Modifying the McKenzie Stretching Theory for Sedimentary Basins to Account for the Depth Dependence of Sediment Density

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ABSTRACT

The McKenzie stretching theory is a simple model for the evolution of sedimentary basins such as basins underlying most continental shelves. McKenzie's model explains basin subsidence as the isostatic response to and subsequent cooling of the lithosphere. In addition to the stretching factor, and the initial thickness of the continental crust, the history of subsidence also depends on the average density of the basin fill (e.g., water or sediment). McKenzie's model requires prior specification of fill density, but it really depends only on the vertical average density of fill. In reality, sediment density varies with depth as it compacts in response to burial. To simulate a varying density, a simple mathematical model for density based on changes in porosity is proposed to be inserted into the McKenzie model. Only after the testing to be sure that the new model matches McKenzie's original findings can the preliminary investigation into the case study commence so as to compare the newly created model with trusted observations. The model we adopt to characterize the density of sedimentary fill is simplistic: we assume fill density is purely a function of depth. But it is a more general formula than McKenzie's, and provides an approximate basis for accommodating spatial (and temporal) variation in sediment density.

INTRODUCTION

The proposal for the evolution of sedimentary basin subsidence due to large scale regional lithospheric stretching inducing subsidence was introduced by McKenzie in his 1978 paper titled "Some Remarks on the Development of Sedimentary Basins". His simple model for the development of basins such as the Great Basin, the North Sea, and the Michigan Basin commenced with large scale "extensive normal faulting and subsidence" was how McKenzie [1] came about using the stretching of the continental crust to model this evolution. It is through this proposed stretching regime that McKenzie [1] provided an alternative to the model of Haxby et al. [2] which used phase changes and mantle diapirs to "intrude and replace the lower part of the lithosphere with rock without…major deformation at the surface".

Prior to McKenzie [1] the majority of models regarding sedimentary basin evolution focused primarily on mating the results gathered during the characterization of oceanic lithosphere mechanics. Haxby et al. [2] did assume like McKenzie [1] that subsidence was caused by lithospheric loading, however Haxby attempted to place a then geologically undiscovered mechanism as to why loading occurred, the intrusions from the asthenosphere shown in Haxby et al. [2] Figure 10, or Figure 22 in this paper. Due to the findings of McKenzie [1] not agreeing with the findings of Haxby et al. [2], the main emphasis of this study will be based upon McKenzie's model for subsidence due to large area stretching.

McKenzie [1] stated that his results, Figure 21, were quantified using a stand-alone fixed density of 2.5 g cm^{-3} in order to allow for the only parameter to change to be the amount of stretching the basin has experienced, or β. His model leaves plenty to be desired and so it was determined that a simple model for density change could be constructed to observe the effects that a varying density such as those present throughout all the basins mentioned by McKenzie would have on his model. The simplest form of density change was first envisioned to be through the acquisition of proprietary drill logs or data from drill core, yet this was later abandoned due to the projected expense and rarity of data. So it was decided that a simple mathematical model for varying density could be obtained by modifying the porosity equation proposed by Athy [3], and used after by the likes of Hoholick et al. [4] to some success, and the equation for bulk density that is a function of porosity. Seeing as this manipulation could potentially lead to a better characterization of a sedimentary basin's subsidence history and so the following mathematical modeling was then undertaken.

To characterize how a basin would look compared to the potential model, a specific sedimentary basin and geologic formation within that basin were necessary in order for use as a case study comparison. It was for this reason that the Michigan Basin was chosen, due to its mention within McKenzie [1], and the availability of an already proposed model for porosity change within the bottom most sedimentary layer of the Michigan Basin, the Mt. Simon Sandstone, within Hoholick et al.[4]. The Mt. Simon

Sandstone as mentioned in Hoholick et al. [4] is the basal feldspathic sandstone directly above the underlying basement rock found in the Michigan Basin and in the Illinois Basin where Hoholick's main observations were made. The model presented herein, the Mt. Simon was assumed to be perfectly sorted pure quartz sandstone in order to produce a best fit situation for the density model.

Goals and Objectives

The overall goal of this research was to modify the basin evolution model created by McKenzie in his 1978 paper, "Some Remarks on the Development of Sedimentary Basins" to account for the varying sediment fill density created through increasing burial and compaction throughout a basin's history. As McKenzie's model was created using only a single sediment fill density it was hypothesized that a density that increases throughout the same time and depth frame would increase the subsidence effects seen in McKenzie's original findings, most significantly with regard to the Post Stretching Subsidence curve. In order to be able to view this effect, a mathematical model that achieved the same results as McKenzie was first needed to be forged and programmed in order to show that his results were achievable. Second, a sedimentary basin, in this case the Michigan Basin, had to be chosen in order to acquire boundary conditions to be used for the parameters of a new mathematically created density model. The basis of this new model would need to use previously published and widely available porosity versus depth models such as that created by Athy [3] and furthered by Hoholick et al. [4] so as to not be reliant upon proprietary and expensive density data that could be provided by industry sources. Once these first two objectives were completed, the modification of McKenzie's model via programming the original McKenzie model code to accept the results of the newly created density model would commence. Testing would need to then be done to ensure the new programming code is self-consistent with the old code that replicates McKenzie's results, after which the new modified McKenzie model that accounts for varying density can be generated and analyzed.

METHODS

Replication of McKenzie's Results

The replication of the McKenzie Stretching Model was the first step in the study of the Stretching Theory as the subsidence profile McKenzie [1] Figure 4, or Figure 21, best models what is to be expected of an intra-cratonic basin's subsidence profile. The profile is due to initial thermally induced stretching subsidence Si and post stretching sediment affiliated subsidence St illustrated by Figure 4. The instantaneous stretching by a factor β and subsequent subsidence from cooling and a long term reversal of the then stretched lithosphere to a new steady state thermal regime are illustrated in McKenzie [1] Figure 24. Coupled with the table of McKenzie's original values from Parsons et al. [5], Table 1, for variables such as mantle density, coefficients of thermal expansion and basin fill McKenzie [1] Table 1 are used as the parameters of this model. A visual representation of the boundary conditions in Figure 24 presents a model for the instantaneous stretching of the lithosphere from an initial length of α to length β granting an upwelling of the asthenosphere. Thermal decay of the asthenospheric intrusion produces the initial thermally induced subsidence due to stretching S_i:

$$S_i = \frac{a[(\rho_0 - \rho_C)\frac{t_c}{a}\left(1 - \alpha T_1 \frac{t_c}{a}\right) - \frac{\alpha T_1 \rho_0}{2}](1 - \frac{1}{\beta})}{\rho_0(1 - \alpha T_1) - \rho_W} \tag{1}$$

with the table of McKenzie's original values McKenzie [1]Table 1 from Parsons et al. [5] used for variable parameters such as mantle density ρ_0, plate thickness a, and basin fill density ρ_f. Post stretching subsidence due to sediment or water infilling of the thermally stretched and subsided basin can be characterized by the equation:

$$S_t = e(0) - e(t) \tag{2}$$

where $e(t)$ is the elevation above the final depth to which the upper surface of the lithosphere sinks:

$$e(t) = K\{\sum_{m=0}^{\infty} \frac{1}{(2m+1)^2}[\frac{\beta}{(2m+1)\pi}\sin\frac{(2m+1)\pi}{\beta}]\exp(-\left(2m+1)^2\frac{t}{\tau}\right)\} \tag{3}$$

and variable K, or the constant that combines all of the constants that appear before the summation of McKenzie's equation for e(t):

$$K = \frac{4}{\pi^2}\left(\frac{a\rho_0\alpha T_1}{\rho_0 - \rho_W}\right) \tag{4}$$

It is through the combination of equations (1) and (2) that we get total subsidence, or S:

$$S = S_i + S_t \tag{5}$$

Through the use of MathWorks MATLAB programming language, an algorithm expressed as a function was created in order to calculate equations (1), (2) and (4), and a MATLAB script was assembled in order to best plot the results, resulting in the MATLAB function DEmckenzie78.m (Appendix A), and DEmckenzie78Script.m (Appendix B). DEmckenzie78.m serves to evaluate the initial subsidence "Si" due to thermally induced subsidence, and post-extension subsidence "Sth" given a vector that represents time in millions of years t, a dimensionless stretching factor β, and a parameter structure p that is a consolidation of the initial parameters presented by McKenzie [1]Table 1 into a MATLAB structure p. This method results in only one variable being called for the calculations of equations (1), (2) and (4) instead of eight, and allows DEmckenzie78.m to be run for interchangeable parameters given different values dependent on specific situations. DEmckenzie78.m also calls a private function mckenzie_e.m that evaluates equation (3) separately until convergence given values of K given by the calculation of equation (4) in the parent function, stretching factor β, characteristic time scale τ, and time vector t. This is due to the complexity of the iterations necessary to achieve convergence of the summation and the need to be able to modify its structure without compromising the mechanics and structure of the rest of the parent MATLAB function. Function DEmckenzie78.m is then able to use mckenzie_e.m for the evaluation of equation (2).

Script DEmckenzie78Script.m plots the results of the evaluations produced by function DEmckenzie78.m. The focus of DEmckenzie78Script.m is on the post stretching subsidence profile and the total subsidence profile given the changing β values for respective water and sediment filled situations with each situation given an arbitrary fixed value for the fill density ρ_f that changes depending on whether the basin is water or sediment filled. The values chosen are the density of water and an arbitrary sediment fill density, with values of 1000 kg m^{-3} and 2500 kg m^{-3} respectively. Different stretching values are used to reveal the shape change due to differences in basin mechanical characteristics during initial thermal and post stretching subsidence, with values of 1.25, 1.5, 2, 4, and 10 being given for both the water and sediment filled situations. Time t was chosen as a vector one hundred and twenty one million years long with 500 intervals in order to have enough data points to show a good representation of the subsidence profile, and to have the time scale of figures to be plotted match that of McKenzie [1] Figure 4 or, Figure 21 as the x-axis is shown as $\sqrt{t}$. The entirety of the first half of DEmckenzie78Script replicates McKenzie [1] Figure 4 (Figure 21) using water and arbitrary sediment densities for sediment fill, Figures 1 and 2, while the second half of the script plots figures for the combination of the initial and post stretching subsidence, Figures 3 and 4.

Bevis-Enriquez Density Model

Arbitrary values for density however are not conclusive when dealing with subsidence as the density increases with depth as shown with a density versus depth curve. However, to evaluate the subsidence profile using multiple formations of varying densities was deemed too extensive, instead the focus was chosen to be on one density variable formation. The lack of density versus depth curves widely available for the Michigan Basin, and the proprietary nature of density versus depth data from drill cores shifted the approach to mathematical analysis. Hoholick et al. [4] delves into the porosity and cementation properties of the Cambrian aged Mt. Simon Sandstone found in the Illinois Basin, which is also present in the Michigan Basin as the lowest sedimentary formation. Given that that Mount Simon is the lowest sedimentary formation, it can be assumed that it has experienced the most variability change with continued subsidence and deposition of sediments above it and the assumption that is has experienced the entirety of the subsidence history of the basin as well. The equation given for porosity versus depth of the Mount Simon by Hoholick [3] is given as:

$$\phi = 31.08\,\exp(0.00026\,d) \qquad (6)$$

where ϕ is porosity, 31.08 is the coefficient that represents the initial starting value for porosity as a percentage, and d is depth in feet. This equation is a variation of the Athy [3] equation for porosity:

$$P = p(e^{-bx}) \qquad (7)$$

where P is the porosity, p is the average porosity at the surface, b, the compaction coefficient is a constant and x is the depth of burial Athy [3]. It was assumed that the constant b for the Mount Simon Formation was 0.00026 despite the region of study being the Michigan Basin and not the Illinois, and that the average porosity at the surface p is 36%. This is the statistical average porosity for a randomly packed, perfectly spherical grained, very well sorted sandstone in order to assume the most perfect depositional and matrix conditions that could possibly apply to the Mt. Simon despite it fitting none of these criteria. In order to calculate bulk density dependent on porosity, the equation:

$$\rho_b = \rho_m - \phi(\rho_m - \rho_W) \qquad (8)$$

where ρ_b is bulk density, ρ_m is matrix density, $\rho_W$is the density of the pore fluid which in this case is assumed to be water, and ϕ is porosity.

The MATLAB function densitymodel_BE1.m (Appendix C) evaluates the average density of the fill above the maximum depth of the basin, in this case being the Michigan Basin, with the focus on the M.t Simon Sandstone. However, densitymodel_BE1.m evaluates an empirical model that was inspired by the fact that combining (7) and (8) results in the form:

$$density(z) = c_1 + c_2 \exp(-c_3 z) \qquad (9)$$

where density is a function of depth z, and three coefficients c_1, c_2 and c_3 which are obtainable by performing least squares fitting for any sediment filled basin's density versus depth. While this formula was motivated by previous work on the reduction of porosity with depth, we adopt (9) as our model, and we can us it without reliance on porosity data. This allows for densitymodel_BE1.m to analyze and characterize mathematically any sediment filled basin based on a density versus depth profile and thus negating the need for proprietary drill core data. The coefficients c_1, c_2 and c_3 represent ρ_m, -a (ρ_m-ρ_W), and $\frac{b}{0.3048}$ respectively, where a is the average porosity at the surface, b is the compaction coefficient, and the 0.3048 in the denominator is the conversion coefficient for the conversion of meters to feet as the Athy [6] equation is characterized in feet and not SI units. Equation (9) is then run through a loop using parameters selected for the Michigan Basin case study, those of the Mt. Simon Sandstone, to calculate the average fill density, the results are then plotted by the MATLAB script densitymodel_BE1Script.m (Appendix D), in order to visualize the density versus depth and average density versus depth curves for the Mt. Simon Sandstone (Figures 5 and 6).

Modifying the McKenzie Theory

In order to evaluate the McKenzie stretching theory in terms of variable densities, new code had to be written in order to achieve self-consistency to ensure that McKenzie's theory is being replicated for the Michigan Basin case study. The eventual strategy was not calculating the McKenzie model using code in series, or first Si and then Sth, but in parallel so Si and Sth are calculated separately and simultaneously. This was done to make the functions simpler and faster to calculate and cleaner. The first function, mckenzie78_Si.m (Appendix E), was coded so as to add a loop to the calculation of Equation 1 in order to allow the old calculation found in mckenzie78.m to use a density parameter that varies. When calculated the new loop allows the function to iterate, or calculate over and over, until the change between two different calculation values converges, or reaches equilibrium. Once the function converges the iterations then stop and the loop is exited resulting in final Si which is then used as the Si value, or the initial shift down the Y-Axis in the Total Subsidence vs. the square root of time plots that represents the initial instantaneous thermal subsidence. New function number two, mckenzie78_St.m (Appendix F), was coded using the same logic as mckenzie78_Si.m in that a new loop structure was needed in order to calculate for the varying density. Hence a new loop was incorporated to iterate until convergence and thus produce the desired Post Stretching subsidence profiles seen after the initial translations down the Y-Axis that indicate the thermally induced stretching subsidence. Testing of these new codes was then delegated to scripts that would assess the validity of both newly created functions by showing that the newly created functions achieve self-consistency using the same parameters as DEmckenzie78.m and DEmckenzie78Script.m.

Test Scripts

MATLAB script TestMcKenzie.m (Appendix G), was coded to run both mckenzie78_Si.m and mckenzie78_St.m in tandem using the same parameters present in DEmckenzie78Script.m to plot Figures 7, 8, 9 and 10 as well as Table 3. MATLAB script Test_mckenzie_Si_VD.m (Appendix I), was written to test the iterations, or looped calculations, of mckenzie_Si_VD.m (Appendix H) versus those of TestMcKenzie.m and Table 2 was produced via this script. A new MATLAB script, Test1000_mckenzie_St_VD.m (Appendix J), was then programmed to test the self-consistency of the densitymodel_BE1.m function within the confines of the newly programmed functions mckenzie78_Si.m and mckenzie78_St.m. Test1000_mckenzie_St.VD.m was programmed to create a density vs. depth plot that has an average density of 1000 kg m^{-3} via modifying the "vector c" to produce Figures 11 and 12. A similar MATLAB script, Test2500_mckenzie_St_VD.m (Appendix K), was created to evaluate the same type of case as Test1000_mckenzie_St_VD.m except using a density value of 2500 kg m^{-3} and leaving the rest of the parameters the same. Figures 13 and 14 were produced using Test2500_mckenzie_St.VD.m but a new test of self-consistency was deemed needed to validate the new code as consistent with McKenzie's model.

Figures 15 and 16 were created using MATLAB script Test2_mckenzie_St_VD.m (Appendix L) while Figures 17 and 18 were produced via script Test2A_mckenzie_St_VD.m (Appendix M) in to test how the newly created code behaves when density varies slightly and not heavily such as in Figures 5 and 6. Once all of the testing was performed the actual model of the newly validated McKenzie Model was confirmed to be self-consistent, the Michigan Basin case study using the parameters chosen for the Mt. Simon Sandstone was programmed as a MATLAB script, TestMichigan_mckenzie_St_VD.m (Appendix N). This script then plots Figure 19, the combined Density and Average Density vs. Depth plot using the same parameters as Figures 5 and 6 except with a larger depth interval of 25 km. Figure 20 is the plot of the Total Subsidence vs. the square root of time plot for the Mt. Simon Sandstone parameters for multiple β values.

RESULTS

Test Results

Figures 1 through 4 were produced through the use of DEmckenzie78Script.m so as to replicate the results found in McKenzie [1], as Figure 1 is identical to McKenzie's Figure 4 (Figure 21) which reveals Post Stretching Subsidence. Figures 2, 3 and 4 also produced using DEmckenzie78Script.m represent the replication of McKenzie's original results. Both go several steps further by producing curves based on changing the density value to 2500 kg m^{-3} (Figure 2), and by combining Equations 1 and 3 (Equation 5) to produce curves for Total Subsidence (Figures 3 and 4) that allows a better overall characteristic of the complete picture of McKenzie's model of basin evolution.

Figures 5 and 6 are the results of the Bevis-Enriquez density model, specifically the density curve (Figure 5) and the average density curve (Figure 6) produced using the vector c parameters specified in densitymodel_BE1Script.m. The curves differ, and were expected to differ in their shape. Figure 6 is the density averaged over the entire depth interval thus resulting in a less extreme curve that does not level out so fast near 5000 meters depth.

Once functions mckenzie78_Si.m and mckenzie78_St.m were programmed, testing commenced to ensure that the results they produce agreed with the previously validated results of DEmckenzie78.m and DEmckenzie78Script.m. To accomplish this, MATLAB script TestMcKenzie.m was coded to use the same parameters as DEmckenzie78Script.m, and in doing so Figures 7, 8, 9 and 10 were produced. The results correlate with their identical case present in DEmckenzie78Script.m leading to Figure 7 being the same as Figure 1, Figure 8 being the same as Figure 2, etc. This production of exact replicas of the previous findings ensures self-consistency between the DEmckenzie78.m, mckenzie78_Si.m and mckenzie78_St.m, allowing for the introduction of the Bevis-Enriquez density model, or densitymodel_BE1.m to be used in place of the singular density values previously used 1000 kg m^{-3} and 2500 kg m^{-3} .

The testing phase of the new and self-consistent McKenzie functions coupled with the Bevis-Enriquez density model needed to produce self-consistent results and to ensure that the density model did not significantly affect the characteristic shape of McKenzie's original findings. This was done in multiple ways, the first being nonvisual in the sense that the iterations were focused on before any figures were to be produced to make sure that the two models reached the same solution, thus resulting in Table 2 and Table 3. It can be seen that the script Test_mckenzie_Si_VD.m, a test script with a modified vector c that produces an average density of 1000 kg m^{-3}, and script TestMcKenzie.m both produce the same Si values for the first iteration and first calculation, both resulting in an initial thermal subsidence value of 767.08 meters, ensuring self-consistency between the two versions of code, old and new between the Si functions.

Tests of the coupling between McKenzie's model and the Bevis-Enriquez model were made to ensure that the two combined models could reproduce the original McKenzie results through fixing the vector c so that it is only a single fixed density. Test1000_mckenzie_St_VD.m and Test2500_mckenzie_St_VD.m use a modified parameter for average density via changing the vector c to produce density values of 1000 kg m^{-3} and

2500 kg m^{-3}, producing Figure 11 and Figure 13. Each curve is a straight vertical line showing constant density over the entire depth interval and the resultant Total Subsidence plots. Figure 12, or the Total Subsidence plot created by Test1000_mckenzie_St_VD.m, is shown to match Figure 3 and 9, and Figure 14, the Total Subsidence plot created using Test2500_mckenzie_St_VD.m matches Figure 4 and Figure 10, thus ensuring visually that the combination of the Bevis-Enriquez density model and McKenzie's model is self-consistent.

The final test determined the effects of changing density upon the McKenzie/Bevis-Enriquez model. A small change in density was settled upon as it would also help verify self-consistency simultaneously. A modification of the vector c within the script Test2_mckenzie_St_VD.m was made to generate Figure 15, a density plot that only consists of a density change of 60 kg m^{-3}. The density values generated by Test2_mckenzie_St_VD.m are considered arbitrary. Their only purpose is to show that possible small changes in density do not result in great change to the Total Subsidence curves.

Michigan Basin Case Study

Testing of the McKenzie stretching theory that was modified to use the Bevis-Enriquez density model for varying density was complete and the modeling of the Michigan Basin case study commenced. Figure 19 is the combination of the Average Density and Density vs. Depth curves shown by Figures 5 and 6, over a larger depth interval. Figure 20 is the model for the Michigan Basin using the McKenzie stretching model combined with the Bevis-Enriquez density model, using the parameters chosen for the Mt. Simon Sandstone. When compared to the previous plots for Total Subsidence vs. the square root of Time for constant density values of 2500 kg m^{-3} such as Figure 14, it can be seen that the factor with the most significant change between the constant and varying density parameters is the S_i value. The overall profile of the Post Stretching Subsidence section of Figure 20 does not seem to change over all, except for perhaps the magnitude of the subsidence, which in this case does not seem to have changed significantly.

DISCUSSION

The mathematical model for porosity created by Athy [3] and coded in this study has been criticized as not being sophisticated enough as those proposed by Fowler [6] in his 1998 paper, "Fast and Slow Compactions in Sedimentary Basins". Fowler [6] argues that complications with Athy [3] such as the occurrence of dewatering in the form of diagenesis of smectite and precipitation of cement within pores which can both act similarly to hydraulic fracturing proppants. The multiple models for basin evolution generally agree with the McKenzie model of subsidence in which thermal stretching occurred and allowed for the ascendance of hot asthenosphere that then cooled and caused flexure of the lithosphere below it. This regime is postulated in Haxby et al. [2], Howell et al. [7], and Ahern et al. [8] that all cover the evolution of the Michigan Basin. Haxby et al. [2] and Ahern et al. [8] show that the ultimate thickness of the elastic lithosphere is not fixed as assumed both in McKenzie's model and herein. Howell et al. [7] also argues that the evolution of the Michigan Basin occurred in several phases, with subsidence transitioning through different regimes and having different spatial orientations due to tilting occurring within the Basin during subsidence by tectonic actions.

The main goal for this study was achieved through modification of the McKenzie stretching theory to account for a non-stationary density. The model achieved self-consistency throughout the testing process, and so the final model was considered valid within McKenzie's theory. McKenzie's theory does allow for what can be considered a first order approximation of basin evolution methodology. In McKenzie [1] several parameters such as lithospheric thickness do not change, the model does not take into account the possibility for multiple instances of subsidence, and stretching is non-instantaneous. The model also over looks several key factors with porosity change that affects density change due to how simple the model is. Whether the new model better quantified the evolution of the Michigan Basin, it remains inconclusive. Factors such as pore fluids resisting compactive overburden pressure, precipitating cements acting as hydraulic fracturing proppants, and pore fluids increasing in total dissolved solids thus experiencing increased density were a few factors not taken into account within this study. It can be seen in Howell et al. [5] Figure 4 C, or Figure 23 that the overall profile of the stratigraphic curve for the Mt. Simon Sandstone does not match any of the curves created by this McKenzie and Bevis-Enriquez hybrid model.

Future work that builds upon the work done in this study requires more sophisticated coding in order to account for the numerous factors that were disregarded, unaccounted for, or simplified greatly. Possible changes to the current model created for this study could include a more thorough approach to the density versus depth curve so that greater emphasis is paid to changes within pores and formational fluids with depth, and also a mathematical model such as that created by

Fowler et al [6]. Further modification would need to be applied to McKenzie's model to account for multiple periods of subsidence that each occurred for different time spans that occur differently over time, and that incorporate several different lithologic units.

CONCLUSIONS

The model produced in this study represents a simple, first order approximation of the basin evolution for the Michigan Basin. This simplified model calculated a density versus depth curve mathematically rather than a profile created empirically through processes such as well logging or analyzing drill cores. McKenzie's original work did not incorporate or account for several factors that can and do occur geologically such as multiple subsidence sequences and increasing lithosphere thickness. As the present study's goal was to see how changing density affected McKenzie's model after successfully replicating and modifying his results, it can be declared a success in that it was shown that the most significant change was within the Thermally Induced Stretching regime and not the Post Stretching Subsidence regime unlike what I had hypothesized at the beginning of this study. Under the terms that this model would prove a better overall approximation for the evolution of the Michigan Basin, this model is considered inconclusive as its results do not match the stratigraphic cross section of the Michigan Basin.

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REFERENCES CITED

1 McKenzie, Dan. "Some Remarks on the Development of Sedimentary Basins." *Earth and Planetary Science Letters*. 40.1 (1978): 25-32.

2 Haxby, W.F., D.L. Turcotte, and J.M. Bird. "Thermal and Mechanical Evolution of the Michigan Basin."*Tectonophysics*. 36.1-3 (1976): 57-75.

3 Athy, L.F. "Density, Porosity, and Compaction of Sedimentary Rocks." *Bulletin of the American Association of Petroleum Geologists* 14 (1930): 1-24.

4 Hoholick, J.D., T. Metarko, and P.E. Potter. "Regional Variations of Porosity and Cement - St. Peter and Mount Simon Sandstones in Illinois Basin." *AAPG Bulliten - American Association of Petroleum Geologists*. 68.6 (1984): 753-764.

5 Parsons, B., and J.G. Sclater. "An Analysis of Variation of Ocean-Floor Bathymetry and Heat-Flow with Age." *Journal of Geophysical Research* 82 (1977): 803-827.

6 Fowler, A.C., and X.S. Yang. "Fast and Slow Compaction in Sedimentary Basins." *SIAM Journal on Applied Mathematics* 59 (1998): 365-385.

7 Howell, P.D., and B.A. van der Pluijm. "Structural sequences and styles of subsidence in the Michigan Basin." *Geological Society of America Bulletin* 111 (1999): 974-991.

8 Ahern, J.L., and P.J. Dikeou. "Evolution of the lithosphere beneath the Michigan Basin."*Earth and Planetary Science Letters* 95 (1989): 73-84.

TABLES

Table 1: McKenzie (1978) Values of Parameters Used

Values of parameters used (mostly taken from Parsons and Sclater [9])

a	=	125 km
ρ_0	=	3.33 g cm^{-3}
ρ_c	=	2.8 g cm^{-3}
ρ_w	=	1.0 g cm^{-3}
α	=	3.28 × 10^{-5} °C^{-1}
T_1	=	1333°C
τ	=	62.8 My
kT_1/a	=	0.8 μcal cm^{-2} s^{-1}
E_0	=	3.2 km

Table 2: Test_mckenzie_Si_VD.m values for Beta=1.25, showing iteration sequence for Si

Iteration	dens (kg/m^3)	avdens (kg/m^3)	Si (meters)	del-Si (meters)
1	1000.00	1000.00	767.08	0.00

Si (meters) =

7.670783102383317e+002

Table 3: TestMcKenzie.m 1st Si value for Beta=1.25 (1st iteration)

Si_1 =

7.670783102383317e+002

FIGURES

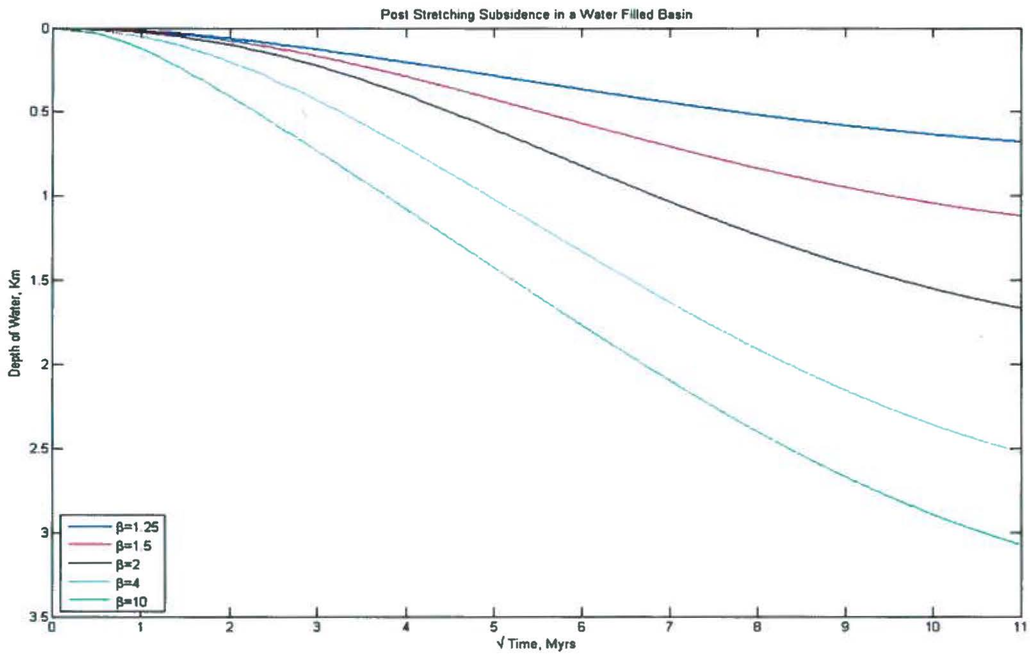

Figure 1: Plot of Post Stretching Subsidence vs. Square Root (Time) for a Water Filled Sedimentary Basin using DEmckenzie78Script.m

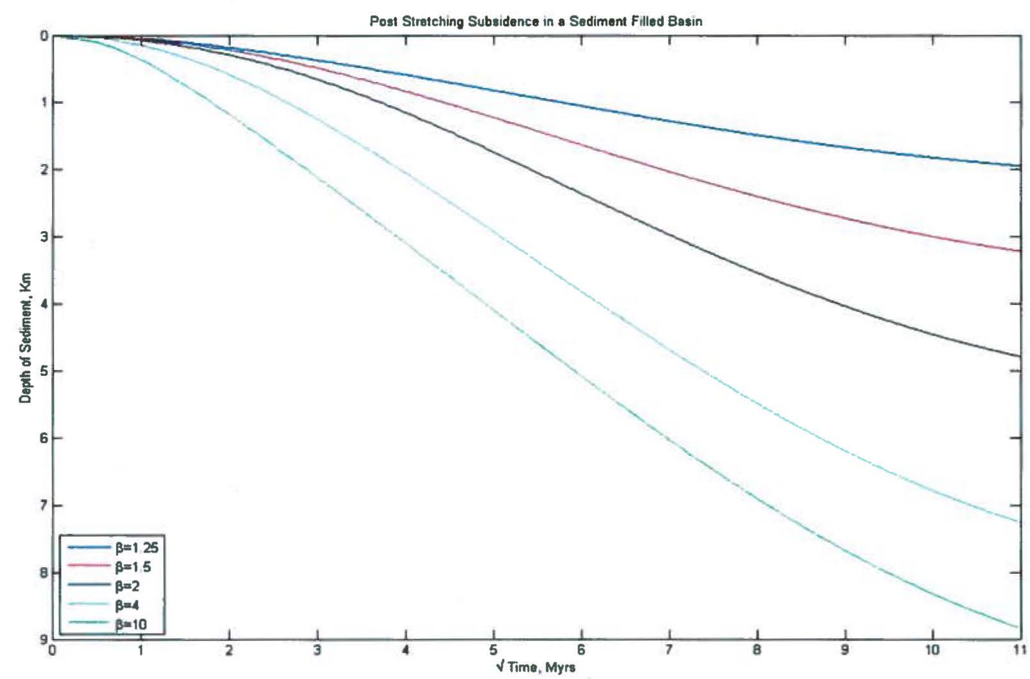

Figure 2: Plot of Post Stretching Subsidence vs. Square Root (Time) for a Sediment Filled Sedimentary Basin, using DEmckenzie78Script.m

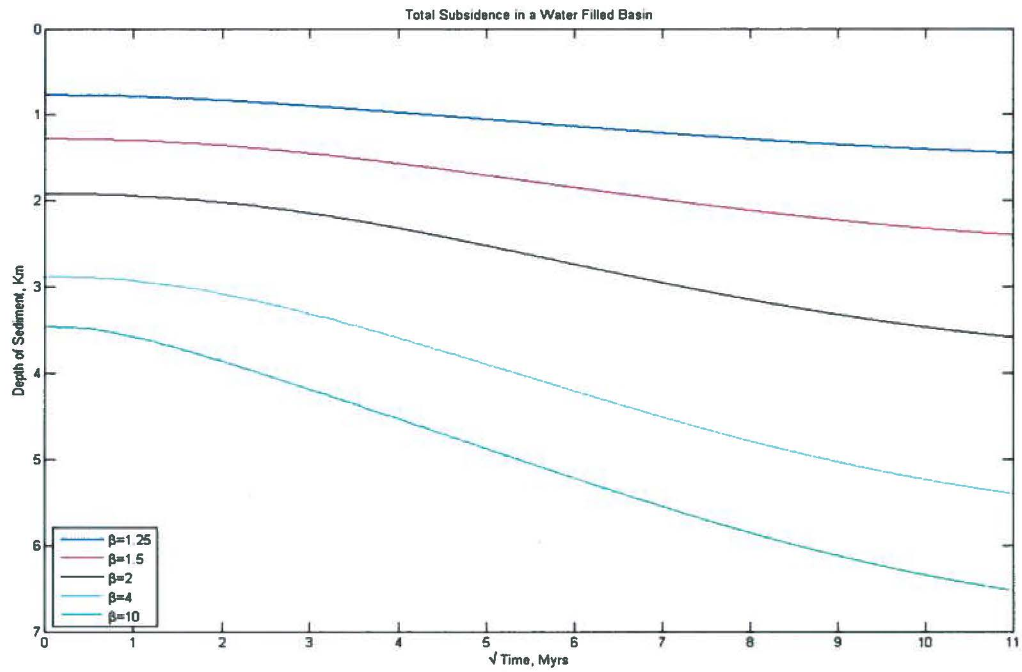

Figure 3: Plot of Total Subsidence vs. Square Root (Time) for a Water Filled Sedimentary Basin using DEmckenzie78Script.m

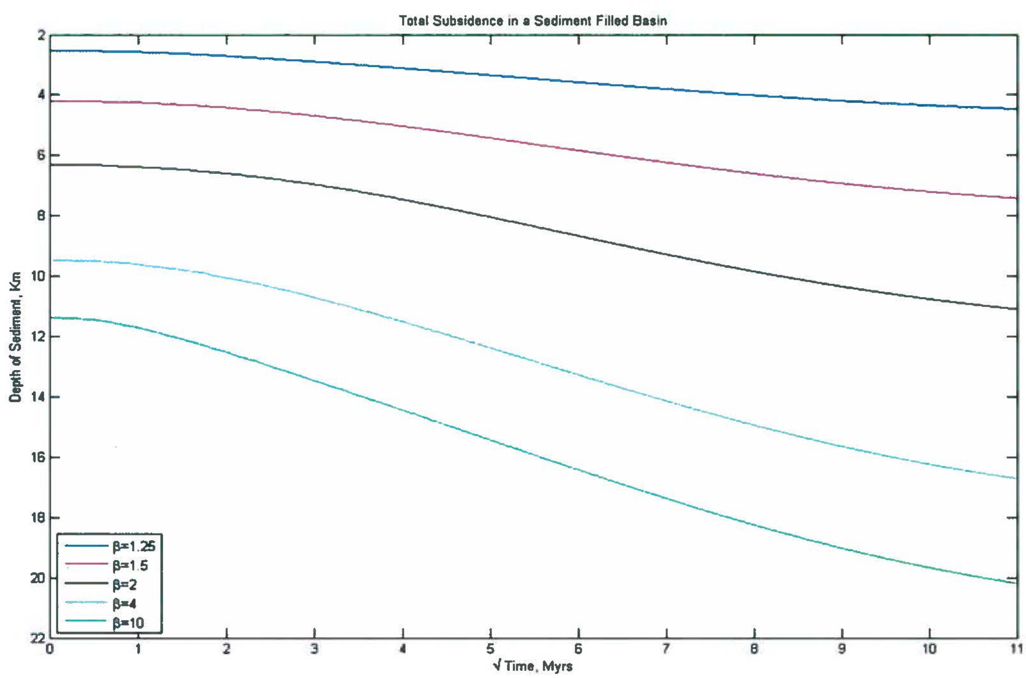

Figure 4: Plot of Total Subsidence vs. Square Root (Time) for a Sediment Filled Sedimentary Basin using DEmckenzie78Script.m

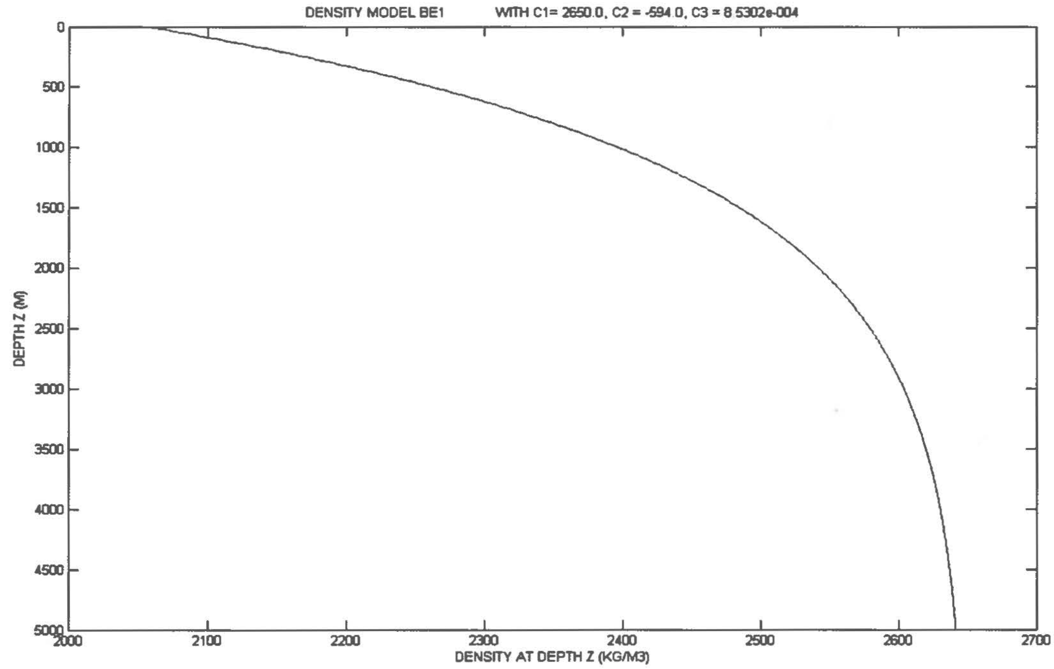

Figure 5: Plot of Density vs. Depth using the parameters selected for the Michigan Basin, produced via densitymodel_BE1Script.m

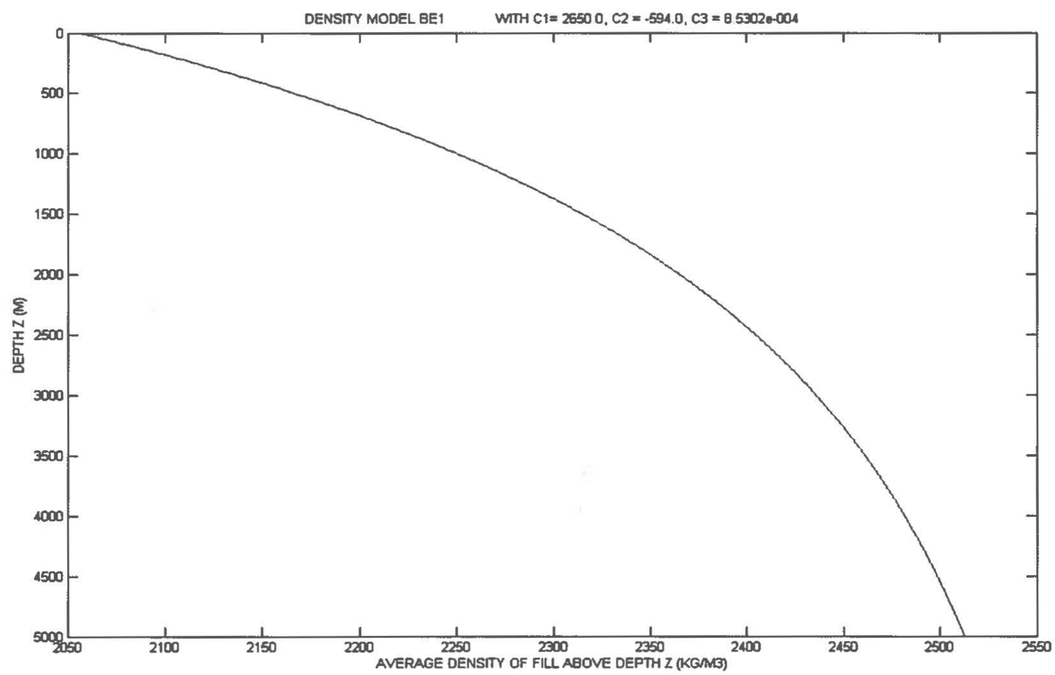

Figure 6: Plot of Average Density vs. Depth using the parameters selected for the Michigan Basin, produced via densitymodel_BE1Script.m

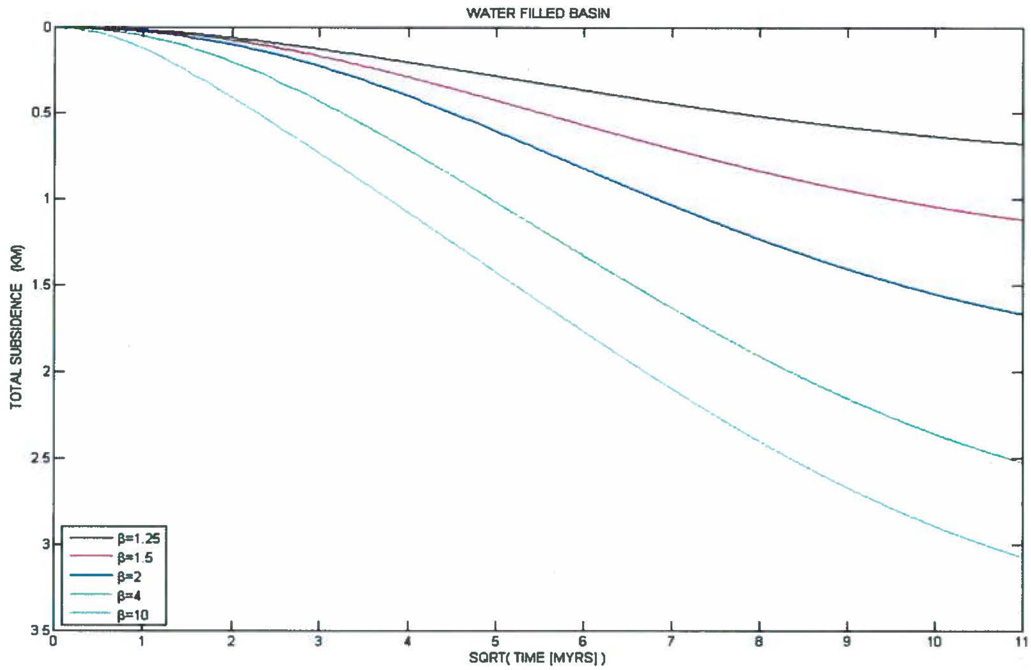

Figure 7: Plot of Post Stretching Subsidence vs. Square Root (Time) for a Water Filled Sedimentary Basin using TestMcKenzie.m, creates self-consistent solution with DEmckenzie78Script.m and Figure 1

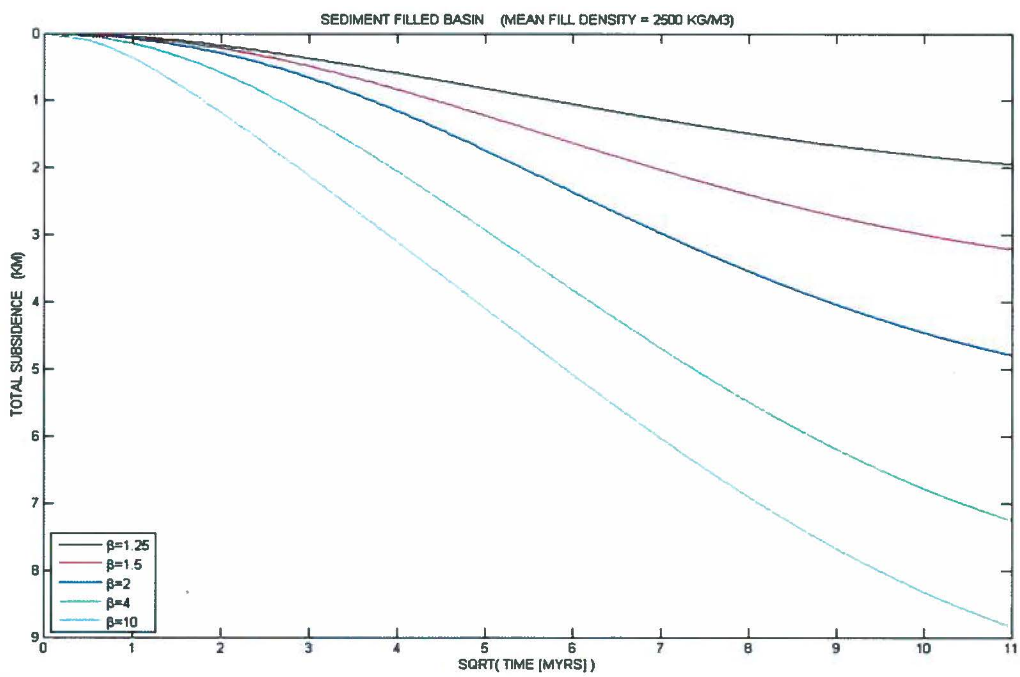

Figure 8: Plot of Post Stretching Subsidence vs. Square Root (Time) for a Sediment Filled Sedimentary Basin using TestMcKenzie.m, creates self-consistent solution with DEmckenzie78Script.m and Figure 2

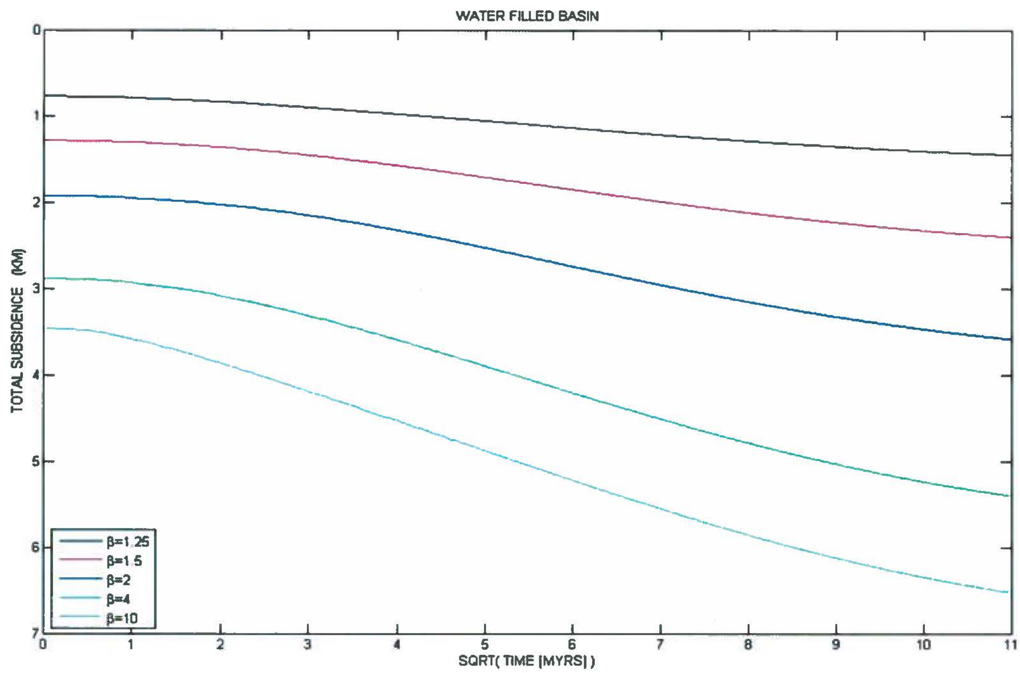

Figure 9: Plot of Total Subsidence vs. Square Root (Time) for a Water Filled Sedimentary Basin using TestMcKenzie.m, creates self-consistent solution with DEmckenzie78Script.m and Figure 3

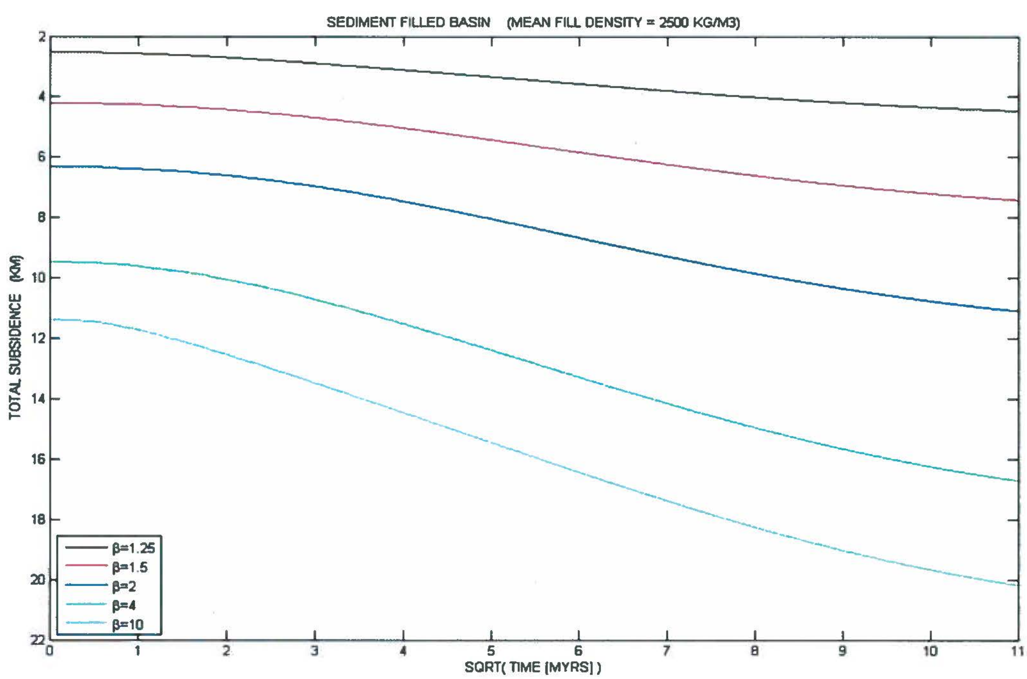

Figure 10: Plot of Total Subsidence vs. Square Root (Time) for a Sediment Filled Sedimentary Basin using TestMcKenzie.m, creates self-consistent solution with DEmckenzie78Script.m and Figure 4

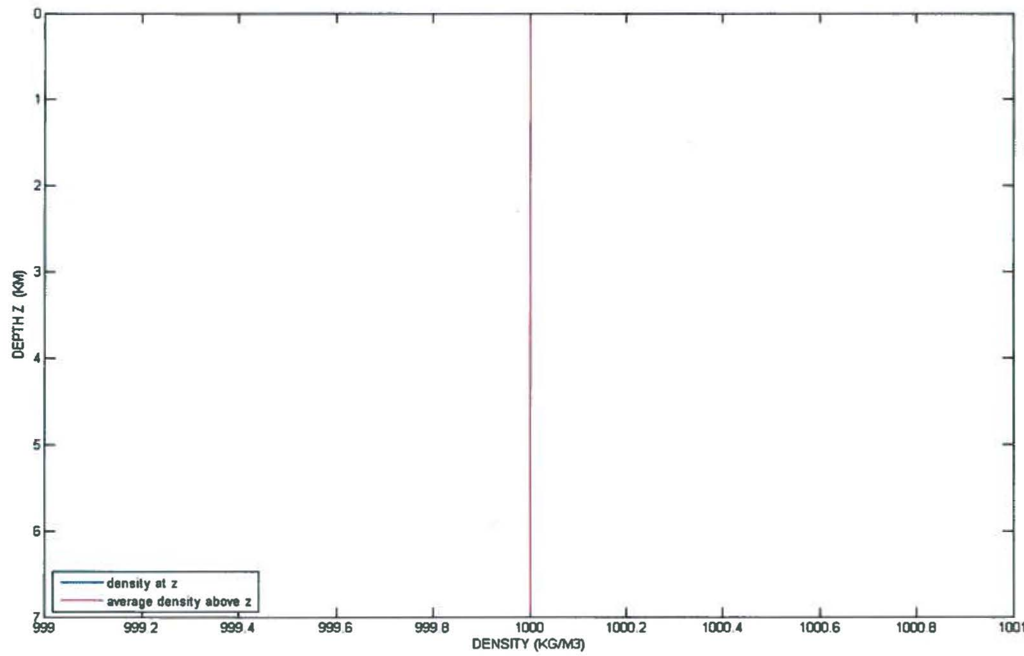

Figure 11: Plot of Density vs. Depth for a Water Filled Sedimentary Basin using the Bevis-Enriquez Density Model to show a constant density equal to 1000 kg/m^3, produced using Test1000_mckenzie_St_VD.m

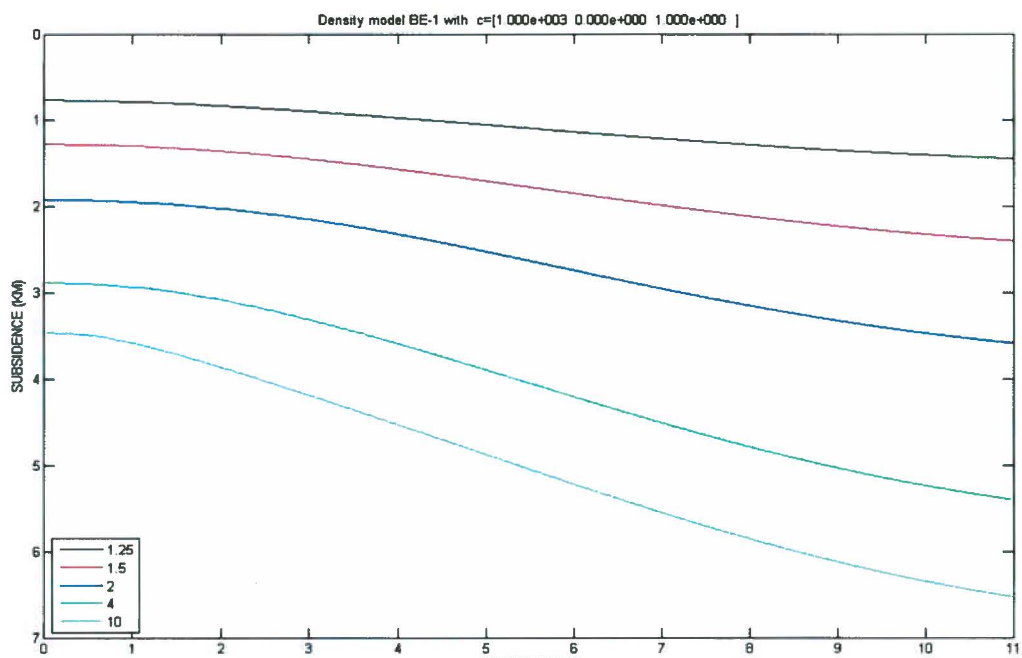

Figure 12: Plot of Total Subsidence vs. Square Root (Time) for a Water Filled Sedimentary Basin using Test1000_mckenzie_St_VD.m (Bevis-Enriquez Density Model in combination with mckenzie78_St_VD.m) to produce a self-consistent solution with Figures 3 and 9

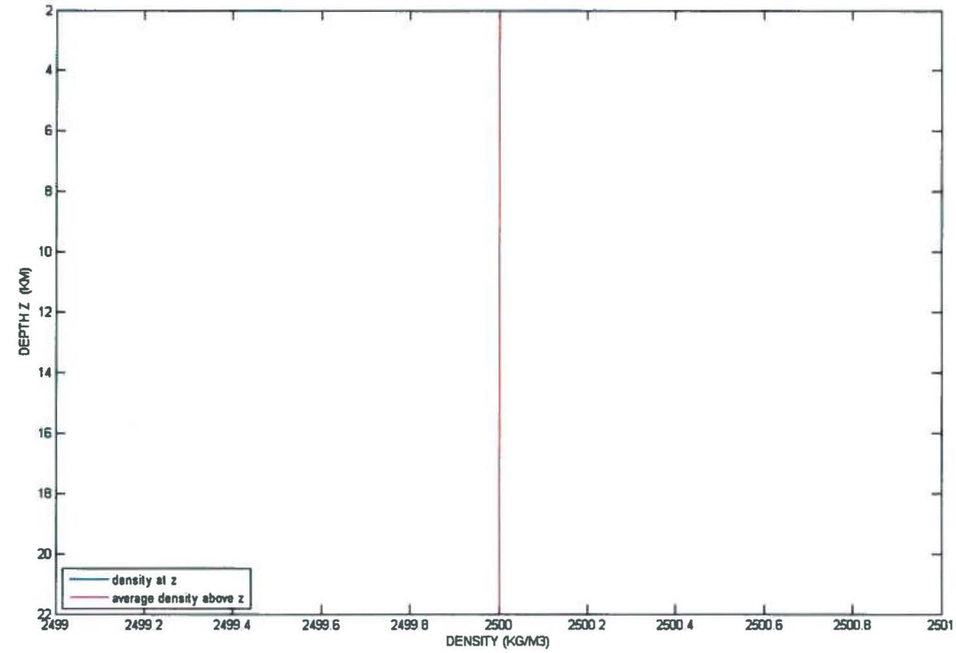

Figure 13: Plot of Density vs. Depth for a Sediment Filled Sedimentary Basin using the Bevis-Enriquez Density Model to show a constant density equal to 2500 kg/m^3, produced using Test2500_mckenzie_St_VD.m (Bevis-Enriquez Model in combination with mckenzie78_St_VD.m)

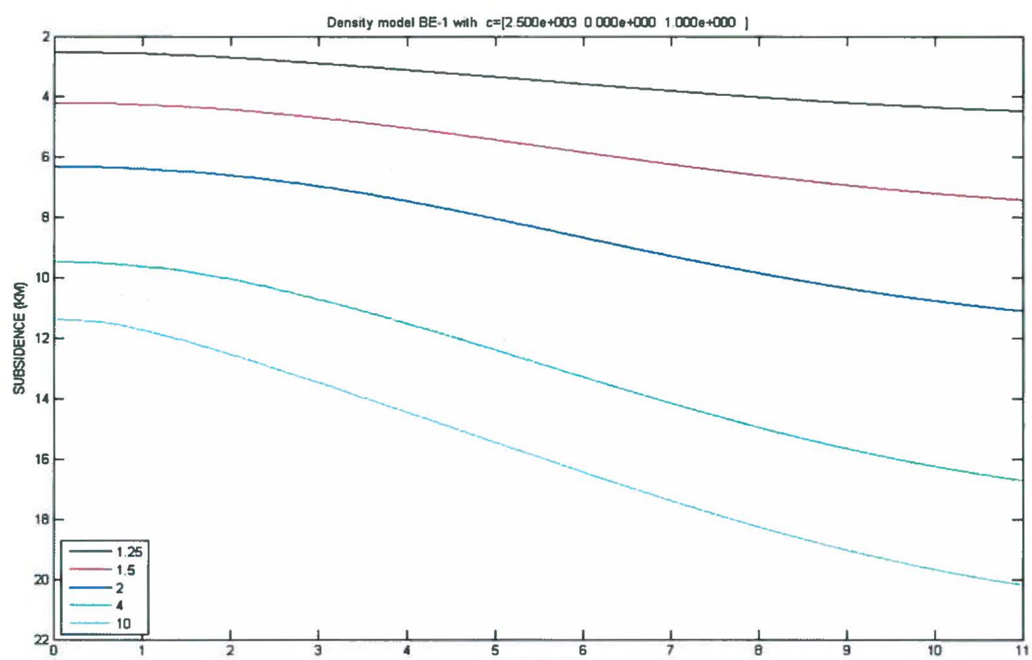

Figure 14: Plot of Total Subsidence vs. Square Root (Time) for a Sediment Filled Sedimentary Basin using Test2500_mckenzie_St_VD.m (Bevis-Enriquez Model in combination with mckenzie78_St_VD.m) to produce a self-consistent solution with Figures 4 and 10

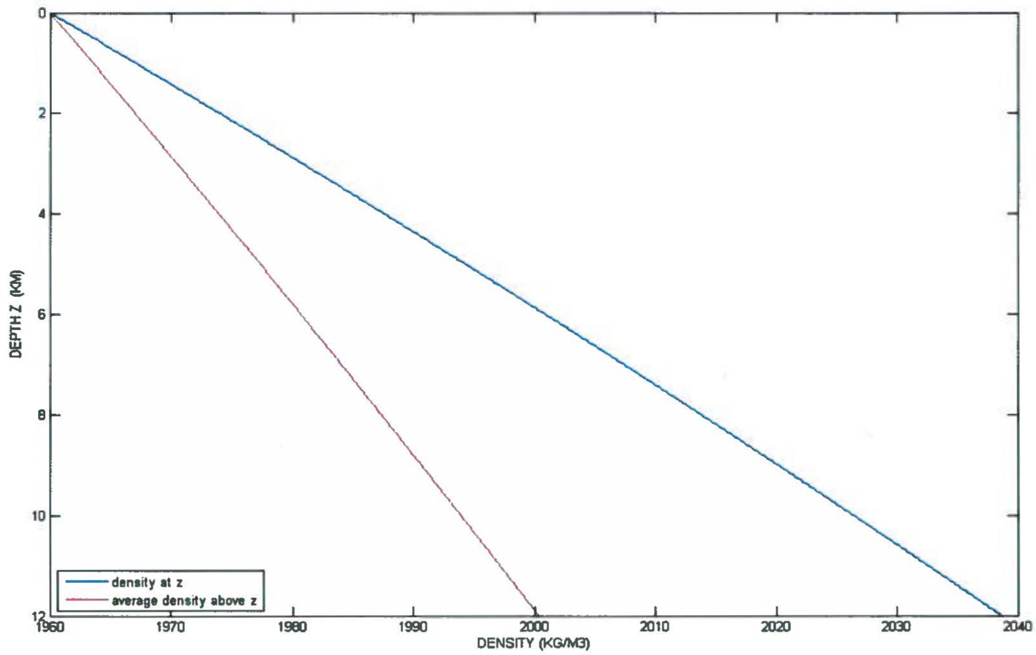

Figure 15: Plot of Density vs. Depth for a Sediment Filled Sedimentary Basin using arbitrary parameters of vector c, to show a situation where the Average Density vs. Depth varies slightly (60 kg/m^3) over the entire Depth to test for the effects of slight variations in Average Densit on the model, produced via Test2_mckenzie_St_VD.m

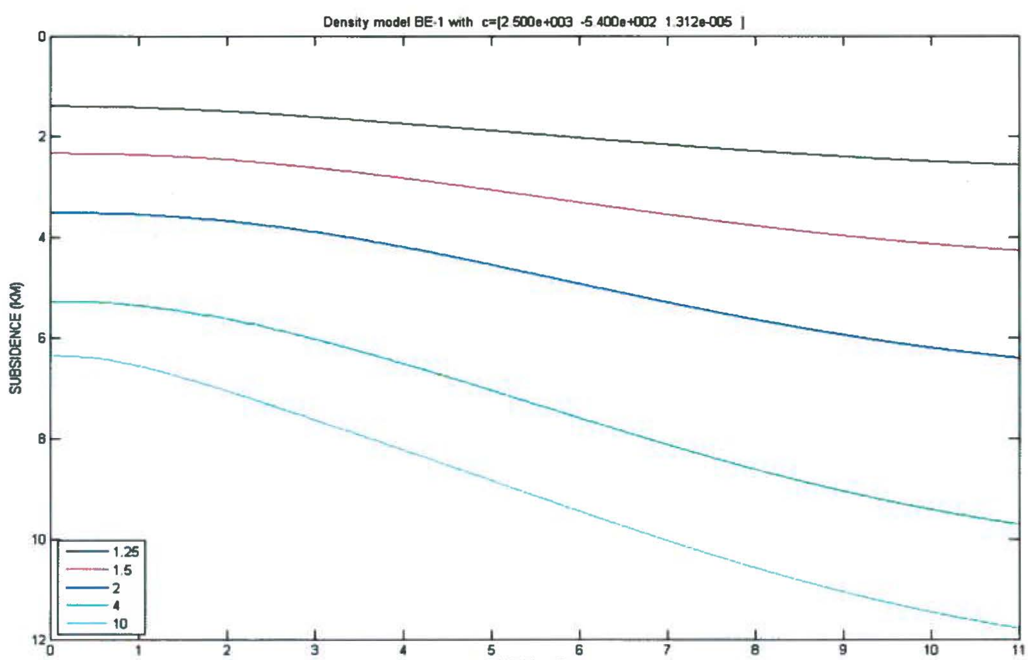

Figure 16: Plot of Total Subsidence vs. Square Root (Time) for a Sediment Filled Sedimentary Basin using arbitrary parameters of vector c specified within Test2_mckenzie_St_VD.m that have the Average Density varying by 60 kg/m^3

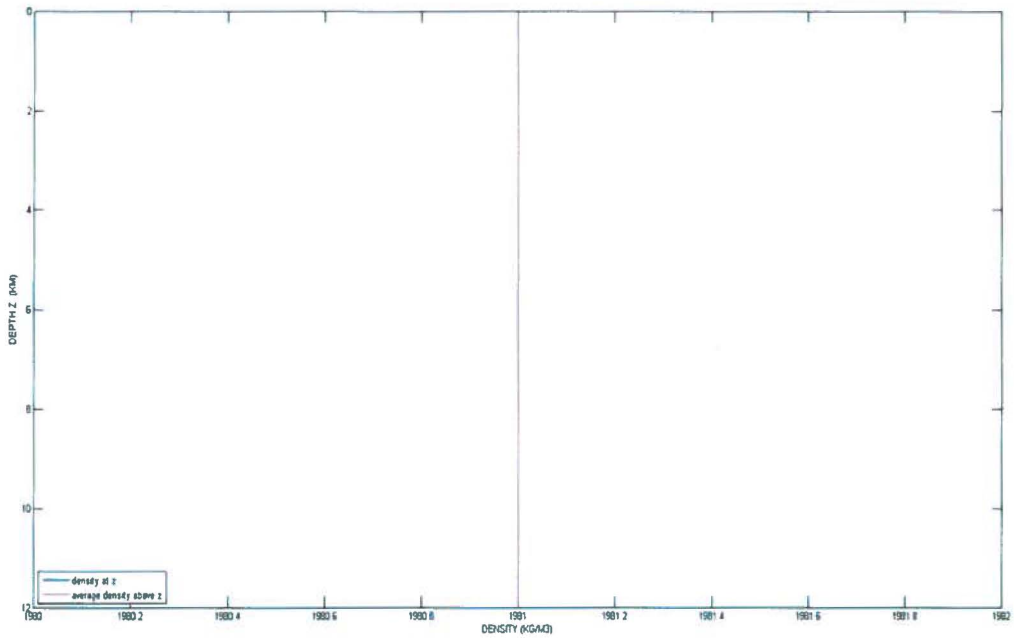

Figure 17: Plot of Density vs. Depth for a Sediment Filled Sedimentary Basin via Test2A_mckenzie_St_VD.m where the Average Density (1981 kg/m^3) is the average value of the Average Density curve (Figure 15), to test the effects of slight variations of Average Density on the McKenzie Model, and to test for self-consistency within the new model

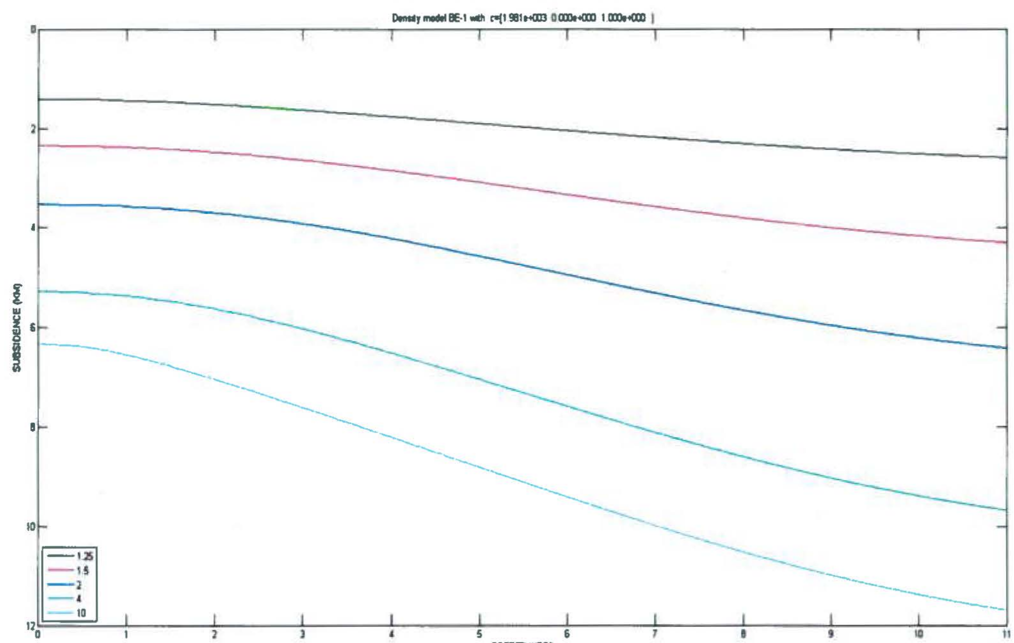

Figure 18: Plot of Total Subsidence vs. Square Root (Time) for a Sediment Filled Sedimentary Basin via Test2A_mckenzie_St_VD.m where the Average Density is 1981 kg/m^3, to show that the McKenzie model is self-consistent under small variations of Average Density like that present in Figure 15 and 16

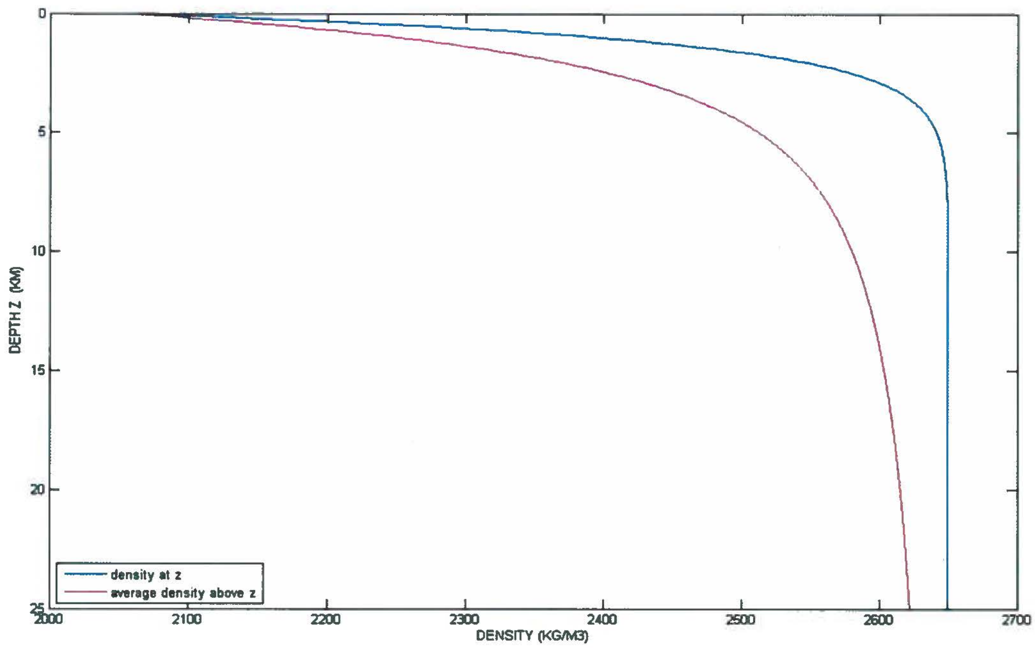

Figure 19: Plot of Density vs. Depth for the Sediment Filled Michigan Basin case study, using parameters chosen for the Michigan Basin (Mt. Simon Sst), calculated using the Bevis-Enriquez Density Model (identical to Figure's 5 &6, with a larger depth interval), produced via TestMichigan_mckenzie_St_VD.m

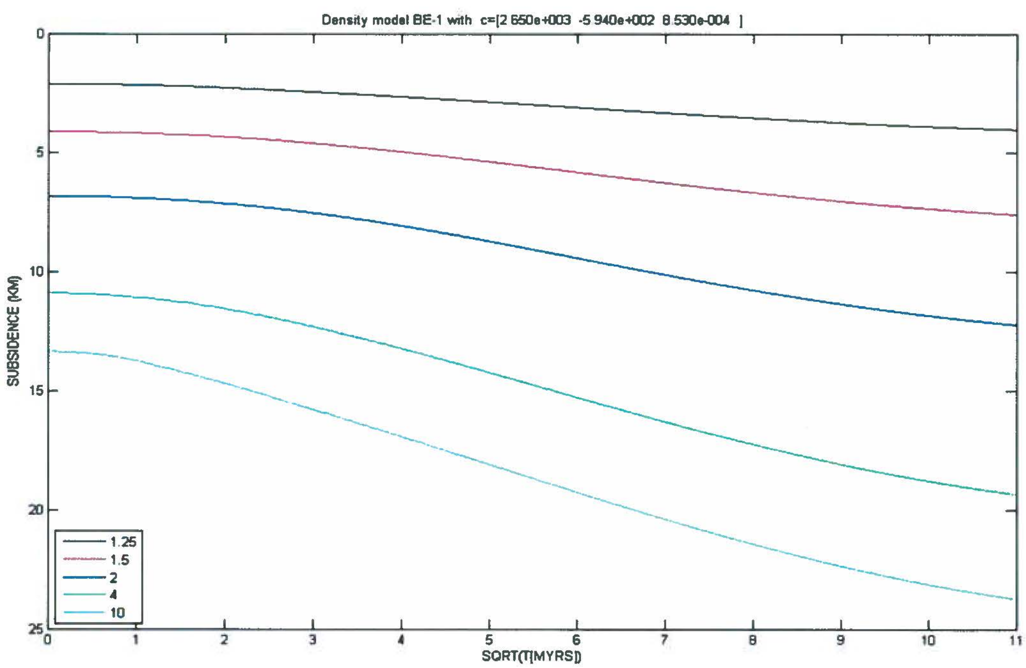

Figure 20: Plot of Total Subsidence vs. Square Root (Time) for the Sediment Filled Michigan Basin case study, using the parameters chosen for the Michigan Basin (Mt. Simon St), in order to model the effects that the changing Average Density with increasing Depth produced via the Bevis-Enriquez Density Model have on the McKenzie Model

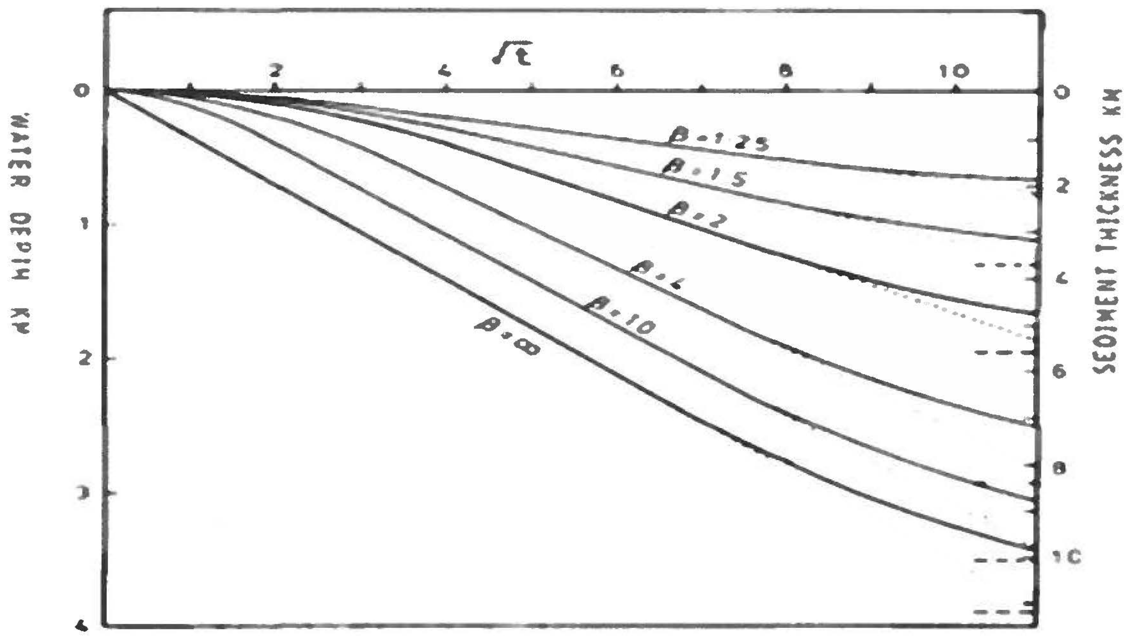

Figure 21: Plot of Post Stretching Subsidence vs. Square Root (Time) produced within McKenzie [1]

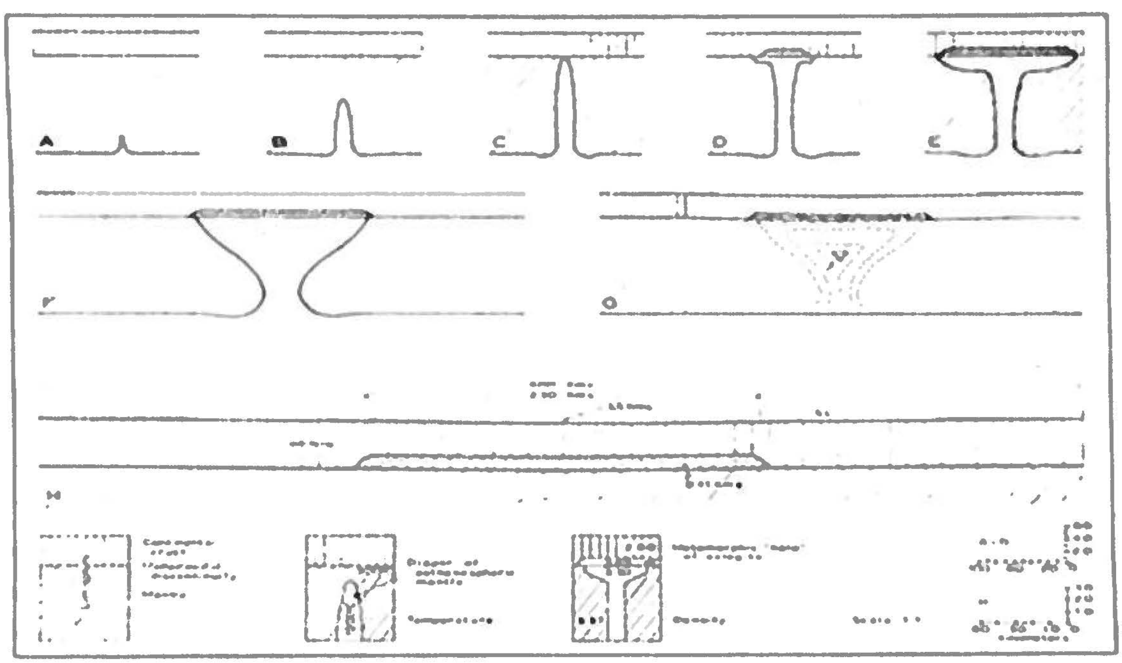

Figure 22: Diagram representing the subsidence caused by diapiric intrusions as shown in Haxby et al. [2]

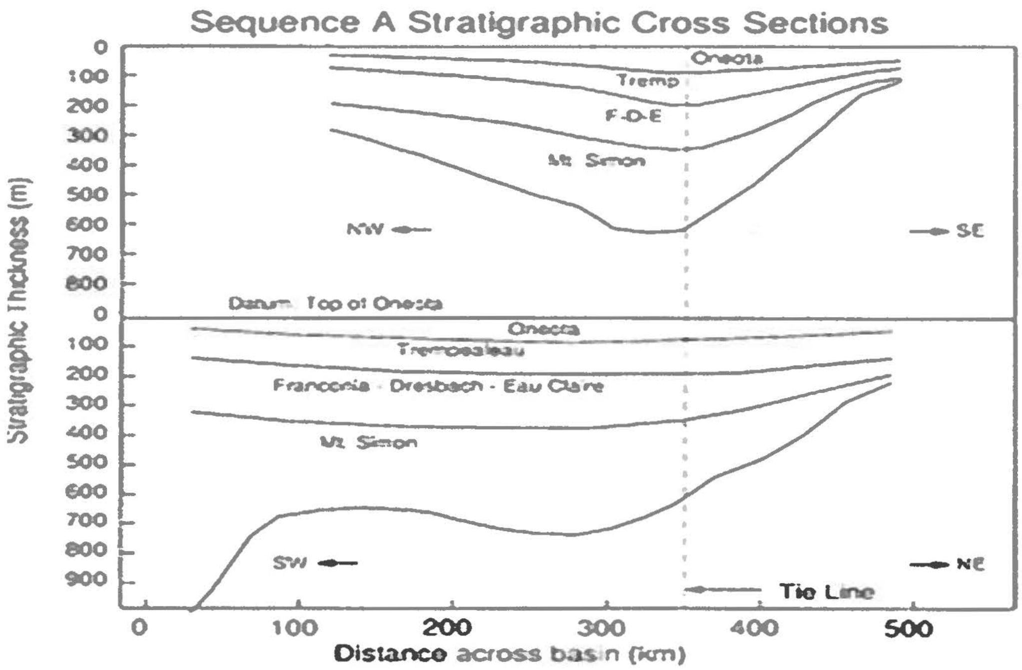

Figure 23: Diagram representing the stratigraphic cross section of the Michigan Basin with regards to the Mt. Simon Sandston as shown in Howell et al. [5]

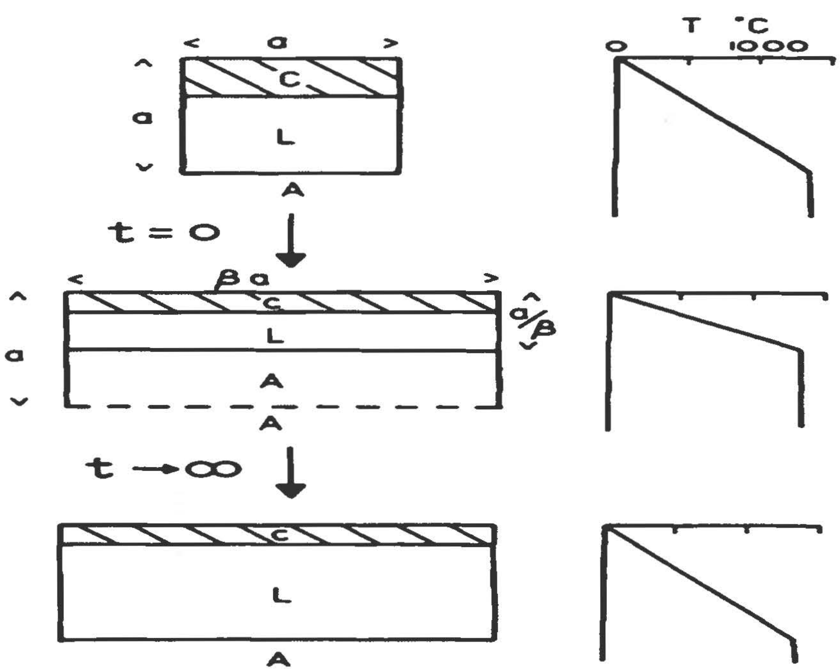

Figure 24: Diagram representing the boundary conditions given by McKenzie [1] for the basis of his model

APPENDIX A

```
function [Sth,Si] = DEmckenzie78(t,p,B)
%
%   Evaluating subsidence Si & post-extension subsidence St, given a vector
%   of times t in Myrs.
%
%Inputs:
%       t    a vector of length n containing time since stretching, in Myrs
%       p    parameter structure such that:
%                p.a= plate thickness (meters)
%                p.rho0= mantle density (kg/m^3)
%                p.rhoc= crust densitty (kg/m^3)
%                p.rhof= basin density (kg/m^3)
%                p.aplha= coefficient of thermal expansion (degC^-1)
%                p.tau=
%                p.tc= initial continental crust thickness (meters)
%       B    dimensionless stretching factor, a salar
%
%Outputs:
%        Si  initial subsidence (kilometers) - a scalar
%        Sth  (post-extension) thermal subsidence (kilometers)
%            a vector of length n
%
%Daniel Enriquez  ES5646   Oct 26th 2012

%Unpack structure
a= p.a;
rho0= p.rho0;
rhoc= p.rhoc;
rhof= p.rhof;
alpha= p.alpha;
T1= p.T1;
tau= p.tau;
tc= p.tc;

K= (4/pi^2)*((a*rho0*alpha*T1)/(rho0-rhof)); %(Kappa)

if nargout>1
  dr=rho0-rhoc;
Si= a*[ (dr*tc/a)*(1-alpha*T1*tc/a) - alpha*T1*rho0/2 ]*(1-1/B);
Si=Si/(rho0*(1-alpha*T1) - rhof);

end

Si= Si*0.001; %Converting meters to kilometers

e0=mckenzie_e(K,B,0,tau);
e0= e0*0.001; %Converting meters to kilometers
et=mckenzie_e(K,B,t,tau);
et= et*0.001; %Converting meters to kilometers

Sth=e0-et;
```

```
function e = mckenzie_e(K,beta,t,tau)
% private function used by function mckenzie78.m
% to evaluate equation [8] in McKenzie(1978)
%
% USAGE:     e = mckenzie_e(K,beta,t,tau)
%
% INPUT:
%     K      a constant that combines all the constants that appear
%            ahead of the summation sign in equation [8], including
%            the 4/pi^2. This constant is computed in SI units, except that
%            temperature T1 is given in degrees C.
%  beta      the stretching factor, a dimensionless number
%     t      a scalar or vector containing time(s) since the instantaneous
%            stretching event occured: must be in same units as tau
%   tau      the characteristic time scale, given in the same units as t
%
% OUTPUT
%     e      a vector of the same length as t
%
% NOTES: I suggest giving t and tau in seconds so that all computations are
% done in SI units, except that K must be computed using T1 stated in degrees
% C. This is because of the way the BCs were formulated [ and because rho0
% is the mantle density at 0 degrees C, not at absolute zero (0 K)] - a
% rather non-standard arrangement employed by McKenzie (1978).


summ=zeros(size(t));   % since t is a vector, summ will be a vector too
i=0;
cnvrg=0;
rtol=5e-5;      % relative accuracy required (to achieve convergence)
maxiter=25;     % maximum number of iterations allowed
while cnvrg==0
    i=i+1;
    m=i-1;
    tmpo=2*m+1;
    expon=exp(-tmpo^2*t/tau);
    term=(1/tmpo^2)*[(beta/(tmpo*pi))*sin(tmpo*pi/beta)*expon];
    summ=summ + term;
    if i>3
        delta=abs(term./summ);
        k=find(delta>rtol);   % find indices i such that convergence not
                              % achieved for t(i)
        if isempty(k)   % if k is empty, convergence occurs for all t(i)
            cnvrg=1;
        end
    end
    if i>maxiter
        error('failed to converge: reduce beta or increase maxiter')
    end
end
e=K*summ;
```

APPENDIX B

```
%Daniel Enriquez
%Feb 1 2014
%DEmckenzie78Script.m

clear all %clc; %close all;

%Inputs of water filled basin
p.a    =125e3;    % ultimate plate thickness, a, in meters. (The cooling
plate model parameter).
p.tc   = 35e3;     % initial thickness of crust, in meters
p.rho0 = 3300;    % density of mantle at zero degrees C, in kg/m3
p.rhoc = 2800;    % density of crystalline continental crust at zero C, in
kg/m3
p.rhof = 1000;    % density of basin fill material kg/m3. For water filled
basin = rhow =1000 kg/m3.
                  % if sediment filled, set rhof to (vertical) average
density of sediments
p.alpha= 3.28e-5; % volume coeff of thermal expansion, for mantle, in K^(-1)
p.T1= 1333;       % basal temperature (in mantle below plate) in C
p.k=  3.1414;     % thermal conductivity of mantle, in W/(m K)    **
McKenzie1978 val
p.C=  1.192e3;    % specific heat of mantle, in J/(kg K)     if k=3.1414,  T1
in C

% option
opt='C';

% computed derived constants
kappa=p.k/(p.rho0*p.C)     % thermal diffusivity
tau=p.a^2/(pi^2*kappa);    % tau in seconds
spMy=(1e6*365.25*24*60*60);
tau_Myrs=tau/spMy
p.tau=tau_Myrs;

t=linspace(0,121,500);


%Waterfilled
B1= 1.25; %values for beta
B2= 1.5;
B3= 2;
B4= 4;
B5= 10;

[Sth1,Si1] = DEmckenzie78(t,p,B1); %Do Sth and Si calculations for each value
beta
[Sth2,Si2] = DEmckenzie78(t,p,B2);
[Sth3,Si3] = DEmckenzie78(t,p,B3);
[Sth4,Si4] = DEmckenzie78(t,p,B4);
[Sth5,Si5] = DEmckenzie78(t,p,B5);

sqt= sqrt(t);
```

```
figure(5)
plot(sqt,Sth1,'b-')
xlabel('\surd Time, Myrs');
ylabel('Depth of Water, Km');
title('Post Stretching Subsidence in a Water Filled Basin');
set(gca,'ydir','rev');
hold on

plot(sqt,Sth2,'r-');
plot(sqt,Sth3,'k-');
plot(sqt,Sth4,'c-');
plot(sqt,Sth5,'g-');

legend('\beta=1.25','\beta=1.5','\beta=2','\beta=4','\beta=10','Location','So
uthWest')
hold off

%Total Subsidence for Water Filled Basin
S1 = Sth1+Si1;
S2 = Sth2+Si2;
S3 = Sth3+Si3;
S4 = Sth4+Si4;
S5 = Sth5+Si5;

figure(6)
plot(sqt,S1,'b-')
xlabel('\surd Time, Myrs');
ylabel('Depth of Sediment, Km');
title('Total Subsidence in a Water Filled Basin');
set(gca,'ydir','rev');
hold on

plot(sqt,S2,'r-');
plot(sqt,S3,'k-');
plot(sqt,S4,'c-');
plot(sqt,S5,'g-');

legend('\beta=1.25','\beta=1.5','\beta=2','\beta=4','\beta=10','Location','So
uthWest')
hold off


%Inputs of Sediment filled basin

%Inputs of water filled basin
p1.a    =125e3;    % ultimate plate thickness, a, in meters. (The cooling
plate model parameter).
p1.tc   = 35e3;     % initial thickness of crust, in meters
p1.rho0 = 3300;    % density of mantle at zero degrees C, in kg/m3
p1.rhoc = 2800;    % density of crystalline continental crust at zero C, in
kg/m3
p1.rhof = 2500;    % density of basin fill material kg/m3. For water filled
basin = rhow =1000 kg/m3.
```

```
                  % if sediment filled, set rhof to (vertical) average
density of sediments
p1.alpha= 3.28e-5; % volume coeff of thermal expansion, for mantle, in K^(-1)
p1.T1= 1333;       % basal temperature (in mantle below plate) in C
p1.k=  3.1414;     % thermal conductivity of mantle, in W/(m K)    **
McKenzie1978 val
p1.C=  1.192e3;    % specific heat of mantle, in J/(kg K)     if k=3.1414,
T1 in C

% option
opt='C';

% computed derived constants
kappa=p.k/(p.rho0*p.C)     % thermal diffusivity
tau=p.a^2/(pi^2*kappa);    % tau in seconds
spMy=(1e6*365.25*24*60*60);
tau_Myrs=tau/spMy
p1.tau=tau_Myrs;

%betaSediment=(1.25,1.5,2,4,10)
[Sths1,Sis1] = DEmckenzie78(t,p1,B1); %Do Sth and Si calculations for each
value beta
[Sths2,Sis2] = DEmckenzie78(t,p1,B2);
[Sths3,Sis3] = DEmckenzie78(t,p1,B3);
[Sths4,Sis4] = DEmckenzie78(t,p1,B4);
[Sths5,Sis5] = DEmckenzie78(t,p1,B5);

sqt1= sqrt(t);

figure(7)
plot(sqt1,Sths1,'b-')
xlabel('\surd Time, Myrs');
ylabel('Depth of Sediment, Km');
title('Post Stretching Subsidence in a Sediment Filled Basin');
set(gca,'ydir','rev');
hold on

plot(sqt1,Sths2,'r-');
plot(sqt1,Sths3,'k-');
plot(sqt1,Sths4,'c-');
plot(sqt1,Sths5,'g-');

legend('\beta=1.25','\beta=1.5','\beta=2','\beta=4','\beta=10','Location','So
uthWest')
hold off

%Total Subsidence for both Water and Sediment filled basins



%Total Subsidence for Sediment Filled Basin
Ss1 = Sths1+Sis1;
Ss2 = Sths2+Sis2;
Ss3 = Sths3+Sis3;
Ss4 = Sths4+Sis4;
```

```
Ss5 = Sths5+Sis5;

figure(8)
plot(sqt1,Ss1,'b-')
xlabel('\surd Time, Myrs');
ylabel('Depth of Sediment, Km');
title('Total Subsidence in a Sediment Filled Basin');
set(gca,'ydir','rev');
hold on

plot(sqt1,Ss2,'r-');
plot(sqt1,Ss3,'k-');
plot(sqt1,Ss4,'c-');
plot(sqt1,Ss5,'g-');

legend('\beta=1.25','\beta=1.5','\beta=2','\beta=4','\beta=10','Location','So
uthWest')
hold off
```

APPENDIX C

```
function [dens, avdens] = densitymodel_BE1(c,z)
%densitymodel_BE1 evaluate the Bevis-Enriquez density model for sediments
% This simple model for self-compaction assumes that the density of
% the sediments in a sediment-filled basin is a function of depth z below
% the surface of the basin, and has form
%
%     density = c1 + c2*exp(-c3*z)                                    [1]
%
% where coefficients c1 and c2 have units of density (e.g. kg/m^3) and
% coefficient c3 has units of inverse length (e.g. m^(-1) ).
%
% This function evaluates density at depth z, and also the average
% density of the sediments between the surface and depth z.
%
% USAGE:      dens = densitymodel_BE1(c,z)
%            [dens, avdens] = densitymodel_BE1(c,z)
%
% INPUT:
%       c    3-vector containing model coefficients c1,c2 and c3
%       z    depth below the surface of the sedimentary basin (meters)
%
% OUTPUT:
%    dens    the density of the sediment at depth z (kg/m3)
%  avdens    the average density of the sediments between the surface
%            and depth z (kg//m3)
%

% version 1.0           M Bevis and D Enriquez         28 Feb 2014
if length(c)~=3
    error('argument c must be a 3-vector')
end
dens=c(1) + c(2)*exp(-c(3)*z);
if nargout>1
    avdens=c(1) - (c(2)./(c(3)*z)).*( exp(-c(3)*z) - 1 );
    j=find(z==0);
    avdens(j)=dens(j);
end
```

APPENDIX D

```
% density_BE1Script.m
% version 1.0          M Bevis and D Enriquez         28 Feb 2014
% This script shows the use of the function densitymodel_BE1.m
% based on the Bevis-Enriquez Model for the density profile in a
% sediment-filled basin. Try the command 'help densitymodel_BE1'
% for more details about this simple empirical model
clear all

% Assign values to the ceofficiemts of the Bevis-Enriquez model (type 1) for
% the for density-depth profile of the fill in a sedimentary basin
a=0.36; b=0.00026;  rhom=2650;  rhow=1000;
c(1)=rhom;            % here we derive c from the porosity relation, but
c(2)=-(rhom-rhow)*a; % we expect the coefficient vector c will normally
c(3)=b/0.3048;        % be obtained by a LS fit to (z,density) data

z=linspace(0,5000,500);  % depth in meters

% evaluate (1) density at depth z, and (2) average density of the
% sediments above depth z based on model BE1
[dens, avdens] = densitymodel_BE1(c,z);

figure(1)
plot(dens,z,'r-')
tit=['DENSITY MODEL BE1            WITH C1= ',sprintf('%5.1f',c(1)),...
    ', C2 = ',sprintf('%6.1f',c(2)),...
    ', C3 = ',sprintf('%6.4e',c(3))];
title(tit)
ylabel('DEPTH Z (M)')
xlabel('DENSITY AT DEPTH Z (KG/M3)')
set(gca,'YDir','reverse')

figure(2)
plot(avdens,z,'r-')
ylabel('DEPTH Z (M)')
xlabel('AVERAGE DENSITY OF FILL ABOVE DEPTH Z (KG/M3)')
title(tit)
set(gca,'YDir','reverse')
```

APPENDIX E

```
function Si = mckenzie78_Si(p,beta)
%mckenzie78_Si instantaneous subsidence according to McKenzie (1978)
% The McKenzie stretching-sibsidence model has two components of
% subsidence, the instantaneous subsidence (Si) produced by instantaneous
% stretching, and the subsequent thermal component of subsidence (St) which
% is a function of time since the stretching event. This function solves
% for Si,and the companion function mckenzie78_St solved for St
%
% USAGE:   Si     = mckenzie78_Si(p,beta)
%
% INPUT:
%    p the parameter structure containing fields
%       p.a;     % ultimate plate thickness, a, in meters.
%       p.tc;    % initial thickness of crust, in meters
%       p.rho0;  % density of mantle at zero degrees C, in kg/m3
%       p.rhoc;  % density of crystalline continental crust
%       p.rhof;  % average density of basin fill, either water or sediment
%       p.alpha; % volume coeff of thermal expansion, for mantle, in K^(-1)
%       p.T1;    % mantle temperature at base of plate (fixed), in deg C
%       p.k;     % thermal conductivity of mantle, in W/(m K)
%       p.C;     % specific heat of mantle, in J/(kg K)
% beta the stretching factor
%
% OUTPUT:
%   Si  the initial or instantaneous subsidence in meters (a scalar)
%
% This function assumes and requires than p.rhof is scalar
%
% See also functions mckenzie78_Si_VD.m, mckenzie78_St.m and
% mckenzie78_St_VD.m

% version 1.0      Michael Bevis and Daniel Enriquez        16 April 2014

%unpack the model paramter structure p
a    =p.a;     % ultimate plate thickness, a, in meters.
tc   =p.tc;    % initial thickness of crust, in meters
rho0 =p.rho0;  % density of mantle at zero degrees C, in kg/m3
rhoc =p.rhoc;    % density of crystalline continental crust
rhof =p.rhof;    % average density of basin fill, either water or sediment
alpha =p.alpha;  % volume coeff of thermal expansion, for mantle, in K^(-1)
T1     =p.T1;       % mantle temperature at base of plate (fixed), in deg C
k      =p.k;        % thermal conductivity of mantle, in W/(m K)
C      =p.C;        % specific heat of mantle, in J/(kg K)


dr=rho0-rhoc;
Si= a*[ (dr*tc/a)*(1-alpha*T1*tc/a) - alpha*T1*rho0/2 ]*(1-1/beta);
Si=Si/(rho0*(1-alpha*T1) - rhof);
```

APPENDIX F

```
function St = mckenzie78_St(t,p,beta)
%mckenzie78_Si thermal subsidence according to McKenzie (1978)
% The McKenzie stretching-sibsidence model has two components of
% subsidence, the instantaneous subsidence (Si) produced by instantaneous
% stretching, and the subsequent thermal component of subsidence (St) which
% is a function of time since the stretching event. This function solves
% for St,and the companion function mckenzie78_Si solves for Si
%
% USAGE:    St     = mckenzie78_St(t,p,beta)
%
% INPUT:
%   t  vector of length nt containing the times (after the stretching
%      event) at which thermal subsudence is to be computed
%   p  the parameter structure containing fields
%        p.a;     % ultimate plate thickness, a, in meters.
%        p.tc;    % initial thickness of crust, in meters
%        p.rho0;  % density of mantle at zero degrees C, in kg/m3
%        p.rhoc;  % density of crystalline continental crust
%        p.rhof;  % average density of basin fill, either water or sediment
%        p.alpha; % volume coeff of thermal expansion, for mantle, in K^(-1)
%        p.T1;    % mantle temperature at base of plate (fixed), in deg C
%        p.k;     % thermal conductivity of mantle, in W/(m K)
%        p.C;     % specific heat of mantle, in J/(kg K)
% beta the stretching factor
%
% OUTPUT:
%   St  a vector of length nt containing thermal subsidence at times t,
%       expressed in meters
%
% This function allows p.rhof to be scalar in which case it is assumed
% to be constant, or a vector of length nt, in which case p.rhof(i) is
% the average fill density at time t(i).
%
% See also functions mckenzie78_Si_VD.m, mckenzie78_St.m and
% mckenzie78_St_VD.m

% version 1.0      Michael Bevis and Daniel Enriquez         16 April 2014
% This function was modified from function mckenzie78 by M. Bevis
% (written in 10/24/2012) which combined the capabilities of
% mckenzie78_Si.m and mckenzie78_St.m




% t arrives in units of years, so convert to seconds
spy=365.25*24*60*60;
t=t*spy;
nt=length(t);

% unpack the model parameter structure p
a    =p.a;     % ultimate plate thickness, a, in meters.
tc   =p.tc;    % initial thickness of crust, in meters
rho0 =p.rho0;  % density of mantle at zero degrees C, in kg/m3
```

```
rhoc =p.rhoc;      % density of crystalline continental crust
rhof =p.rhof;      % average density of basin fill, either water or sediment
alpha =p.alpha;    % volume coeff of thermal expansion, for mantle, in K^(-1)
T1    =p.T1;          % mantle temperature at base of plate (fixed), in deg C
k     =p.k;           % thermal conductivity of mantle, in W/(m K)
C     =p.C;           % specific heat of mantle, in J/(kg K)

if length(rhof)~=1 & length(rhof)~=nt
    error('p.rhof must be a scalar or match the length if argument t')
end
if length(rhof)==1
    rhof=rhof*ones(1,nt);
end

kappa=p.k/(p.rho0*p.C);
tau=p.a^2/(pi^2*kappa);    % tau in seconds
spMy=(1e6*365.25*24*60*60);
tau_Myrs=tau/spMy;

% normally one would convert temperatures, including T1, from Centrigrade
% to Kelvin thus:
%  T1=T1+273;
% But in this case one does not, because rho0 is tied to a reference
% temperature of zero Centrigrade.

% compute St
K= 4*a*rho0*alpha*T1./(pi^2*(rho0-rhof));
E0= mckenzie_e(K,beta,0,tau);
Et= mckenzie_e(K,beta,t,tau);
St=E0-Et;

if nargout==2    % compute Si
    dr=rho0-rhoc;
    Si= a*[ (dr*tc/a)*(1-alpha*T1*tc/a) - alpha*T1*rho0/2 ]*(1-1/beta);
    Si=Si/(rho0*(1-alpha*T1) - rhof);
end




function e = mckenzie_e(K,beta,t,tau)
% private function used by function mckenzie78.m
% to evaluate equation [8] in McKenzie(1978)
%
% USAGE:      e = mckenzie_e(K,beta,t,tau)
%
% INPUT:
%     K       a constant that combines all the constants that appear
%             ahead of the summation sign in equation [8], including
%             the 4/pi^2. This constant is computed in SI units, except that
%             temperature T1 is given in degrees C.
%  beta       the stretching factor, a dimensionless number
%     t       a scalar or vector containing time(s) since the instantaneous
%             stretching event occured: must be in same units as tau
%   tau       the characteristic time scale, given in the same units as t
```

```
%
% OUTPUT
%     e      a vector of the same length as t
%
% NOTES: I suggest giving t and tau in seconds so that all computations are
% done in SI units, except that K must be computed using T1 stated in degrees
% C. This is because of the way the BCs were formulated [ and because rho0
% is the mantle density at 0 degrees C, not at absolute zero (0 K)] - a
% rather non-standard arrangement employed by McKenzie (1978).

summ=zeros(size(t));    % since t is a vector, summ will be a vector too
i=0;
cnvrg=0;
rtol=5e-5;      % relative accuracy required (to achieve convergence)
maxiter=25;     % maximum number of iterations allowed
while cnvrg==0
    i=i+1;
    m=i-1;
    tmpo=2*m+1;
    expon=exp(-tmpo^2*t/tau);
    term=(1/tmpo^2)*[(beta/(tmpo*pi))*sin(tmpo*pi/beta)*expon];
    summ=summ + term;
    if i>3
        delta=abs(term./summ);
        k=find(delta>rtol);    % find indices i such that convergence not
                               % achieved for t(i)
        if isempty(k)   % if k is empty, convergence occurs for all t(i)
            cnvrg=1;
        end
    end
    if i>maxiter
        error('failed to converge: reduce beta or increase maxiter')
    end
end
e=K.*summ;
```

APPENDIX G

```
% TestMcKenzie.m   test functions mckenzie78_Si.m and mckenzie78_St.m
% version 1.0      Michael Bevis and Daniel Enriquez         16 April 2014


clear all; close all; clc;

%
p.a    =125e3;     % ultimate plate thickness, a, in meters. (The cooling
plate model parameter).
p.tc   = 35e3;      % initial thickness of crust, in meters
p.rho0 = 3300;     % density of mantle at zero degrees C, in kg/m3
p.rhoc = 2800;     % density of crystalline continental crust at zero C, in
kg/m3
p.rhof = 1000;     % density of basin fill material kg/m3. For water filled
basin = rhow =1000 kg/m3.
                   % if sediment filled, set rhof to (vertical) average
density of sediments
p.alpha= 3.28e-5; % volume coeff of thermal expansion, for mantle, in K^(-1)
p.T1= 1333;        % basal temperature (in mantle below plate) in C
p.k=  3.1414;      % thermal conductivity of mantle, in W/(m K)    **
McKenzie1978 val
p.C=  1.192e3;     % specific heat of mantle, in J/(kg K)      if k=3.1414,  T1
in C



% computed derived constants
kappa=p.k/(p.rho0*p.C);    % thermal diffusivity
tau=p.a^2/(pi^2*kappa);   % tau in seconds
spMy=(1e6*365.25*24*60*60);
tau_Myrs=tau/spMy;

% compute times t
tMyrs=linspace(0,144,600);
tMyrs=linspace(0,121,500);
t_yrs=tMyrs*1e6;
rT=sqrt(tMyrs);



% WATER FILLED BASIN

beta=1.25;
%[St,Si_1] = mckenzie78(t_yrs, p, beta);
Si_1 = mckenzie78_Si(p, beta)
St   = mckenzie78_St(t_yrs, p, beta);
S=Si_1+St;
figure(1)
plot(rT,S/1000,'k-')
set(gca,'YDir','rev')
set(gca,'XLim',[min(rT) max(rT)])
hold on
```

```
beta=1.5;
%[St,Si_2] = mckenzie78(t_yrs, p, beta);
Si_2 = mckenzie78_Si(p, beta);
St   = mckenzie78_St(t_yrs, p, beta);
S=Si_2+St;
plot(sqrt(tMyrs),S/1000,'r-')
beta=2;
%[St,Si_3] = mckenzie78(t_yrs, p, beta);
Si_3 = mckenzie78_Si(p, beta);
St   = mckenzie78_St(t_yrs, p, beta);
S=Si_3+St;
plot(sqrt(tMyrs),S/1000,'b-')
beta=4;
%[St,Si_4] = mckenzie78(t_yrs, p, beta);
Si_4 = mckenzie78_Si(p, beta);
St   = mckenzie78_St(t_yrs, p, beta);
S=Si_4+St;
plot(sqrt(tMyrs),S/1000,'g-')
beta=10;
%[St,Si_5] = mckenzie78(t_yrs, p, beta);
Si_5 = mckenzie78_Si(p, beta);
St   = mckenzie78_St(t_yrs, p, beta);
S=Si_5+St;
plot(sqrt(tMyrs),S/1000,'c-')
hold off
xlabel('SQRT( TIME [MYRS] )')
ylabel('TOTAL SUBSIDENCE   (KM)')
title('WATER FILLED BASIN')
legend('\beta=1.25','\beta=1.5','\beta=2','\beta=4','\beta=10','Location','So
uthWest')
hold off

% SEDIMENT FILLED BASIN
p.rhof = 2500;

beta=1.25;
%[St,Si_1s] = mckenzie78(t_yrs, p, beta);
Si_1s = mckenzie78_Si(p, beta);
St   = mckenzie78_St(t_yrs, p, beta);
S=Si_1s+St;
figure(2)
plot(rT,S/1000,'k-')
set(gca,'YDir','rev')
set(gca,'XLim',[min(rT) max(rT)])
hold on
beta=1.5;
%[St,Si_2s] = mckenzie78(t_yrs, p, beta);
Si_2s = mckenzie78_Si(p, beta);
St   = mckenzie78_St(t_yrs, p, beta);
S=Si_2s+St;
plot(sqrt(tMyrs),S/1000,'r-')
beta=2;
%[St,Si_3s] = mckenzie78(t_yrs, p, beta);
Si_3s = mckenzie78_Si(p, beta);
St   = mckenzie78_St(t_yrs, p, beta);
S=Si_3s+St;
```

```
plot(sqrt(tMyrs),S/1000,'b-')
beta=4;
%[St,Si_4s] = mckenzie78(t_yrs, p, beta);
Si_4s = mckenzie78_Si(p, beta);
St    = mckenzie78_St(t_yrs, p, beta);
S=Si_4s+St;
plot(sqrt(tMyrs),S/1000,'g-')
beta=10;
%[St,Si_5s] = mckenzie78(t_yrs, p, beta);
Si_5s = mckenzie78_Si(p, beta);
St    = mckenzie78_St(t_yrs, p, beta);
S=Si_5s+St;
plot(sqrt(tMyrs),S/1000,'c-')
hold off
xlabel('SQRT( TIME [MYRS] )')
ylabel('TOTAL SUBSIDENCE   (KM)')
MFD=num2str(p.rhof);
title(['SEDIMENT FILLED BASIN    (MEAN FILL DENSITY = ',MFD,' KG/M3)'])
legend('\beta=1.25','\beta=1.5','\beta=2','\beta=4','\beta=10','Location','So
uthWest')
hold off


%PostStretching Subsidence
% WATER FILLED BASIN
p.rhof=1000;

beta=1.25;
%[St,Si_1] = mckenzie78(t_yrs, p, beta);
Si_1 = mckenzie78_Si(p, beta);
St    = mckenzie78_St(t_yrs, p, beta);
S=Si_1+St;
figure(3)
plot(rT,St/1000,'k-')
set(gca,'YDir','rev')
set(gca,'XLim',[min(rT) max(rT)])
hold on
beta=1.5;
%[St,Si_2] = mckenzie78(t_yrs, p, beta);
Si_2 = mckenzie78_Si(p, beta);
St    = mckenzie78_St(t_yrs, p, beta);
S=Si_2+St;
plot(sqrt(tMyrs),St/1000,'r-')
beta=2;
%[St,Si_3] = mckenzie78(t_yrs, p, beta);
Si_3 = mckenzie78_Si(p, beta);
St    = mckenzie78_St(t_yrs, p, beta);
S=Si_3+St;
plot(sqrt(tMyrs),St/1000,'b-')
beta=4;
%[St,Si_4] = mckenzie78(t_yrs, p, beta);
Si_4 = mckenzie78_Si(p, beta);
St    = mckenzie78_St(t_yrs, p, beta);
S=Si_4+St;
plot(sqrt(tMyrs),St/1000,'g-')
beta=10;
```

```
%[St,Si_5] = mckenzie78(t_yrs, p, beta);
Si_5 = mckenzie78_Si(p, beta);
St   = mckenzie78_St(t_yrs, p, beta);
S=Si_5+St;
plot(sqrt(tMyrs),St/1000,'c-')
hold off
xlabel('SQRT( TIME [MYRS] )')
ylabel('TOTAL SUBSIDENCE   (KM)')
title('WATER FILLED BASIN')
legend('\beta=1.25','\beta=1.5','\beta=2','\beta=4','\beta=10','Location','So
uthWest')
hold off

% SEDIMENT FILLED BASIN
p.rhof = 2500;

beta=1.25;
%[St,Si_1s] = mckenzie78(t_yrs, p, beta);
Si_1s = mckenzie78_Si(p, beta);
St   = mckenzie78_St(t_yrs, p, beta);
S=Si_1s+St;
figure(4)
plot(rT,St/1000,'k-')
set(gca,'YDir','rev')
set(gca,'XLim',[min(rT) max(rT)])
hold on
beta=1.5;
%[St,Si_2s] = mckenzie78(t_yrs, p, beta);
Si_2s = mckenzie78_Si(p, beta);
St   = mckenzie78_St(t_yrs, p, beta);
S=Si_2s+St;
plot(sqrt(tMyrs),St/1000,'r-')
beta=2;
%[St,Si_3s] = mckenzie78(t_yrs, p, beta);
Si_3s = mckenzie78_Si(p, beta);
St   = mckenzie78_St(t_yrs, p, beta);
S=Si_3s+St;
plot(sqrt(tMyrs),St/1000,'b-')
beta=4;
%[St,Si_4s] = mckenzie78(t_yrs, p, beta);
Si_4s = mckenzie78_Si(p, beta);
St   = mckenzie78_St(t_yrs, p, beta);
S=Si_4s+St;
plot(sqrt(tMyrs),St/1000,'g-')
beta=10;
%[St,Si_5s] = mckenzie78(t_yrs, p, beta);
Si_5s = mckenzie78_Si(p, beta);
St   = mckenzie78_St(t_yrs, p, beta);
S=Si_5s+St;
plot(sqrt(tMyrs),St/1000,'c-')
hold off
xlabel('SQRT( TIME [MYRS] )')
ylabel('TOTAL SUBSIDENCE   (KM)')
MFD=num2str(p.rhof);
title(['SEDIMENT FILLED BASIN    (MEAN FILL DENSITY = ',MFD,' KG/M3)'])
```

```
legend('\beta=1.25','\beta=1.5','\beta=2','\beta=4','\beta=10','Location','So
uthWest')
hold off
```

APPENDIX H

```
function [Si,rhof] = mckenzie78_Si_VD(p,beta,c)
%mckenzie78_Si instantaneous subsidence using a variable density model
% The McKenzie stretching-subsidence model has two components of
% subsidence, the instantaneous subsidence (Si) produced by instantaneous
% stretching, and the subsequent thermal component of subsidence (St) which
% is a function of time since the stretching event. This function solves
% for Si, but unlike function mckenzie78_Si the average basin fill density
% rhof is not preassigned, but is computed using the BE1 density model
% (enter command "help densitymodel_BE1" for more details.
%
% USAGE:    [Si,rhof]  = mckenzie78_Si_VD(p,beta,c)
%
% INPUT:
%     p the parameter structure containing fields
%        p.a;     % ultimate plate thickness, a, in meters.
%        p.tc;    % initial thickness of crust, in meters
%        p.rho0;  % density of mantle at zero degrees C, in kg/m3
%        p.rhoc;  % density of crystalline continental crust
%        p.alpha; % volume coeff of thermal expansion, for mantle, in K^(-1)
%        p.T1;    % mantle temperature at base of plate (fixed), in deg C
%        p.k;     % thermal conductivity of mantle, in W/(m K)
%        p.C;     % specific heat of mantle, in J/(kg K)
% beta the stretching factor
%     c  the coefficient vector giving basin fill density as a function of
%        of depth, as well as average basin fill density above a given depth
%
% OUTPUT:
%    Si  the initial or instantaneous subsidence in meters (a scalar)
%
% This function assumes and requires than p.rhof is scalar
%
% See also functions mckenzie78_Si_VD.m, mckenzie78_St.m and
% mckenzie78_St_VD.m

% version 1.0            Michael Bevis            16 April 2014
% This function was modified from function mckenzie78 by M. Bevis
% (written in 10/24/2012) which combined the capabilities of
% mckenzie78_Si.m and mckenzie78_St.m

%unpack the model paramter structure p
a     =p.a;      % ultimate plate thickness, a, in meters.
tc    =p.tc;     % initial thickness of crust, in meters
rho0 =p.rho0;  % density of mantle at zero degrees C, in kg/m3
rhoc =p.rhoc;    % density of crystalline continental crust
alpha =p.alpha;  % volume coeff of thermal expansion, for mantle, in K^(-1)
T1     =p.T1;        % mantle temperature at base of plate (fixed), in deg C
k      =p.k;         % thermal conductivity of mantle, in W/(m K)
C      =p.C;         % specific heat of mantle, in J/(kg K)

maxit=9;
p.rhof=1000;
Siw = mckenzie78_Si(p,beta)
Si = Siw;
oldSi=Si;
```

```
cnvrgd=0;
it=0;
fprintf(1,'iter   dens     avdens     Si        del-Si\n');
while cnvrgd==0
    it=it+1;
    if it>maxit
        error('maximum number of iteratios exceeded')
    end
    [dens, avdens] = densitymodel_BE1(c,Si);
    p.rhof=avdens;
    Si = mckenzie78_Si(p,beta);
    dS=abs(Si-oldSi);
    fprintf(1,'%4i  ',it);
    fprintf(1,'%6.2f  ',dens);
    fprintf(1,'%6.2f  ',avdens);
    fprintf(1,'%9.2f  ',Si);
    fprintf(1,'%9.2f\n',dS);

    if dS<1  % if Si changed by <1 meter
        cnvrgd=1;
    else
        oldSi=Si;
    end
end
rhof=avdens;
```

APPENDIX I

```
%Test_mckenzie_Si_VD.m
% version 1.0            Michael Bevis            16 April 2014
% This function was modified from function mckenzie78 by M. Bevis
% (written in 10/24/2012) which combined the capabilities of
% mckenzie78_Si.m and mckenzie78_St.m
clear all

%
p.a    =125e3;     % ultimate plate thickness, a, in meters. (The cooling
plate model parameter).
p.tc   = 35e3;      % initial thickness of crust, in meters
p.rho0 = 3300;     % density of mantle at zero degrees C, in kg/m3
p.rhoc = 2800;     % density of crystalline continental crust at zero C, in
kg/m3
p.rhof = 1000;     % density of basin fill material kg/m3. For water filled
basin = rhow =1000 kg/m3.
                   % if sediment filled, set rhof to (vertical) average
density of sediments
p.alpha= 3.28e-5; % volume coeff of thermal expansion, for mantle, in K^(-1)
p.T1= 1333;        % basal temperature (in mantle below plate) in C
p.k=  3.1414;      % thermal conductivity of mantle, in W/(m K)    **
McKenzie1978 val
p.C=  1.192e3;

% Assign values to the ceofficiemts of the Bevis-Enriquez model (type 1) for
% the for density-depth profile of the fill in a sedimentary basin
a=0.36; b=0.00026;  rhom=2650;  rhow=1000;
c(1)=rhom;            % here we derive c from the porosity relation, but
c(2)=-(rhom-rhow)*a; % we expect the coefficient vector c will normally
c(3)=b/0.3048;        % be obtained by a LS fit to (z,density) data

c(1)=1000;            % here we derive c from the porosity relation, but
c(2)=0; % we expect the coefficient vector c will normally
c(3)=1;        % be obtained by a LS fit to (z,density) data


beta=1.25;
[Si,rhof] = mckenzie78_Si_VD(p,beta,c)
```

APPENDIX J

```
%Daniel Enriquez
%4-18-2014
%Undergrad Senior Thesis
%Test Scenerio 1: McKenzie_St_VD for avdens of 1000

clear all; %close all; %clc;

%
p.a    =125e3;     % ultimate plate thickness, a, in meters. (The cooling
plate model parameter).
p.tc   = 35e3;      % initial thickness of crust, in meters
p.rho0 = 3300;     % density of mantle at zero degrees C, in kg/m3
p.rhoc = 2800;     % density of crystalline continental crust at zero C, in
kg/m3
p.rhof = 1000;     % density of basin fill material kg/m3. For water filled
basin = rhow =1000 kg/m3.
                   % if sediment filled, set rhof to (vertical) average
density of sediments
p.alpha= 3.28e-5; % volume coeff of thermal expansion, for mantle, in K^(-1)
p.T1= 1333;        % basal temperature (in mantle below plate) in C
p.k=  3.1414;      % thermal conductivity of mantle, in W/(m K)    **
McKenzie1978 val
p.C=  1.192e3;

c=[1000,0,1];      %c values for constant density of 2500 kg/m^3

%c(1)=1000; c(2)=0; c(3)=1;

% compute times t
tMyrs=linspace(0,144,600);
tMyrs=linspace(0,121,600);
t_yrs=tMyrs*1e6;
rootT=sqrt(tMyrs);  %McKenzie likes to plot sqrt(tMyrs)

betas=[1.25 1.5 2 4 10];
clr=['krbgc'];
nbeta=length(betas);
for k=1:nbeta
    betastr{k}=num2str(betas(k));
end

for i=1:nbeta
    beta=betas(i);
    [St,Si,rhof] = mckenzie78_St_VD(t_yrs,p,beta,c);
    S=Si+St;
    Skm=S/1000;

    figure(5)
    plot(rootT,Skm,'-','Color',clr(i));
    if i==1
        set(gca,'YDir','rev');
        hold on
    end
```

```
end
hold off
set(gca,'XLim',[0 max(rootT)])
xlabel('SQRT(T[MYRS])')
ylabel('SUBSIDENCE (KM)')
cvals=sprintf('%6.3e  ',c);
title(['Density model BE-1 with  c=[',cvals,']'])
legend(betastr,'Location','SouthWest')
ylm=get(gca,'YLim');

figure(6)
zkm=linspace(ylm(1),ylm(2),500);
zm=zkm*1e3;
[dens, avdens] = densitymodel_BE1(c,zm);
plot(dens,zkm,'b-',avdens,zkm,'r-')
set(gca,'YDir','rev');
legend('density at z','average density above z','Location','SouthWest')
ylabel('DEPTH Z  (KM)')
xlabel('DENSITY (KG/M3)')

x=mean(avdens)
```

APPENDIX K

```
%Daniel Enriquez
%4-18-2014
%Undergrad Senior Thesis
%Test Scenerio 1: McKenzie_St_VD for avdens of 2500

clear all; close all; clc;

%
p.a    =125e3;    % ultimate plate thickness, a, in meters. (The cooling
plate model parameter).
p.tc   = 35e3;     % initial thickness of crust, in meters
p.rho0 = 3300;    % density of mantle at zero degrees C, in kg/m3
p.rhoc = 2800;    % density of crystalline continental crust at zero C, in
kg/m3
p.rhof = 1000;    % density of basin fill material kg/m3. For water filled
basin = rhow =1000 kg/m3.
                  % if sediment filled, set rhof to (vertical) average
density of sediments
p.alpha= 3.28e-5; % volume coeff of thermal expansion, for mantle, in K^(-1)
p.T1= 1333;       % basal temperature (in mantle below plate) in C
p.k=  3.1414;     % thermal conductivity of mantle, in W/(m K)    **
McKenzie1978 val
p.C=  1.192e3;

c=[2500,0,1];     %c values for constant density of 2500 kg/m^3

%c(1)=1000; c(2)=0; c(3)=1;

% compute times t
tMyrs=linspace(0,144,600);
tMyrs=linspace(0,121,600);
t_yrs=tMyrs*1e6;
rootT=sqrt(tMyrs);  %McKenzie likes to plot sqrt(tMyrs)

betas=[1.25 1.5 2 4 10];
clr=['krbgc'];
nbeta=length(betas);
for k=1:nbeta
    betastr{k}=num2str(betas(k));
end

for i=1:nbeta
    beta=betas(i);
    [St,Si,rhof] = mckenzie78_St_VD(t_yrs,p,beta,c);
    S=Si+St;
    Skm=S/1000;

    figure(3)
    plot(rootT,Skm,'-','Color',clr(i));
    if i==1
        set(gca,'YDir','rev');
        hold on
    end
```

```
end
hold off
set(gca,'XLim',[0 max(rootT)])
xlabel('SQRT(T[MYRS])')
ylabel('SUBSIDENCE (KM)')
cvals=sprintf('%6.3e  ',c);
title(['Density model BE-1 with  c=[',cvals,']'])
legend(betastr,'Location','SouthWest')
ylm=get(gca,'YLim');

figure(4)
zkm=linspace(ylm(1),ylm(2),500);
zm=zkm*1e3;
[dens, avdens] = densitymodel_BE1(c,zm);
plot(dens,zkm,'b-',avdens,zkm,'r-')
set(gca,'YDir','rev');
legend('density at z','average density above z','Location','SouthWest')
ylabel('DEPTH Z  (KM)')
xlabel('DENSITY (KG/M3)')

x=mean(avdens)
```

APPENDIX L

```
%Daniel Enriquez
%4-18-2014
%Undergrad Senior Thesis
%Test Scenerio 1: McKenzie_St_VD for Case where mean density slightly
%varies


clear all; close all; clc;

%
p.a    =125e3;    % ultimate plate thickness, a, in meters. (The cooling
plate model parameter).
p.tc   = 35e3;     % initial thickness of crust, in meters
p.rho0 = 3300;    % density of mantle at zero degrees C, in kg/m3
p.rhoc = 2800;    % density of crystalline continental crust at zero C, in
kg/m3
p.rhof = 1000;    % density of basin fill material kg/m3. For water filled
basin = rhow =1000 kg/m3.
                  % if sediment filled, set rhof to (vertical) average
density of sediments
p.alpha= 3.28e-5; % volume coeff of thermal expansion, for mantle, in K^(-1)
p.T1= 1333;       % basal temperature (in mantle below plate) in C
p.k=  3.1414;     % thermal conductivity of mantle, in W/(m K)    **
McKenzie1978 val
p.C=  1.192e3;

% Assign values to the ceofficiemts of the Bevis-Enriquez model (type 1) for
% the for density-depth profile of the fill in a sedimentary basin
a=0.36; b=0.000004;  rhom=2500;  rhow=1000;
c(1)=rhom;             % here we derive c from the porosity relation, but
c(2)=-(rhom-rhow)*a; % we expect the coefficient vector c will normally
c(3)=b/0.3048;         % be obtained by a LS fit to (z,density) data

%c(1)=1000; c(2)=0; c(3)=1;

% compute times t
tMyrs=linspace(0,144,600);
tMyrs=linspace(0,121,600);
t_yrs=tMyrs*1e6;
rootT=sqrt(tMyrs);  %McKenzie likes to plot sqrt(tMyrs)

betas=[1.25 1.5 2 4 10];
clr=['krbgc'];
nbeta=length(betas);
for k=1:nbeta
    betastr{k}=num2str(betas(k));
end

for i=1:nbeta
    beta=betas(i);
    [St,Si,rhof] = mckenzie78_St_VD(t_yrs,p,beta,c);
    S=Si+St;
    Skm=S/1000;
```

```
    figure(1)
    plot(rootT,Skm,'-','Color',clr(i));
    if i==1
        set(gca,'YDir','rev');
        hold on
    end
end
hold off
set(gca,'XLim',[0 max(rootT)])
xlabel('SQRT(T[MYRS])')
ylabel('SUBSIDENCE (KM)')
cvals=sprintf('%6.3e  ',c);
title(['Density model BE-1 with  c=[',cvals,']'])
legend(betastr,'Location','SouthWest')
ylm=get(gca,'YLim');

figure(2)
zkm=linspace(ylm(1),ylm(2),500);
zm=zkm*1e3;
[dens, avdens] = densitymodel_BE1(c,zm);
plot(dens,zkm,'b-',avdens,zkm,'r-')
set(gca,'YDir','rev');
legend('density at z','average density above z','Location','SouthWest')
ylabel('DEPTH Z  (KM)')
xlabel('DENSITY (KG/M3)')

x=mean(avdens)
```

APPENDIX M

```
%Daniel Enriquez
%4-18-2014
%Undergrad Senior Thesis
%Test Scenerio 1: McKenzie_St_VD for avdens of Test2

%clear all; close all; clc;

%
p.a    =125e3;     % ultimate plate thickness, a, in meters. (The cooling
plate model parameter).
p.tc   = 35e3;      % initial thickness of crust, in meters
p.rho0 = 3300;     % density of mantle at zero degrees C, in kg/m3
p.rhoc = 2800;     % density of crystalline continental crust at zero C, in
kg/m3
p.rhof = 1000;     % density of basin fill material kg/m3. For water filled
basin = rhow =1000 kg/m3.
                   % if sediment filled, set rhof to (vertical) average
density of sediments
p.alpha= 3.28e-5; % volume coeff of thermal expansion, for mantle, in K^(-1)
p.T1= 1333;        % basal temperature (in mantle below plate) in C
p.k=  3.1414;      % thermal conductivity of mantle, in W/(m K)    **
McKenzie1978 val
p.C=  1.192e3;

c=[1981,0,1];      %c values for constant density of 2500 kg/m^3

%c(1)=1000; c(2)=0; c(3)=1;

% compute times t
tMyrs=linspace(0,144,600);
tMyrs=linspace(0,121,600);
t_yrs=tMyrs*1e6;
rootT=sqrt(tMyrs);  %McKenzie likes to plot sqrt(tMyrs)

betas=[1.25 1.5 2 4 10];
clr=['krbgc'];
nbeta=length(betas);
for k=1:nbeta
    betastr{k}=num2str(betas(k));
end

for i=1:nbeta
    beta=betas(i);
    [St,Si,rhof] = mckenzie78_St_VD(t_yrs,p,beta,c);
    S=Si+St;
    Skm=S/1000;

    figure(3)
    plot(rootT,Skm,'-','Color',clr(i));
    if i==1
        set(gca,'YDir','rev');
        hold on
    end
```

```
end
hold off
set(gca,'XLim',[0 max(rootT)])
xlabel('SQRT(T[MYRS])')
ylabel('SUBSIDENCE (KM)')
cvals=sprintf('%6.3e  ',c);
title(['Density model BE-1 with  c=[',cvals,']'])
legend(betastr,'Location','SouthWest')
ylm=get(gca,'YLim');

figure(4)
zkm=linspace(ylm(1),ylm(2),500);
zm=zkm*1e3;
[dens, avdens] = densitymodel_BE1(c,zm);
plot(dens,zkm,'b-',avdens,zkm,'r-')
set(gca,'YDir','rev');
legend('density at z','average density above z','Location','SouthWest')
ylabel('DEPTH Z  (KM)')
xlabel('DENSITY (KG/M3)')

x=mean(avdens)
```

APPENDIX N

```
%Daniel Enriquez
%4-18-2014
%Undergrad Senior Thesis
%Test Scenerio 1: McKenzie_St_VD for Mt. Simon Sandstone Michigan Basin
%Case

clear all; close all; clc;

%
p.a    =125e3;     % ultimate plate thickness, a, in meters. (The cooling
plate model parameter).
p.tc   = 35e3;      % initial thickness of crust, in meters
p.rho0 = 3300;     % density of mantle at zero degrees C, in kg/m3
p.rhoc = 2800;     % density of crystalline continental crust at zero C, in
kg/m3
p.rhof = 1000;     % density of basin fill material kg/m3. For water filled
basin = rhow =1000 kg/m3.
                   % if sediment filled, set rhof to (vertical) average
density of sediments
p.alpha= 3.28e-5; % volume coeff of thermal expansion, for mantle, in K^(-1)
p.T1= 1333;        % basal temperature (in mantle below plate) in C
p.k=  3.1414;      % thermal conductivity of mantle, in W/(m K)    **
McKenzie1978 val
p.C=  1.192e3;

% Assign values to the ceofficiemts of the Bevis-Enriquez model (type 1) for
% the for density-depth profile of the fill in a sedimentary basin
a=0.36; b=0.00026;  rhom=2650;  rhow=1000;
c(1)=rhom;            % here we derive c from the porosity relation, but
c(2)=-(rhom-rhow)*a; % we expect the coefficient vector c will normally
c(3)=b/0.3048;        % be obtained by a LS fit to (z,density) data

%c(1)=1000; c(2)=0; c(3)=1;

% compute times t
tMyrs=linspace(0,144,600);
tMyrs=linspace(0,121,600);
t_yrs=tMyrs*1e6;
rootT=sqrt(tMyrs);  %McKenzie likes to plot sqrt(tMyrs)

betas=[1.25 1.5 2 4 10];
clr=['krbgc'];
nbeta=length(betas);
for k=1:nbeta
    betastr{k}=num2str(betas(k));
end

for i=1:nbeta
    beta=betas(i);
    [St,Si,rhof] = mckenzie78_St_VD(t_yrs,p,beta,c);
    S=Si+St;
    Skm=S/1000;
```

```
    figure(3)
    plot(rootT,Skm,'-','Color',clr(i));
    if i==1
        set(gca,'YDir','rev');
        hold on
    end
end
hold off
set(gca,'XLim',[0 max(rootT)])
xlabel('SQRT(T[MYRS])')
ylabel('SUBSIDENCE (KM)')
cvals=sprintf('%6.3e  ',c);
title(['Density model BE-1 with  c=[',cvals,']'])
legend(betastr,'Location','SouthWest')
ylm=get(gca,'YLim');

figure(4)
zkm=linspace(ylm(1),ylm(2),500);
zm=zkm*1e3;
[dens, avdens] = densitymodel_BE1(c,zm);
plot(dens,zkm,'b-',avdens,zkm,'r-')
set(gca,'YDir','rev');
legend('density at z','average density above z','Location','SouthWest')
ylabel('DEPTH Z  (KM)')
xlabel('DENSITY (KG/M3)')

x=mean(avdens)
```