

FINITE TEMPERATURE QCD

J. Engels
Fakultät für Physik
Universität Bielefeld
Germany

Abstract:

A review of finite temperature lattice calculations for quantum chromodynamics is given. We show how the thermodynamic quantities can be evaluated by Monte Carlo methods, once finite temperature field theory has been formulated on a lattice. The existing results for chemical potential zero and in quenched approximation are discussed. They exhibit a clear first order transition for SU(3) lattice QCD and probably a second order transition for SU(2) lattice QCD. The chiral and deconfinement transitions are coinciding in the quenched approximation.

I. Introduction

The composite nature of hadrons, as assumed by the quark model or quantum chromodynamics (QCD), leads to the immediate conjecture, that at sufficiently high temperatures or densities strongly interacting matter undergoes a phase transition from a state of interacting hadrons to a quark-gluon plasma¹⁾. The phase transition may eventually proceed in two distinct steps²⁾: one, where the quarks and gluons bound in hadrons deconfine into massive quarks and gluons and a second transition, in which chiral symmetry is restored and the quarks and gluons become massless.

Since, with quantum chromodynamics we hope to have the basic theory of strong interactions, we should be able to show that QCD indeed leads to a phase diagram for strong-interaction matter like the one depicted in fig. 1. It is still unclear, however, whether the chiral and deconfinement transitions occur at different critical temperatures and if so, which one is lower.

The Monte Carlo evaluation of lattice QCD³⁾ gives us in principle the possibility to cover the whole range of physical temperatures T (and chemical potentials μ). Because of technical limitations at present only parts of the phase diagram are investigated. There exist detailed calculations for zero chemical potential⁴⁻¹¹⁾ - in the "quenched

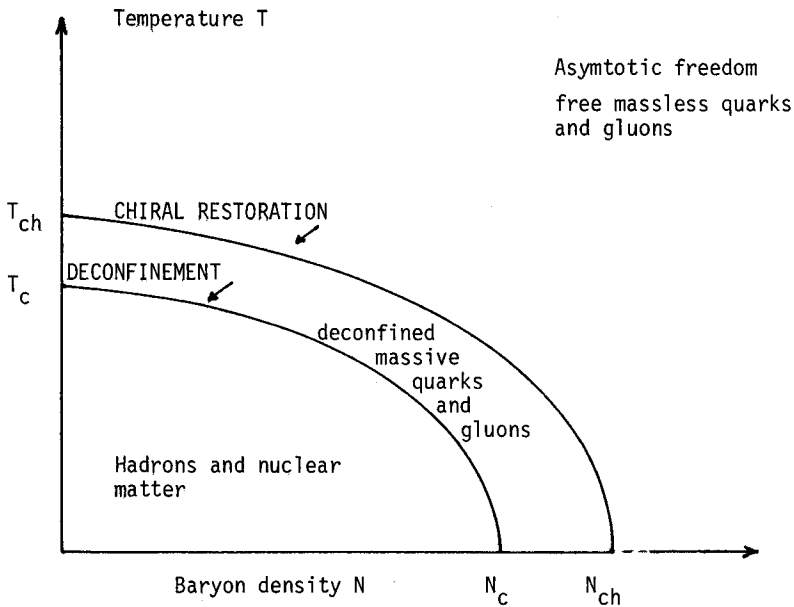


Figure 1 : The phase diagram for strongly interacting matter: confined, deconfined with spontaneously broken chiral symmetry, and deconfined with restored chiral symmetry.

approximation, where virtual quark loops are neglected, the essential finite temperature work is done. We shall report on these results. For $\mu \neq 0$ first, exploratory lattice calculations have been carried out by Kogut et al.¹²⁾ confirming the general picture.

The order of a transition and the transition temperature can be determined in two ways:

- i) the thermodynamic quantities like energy density, specific heat, pressure etc. are calculated as a function of temperature. Discontinuous or singular behaviour of any of these variables is then a sign for a transition⁷⁾;
- ii) one calculates order parameters associated with the corresponding transition, e.g. the thermal Wilson loop $\langle L \rangle$ is an order parameter for the deconfinement transition in pure Yang-Mills theory^{4,5,6)}, $\langle \bar{\Psi}\Psi \rangle$ is an order parameter for the chiral transition in QCD^{8,9,13,14)}.

In the next section, we shall review the relevant equations for finite temperature field theory and their application to QCD on the lattice. The following section presents the results for the pure Yang-Mills system, i.e. QCD without fermions. The contribution of the fermions and the chiral transition are then discussed in section IV. In section V we

give a summary and outlook on possible future progress in this field.

II. QCD for Finite Physical Temperatures on the Lattice

The formulation of quantum field theory for finite physical temperatures starts with the definition of the partition function

$$Z = \text{Tr} e^{-\beta(H - \mu N)} \quad , \quad (1)$$

where β is the inverse of the temperature, $\beta = T^{-1}$, μ the chemical potential, H the Hamiltonian of the system and N the particle density (the difference of the quark and antiquark densities). For simplicity and because we do not report on non-zero μ results we omit in the following all terms proportional to μ . The partition function can also be written as a Feynman path integral¹⁵⁾

$$Z = \int [d\phi] \exp \left\{ \int_0^\beta d\tau \int_V d^3x \mathcal{L}(\phi, i\dot{\phi}) \right\} \quad , \quad (2)$$

where \mathcal{L} is the Lagrangian density, $\tau = it$ and ϕ denotes all kinds of classical fields. Because of the trace operation in eq. (1) only periodic (antiperiodic) configurations

$$\phi(\vec{x}, 0) = \pm \phi(\vec{x}, \beta) \quad (3)$$

for Bose (Fermion) fields are to be considered in the functional integral. Since the functional-integral formalism does not include normal ordering, results obtained from eq. (2) have to be corrected for possible ground state contributions. In terms of the Euclidean action

$$S = - \int_0^\beta d\tau \int_V d^3x \mathcal{L} \quad , \quad (4)$$

the partition function becomes

$$Z(\beta, V) = \int [d\phi] e^{-S} \quad . \quad (5)$$

The Euclidean form thus contains a four-dimensional integral - the action, which is asymmetric in the space and temperature directions.

To obtain a momentum cut-off in the theory and to perform the $x - \tau$ integrations in eq. (4) we introduce a lattice¹⁶⁾. It is an asymmetric one, with N_σ sites and spacing a_σ for all spatial directions and N_β sites and spacing a_β for the thermal direction. Moreover, the lattice must be periodic in the thermal direction. Though it is not required, one usually takes periodic boundary conditions in the spatial directions too. Volume and temperature are then given by

$$V = (N_\sigma a_\sigma)^3 \quad , \quad \beta = N_\beta a_\beta = T^{-1} \quad . \quad (6)$$

Only the choice of asymmetric lattices allows for independent variations of volume and temperature. This is necessary for two reasons:

- i) the integration boundaries in the action integral are different for thermal and spatial directions and
- ii) at fixed N_σ and N_β , i.e. for a lattice with a fixed number of points, one wants to take partial derivatives with respect to V and β , because the energy density is given by

$$\epsilon = - \frac{1}{V} \left. \frac{\partial \ln Z}{\partial \beta} \right|_V \quad (7)$$

and the pressure by

$$P = \frac{1}{\beta} \left. \frac{\partial \ln Z}{\partial V} \right|_\beta \quad (8)$$

In the actual calculations⁷⁾ these derivatives are replaced by derivatives with respect to a_σ and the asymmetry variable $\xi = a_\sigma/a_\beta$. Then one obtains e.g. instead of eq. (7)

$$\epsilon = \frac{\xi^2}{N_\sigma^3 N_\beta^4 a_\sigma^4} \left. \frac{\partial \ln Z}{\partial \xi} \right|_{a_\sigma} \quad (9)$$

Let us now go from the general case to QCD. The corresponding Lagrangian density is

$$\mathcal{L} = - \frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu} + \bar{\Psi}(i\not{\partial} - g\not{A}_a \lambda^a)\Psi = \mathcal{L}_{YM}(A) + \mathcal{L}_F(A, \Psi, \bar{\Psi}) \quad (10)$$

with

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g f_{bc}^a A_\mu^b A_\nu^c \quad (11)$$

The f^{abc} are the $SU(N_c)$ structure functions, g the coupling constant and the colour indices a, b, c run from 1 to $N_c^2 - 1$. On the lattice the functional integration variables A , the gauge fields, are substituted by link variables

$$U_{xy} = \exp \{-i(x-y)^\mu A_\mu(\frac{x+y}{2})\} \quad (12)$$

where x and y denote adjacent sites on the lattice and $A_\mu = A_\mu^a \lambda_a$, the λ_a are the generators of the $SU(N_c)$ group. The spinor fields are attached to the sites of the lattice, they become site variables $\Psi_S, \bar{\Psi}_S$. The partition function can then be written as

$$Z = \int_{\text{links}} \Pi dU \int_{\text{sites}} \Pi d\Psi d\bar{\Psi} \exp \{-S^G(U) - S^F(U, \bar{\Psi}, \Psi)\} \quad (13)$$

where in thermal direction the bosonic integration variables U obey periodic boundary conditions and the spinor fields $\bar{\Psi}, \Psi$ antiperiodic boundary conditions. The action $S^G(U)$ describes the pure Yang-Mills part, $S^F(U, \bar{\Psi}, \Psi)$ the fermionic part of the full QCD action. There are now many possibilities for the dependence of the action on the lattice

variables. Their only requirement to be satisfied is, that in the continuum limit, where

$$N_\sigma, N_\beta \rightarrow \infty \quad ; \quad a_\sigma, a_\beta \rightarrow 0$$

with V and β fixed, one has to recover the correct classical continuum action. The most widely used action for the pure gauge field part is the one of Wilson¹⁶⁾:

$$S^G(U) = 2N_c \left[K_\sigma^G \sum_{\{P_\sigma\}} \left(1 - \frac{1}{N_c} \text{Re Tr } U_{ij} U_{jk} U_{kl} U_{li} \right) + K_\beta^G \sum_{\{P_\beta\}} \left(1 - \frac{1}{N_c} \text{Re Tr } U_{ij} U_{jk} U_{kl} U_{li} \right) \right] \quad (14)$$

Here the sum $\{P_\sigma\}$ runs over all plaquettes with only spatial links, the sum $\{P_\beta\}$ over all plaquettes with two spatial and two temporal links.

As a consequence of the asymmetry of the lattice spacings two coupling constants g_σ and g_β appear in the action¹⁷⁾, namely via

$$K_\sigma^G = \frac{1}{g_\sigma^2} \xi^{-1} \quad ; \quad K_\beta^G = \frac{1}{g_\beta^2} \xi \quad (15)$$

One needs two of them for $a_\sigma \neq a_\beta$ ($\xi \neq 1$), because then independent variations in a_σ and a_β or ξ can be compensated by changing g_σ and g_β correspondingly, such that physically measurable quantities remain the same. So, g_σ and g_β become functions of a_σ and ξ . For $\xi = 1$, i.e. $a_\sigma = a_\beta = a$, there is, of course, only one coupling constant

$$g(a) = g_\sigma(a_\sigma, 1) = g_\beta(a_\sigma, 1) \quad ; \quad K_{\sigma,\beta}^G(\xi = 1) = g^{-2} \quad (16)$$

Choosing Wilson fermions¹⁶⁾, the fermionic part of the action is (for one quark flavour)

$$S^F = \sum_{n,m} \bar{\Psi}_n Q_{nm} \Psi_m \quad (17)$$

where

$$Q_{nm} = 11 - K_\beta^F M_{0,nm} - K_\sigma^F \sum_{\mu=1}^3 M_{\mu,nm} \quad (18)$$

$$M_{\mu,nm} = (1 - \gamma_\mu) U_{nm} \delta_{n,m-\hat{\mu}} + (1 + \gamma_\mu) U_{nm}^+ \delta_{n,m+\hat{\mu}} \quad (19)$$

In the last three equations colour and spinor indices have been suppressed. Because of the antiperiodic boundary conditions for fermion variables in temperature direction at the upper boundary $\delta_{n,m\pm\hat{0}}$ has to be replaced by $-\delta_{n,m\pm\hat{0}}$. Notice that, as S^G , also the fermionic part S^F depends on two different coupling constants (here called hopping parameters) K_σ^F and K_β^F for asymmetric lattice spacings:

$$K_{\sigma}^F = \frac{4}{3+\xi} k_{\sigma}(a_{\sigma}, \xi) \quad ; \quad K_{\beta}^F = \frac{4\xi}{3+\xi} k_{\beta}(a_{\sigma}, \xi) \quad . \quad (20)$$

For $\xi = 1$ we have

$$K_{\sigma, \beta}^F(\xi = 1) = k(a) \quad , \quad (21)$$

which is the usual hopping parameter¹⁶⁾.

The integral in eq. (13) over the fermionic degrees of freedom $\bar{\Psi}, \Psi$ can be performed and yields¹⁸⁾

$$Z = \int \prod_{\text{links}} dU e^{-S^G(U)} \det Q \quad . \quad (22)$$

Evaluating formula (9) leads then to two terms for the energy density¹⁴⁾

$$\epsilon = \epsilon^G + \epsilon^F \quad , \quad (23)$$

with

$$\begin{aligned} \epsilon^G a_{\sigma}^4 &= - \frac{2N_c \xi^2}{N_{\sigma}^3 N_{\beta}} Z^{-1} \int \Pi dU e^{-S^G} \det Q \times \\ &\times \left\{ \frac{\partial K_{\sigma}^G}{\partial \xi} \sum_{\{P_{\sigma}\}} \left(1 - \frac{1}{N_c} \text{Re Tr } UUUU\right) \right. \\ &\left. + \frac{\partial K_{\beta}^G}{\partial \xi} \sum_{\{P_{\beta}\}} \left(1 - \frac{1}{N_c} \text{Re Tr } UUUU\right) \right\} \quad (24) \end{aligned}$$

for the gauge field contribution, and

$$\begin{aligned} \epsilon^F a_{\sigma}^4 &= - \frac{2N_c \xi^2}{N_{\sigma}^3 N_{\beta}} Z^{-1} \int \Pi dU e^{-S^G} \det Q \times \\ &\times \left\{ \frac{\partial K_{\sigma}^F}{\partial \xi} \sum_{\mu=1}^3 \text{Tr}(M_{\mu} Q^{-1}) + \frac{\partial K_{\beta}^F}{\partial \xi} \text{Tr}(M_0 Q^{-1}) \right\} \quad (25) \end{aligned}$$

for the fermion contribution.

Compared to the pure Yang-Mills theory the computational problem introduced by the fermions is to determine $\det Q$ and Q^{-1} ; e.g. in ϵ^G the only influence of the fermions is contained in the factor $\det Q$, which essentially renormalizes g . Since the calculation of the determinant of Q presents the most serious difficulties, one usually takes the so-called quenched approximation¹⁹⁾, which consists in setting in all equations

$$\det Q = 1 \quad . \quad (26)$$

This approximation corresponds to the neglect of all virtual quark loops. The results, which we show in the following, were all obtained using eq. (26).

III. Results for the Yang-Mills System

The Monte Carlo evaluation of the above formulae has to be carried out on a spatial lattice as large as possible - one would like to come close to the thermodynamic limit $V \rightarrow \infty$ ($N_{\sigma} \rightarrow \infty$). In the cases reported here, $\xi = 1$, i.e. $a_{\sigma} = a_{\beta} = a$ was used. For the Yang-Mills system the only parameter is then the coupling constant g . In the continuum limit, g and the lattice spacing a are related by the renormalization group equation (RGE)

$$a \Lambda_L = \exp \left\{ -\frac{24\pi^2}{11N_c g^2} - \frac{51}{121} \ln \frac{11N_c g^2}{48\pi^2} \right\}, \quad (27)$$

where Λ_L is the lattice scale parameter. All physical units on the lattice are appropriate powers of a or Λ_L^{-1} . The lattice scale parameter can be expressed in conventional units (MeV), once a known physical quantity (usually the string tension³⁾) is measured on the lattice. At fixed g^2 , the temperature is then obtained from

$$T = 1 / (N_{\beta} a (g^2)) \quad (28)$$

The thermodynamic average of a quantity X is given by

$$\langle X \rangle = \int \Pi dU e^{-S(U)} X(U) / \int \Pi dU e^{-S(U)}, \quad (29)$$

$$S(U) = S^G(U) + \ln \det \Omega(U) \quad (30)$$

For the pure Yang-Mills case or for full QCD in quenched approximation S is equal to S^G .

The calculation of the energy density ϵ^G from eq. (24) essentially requires the determination of plaquette averages

$$\bar{P} = 1 - \frac{1}{N_c} \text{Re} \langle \text{Tr} UUUU \rangle, \quad (31)$$

because for $\xi = 1$ we have

$$\epsilon^G a^4 = 6N_c \left\{ g^{-2} (\bar{P}_{\sigma} - \bar{P}_{\beta}) - \frac{\partial g_{\sigma}^{-2}}{\partial \xi} \Big|_{\xi=1} (\bar{P}_{\sigma} - \bar{P}_{\text{sym}}) - \frac{\partial g_{\beta}^{-2}}{\partial \xi} \Big|_{\xi=1} (\bar{P}_{\beta} - \bar{P}_{\text{sym}}) \right\} \quad (32)$$

The $\bar{P}_{\sigma, \beta}$ denote the averages of spatial and thermal plaquettes, \bar{P}_{sym} is the average plaquette value on a large symmetric ($N_{\sigma} = N_{\beta}$) lattice. Subtraction of \bar{P}_{sym} in eq. (32) just takes away the vacuum contribution, since a large symmetric lattice simulates a zero temperature system. The derivatives of $g_{\sigma, \beta}^{-2}$ with respect to ξ are known constants in the limit $g^2 \rightarrow 0$ (20).

What is the expected behaviour as a function of temperature of the gauge field energy density? Below the critical temperature T_c one

expects to find the energy density of a gas of hadrons. For a pure Yang-Mills system these hadrons must be glueballs and their excitations. At T_C a phase transition - the deconfining transition - should occur, signalled by a discontinuity for a first order transition or a singularity in the specific heat for a second order transition. For large temperatures ϵ^G should approach the energy density of a gas of free gluons, which is given by the Stefan-Boltzmann law

$$\epsilon_{SB}^G = \frac{\pi^2}{30} 2(N_C^2 - 1) T^4 \quad . \quad (33)$$

As can be seen in figs. 2 and 3 these expectations are in fact borne out by actual Monte Carlo simulations of the Yang-Mills system^{6,7)}.

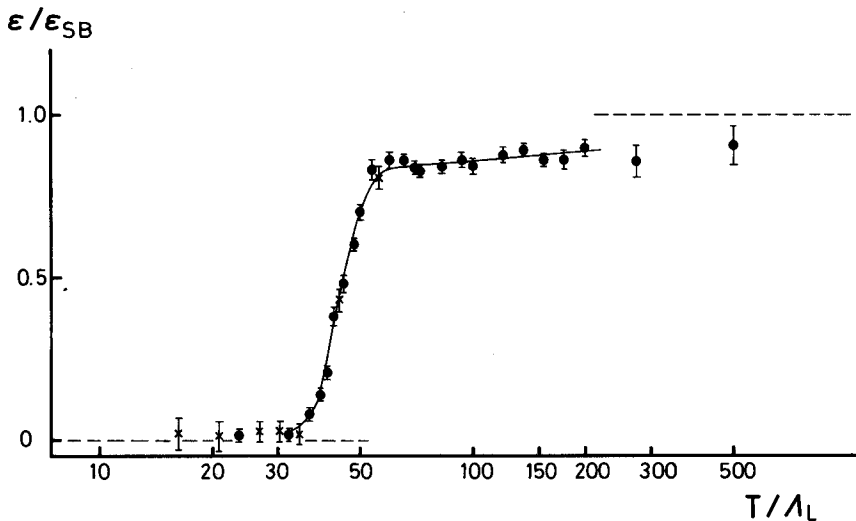


Figure 2 : The ratio $\epsilon^G/\epsilon_{SB}^G$ for the pure SU(3) Yang-Mills system calculated on a $10^3 \times 3$ lattice versus temperature from ref.7.

Obviously, both for SU(2) and SU(3), there is a phase transition, probably of second order for SU(2). First results⁶⁾ for SU(3) suggest a first order transition in that case. However, to confirm this assumption a more accurate and detailed investigation of the transition region is necessary. To this end one evaluates the thermal Wilson loop L

$$L = \frac{1}{N_C} \text{Tr} \prod_{\tau=1}^{N_\beta} U_{\vec{x}; \tau, \tau+1} \quad . \quad (34)$$

It is related to the free energy F_q of an isolated colour charge⁴⁾

$$\langle L \rangle \sim e^{-\beta F_q} \quad . \quad (35)$$

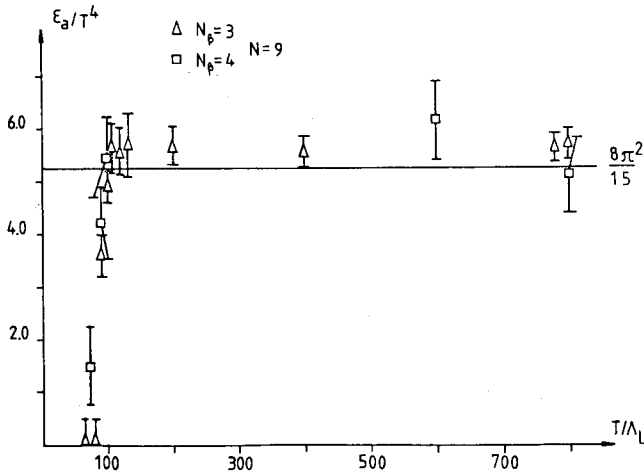


Figure 3 : The quantity ϵ_a^G/T^4 for the pure SU(3) Yang-Mills system calculated on lattices with $N_\sigma = 9$, $N_\beta = 3, 4$ ⁶⁾; ϵ_a^G contains only the main contribution to ϵ^G , which is proportional to g^{-2} . The horizontal line is the Stefan-Boltzmann result.

On an infinite lattice $\langle L \rangle$ vanishes in the confined phase, is non-zero above the deconfinement temperature and therefore an order parameter for the deconfinement transition. If we have a first order transition, then there must be a critical coupling g_c^2 , where two different states of the lattice exist, one of them having a non-zero $\langle L \rangle$ -value. We can make use of the Monte Carlo (MC) (Metropolis) method itself to obtain these distinct states. A normal MC calculation would start with either a completely ordered state of the lattice, i.e. all links being equal to the unit matrix (that corresponds to $g^2 \rightarrow 0$, high temperature) or a random (hot) configuration of links (corresponding to $g^2 \rightarrow \infty$, low temperature). During the MC procedure the lattice will thermalize, i.e. reach its thermal equilibrium. At the critical coupling the ordered start will then lead to the non-zero $\langle L \rangle$ -value, the hot start to a close to zero $\langle L \rangle$ -value. If the spatial volume V of the lattice is too small, there will be flips between the two phases, but when the spatial volume is large enough the two distinct states will persist for a large number of iterations. This enables one to determine the critical coupling g_c (and with eq. (28) T_c) with great accuracy. In fig. 4 such a search for the first order transition in SU(3) Yang-Mills theory on an $8^3 \times 3$ lattice is shown ¹⁰⁾. At $6/g^2 = 5.5531$ clearly two coexistent states are found. The energy densities of the two states are of course also different; the complete energy density as a function of

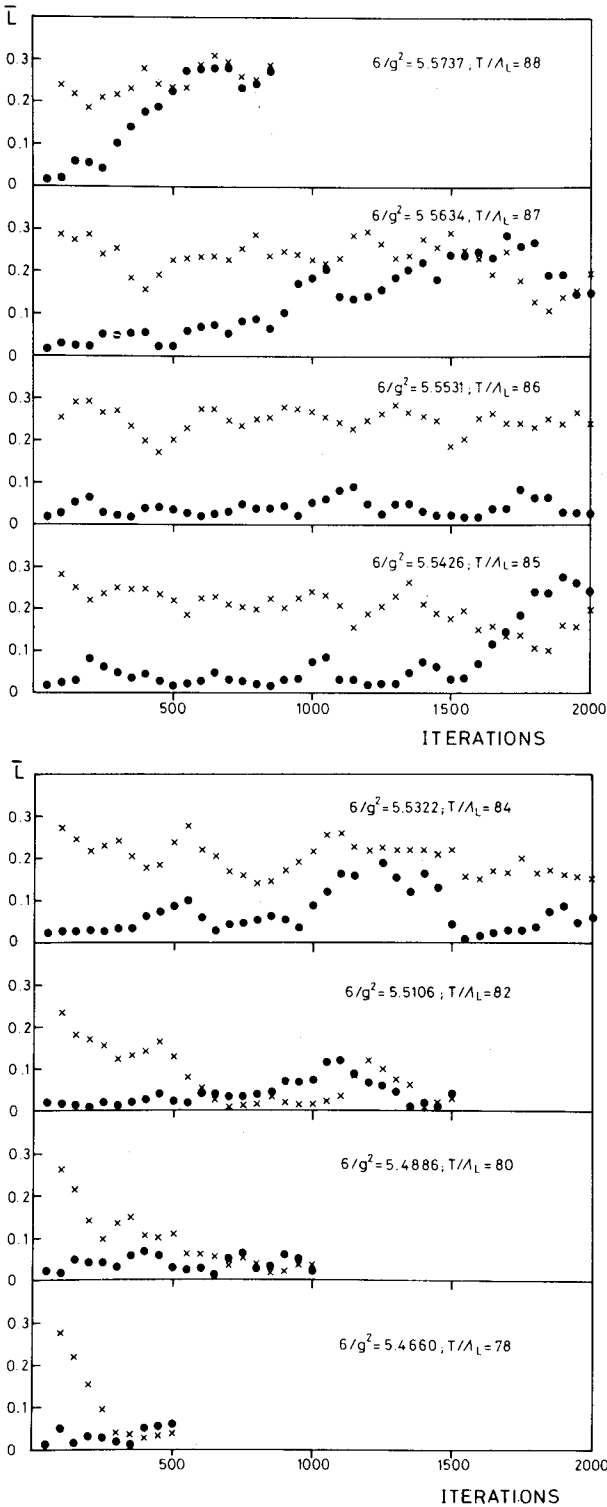


Figure 4 :

The average \bar{L} over 50 successive iterations of the order parameter L for the SU(3) Yang-Mills system, as a function of the total number of iterations after ordered (x) and random (o) starts, calculated on an $8^3 \times 3$ lattice¹⁰⁾ for various values of the coupling $6/g^2$; also shown is the associated temperature, using the RGE, eq. (27).

temperature is shown in fig. 5.

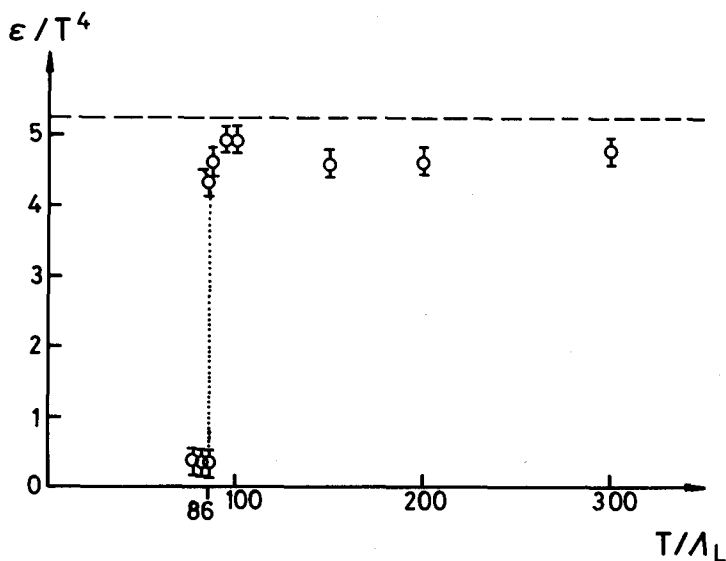


Figure 5 : The energy density ϵ^G/T^4 for the SU(3) Yang-Mills system as function of temperature, calculated on an $8^3 \times 3$ lattice¹⁰⁾.

In table 1 the results for the deconfinement temperature as obtained from calculations on lattices with different N_β are compared¹⁰⁾.

Table 1: The deconfinement temperature

| N_β | $6/g_c^2$ | $T_c[\Lambda_L]$ | $T_c[\sqrt{\sigma}]$ |
|-----------|-----------------|------------------|--------------------------------|
| 2 | 5.11 ± 0.01 | 78 ± 1 | |
| 3 | 5.55 ± 0.01 | 86 ± 1 | 0.519 ± 0.015 |
| 4 | 5.70 ± 0.01 | 76 ± 1 | 0.519 ± 0.050 $- 0.030$ |

The deviations, when T_c is expressed in units of Λ_L via eqs. (27) and (28), can be explained by higher order terms in the renormalization group relation, without any violation of general scaling behaviour. Indeed, when T_c is given in units of $\sqrt{\sigma}$ ²¹⁾ (σ is the string tension) no measurable deviations remain and

$$T_c = (0.519 \pm 0.050) \sqrt{\sigma} \approx 208 \pm 20 \text{ MeV} \quad , \quad (36)$$

which is agreeing well with the value obtained in the SU(2) case⁷⁾.

Since for SU(3) we have a first order transition, we can calculate

the latent heat - the difference in energy density between the two states at the transition. In fig. 6 we show $\Delta\epsilon^G/T_C^4$, which is a dimensionless quantity and should scale.

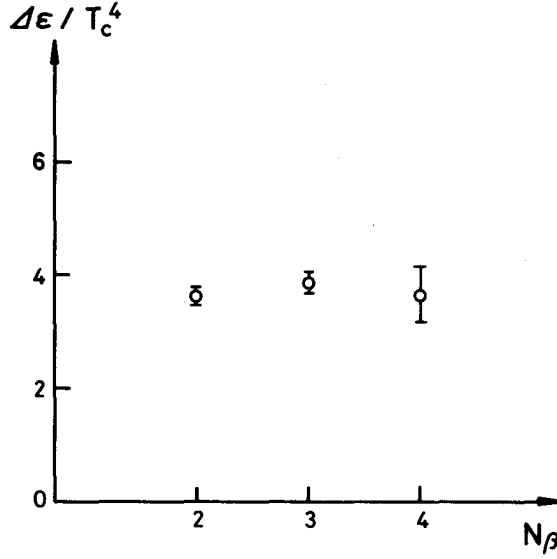


Figure 6 : The latent heat $\Delta\epsilon^G/T_C^4$ as function of temperature lattice size N_β from ref. 10.

In fact, the result¹⁰⁾ is independent of N_β and therefore of g_c (see table 1):

$$\Delta\epsilon^G/T_C^4 = 3.75 \pm 0.25 \quad , \quad (37)$$

$$\Delta\epsilon^G = 875 \pm 80 \text{ MeV/fm}^3 \quad . \quad (38)$$

The error in the last equation does not include the error in T_C .

IV. The Contribution of the Fermions in the Quenched Approximation

Evaluating eq. (25) for the fermion contribution ϵ^F to the energy density at $\xi = 1$ requires the derivatives $\partial K_{\sigma,\beta}^F/\partial\xi$. If we neglect the ξ -dependence of k_σ and k_β in eq. (20), we obtain

$$\left. \frac{\partial K_\sigma^F}{\partial\xi} \right|_{\xi=1} = -\frac{1}{4} k \quad , \quad \left. \frac{\partial K_\beta^F}{\partial\xi} \right|_{\xi=1} = \frac{3}{4} k \quad . \quad (39)$$

Also, at $\xi = 1$ the matrix Q from eq. (18) simplifies to

$$Q = 11 - kM \quad , \quad (40)$$

where

$$M = \sum_{\mu=0}^3 M_{\mu} . \quad (41)$$

The inverse of Q is then readily computed via a hopping parameter expansion (HPE) ^{13,22,23)}

$$Q^{-1} = \sum_{l=0}^{\infty} k^l M^l . \quad (42)$$

Insertion of the last equation into eq. (25) leads then - in quenched approximation - to a HPE for the fermion energy density

$$\begin{aligned} \epsilon^F a^4 = & - \frac{3}{4N_{\sigma}^3 N_{\beta}} \sum_{l=0}^{\infty} k^{l+1} \times \\ & \times \left\{ \langle \text{Tr } M_O M^l \rangle - \frac{1}{3} \sum_{\nu=1}^3 \langle \text{Tr } M_{\nu} M^l \rangle \right\} . \quad (43) \end{aligned}$$

Note, that because of the approximation in eq. (39), on a symmetric lattice ($N_{\sigma} = N_{\beta}$) ϵ^F is zero, i.e. no additional vacuum correction has to be introduced. As a consequence of the Kronecker deltas in eq. (19), only closed loops of length $l+1$ contribute to $\langle \text{Tr } M_{\mu} M^l \rangle$. On an infinite lattice $l+1 = 2n$ and the first non-vanishing loop has length 4, it is proportional to a plaquette. For $N_{\beta} = 2, 3$ however, the loop can be closed in temperature direction already with length 2, 3 because of the (anti)periodic boundary conditions. The loop is then proportional to the thermal Wilson loop $\langle L \rangle$ ¹⁴⁾.

The size of the hopping parameter $k(a)$ has to be determined by a separate calculation, either by requiring that the pion mass is zero ²³⁾, or by determining the convergence radius of the HPE for $\langle \bar{\Psi}\Psi \rangle$ ⁸⁾. Taking Susskind fermions instead of Wilson fermions, where the bare quark mass is the parameter corresponding to the hopping parameter, involves a comparable problem: one must compute the results for finite quark mass and then extrapolate to zero quark mass ⁹⁾.

In fig. 7 the fermion energy density ϵ^F/T^4 for one quark flavour and SU(2) is shown as a function of temperature. The energy density ϵ^F approaches the value of the free theory - the Stefan-Boltzmann limit on a lattice of the same size (indicated by a dashed line) - after a sudden jump at around the same temperature, where (see fig. 2) the gluon energy density ϵ^G has presumably a second order transition. That this indeed must happen - at least in the quenched approximation - is clear from the expansion in eq. (43), since the first non-vanishing term is proportional to $\langle L \rangle$, which stays an order parameter in the quenched approximation,

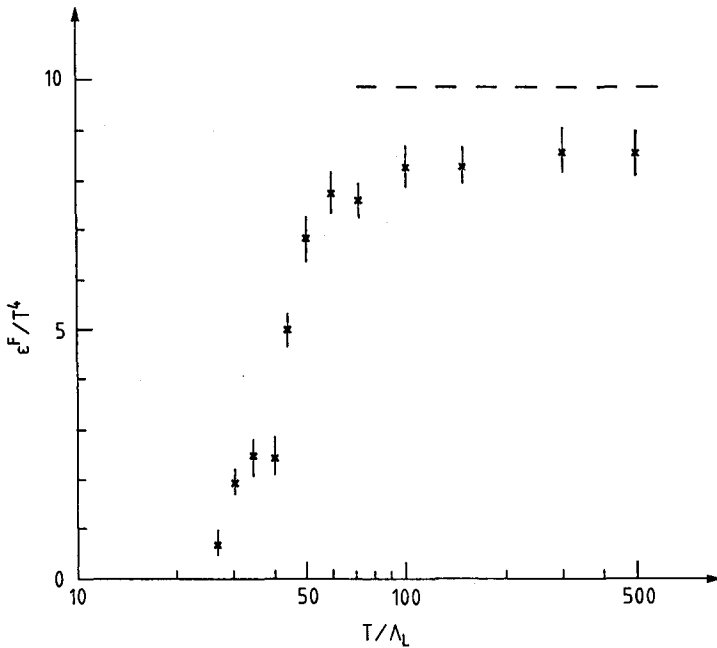


Figure 7 : The fermionic contribution ϵ^F to the energy density of SU(2) lattice QCD divided by the fourth power of temperature as a function of temperature, calculated on an $8^3 \times 3$ lattice with a 46th order hopping parameter expansion for massless quarks⁸⁾. The dashed line is the Stefan-Boltzmann result on a lattice of the same size.

and since the fourth order term, which is proportional to $\bar{P}_\sigma - \bar{P}_\beta$, behaves at the transition essentially like ϵ^G . For a first order transition like in SU(3) the coincidence becomes even more obvious, as can be seen from fig. 8. Kogut et al.¹¹⁾ estimate for the latent heat of the total system and four flavours

$$\Delta(\epsilon^G + \epsilon^F) = 1.50 \pm 0.5 \text{ GeV/fm}^3 \quad . \quad (44)$$

Finally let us comment on the problem of chiral symmetry restoration. This question is in the case of Wilson fermions particularly complex, since chiral symmetry is by construction broken on the lattice. Even a system of non-interacting massless fermions leads to a non-zero $\langle \bar{\Psi}\Psi \rangle_{\text{SB}}$. To study chiral symmetry restoration, one would therefore first have to show that $\langle \bar{\Psi}\Psi \rangle$, after subtraction of an adequate term, exhibits scaling behaviour and then check at what T it leads to a vanishing expectation value indicating chiral symmetry. A first step in the right direction consists in simply considering¹³⁾

$$\langle \bar{\Psi}\Psi \rangle - \langle \bar{\Psi}\Psi \rangle_{SB} \quad (45)$$

as an order parameter of chiral symmetry for Wilson fermions.

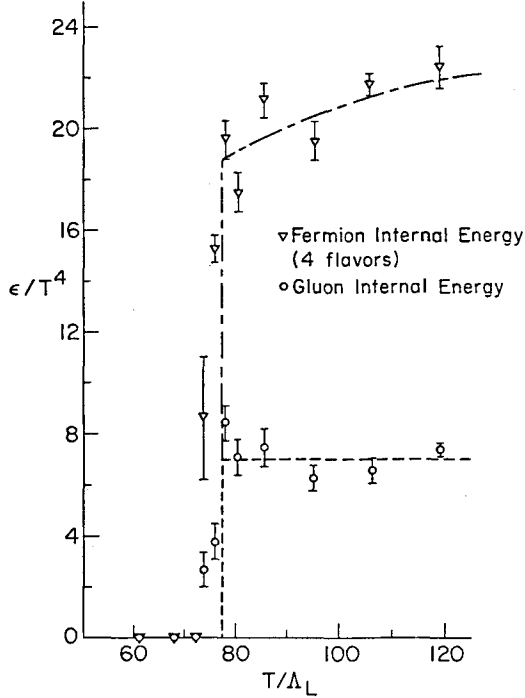


Figure 8 : The gluon and fermion energy densities ϵ^G/T^4 and ϵ^F/T^4 for 4 Susskind fermions versus temperature, calculated on an $8^3 \times 4$ lattice from ref. 11.

In fig. 9 this quantity is shown for SU(2) as function of temperature⁸⁾. At the transition ($T_c \sim 40 \Lambda_L$), expression (45) is still finite, suggesting $T_c < T_{ch}$ (the chiral transition temperature). The same result was found by Kogut et al.⁹⁾ for SU(2) Susskind fermions. However, the impression, that T_c and T_{ch} are different for SU(2) may be due to the second order nature of the transition and of course the approximations in the calculation. A clear picture is found for SU(3)⁹⁾. As can be seen in fig. 10 both the order parameter $\langle L \rangle$ for the deconfinement transition and the order parameter $\langle \bar{\Psi}\Psi \rangle$ for Susskind fermions and the chiral transition change their behaviour drastically at the same critical coupling. Like in the case of ϵ^G and ϵ^F also the first order deconfinement and chiral transitions must coincide in the quenched approximation, because

$$\langle \bar{\Psi}\Psi \rangle = \langle \text{Tr} (\text{colour} + \text{spin}) Q^{-1} \rangle \quad (46)$$

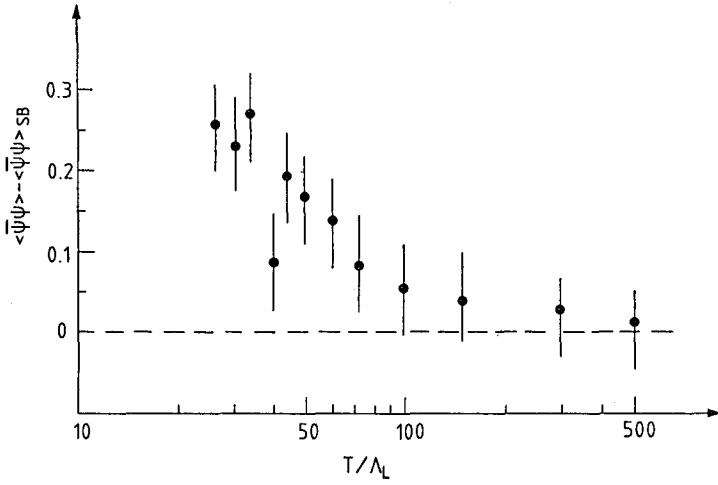


Figure 9 : The quantity $\langle \bar{\psi}\psi \rangle - \langle \bar{\psi}\psi \rangle_{SB}$ versus temperature for SU(2) and Wilson fermions from ref. 8, calculated on an $8^3 \times 3$ lattice.

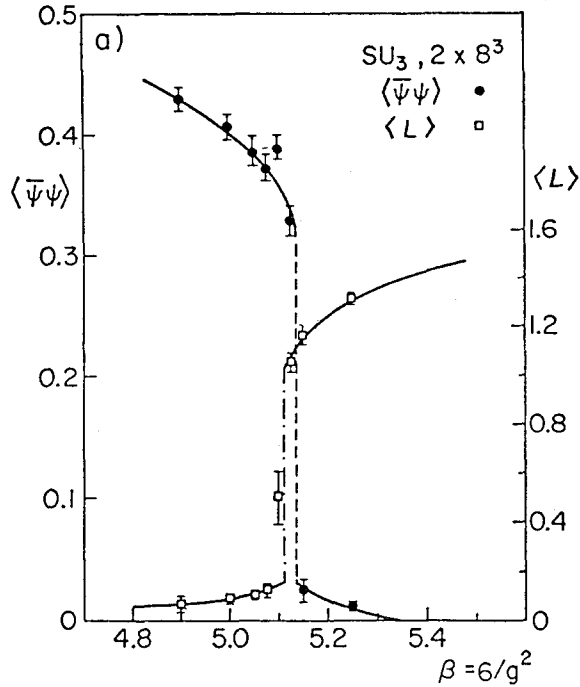


Figure 10: The quantity $\langle \bar{\psi}\psi \rangle$ for Susskind fermions and the thermal Wilson loop $\langle L \rangle$ for SU(3) versus $6/g^2$ from ref. 9, calculated on an $8^3 \times 2$ lattice.

and the corresponding hopping parameter expansion contains a term proportional to $\langle L \rangle$.

V. Summary and Outlook

In summarizing the results of Monte Carlo evaluations of lattice QCD in the quenched approximation and for chemical potential $\mu = 0$, one finds for SU(3)

- i) a first order deconfinement transition at $T_c = 208 \pm 20$ MeV for both the pure gauge field and the fermion contributions
- ii) the latent heat at the transition is

$$\Delta\epsilon^G = 875 \pm 80 \text{ MeV/fm}^3$$

or

$$\Delta(\epsilon^G + \epsilon^F) \approx 1.5 \pm 0.5 \text{ GeV/fm}^3 \quad \text{for } N_f = 4$$

- iii) the chiral transition temperature is the same as the deconfinement temperature.

For SU(2) the deconfinement transition is probably of second order, the critical temperature has about the same value as for SU(3).

The removal of the quenched approximation, i.e. the introduction of virtual quark loops into the calculation may change the above results considerably. First, exploratory calculations indicate^{24,25)}, that with decreasing quark mass the first order transition is replaced by a second order transition, which may even eventually disappear, when the quark mass becomes zero. Also, because the thermal Wilson loop $\langle L \rangle$ is no longer an order parameter in the full theory, the conclusion from the HPE, that the chiral and deconfinement transitions should coincide, if they are of first order, is no longer true. Should these questions be settled, the next step would be the investigation of the phase diagram (fig. 1) for non-zero chemical potential. There is a lot of work still to be done.

References

- 1) E.V. Shuryak, Phys. Reports 61 (1980) 71;
D.J. Gross, R.D. Pisarski and L.G. Yaffe, Rev. Mod. Phys. 53 (1981) 43;
H. Satz, Proc. 5th High energy heavy ion study, Berkeley/Cal. 1981.
- 2) G. Baym, Proc. of the Bielefeld Workshop 1982, H. Satz and J. Jacob eds., World Scientific, Singapore, 1982.
- 3) M. Creutz, Phys. Rev. D21 (1980) 2308; Phys. Rev. Lett. 45 (1980) 313.
- 4) L. McLerran and B. Svetitsky, Phys. Lett. 98B (1981) 195;

- J. Kuti, J. Polónyi and K. Szlachányi, Phys. Lett. 98B (1981) 199;
 J. Engels, F. Karsch, I. Montvay and H. Satz, Phys. Lett. 101B (1981) 89.
- 5) K. Kajantie, C. Montonen and E. Pietarinen, Z. Phys. C9 (1981) 253.
 - 6) I. Montvay and E. Pietarinen, Phys. Lett. 110B (1982) 148; 115B (1982) 151.
 - 7) J. Engels, F. Karsch, I. Montvay and H. Satz, Nucl. Phys. B205 [FS5] (1982) 545.
 - 8) J. Engels and F. Karsch, Phys. Lett. 125B (1983) 481.
 - 9) J. Kogut et al., Phys. Rev. Lett. 50 (1983) 393.
 - 10) T. Çelik, J. Engels and H. Satz, Phys. Lett. 125B (1983) 411; 129B (1983) 323.
 - 11) J. Kogut et al., Phys. Rev. Lett. 51 (1983) 869.
 - 12) J. Kogut et al., Illinois preprint ILL-(TH)-83-10, April 1983.
 - 13) C.B. Lang and H. Nicolai, Nucl. Phys. B200 [FS4] (1982) 135.
 - 14) J. Engels, F. Karsch and H. Satz, Phys. Lett. 113B (1982) 398.
 - 15) C. Bernard, Phys. Rev. D9 (1974) 3312.
 - 16) K. Wilson, Phys. Rev. D10 (1974) 2445; in "New Phenomena in Sub-nuclear Physics", ed. A. Zichichi, Plenum Press, New York 1977 (Erice 1975)
 - 17) A. Hasenfratz and P. Hasenfratz, Nucl. Phys. B193 (1981) 210.
 - 18) T. Matthews and A. Salam, Nuovo Cim. 12 (1954) 563; 2 (1955) 120.
 - 19) H. Hamber and G. Parisi, Phys. Rev. Lett. 47 (1981) 1792;
 E. Marinari, G. Parisi and C. Rebbi, Phys. Rev. Lett. 47 (1981) 1795;
 D. Weingarten, Phys. Lett. 109B (1982) 57.
 - 20) F. Karsch, Nucl. Phys. B205 [FS5] (1982) 285.
 - 21) F. Gutbrod, P. Hasenfratz, Z. Kunszt and I. Montvay, CERN preprint TH 3591, May 1983.
 - 22) A. Hasenfratz and P. Hasenfratz, Phys. Lett. 104B (1981) 489.
 - 23) A. Hasenfratz, P. Hasenfratz, Z. Kunszt and C.B. Lang, Phys. Lett. 110B (1982) 289.
 - 24) P. Hasenfratz, F. Karsch and I.O. Stamatescu, CERN preprint TH 3636 (1983).
 - 25) T. Çelik, J. Engels and H. Satz, Bielefeld preprint BI-TP 83/15, August 1983 (Phys. Lett. in press).