good). One-dimensional CIs are listed in the third column of Table 1. Because of the longer ages at low Ω_M (high Ω_Λ), we can now accept Ω_o as a fifth constraint (although we remind the reader that the constraint $\Omega \approx 0.2$ may not be appropriate to a ΛCDM model [§ 2.5]); the $\chi^2_{\rm min}$ is 5.4 for 2 dof (93%, barely acceptable). In this case large Ω_M and small h are now excluded while $\eta_{10} > 4$ is still favored strongly.

We have not imposed any direct constraint on Ω_Λ. There are claims that, for a ΛCDM model, $\Omega_\Lambda < 0.51$ (based on limited statistics of seven supernovae; Perlmutter et al. 1997) and $\Omega_\Lambda < 0.66$ (based on a paucity of lensing events; Kochanek 1996). The lensing constraint has been in dispute because of absorption, but recent work indicates that absorption is probably unimportant (Kochanek 1996; Falco, Kochanek, & Munoz 1997). These Ω_Λ constraints agree in a general way with our result $\Omega_M \gtrsim 0.4$ (Fig. 13).

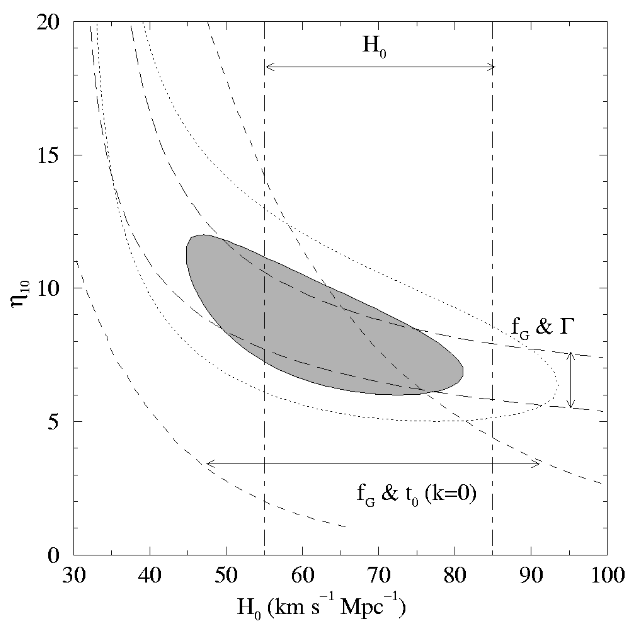

FIG. 14.—Same as Fig. 2, but for ΛCDM (flat) models

5. CONCLUSIONS

If BBN constraints on the baryon density are removed (or relaxed), the interaction among the shape-parameter (Γ) constraint, the f_G (cosmic baryon fraction) constraint, and the value of η_{10} assumes critical importance. These constraints still permit a flat CDM model, but only as long as $h < 0.5$ is allowed by observations of h. The f_G constraint means that large Ω_M implies fairly large Ω_B. Therefore, the exponential term in Γ becomes important, allowing $\Omega_M = 1$ to satisfy the Γ constraint. Values of $\eta_{10} \approx$ 8–15 are required (Fig. 4). The best-fit SCDM model has $h \approx 0.43$ and $\eta_{10} \approx 13$, which is grossly inconsistent with the predictions of BBN and the observed abundances of D, ^{4}He, and ^{7}Li. For $h > 0.5$ a fit to SCDM is no longer feasible (Fig. 4). The SCDM model is severely challenged.

The Γ and age constraints also challenge low-density CDM models. The Γ constraint permits $\Omega_M < 0.4$ only for high h, while the age constraint forbids high h, so $\Omega_M \gtrsim 0.4$ is required. Values $\eta_{10} \gtrsim 6$ are favored strongly over $\eta_{10} \lesssim 2$. The bound $\Omega_M \gtrsim 0.4$ conflicts with the added cluster constraint $\Omega_o = 0.2 \pm 0.1$ at the 98% CL, suggesting strongly that there is additional mass not traced by light.

Although a few plausible variations on the CDM models do not affect the constraints very much (Figs. 7–10), removing the Γ constraint would have a dramatic effect. Both high and low values of Ω_M would then be permitted.

Adopting a smaller observed $\Gamma_o \approx 0.15$ from the *IRAS* redshift survey also makes a difference. Values $\Omega_M \approx 0.3$ and $\eta_{10} \approx 5$ are then favored, but even $\eta_{10} \approx 2$ is not excluded.

At either low or high density, the situation remains about the same for the ΛCDM models (Figs. 13 and 14). Because the ages are longer, we can tolerate $\Omega_M \approx 0.3$ for $h = 0.85$. The ΛCDM model therefore accepts (barely) the added constraint $\Omega_o = 0.2 \pm 0.1$ at the 7% CL, even with the larger $\Gamma_o \approx 0.255$. Improved future constraints on Ω_Λ will come into play here.

Having bounded the baryon density using data independent of constraints from BBN, we may explore the consequences for the light element abundances. In general, our fits favor large values of η_{10} ($\gtrsim 5$) over small values ($\lesssim 2$). While such large values of the baryon density are consistent with estimates from the Lyα forest, they may create some tension for BBN. For deuterium there is no problem, since for $\eta_{10} \gtrsim 5$ the BBN-predicted abundance, $(\mathrm{D/H})_P \lesssim 4 \times 10^{-5}$ ($2\ \sigma$), is entirely consistent with the low abundance inferred for some of the observed QSO absorbers (Tytler et al. 1996; Burles & Tytler 1996; Burles & Tytler 1997, 1998a, 1998b). Similarly, the BBN-predicted lithium abundance, $(\mathrm{Li/H})_P \gtrsim 1.7 \times 10^{-10}$, is consistent with the observed surface lithium abundances in the old, metal-poor stars (including, perhaps, some minimal destruction or dilution of the prestellar lithium). However, the real challenge comes from ^{4}He where the BBN prediction for $\eta_{10} \gtrsim 5$, $Y_P \gtrsim 0.246$ ($2\ \sigma$), is to be contrasted with the H II region data that suggest $Y_P \lesssim 0.238$ (OS; OSS).

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While preparing the final version of our manuscript, we saw the paper by Lineweaver & Barbosa (1998), which has some overlap with our work. Their analysis in § 4 has some similarities to our calculations, but there are some important differences. For example, they have only two free parameters since η_{10} is incorporated by a questionable procedure relying on BBN and is not free, and they use a different theoretical expression and different error bars for the shape parameter Γ. Their §§ 1–3 are of more interest, since they apply entirely different constraints from the CMB angular power spectrum. It is gratifying that their resulting CRs, based on independent data, are rather similar to ours (e.g., compare their Fig. 3 with our Fig. 9).

REFERENCES

Bahcall, N. A., Lubin, L. M., & Dorman, V. 1995, ApJ, 447, L81

Bardeen, J. M., Bond, J. R., Kaiser, N., & Szalay, A. S. 1986, ApJ, 304, 15

Barnett, R. M., et al. 1996, Phys. Rev. D, 54, 27

Bevington, P. R., & Robinson, D. K. 1992, Data Reduction and Error Analysis for the Physical Sciences (2d ed.; New York: McGraw-Hill), § 11.5

Bi, H., & Davidsen, A. F. 1997, ApJ, 479, 523

Bludman, S. A. 1997, ApJ, submitted (astro-ph/9706047)

Bolte, M., & Hogan, C. J. 1995, Nature, 376, 399

Borgani, S., et al. 1997, New Astron., 1, 321

Briel, U. G., Henry, J. P., & Böhringer, H. 1992, A&A, 259, L31

Bureau, M., Mould, J. R., & Staveley-Smith, L. 1996, ApJ, 463, 60

Burles, S., & Tytler, D. 1996, ApJ, 460, 584

———. 1997, AJ, 114, 1330

———. 1998a, ApJ, 499, 699

———. 1998b, ApJ, in press

Carlberg, R. G., Yee, H. K. C., & Ellingson, E. 1997, ApJ, 478, 462

Carlberg, R. G., Yee, H. K. C., Ellingson, E., Abraham, R., Gravel, P., Morris, S., & Pritchet, C. J. 1996, ApJ, 462, 32

Carr, B. 1993, Nature, 363, 16

Carroll, S. M., Press, W. H., & Turner, E. L. 1992, ARA&A, 30, 499

Carswell, R. F., Rauch, M., Weymann, R. J., Cooke, A. J., & Webb, J. K. 1994, MNRAS, 268, L1

Cen, R. 1997, ApJ, 485, 39

Cen, R., & Ostriker, J. P. 1993, ApJ, 417, 404

Chaboyer, B., Demarque, P., Kernan, P. J., & Krauss, L. M. 1998, ApJ, 494, 96

Chabrier, G., Segretain, L., & Méra, D. 1996, ApJ, 468, L21

Copi, C. J., Schramm, D. N., & Turner, M. S. 1995a, Science, 267, 192

———. 1995b, Phys. Rev. Lett., 75, 3981

Cowan, J. J., McWilliam, A., Sneden, C., & Burris, D. L. 1997, ApJ, 480, 246

Cramer, H. 1946, Mathematical Methods of Statistics (Princeton: Princeton Univ. Press), § 30.3 et passim

Croft, R. A. C., Weinberg, D. H., Katz, N., & Hernquist, L. 1997, ApJ, 488, 532

D'Antona, F., Caloi, V., & Mazzitelli, I. 1997, ApJ, 477, 519

Dar, A. 1995, ApJ, 449, 550

Dekel, A. 1997, in Galaxy Scaling Relations: Origins, Evolution and Applications, ed. L. N. da Costa & A. Renzini (Berlin: Springer), 245

Dekel, A., & Rees, M. J. 1994, ApJ, 422, L1

Edmunds, M. G. 1994, MNRAS, 270, L37

Efstathiou, G., Ellis, R. S., & Peterson, B. A. 1988, MNRAS, 232, 431

Evrard, A. E. 1997, MNRAS, 292, 289

Evrard, A. E., Metzler, C. A., & Navarro, J. F. 1996, ApJ, 469, 494

Fabian, A. C. 1991, MNRAS, 253, 29P

Falco, E. E., Kochanek, C. S., & Munoz, J. M. 1998, ApJ, 494, 47

Fan, X., Bahcall, N. A., & Cen, R. 1997, ApJ, 490, L123

Felten, J. E. 1996, in ASP Conf. Ser. 88, Clusters, Lensing, and the Future of the Universe, ed. V. Trimble & A. Reisenegger (San Francisco: ASP), 271

Fields, B. D., Kainulainen, K., Olive, K. A., & Thomas, D. 1996, New Astron., 1, 77

Fields, B. D., Mathews, G. J., & Schramm, D. N. 1997, ApJ, 483, 625

Fisher, K. B., Scharf, C. A., & Lahav, O. 1994, MNRAS, 266, 219

Fixsen, D. J., Cheng, E. S., Gales, J. M., Mather, J. C., Shafer, R. A., & Wright, E. L. 1996, ApJ, 473, 576

Gould, A. 1995, ApJ, 455, 44

Gratton, R. G., Fusi Pecci, F., Carretta, E., Clementini, G., Corsi, C. E., & Lattanzi, M. 1997, ApJ, 491, 749

Hancock, S., Davies, R. D., Lasenby, A. N., Gutierrez de la Cruz, C. M., Watson, R. A., Rebolo, R., & Beckman, J. E. 1994, Nature, 367, 333

Hata, N., Scherrer, R. J., Steigman, G., Thomas, D., & Walker, T. P. 1996, ApJ, 458, 637

Hata, N., Scherrer, R. J., Steigman, G., Thomas, D., Walker, T. P., Bludman, S., & Langacker, P. 1995, Phys. Rev. Lett., 75, 3977

Hata, N., Steigman, G., Bludman, S., & Langacker, P. 1997, Phys. Rev. D, 55, 540

Hernquist, L., Katz, N., Weinberg, D. H., & Miralda-Escudé, J. 1996, ApJ, 457, L51

Hogan, C. J. 1997, in Proc. 18th Texas Symposium on Relativistic Astrophysics, in press (astro-ph/9702044)

Hu, W., & Sugiyama, N. 1996, ApJ, 471, 542

Izotov, Y. I., Thuan, T. X., & Lipovetsky, V. A. 1994, ApJ, 435, 647

———. 1997, ApJS, 108, 1

Jimenez, R. 1997, Invited Lecture at the Cosmology School in Casablanca 97 (astro-ph/9701222)

Kolatt, T., & Dekel, A. 1997, ApJ, 479, 592

Kochanek, C. S. 1996, ApJ, 466, 638

Kundić, T., et al. 1997, ApJ, 482, 75

Liddle, A. R., Lyth, D. H., Roberts, D., & Viana, P. T. P. 1996a, MNRAS, 278, 644

Liddle, A. R., Lyth, D. H., Viana, P. T. P., & White, M. 1996b, MNRAS, 282, 281

Liddle, A. R., & Viana, P. T. P. 1997, in Proc. International Workshop on Dark Matter in Astro- and Particle Physics: Dark '96 (Heidelberg, 1996 September), ed. H. V. Klapdor-Kleingrothaus & Y. Ramachers (Singapore: World Scientific), 147

Lineweaver, C. H., & Barbosa, D. 1998, ApJ, 496, 624

Lineweaver, C. H., Barbosa, D., Blanchard, A., & Bartlett, J. G. 1997, A&A, 322, 365

Linsky, J. L., et al. 1993, ApJ, 402, 694

Loeb, A., & Mao, S. 1994, ApJ, 435, L109

Loewenstein, M., & Mushotzky, R. F. 1996, ApJ, 471, L83

Lubin, L. M., Cen, R., Bahcall, N. A., & Ostriker, J. P. 1996, ApJ, 460, 10

Maddox, S. J., Efstathiou, G., & Sutherland, W. J. 1996, MNRAS, 283, 1227

Martel, H., Shapiro, P. R., & Weinberg, S. 1998, ApJ, 492, 29

Miralda-Escudé, J., Cen, R., Ostriker, J. P., & Rauch, M. 1996, ApJ, 471, 582

Mohr, J. J., Geller, M. J., & Wegner, G. 1996, AJ, 112, 1816

Mushotzky, R. 1993, in Ann. NY Acad. Sci., 688, Texas/PASCOS '92: Relativistic Astrophysics and Particle Cosmology, ed. C. W. Akerlof & M. A. Srednicki (New York: New York Acad. Sci.), 184

Myers, S. T., Baker, J. E., Readhead, A. C. S., Leitch, E. M., & Herbig, T. 1997, ApJ, 485, 1

Natarajan, P., Kneib, J.-P., Smail, I., & Ellis, R. S. 1997, ApJ, 499, 600

Nittler, R. L., & Cowsik, R. 1997, Phys. Rev. Lett., 78, 175

Olive, K. A., Skillman E. D., & Steigman, G. 1997, ApJ, 483, 788 (OSS)
Olive, K. A., & Steigman, G. 1995, ApJS, 97, 49 (OS)
Olive, K. A., & Thomas, D. 1997, Astropart. Phys., 7, 27
Ostriker, J. P., & Steinhardt, P. J. 1995, Nature, 377, 600
Peacock, J. A. 1997, MNRAS, 284, 885
Peacock, J. A., & Dodds, S. J. 1994, MNRAS, 267, 1020
Perlmutter, S., et al. 1997, ApJ, 483, 565
Pont, F., Mayor, M., Turon, C., & VandenBerg, D. A. 1998, A&A, 329, 87
Prantzos, N. 1996, A&A, 310, 106
Press, W. H., Teukolsky, S. A., Vetterling, W. T., & Flannery, B. P. 1992, Numerical Recipes in Fortran: The Art of Scientific Computing (2d ed.; Cambridge: Cambridge Univ. Press), § 15.6
Rauch, M., Haehnelt, M. G., & Steinmetz, M. 1997, ApJ, 481, 601
Reid, I. N. 1997, AJ, 114, 161
Rugers, M., & Hogan, C. J. 1996, ApJ, 459, L1
Smail, I., Ellis, R. S., Dressler, A., Couch, W. J., Oemler, A., Jr., Sharples, R. M., & Butcher, H. 1997, ApJ, 479, 70
Smith, M. S., Kawano, L. H., & Malaney, R. A. 1993, ApJS, 85, 219
Songaila, A. 1997, in Structure and Evolution of the Intergalactic Medium from QSO Absorption Line Systems, ed. P. Petitjean & S. Charlot (Paris: Editions Frontières), 339
Songaila, A., Cowie, L. L., Hogan, C. J., & Rugers, M. 1994, Nature, 368, 599
Songaila, A., Wampler, E. J., & Cowie, L. L. 1997, Nature, 385, 137
Steigman, G., & Felten, J. E. 1995, Space Sci. Rev., 74, 245
Steigman, G., & Tosi, M. 1992, ApJ, 401, 150
———. 1995, ApJ, 453, 173
Sugiyama, N. 1995, ApJS, 100, 281
Tammann, G. A., & Federspiel, M. 1997, in The Extragalactic Distance Scale, ed. M. Livio, M. Donahue, & N. Panagia (Cambridge: Cambridge Univ. Press), 137
Tonry, J. L., Blakeslee, J. P., Ajhar, E. A., & Dressler, A. 1997, ApJ, 475, 399
Tosi, M. 1996, in ASP Conf. Ser. 98, From Stars to Galaxies: The Impact of Stellar Physics on Galaxy Evolution, ed. C. Leitherer, U. Fritze-von Alvensleben, & J. Huchra (San Francisco: ASP), 299
Trimble, V. 1987, ARA&A, 25, 425
Tytler, D., Burles, S., & Kirkman, D. 1997, ApJ, submitted (astro-ph/9612121)
Tytler, D., Fan, X.-M., & Burles, S. 1996, Nature, 381, 207
van der Marel, R. P. 1991, MNRAS, 253, 710
Walker, T. P., Steigman, G., Schramm, D. N., Olive, K. A., & Kang, H.-S. 1991, ApJ, 376, 51
Wampler, E. J. 1996, Nature, 383, 308
Webb, J. K., Carswell, R. F., Lanzetta, K. M., Ferlet, R., Lemoine, M., Vidal-Madjar, A., & Bowen, D. V. 1997, Nature, 388, 250
Webster, M., Hobson, M. P., Lasenby, A. N., Lahav, O., & Rocha, G. 1998, ApJ, submitted (astro-ph/9802109v1)
Weinberg, S. 1972, Gravitation and Cosmology (New York: Wiley)
Weinberg, D. H., Miralda-Escudé, J., Hernquist, L., & Katz, N. 1997, ApJ, 490, 564
White, D. A., & Fabian, A. C. 1995, MNRAS, 273, 72
White, S. D. M., & Frenk, C. S. 1991, ApJ, 379, 52
White, S. D. M., Navarro, J. F., Evrard, A. E., & Frenk, C. S. 1993, Nature, 366, 429
White, M., & Silk, J. I. 1996, Phys. Rev. Lett., 77, 4704 (erratum 78, 3799)
White, M., Viana, P. T. P., Liddle, A. R., & Scott, D. 1996, MNRAS, 283, 107
Yang, J., Turner, M. S., Steigman, G., Schramm, D. N., & Olive, K. A. 1984, ApJ, 281, 493
Zaroubi, S., Sugiyama, N., Silk, J., Hoffman, Y., & Dekel, A. 1997a, ApJ, 490, 473
Zaroubi, S., Zehavi, I., Dekel, A., Hoffman, Y., & Kolatt, T. 1997b, ApJ, 486, 21