TREE VOLUME AND INCREMENT MODELS

FOR

RADIATA PINE THINNINGS

by

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The investigation of Reineke's Stand Density Index was a joint study with Dr. I.S. Ferguson. Except for this section of chapter IV, and where recognised, this thesis is my own original work.

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ABSTRACT

Linear regression analysis is used to estimate models suitable for predicting the volume and increment of trees to be thinned from radiata pine stands in the south east of South Australia.

The volume model predicts the volume of the trees selected for removal in thinning, from measurements made at time of thinning. The model is an extension of the combined variable equation including stand density, site potential, age and thinning variables.

The increment model predicts the increment that the tree will put on between inventory and time of thinning between one and six years later. The model predicts increment from relative tree size, site potential age, stand density and thinning variables, and the estimated volume of the tree at time of inventory.

Data were derived from 157 thinning operations, the volume model being based on 1418 observations, the increment model 3035.

The models are biologically sound and have been extensively tested to ensure that the assumptions underlying linear regression analysis are met; and have been tested against independent test data.

A number of indices of stand density were evaluated, basal area being marginally better than the ratio of basal area to

the maximum basal area the site can sustain, as derived from a model of stand dynamics.

A site potential measure in South Australia, based on volume production, site quality, was marginally better than site index, a measure based on upper stand height.

I. INTRODUCTION

Metrication

Notation

I. INTRODUCTION

South Australia is the driest state in the Commonwealth, with only 1.2% of the area receiving over 25 inches (635 mm) of rainfall per year (Bednall, 1957). Intensive plantation forestry is generally limited to these higher rainfall areas, the largest zone of which is in the south-east of the state.

The main plantation area is situated in the south-east of the state and has been described by Douglas (1970) and Bednall (1957). The regional resource (including adjacent Victorian plantations) consists of some 232,000 acres (93,900 ha) of softwood plantations and is mainly controlled by four organizations. The Woods and Forests Department of South Australia controls 59%, Softwood Holdings Ltd. 16%, Southern Australia Perpetual Forests Ltd. 16%, and the Forest Commission of Victoria 9%. The Woods and Forests plantations are primarily (90%) radiata pine, *Pinus radiata* (D.Don), and this area currently provides about 75% of the raw material supply for the regions integrated wood based industries. The industry in the region is fully utilizing the present allowable cut from the radiata pine plantations of the Woods and Forests Department. To enable management to make sound policy decisions regarding future usage of this forest resource it is essential that the techniques of yield prediction be soundly based, and of a high order of accuracy.

This thesis is concerned with the short term volume and increment prediction techniques necessary to enable silvicultural operations to be scheduled for a five year planning horizon.

Metrication

All calculations and results are reported in imperial units. All the data used are currently recorded in imperial units. Metric conversion of these data for this study was not undertaken, as conversion would only be practical after a computer based data storage and retrieval system has been designed and implemented, that will include all permanent sample plot data.

To facilitate application of the results to metric field data which will be collected in future inventory work, the metric equivalents of some of the more important equations are reported but with the addition of subscript (m) to the equation number to show that it is metric.

Within the text some of the more important definitions and measurements are converted to the metric equivalent and recorded in brackets following the imperial definition or measurement. The conversions are not necessarily mathematically exact but represent the practical conversion of the imperial quantity. Thus volume to a four inch top diameter underbark will be equivalent to the volume to a 10 cm top diameter underbark and not 10.16 cm; the conversion error incurred being considered inconsequential.

Notation

The more commonly used variables and parameters have been abbreviated when used in the text and in equations. Definitions of these have been summarised in Appendix 3. Abbreviations of less common variables are defined below the relevant equations in the text.

II. YIELD PREDICTION: PRACTICE AND PROBLEMS

CURRENT PRACTICE

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DISCUSSION OF THE PROBLEM

II. YIELD PREDICTION: PRACTICE AND PROBLEMS

CURRENT PRACTICE

The mensuration and management practice in South Australia has been described by Lewis (1957) and Keeves (1970). However it is necessary to reiterate the major points so that the problem to be investigated can be set out clearly.

Stratification

In South Australia it has been found that stratification of the forest into volume productivity classes is more effective than stratification based solely on some convenient measure of upper stand height (Keeves, 1970). This has led to the development of a Site Quality assessment technique, based on total volume production to a 4 inch top diameter underbark at age 9½. This technique described by Lewis in 1954 and by Keeves (1970) provides a detailed stratification of the forest that can be used at the time of subsequent inventory.

On all areas of radiata pine to be assessed, temporary one-tenth acre (0.05 ha) plots are first located subjectively covering the range of forest types to be found in the area. The sampling rate varies from locality to locality but approximately 150 plots are measured over the current annual assessment area of approximately 5,000 acres (2,000 ha). These plots are measured for predominant height and tree diameter, the height to the base of the green crown being measured on 12 randomly selected sample trees. From these measurements volume to a four inch top diameter underbark is estimated through a predominant height tarif relationship. A number of useful indices such as mean diameter, stocking, average height to the base of the green crown, the range of tree diameters, basal area and predominant height are also calculated.

These plots are inspected before assessment commences in each area and this inspection ensures that the assessment is consistent between assessors. Reference to the yield table at age 9½ ensures consistency between widely separated areas, and between plantations with different years of planting.

Assessment is by parallel strips 3 chains (60 m) apart, with each assessor mapping 1½ chains (30 m) on either side of his strip. A considerable amount of accessory information on live stocking, mortality, initial planting spacing and the proportion of ineffective trees is obtained from a systematic 3x5 chain (60x100 m) grid system of .05 acre (.02 ha) plots superimposed on these strips.

Following the assessment, site quality maps are prepared by joining the strip maps together and copies are provided for the District Forester of each Forest Reserve and for Head Office. Reports are also prepared giving for each area details of stocking, spacing, form, access, thinning capabilities and any other information considered relevant. The site quality plans form the basis of any stratification for inventory purposes.

Permanent Sample Plots

The first forest inventory was made in the south-east under the direction of E.H.F. Swain in 1934. Following this work the Department established and maintained a series of permanent sample plots which have been gradually augmented so that at present there are some 320 plots in radiata pine plantations in the south-east of South Australia. The plots have been re-measured at various intervals and provide a basic pool of data unparalleled in Australia. The primary objective of these plots is to determine the following information.

(1) The thinning treatments that will provide a maximum sustained yield of timber.

(2) The thinning treatments that will provide the product mix in terms of pulpwood and sawlogs of various sizes and types that is desired by the integrated industry in the region.

(3) The thinning treatments that yield the highest monetary return consistent with (1) and (2) above.

The standing volume of each plot is estimated from the measurement of sample trees, chosen at random from the range of diameter classes within the plot. Sample tree volumes are derived either by the 10 foot sectional method or by the use of the Regional Volume Table (Lewis and McIntyre, 1963). The number of trees measured is determined from a graph derived by Keeves (1961) which aims to keep the confidence limits of the error in volume due to sampling to within 3%. Work by Keeves (1961) and Jolly (1950) demonstrated the relationship between volume and basal area of trees in radiata pine stands in South Australia to be linear over a wide range of conditions, and it is through the use of this relationship that standing plot volumes are computed. Lewis (1963) has used the permanent plot data to develop a flexible thinning guide, which defines the range of thinning treatments which will satisfy the management objectives of the Woods and Forests Department.

Lewis and Keeves have prepared a yield table, as yet unpublished, for thinned stands of radiata pine based on the data from the permanent plots showing the average yields per acre in various size assortments, that may be expected from the thinnings and clear falling given the site quality and the thinning regime.

Inventory data

In South Australia five year plans are prepared prescribing where logging operations are to be carried out, and the residual stockings to be maintained after thinning. These plans are prepared by the Working Plans Branch after consultation with the foresters stationed on the Forest Reserve. These plans are based on an inventory generally carried out in the year preceding the initial year of the working plan period.

Within each year of planting on a Forest Reserve the area is divided into a number of logging classes. Each logging class is a group of compartments or sub-compartments that have received similar silvicultural thinning treatment in the past and which can receive similar treatment in the future. It is therefore a uniform area for logging purposes but not necessarily for yield prediction purposes.

Logging classes average approximately 65 acres in area and are further stratified by site quality and if necessary by stocking. Randomly located one-fifth acre plots are established such that on the average logging class five plots are established, the number of plots in each strata being proportional to the area of each strata. Unpublished work by Dundon (Keeves, 1970) indicates that this is a minimum sampling intensity and extra plots are generally established in logging classes with a wide range of strata or which are due to be clear felled.

This inventory is carried out in all areas due to receive a second or subsequent thinning, or which are due to be clear felled during the five year Working Plan period. Yields from areas to be first thinned are estimated by interpolation within the unpublished thinned stand yield table prepared by Lewis. No plots are established in them.

For each plot a number of basic parameters are either measured or derived from records.

(1) Plot area; calculated from measurements of the sides of the plot, assuming the plot to be trapezoidal.

(2) Extraction row frequency; the ratio between the number of extraction rows removed from the plot and the number of rows in the plot, including extraction rows.

(3) Age; the current year of measurement minus the year of plantation establishment.

(4) Site Quality; derived from the site quality assessment carried out at age 9½.

(5) Predominant height; derived from Lewis's unpublished yield table for a given age and site quality. If this does not appear to be correct from field observation then the predominant height is estimated from the mean height of the six largest trees on the one-fifth acre plot using an unpublished relationship developed by Keeves.

(6) Diameter breast height overbark; measured on all trees on the plot.

The number of trees to be left after the next thinning is then estimated from the thinning range of Lewis (1963) and the trees to be removed are marked on the ground. The thinnings elect are selected on practical silvicultural considerations which may vary from locality to locality although the thinning can generally be categorised as being a thinning from below, or a low thinning. The diameter of each tree to be thinned is recorded. Careful supervision of the technical staff who mark the thinnings elect in the inventory plot, and the staff who in practice mark the trees to be removed in commercial thinning, ensures that they are consistent one with the other and with desired practice.

Short Term Yield Prediction

From these data the volume of the thinnings to be removed is estimated through the volume - basal area line of the thinnings. Predominant height, age and the percentage number of trees to be removed are the independent variables used to estimate the coefficients of the volume-basal area line (Keeves, 1970). An expression of thinning type was tried in the original calculation of the regressions, but was found to be not significant, probably due to the narrow range of thinning type in the data used. For plots to be clear felled the volume-basal area line coefficients are related to predominant height alone.

Size assortments are estimated from unpublished tree size assortment tables which show the percentage of the volume to four inches top diameter that is within various top diameter limits, given the tree diameter at breast height. The relationship was developed by Lewis who found that the percentage was independent of the tree height.

These calculations enable an estimate to be made of the volume of each tree at time of inventory, which can then be aggregated to provide an estimate of the volume available from the thinnings elect sub-population. As the stands may grow from one to six years before they are felled it has been found essential to incorporate an estimate of this increment in the calculations.

In the case of stands due to be clear felled this increment is estimated from the unpublished thinned stand yield table of Lewis and Keeves. This approach cannot be used for the thinnings elect sub-population because the thinnings elect will have a lower increment than the main crop, being predominantly a thinning from below.

Leech has developed an unpublished increment function which estimates the current annual increment percent for the thinnings elect sub-population. This function uses predominant height, thinning intensity and a measure of stand density, as the independent variables. The function does not include a measure of site potential and in fact assumes an average site quality for each logging class. This leads to some anomalous results which although of little consequence in the estimation of the increment on the thinnings elect from a Forest Reserve, should be investigated.

The calculations of volume and increment are incorporated in a computer system which processes the inventory plot data to the stage where a proposed list of logging operations is prepared by logging classes, for each year of the Working Plan period.

DISCUSSION OF THE PROBLEM

The strength of the South Australian practice currently lies in the marking on the ground of the trees that are considered likely to be removed in the next thinning. This avoids the necessity for averaging stands to determine an average thinning interval, thinning type and thinning intensity and enables the stand to be treated in the manner which past experience has shown to be the most effective. Each stand is marked for thinning on its merits within the framework of Lewis's thinning regime (1963). By using this regime allowance can be made for changes in stand conditions between inventory and thinning, as the stocking after thinning is determined from estimated predominant height at time of thinning.

The trees to be thinned, or thinnings elect, having been identified individually, are then aggregated within the plot and within the stand to provide an estimate of the standing volume of the trees to be thinned. The increment on these trees, between time of inventory and time of thinning, must then be estimated so that an unbiassed estimate of the volume of thinnings can be used in the compilation of the five year plan.

The increment on the thinnings elect subpopulation can be considerable. A study of permanent sample plot information showed that on some plots on high site quality sites the increment over a five year interval, on the second thinnings elect, was equal to the volume of those thinnings elect at the start of the five year period. This volume obviously cannot be ignored if a high standard of management is to be attained and maintained.

In an average five year Working Plan the volume currently estimated as available from thinnings from the combination of all the logging classes during the plan period is approximately 15% higher than the standing volume of these thinnings elect at time of inventory, further emphasizing the need for an accurate estimate of increment.

The aim of this thesis is to develop models for radiata pine that will predict volume and short term increment on the individual trees of the thinnings elect sub-population.

It is desirable that the increment model be redeveloped along sound biological and statistical lines and that the increment model be developed along lines compatible with the volume model (Clutter, 1963).

Only parameters currently available from inventory plot measurements are to be used. The models should estimate tree volume and tree increment so that later estimation of assortments is facilitated, the estimates for the subpopulation of thinnings elect being obtained by aggregating the estimates for each tree.

III. STATISTICAL ANALYSIS

III. STATISTICAL ANALYSIS

The estimation of the relationship between one variable and a number of others is a common problem in forestry to which the technique of multiple linear regression analysis can be applied. Linear regression refers to the linearity of the coefficients of the independent variables and contrasts with non-linear regression analysis in which the coefficients to be estimated may be the power to which a variable is raised.

The analytical techniques used in non-linear regression analysis are still being developed and evaluated and statistical inference is still in a primitive state.

On the other hand the theory underlying linear regression analysis is well established (Johnston, 1960). Freese (1964) contains a concise development of the technique with specific reference to forestry applications. Johnston (1960), Acton (1959) and Sokal and Rohlf (1969) contain a more general development of both the theory and the application. Because non-linear techniques are not well developed it was decided to use linear regression analysis in this study.

ASSUMPTIONS IN LEAST SQUARES LINEAR REGRESSION ANALYSIS

A number of assumptions are made in linear regression analysis.

Homogeneity of the variance

The variance of the residual or error term of the regression is assumed to be constant over the range of the regression data and therefore independent of the magnitude of the dependent or independent variables.

If the variance is heterogeneous then the estimates of the coefficients in the regression will be not as precise as they would have been if the variance was homogeneous. Nevertheless the estimates will still be unbiassed (Johnston, 1960).

There are three commonly used tests of homogeneity of the variance. In all these tests the data is partitioned over the range of the dependent variable and the variance of the residuals within each of these cells is calculated. The statistics used in the three different tests can then be calculated from the cell variances. Hartley's (1950) maximum F-ratio tests the ratio of the largest cell variance to the smallest cell variance. Cochrans test (1941) uses the ratio of the largest cell variance to the pooled variances for all cells. Both these tests are only applicable if the number of observations used to compute the variance is the same for each cell. However, Hartley believes that the sensitivity of the test is not seriously dependent on this assumption and suggests the use of the statistic as a rough test even when the number of observations are different. Bartlett's test (1937) is the only one that allows for differing numbers of observations. The test involves the calculation of a more complex statistic (see Freese (1967) for details) which is tested against the statistic Chi-square.

Acton (1959) considers that none of these three tests are robust, all being sensitive to non-normality in the underlying distributions. However there appears to be general agreement (Acton, 1959; Sokal and Rohlf, 1969) that Bartlett's test is the most robust and most appropriate of these tests for testing for homogeneity of the variance.

If Bartlett's test indicates that the variance is not homogeneous then weighting (Cunia, 1964; Frayer, 1966; Freese, 1964) can be used to eliminate heterogeneity or reduce it to acceptable levels. In some cases this may also be achieved by transforming the dependent variable.

Gerrard (1966) partitioned his tree data into cells of one inch diameter and five feet in height, calculated the variance of each cell and then estimated the function relating the logarithm of variance to the mean tree diameter and mean tree height of each cell. The weighting function used was the reciprocal of the expected value of the variance. However, Cunia (1964) considered that partitioning the data on D^2Ht was satisfactory and found that variance could be satisfactorily estimated as a function of $(D^2Ht)^2$.

In practice then, it is desirable to conduct a preliminary examination of the variance, develop if necessary a weighting function and develop the regression models using this weighting function. When a statistically and biologically satisfactory model has been formulated the data is ordered according to the expected value of the dependent variable, partitioned into approximately equal cells and the variance of each cell tested using Bartlett's test (1937). If the weighting function is adequate then Bartlett's test should be non-significant. If however the test indicates significant heterogeneity of the variance, then a better weighting function should be estimated and the cycle of operations continued until Bartlett's test is non-significant.

Measurement error

In linear regression analysis one of the assumptions that must be met if efficient estimates are to be made of regression coefficients and confidence limits is that the variables are measured without error.

If the dependent variable is measured with error, but the error is unbiassed, then the mean square residual will be inflated resulting in a reduced level of significance in the analysis of variance. Provided that the regression explains a large amount of the variation then this problem is relatively minor.

The dependent variable, volume of the tree to four inches top diameter underbark, includes errors caused by faulty use of the girth tape and the bark guage and by technique errors associated with the use of the ten foot sectional method. The latter can be ignored as they are relatively small, and are consistent in all data used in South Australia. Errors due to the incorrect use of the girth tape are biassed but seem likely to be less than the other errors associated with the measurement of volume, which are generally unbiassed.

If the independent variables are measured with error then there is little effect provided that they are unbiassed and provided that the completed regression model will be applied to data measured with the same source, frequency and degree of error as the data used to develop the model.

The independent variables are, with the exception of errors in diameter through faulty use of the girth tape, estimated without bias. They are all consistent in that similar errors are included in the basic data as are likely to be included in the measurement of inventory plots. This is because the same operators are responsible for both measurements, working to essentially the same procedures.

The effect of measurement errors on this analysis can be considered to be of little consequence.

Serial correlation

If correlation exists between the residuals when a regression model is fitted to successive observations then serial correlation or auto-correlation is said to exist. If serial correlation exists then although the estimates of the regression coefficients are unbiassed, the predictions of the dependent variable will be inefficient and will have needlessly large sampling variances (Johnston, 1960).

The volume and increment models developed may be expected to suffer from serial correlation because there are a number of trees chosen from each plot for each thinning, and these trees will share the same values of the stand parameters. To facilitate the testing of the more important models the data were arranged in order.

For the volume model all the trees from each thinning in each plot were grouped together.

For the increment model, all the trees from each plot were grouped together. Also all the measurements from the same tree (different number of years before thinning) were grouped together.

To test whether serial correlation was a problem, the Durbin-Watson "d" statistic (Durbin and Watson, 1950, 1951; Theil and Nagar, 1961) was calculated. Because of the large number of observations the "d" statistic of Theil and Nagar (1961) using the Von Neumann ratio had to be used.

Testing the models for serial correlation should follow tests of homogeneity of variance. In this study, if significant serial correlation does occur then the observations from the same tree or plot may be randomly culled in an attempt to remove the problem since in general an ample number of observations and degrees of freedom are available.

Normality of residuals

In linear regression analysis the residual or error term of the regression is assumed to be normally distributed. However it is rare to find in the literature covering the derivation of mathematical models of forest growth, statistical tests used to prove that the residuals are normally distributed, although Johnston (1960) considers such a test should be made. Sokal and Rohlf (1969) consider that the consequences of non-normality are not too serious. Only a very skewed distribution would have a marked effect on the significance level of the analysis of variance, but Sokal and Rohlf recognise that it should be tested and corrected where possible by suitable transformation of the data.

Cochran and Cox (1957) suggest that the normality of the residuals should be tested by a Chi-square test, although they point out that this test is not specific and does not indicate whether skewness or kurtosis is the problem. They and others (Sokal and Rohlf, 1969; Snedecor and Cochran, 1967) describe techniques for estimating moment statistics of kurtosis and skewness. These two statistics are then compared with "t" (two tailed test) for infinite degrees of freedom. The latter test is readily applied and as it indicates the type of departure from normality it was chosen as the appropriate test.

TECHNIQUE USED

A computer program REX written by Grosenbaugh (1967) was used to calculate the linear regressions. This program is extremely flexible and produces all necessary statistics to enable an analysis of variance to be calculated. Weighted regressions can be calculated and regressions can be conditioned to pass either through the origin or the mean. A correlation matrix for all variables used in a model can also be calculated.

Choice of the level of significance to be used

In classical theory of statistics it is difficult to derive quantitatively the level of significance that should be used in the statistical tests associated with model development. Two types of errors must be considered. Type I errors are occuring, when a true null hypothesis is rejected; type II errors arise when a false null hypothesis is accepted (Sokal and Rohlf, 1969; Dixon and Massey, 1957). It is desirable that both type I and type II errors should be reduced to the minimum. However, since reducing the probability of a type I error increases the probability of a type II error it is more appropriate to strive for a compromise between each type.

For the volume model development it was expected that regressions would explain a high proportion of the variability of the data, especially as there are 1418 observations from 157 plots. For this phase the significance level chosen was $p = .01$.

For the increment model development, although there were more observations, 3035, the regression models were not considered likely to explain as high a proportion of the variation as the volume models. Whereas errors of measurement are small relative to volume, they are large relative to increment. Seasonal fluctuations in growth are also likely to affect increment more than volume. Because of this a lower significance level was more applicable for the increment model, the level selected being $p = .05$.

Summary

The procedure adopted can be summarised as follows:

(1) Formulate the linear models to be tested.

(2) Examine the variance of the dependent variable.

(3) If necessary, develop an equation to predict variance and derive a weighting function.

(4) Fit the models to the data, using weights if necessary.

(5) Evaluate the models chosing the most acceptable model on statistical and biological grounds.

(6) Test for homogeneity of the variance. If the variance is heterogeneous then re-estimate weighting function and repeat steps (4) to (6).

(7) Test for normality of the residuals. If significant non-normality then transform the dependent variable, recalculate the regressions and retest.

(8) Test for serial correlation. If there is significant serial correlation then fit the accepted model to a reduced data base and re-evaluate. Repeat until serial correlation is not significant.

(9) When all tests are satisfactorily completed test the model on independent data.

PART 1 A CONSIDERATION OF STAND DENSITY

IV. STAND DENSITY INDICES

IV. STAND DENSITY INDICES

INTRODUCTION

"Forestry is bedevilled by a wide range of parameters of density, the only common feature among them being their general ineffectiveness." Baskerville (1962) in noting this focuses attention on one of the perplexing problems of forest mensuration and management, for if a meaningful index of the level of competition or density of the stand can be evolved then it is likely to be a significant variable in any model predicting growth or yield.

For a variable to be included in a hypothesized model it must be capable of being measured with some degree of accuracy. The concept of stand density or competition is an abstract quality that seems incapable of precise definition, so it is logical that all so called measures of density be regarded simply as indices. Although some indices are capable of precise unbiassed measurement they are only a proxy for the intangible concept referred to as stand density.

A number of attempts have been made to define stand density but they are not precise. Spurr (1952) thinks of stand density as the degree to which the area is utilized by trees. Two of the world's leading forestry associations define stand (or crop) density as the density of stocking expressed in number of trees, basal area, volume or other criteria, on a per acre basis, (Soc. Am.For., 1958; Empire Forestry Association, 1953). Curtin (1968) presents another definition in which stand density is defined as the average intensity of competition which is occurring between individual trees within a stand.

Although it seems unlikely that a universal and precise definition of stand density will ever be achieved there are some general requirements that are considered desirable. Whatever the prime defects the index should be clear, consistent, objective and easy to apply (Bickford, Baker and Wilson, 1957). Spurr (1952) considers that it should not be related to age or site productivity and this test has often been used as one of the criteria for evaluating the effectiveness of a stand density index. As noted before, Curtin (1968) considers that the prime requirement of a measure of density is that it should express the intensity of competition. Baskerville (1962) states that "... any meaningful measure of density must assess size and number simultaneously and relate this to a range which constitutes full occupancy for a given site", a point of view that is also held by Baker (1950).

A number of indices can be selected and tested in the development of the models so that the index finally chosen will be the one that is statistically the most effective in the model. The ranking of the indices may not be valid for other management models but it will be satisfactory for the prediction models for radiata pine thinnings.

The common indices of stand density can be classified according to the variables used in their estimation into one of four categories (Curtin, 1968; Vezina, 1964).

(1) Number of Trees.

(2) Number of Trees and Diameter.

(3) Number of Trees and Height.

(4) Number of Trees, Diameter and Height.

NUMBER OF TREES AS AN INDEX OF DENSITY

The simplest index of stand density is number of trees per unit area but its effectiveness is limited unless mean diameter, height or age are held constant and therefore by inference, taken into account. The use of the single parameter number of trees was found by Nelson and Brender (1963) to be too inefficient to be seriously considered as an index of density. Although ignored by most workers it is the simplest index to measure and should be at least tested.

REINEKE'S STAND DENSITY INDEX[1]

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(1) The research reported in this section is the result of joint work by Dr. I.S. Ferguson of the Australian National University and the author.

Reineke (1933) examined data from evenaged stands of full density and concluded that the number of trees per acre N was a function of the quadratic mean diameter D_i, the diameter corresponding to the mean basal area per tree.

$$N = b_0 \, D_i^{\,b_1} \quad \text{.................................. Equation IV.1}$$

The constants b_0 and b_1 in equation IV.1 were estimated by regression analysis following a logarithmic transformation to convert the equation to a linear form.

$$\log_{10}(N) = \log_{10}(b_0) + b_1 \log_{10}(D_i) \quad \text{.......... Equation IV.2}$$

The slope coefficient b_1 appeared to be constant (-1.605) for 12 of the 14 species examined.

Reineke used equation IV.2 to define an index of stand density on the assumption that the same slope would hold for the relationship between the logarithms of the number of stems per acre (N_i) and quadratic mean diameter (D_i) in stands which had not reached full density. The stand density index SDI was arbitrarily defined as the number of trees per acre in a stand having a quadratic mean diameter of 10.0 inches which was of equal density to the stand in question. This latter figure reduced the computation involved in calculating the stand density index where logarithms to base 10 were used.

Substitution and transposition of the elements involved in the definition yielded equation IV.3.

$$\log_{10}(B) = \log_{10}(N_i) - b_1 \log_{10}(D_i) + b_1 \text{ ...... Equation IV.3}$$

This expression has been used widely to estimate stand density, often using the same constant (-1.605) which Reineke established for most of the species he examined.

In view of the use of this index it is strange that the assumptions underlying the biological processes involved have largely been ignored. The index should therefore be critically examined in relation to stand dynamics in plantations of radiata pine.

The data

In selecting stands which had reached "full density", Reineke simply plotted all the available data (mainly from temporary plots) on double-log graph paper and confined his regression analysis to those points which fell on the extreme right of the scatter. This is unsatisfactory since there is no guarantee that the plots selected were uniformly of the same density.

Curtin (1968), in defining density as the average intensity of competition between the individual trees in the stand, pointed out that substantial natural mortality provided a clear indication of when a stand had reached maximum density. Admittedly, seasonal climatic fluctuations from year to year introduced a stochastic element into this condition of maximum density, since the resources available to support the stand will change in accord with the climate. Nevertheless, this concept of maximum density is probably as uniform a condition as one can hope to achieve.

The existence of natural mortality, however, may not be a sufficient condition to identify stands which have reached maximum density. Mortality can arise from causes other than intense competition. These other causes, such as insect and pathogen attack, are often superimposed on, or interact with, climatic fluctuations and thus mortality can never provide a completely unambiguous criterion of maximum density. Nevertheless the existence of substantial and continuing mortality provides a reasonably consistent and objective means of ensuring that stands have reached equivalent conditions of competition and density.

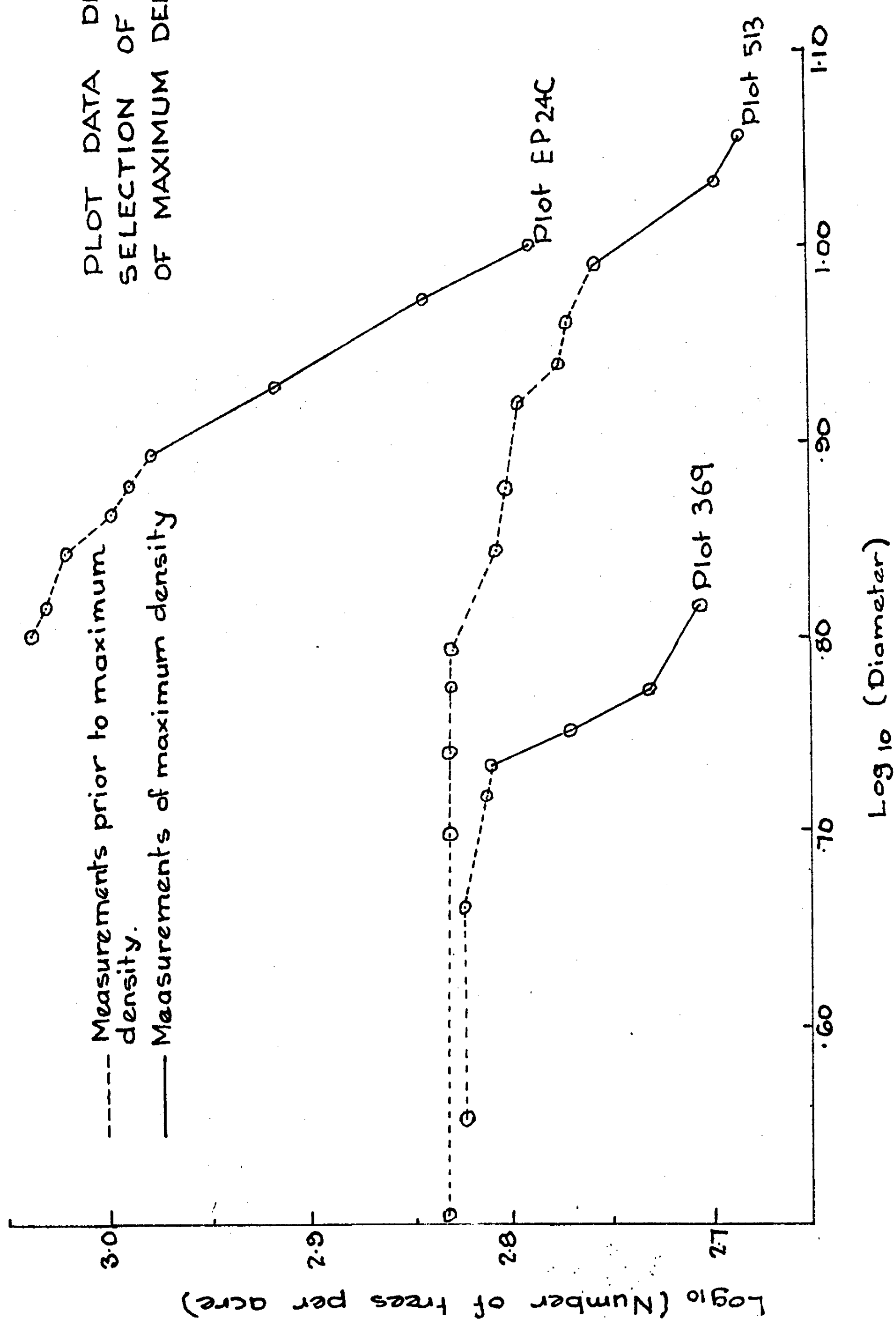
As noted in chapter II there are some 320 permanent sample plots in radiata pine plantations and among these are a number of unthinned plots that provide an opportunity to select stands which have reached maximum density. To this data were added data from a number of plots that are not permanent sample plots, but which have been left unthinned.

The data shown in figure IV.1 have been chosen to illustrate the selection of these observations which had reached maximum density.

The plots included in figure IV.1 cover a wide range of trends in mortality. Plot 369 for example, presented no problems in determining the onset of maximum density. Others,

FIGURE IV.1

PLOT DATA DEMONSTRATING SELECTION OF POINT OF ONSET OF MAXIMUM DENSITY



like Plot 513 provided some difficulty in deciding which of the observations represented the start of maximum density by the criterion of substantial mortality. In such cases, which were typical of the majority of the data examined, a conservative approach was adopted and the earlier observations were excluded. For a few plots, such as Plot EP24C, there was much greater uncertainty but again a conservative approach was adopted in selecting observations.

Some 160 observations from 34 plots were selected as clearly representing conditions of maximum density. These data were further culled by selecting only the first and last observations in each plot. These measures reduced the correlation between successive measurements and the statistical problems which otherwise arise. The 59 observations in the final set of data were used to examine the empirical nature of stand dynamics under conditions of maximum density.

Growth

As many previous studies have shown, the basal area per acre of "fully stocked" stands is a function of age, along with other variables. Hopefully, the data used in this study are based on a more precise and consistent definition of density than the highly subjective definitions which characterized many of these studies.

Given the type of data available it is somewhat easier to estimate the yield function for basal area than the growth function. Equation IV.4 relates basal area B to age A, site potential SI and initial stocking after establishment E, (table IV.1).

$$\log_{10}(B) = 0.7569 - 4.584 \frac{1}{A} + 0.0185\ SI$$

$$- 0.635\ 10^{-4}\ SI^2 + 0.243 \log_{10}(E) \ \ldots\ldots \text{ Equation IV.4}$$

The multiple coefficient of determination of 0.92 indicates that the model is satisfactory. Several other models with different curvilinear forms in SI were tested but equation IV.4 was significantly better. Basal area reaches a maximum corresponding to a site index of 148 feet which is well above the best site found in South Australian radiata pine plantations. The Durbin-Watson "d" statistic was computed and the test was inconclusive, but it seems likely that the effect of any serial correlation would be small and would have little impact on the estimated regression coefficients and their standard errors.

It is proposed to examine the data further using other non-linear models and non-linear regression techniques. However, this model is adequate to examine the dynamic aspects of stand behaviour which underlie Reineke's index.

Basal area per acre is related to the number of stems per acre and quadratic mean diameter as in the indentity, equation IV.5.

$$B \equiv \frac{\pi N_i D_i^2}{576}$$

$$\log_{10}(B) \equiv \log_{10}(N_i) - 2.0 \log_{10}(D_i) + \log(\pi/576) \ \ldots\ldots$$

$$\ldots\ldots \text{ Equation IV.5}$$

TABLE IV.1

REGRESSION STATISTICS EQUATION IV.4

$$\log_{10}(B) = 0.7569 - 4.584\frac{1}{A} + 0.0185\,SI - 0.635\,10^{-4}\,SI^2 + 0.243\,\log_{10}(E)$$

where E = initial stocking after establishment

$\log_{10}$ = logarithms to the base 10

Source of Variation	D.F.	S.S.	M.S.
Regression	4	0.76795	0.19199
Residuals	54	0.06636	0.00123
Total	58	0.83431	

$F_{4/54} = 156$ i.e. significant at $p = .001$

Substituting equation IV.5 in equation IV.4 the following relationship between the number of stems per acre, quadratic mean diameter and other variables is obtained for stands of maximum density:-

$$\log_{10}(N_i) = 3.0202 - 2.0 \log_{10}(D_i) - 4.584 \frac{1}{A} + 0.0185 \text{ SI} - 0.635$$
$$10^{-4} \text{ SI}^2 + 0.243 \log_{10} E \text{ .... Equation IV.6}$$

The difference between the slope coefficient for quadratic mean diameter in equation IV.6 compared with Reineke's result (-1.605) raises some doubts about Reineke's function, but discussion of these is best left until the model of stand dynamics has been completed.

Equation IV.6 defines the locus of all possible combinations of initial stocking after establishment E and quadratic mean diameter D_i for a given age, site index and initial stocking. It does not define the dynamic path which a particular stand would take in terms of the values of N_i and D_i at each point in time.

Mortality

In order to examine the dynamic path of the values of N_i and D_i for a particular stand, it is necessary to examine the other dynamic process involved, mortality. To be consistent with the manner of measurement and the nature of the process, the mortality variable has been defined as the geometric mean annual survival ratio.

$$P = t\sqrt{N/N(t)} \quad \text{.........................} \quad \text{Equation IV.7}$$

where P denotes the geometric mean annual survival ratio,

t denotes the number of years between the observation used in the final data set and the preceding measurement of that plot,

N denotes the number of living stems per acre of the observation in the final data set,

N(t) denotes the number of living stems per acre at the time of the preceding measurement.

This definition is relevant because survival is a multiplicative process, a 90% survival in one year followed by a 70% survival in the second year gives an overall survival after two years of 63%. Successive measurements on any one plot were made at irregular intervals varying from 2 to 6 years so that explicit recognition of the time between measurements was essential.

Various forms of the relationship between survival, site and age were tested. Equation IV.8 appears to be the most satisfactory, table IV.2.

$$\log_{10}(N) - \log_{10}(N(t)) = -0.0092\ t \quad \text{..............} \quad \text{Equation IV.8}$$

where N, N(t) and t are defined as for equation IV.7.

The coefficient of multiple determination for the regression was 0.79. Other models incorporating site potential and age

TABLE IV.2

REGRESSION STATISTICS EQUATION IV.8

$$\log_{10} (N) - \log_{10} (N(t)) = -0.0092\ t$$

where

t = the number of years between the observation used in the final data set and the preceding measurement of that plot,

N = the number of living stems per acre of the observation in the final data set,

N(t) = the number of living stems per acre at the time of preceding measurement.

Source of Variation	D.F.	S.S.	M.S.
Regression	1	0.07363	0.07363
Residuals	58	0.01955	0.00034
Total	59	0.09318	

$F_{1/58} = 217$ i.e. significant at $p = .001$

were not significantly different from equation IV.8. Survival was apparently constant over the range of ages and sites in the sample data, the estimated mean annual survival ratio being 0.84. This is somewhat surprising since other studies (for example Meyer (1938)) suggest that survival increases with increasing age, and to a lesser degree, with increasing site. However, Meyer's trends were based on graphical analyses and on data from a much wider range of ages and sites.

The second term in equation IV.8 is transposed to the right hand side to provide equation IV.9, the second part of the model of stand dynamics.

$$\log_{10}(N) = \log_{10}(N(t)) - 0.0092\ t \qquad \text{Equation IV.9}$$

Given the stocking existing at some previous point of time under conditions of maximum density, equation IV.9 enables current stocking to be predicted.

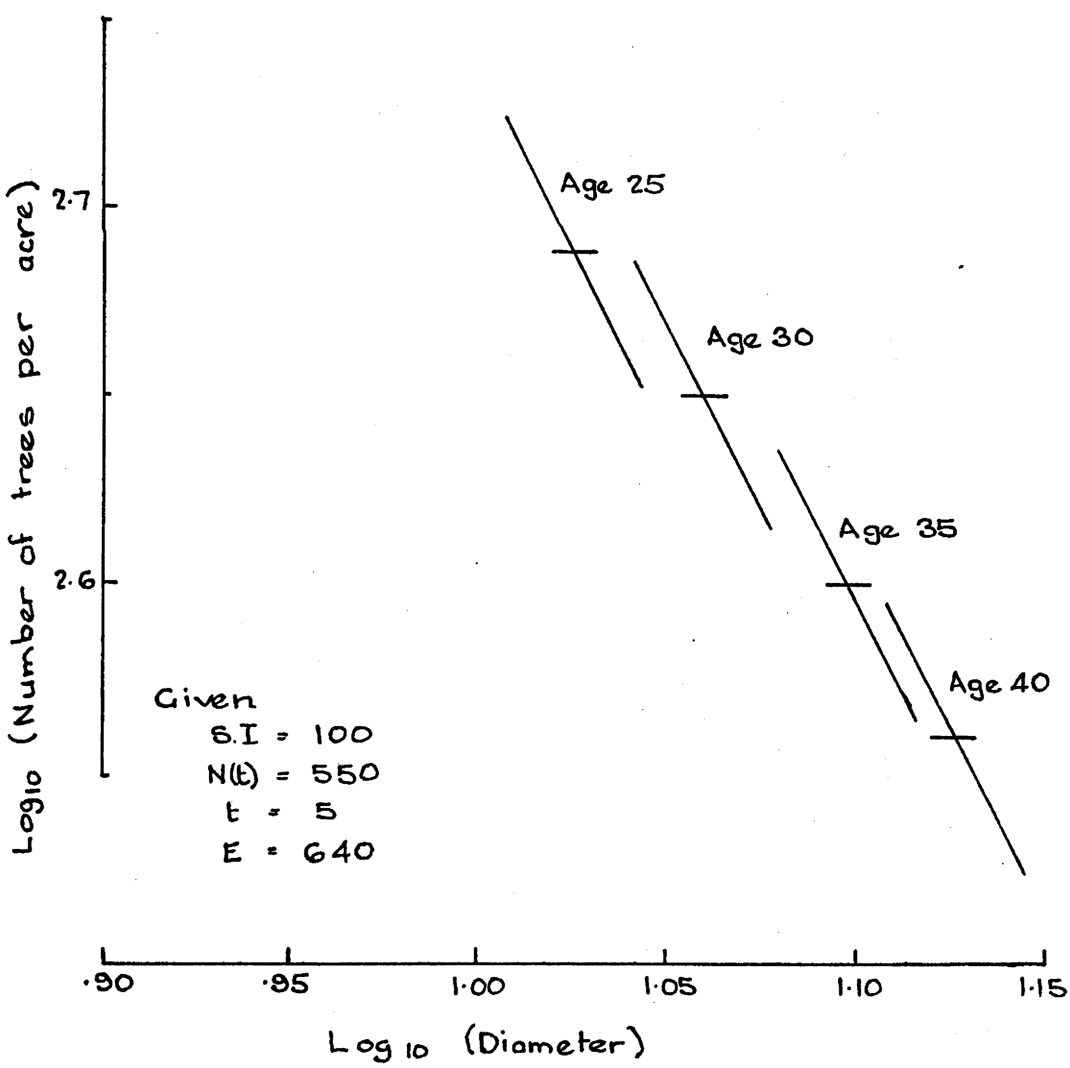
A Simple Model of Stand Dynamics

Equations IV.6 and IV.9 are simultaneous processes whose interaction traces out the dynamic path of the relationship between the number of stems per acre and quadratic mean diameter, over time. This is illustrated for a given site index and initial stocking in figure IV.2.

The sloping lines are the loci traced out by equation IV.9 for those same points in time. The intersection points show the

FIGURE IV. 2

EXAMPLE OF THE DYNAMIC RELATIONSHIP BETWEEN NUMBER OF TREES PER ACRE AND MEAN DIAMETER



dynamic path. Comparison of the trends in figure IV.2 with the observations classed as representing maximum density in figure IV.1 shows that the estimated trends conform with the general behaviour of the data.

Figure IV.3 provides a specific comparison of the actual and estimated intersection points for three of the plots with larger number of measurements during maximum density.

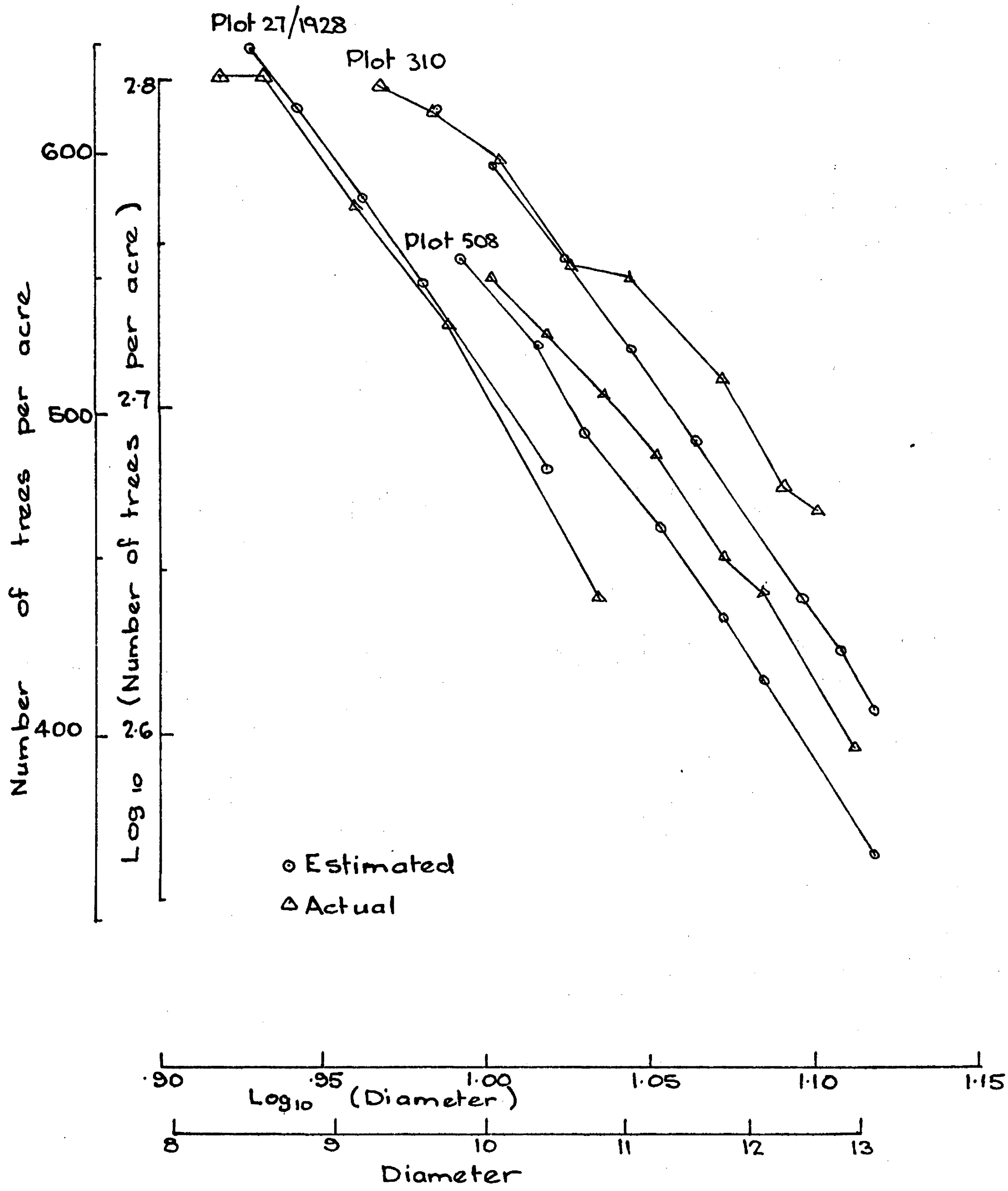
Although an objective statistical test of the difference between the actual and estimated points in figure IV.3 is not possible, because some of the data were used to estimate the dynamic relationships, the visual agreement is sufficient to demonstrate that the concept of two interacting processes is soundly based.

Stand density index

The model of stand dynamics has so far omitted any explicit recognition of Reineke's equation, IV.2, which is called the "reference curve" for stands of full density. If equation IV.2 is added to the model based on equations IV.6 and IV.9 then the system has three equations in only two endogenous variables (N and D_i). Such a system is inconsistent because the three equations cannot be jointly satisfied by a single pair of values of the endogenous variables under any reasonable assumptions about the nature of the system. Plainly, Reineke's equation IV.2 is at fault since it is hard to envisage a system which lacks either a growth or a mortality process.

FIGURE IV.3

COMPARISON OF ESTIMATED AND ACTUAL PLOT TRENDS



Reineke's equation IV.2 can therefore be considered an artifact created by fitting a static function to data which reflect two simultaneous dynamic processes, growth and mortality, and the stand density index therefore lacks any dynamic biological foundation. The slope coefficient for Reineke's equation (-1.605) will nearly always differ from the true value (-2.0) because the true relation between number of stems and diameter is confounded by the dynamic processes which have been ignored. Figure IV.2 illustrates the dynamic processes for a permanent plot.

It is concluded that Reineke's stand density index should not be used as an index of stand density because it has no sound biological foundation. On the other hand the ratio of standing basal area to the theoretical maximum basal area is soundly based and should be evaluated in the volume and increment models as an index of stand density.

Where the observations used are from temporary plots it is more difficult to describe the source of the confounding of Reineke's index, but the same principle holds. For a given species in natural stands of full density, the number of stems per acre will tend to be inversely correlated with age by the very nature of the processes of regeneration, growth and mortality. Hence the observations which Reineke selected from the extreme right of the scatter will be subject to the same confounding apparent if the dynamic processes were to be ignored in figure IV.2. Moreover the reference curve will tend to be based on stands of high site potential rather than low site potential because the latter tend to lie to the left of those from sites of high potential.

However, equation IV.4 provides a means of estimating, for any area of forest, the theoretical maximum basal area that the site could sustain.

In practical inventory the use of the measure would need the initial plantation spacing. This can generally be estimated, or is available from planting records although the order of accuracy is low. The ratio of the standing basal area to the theoretical maximum basal area appears a logical measure although the difference between the logarithms to the base 10 of the two basal area components could be used, the choice of the measure depending on the transformations, if any, of the dependent variable in the volume and increment models.

OTHER INDICES USING NUMBER OF TREES AND DIAMETER

The most commonly used indices of stand density relate the number of trees per acre to the mean diameter of the stand, or some other measure of diameter. However, these indices are affected by previous thinning history which led Hummel (1953) to argue in favour of a measure using height rather than diameter. On the other hand, the fact that number of trees and diameter are affected by thinning may enable a very useful parameter to be derived, using them in combination, to describe the differences in density due to past thinning history.

Basal area per acre

This is the most widely used measure of stand density and has the advantage of being readily measured. Assmann (Vezina, 1963) found that current annual increment of volume can be related to relative basal area, but this meant that an optimum value of basal area had to be set, and a critical level chosen at which incremental loss was an arbitrary 5%. This method depended on the establishment of an optimal basal area which is very difficult to do in practice, and the method has in consequence not found widespread acceptance.

Work in New South Wales (Gentle, Henry and Shepherd, 1962) indicates that the minimum basal area that will avoid incremental loss increases with age. As it may be argued that the minimum basal area, below which incremental losses occur, represents a constant level of competition, this suggests that basal area may not be a particularly good index of density at least at that particular level, and that basal area to some power less than one may be more appropriate.

However, basal area does reflect the level of competition, at least if height is constant, and it should be considered as a variable in the growth and yield model.

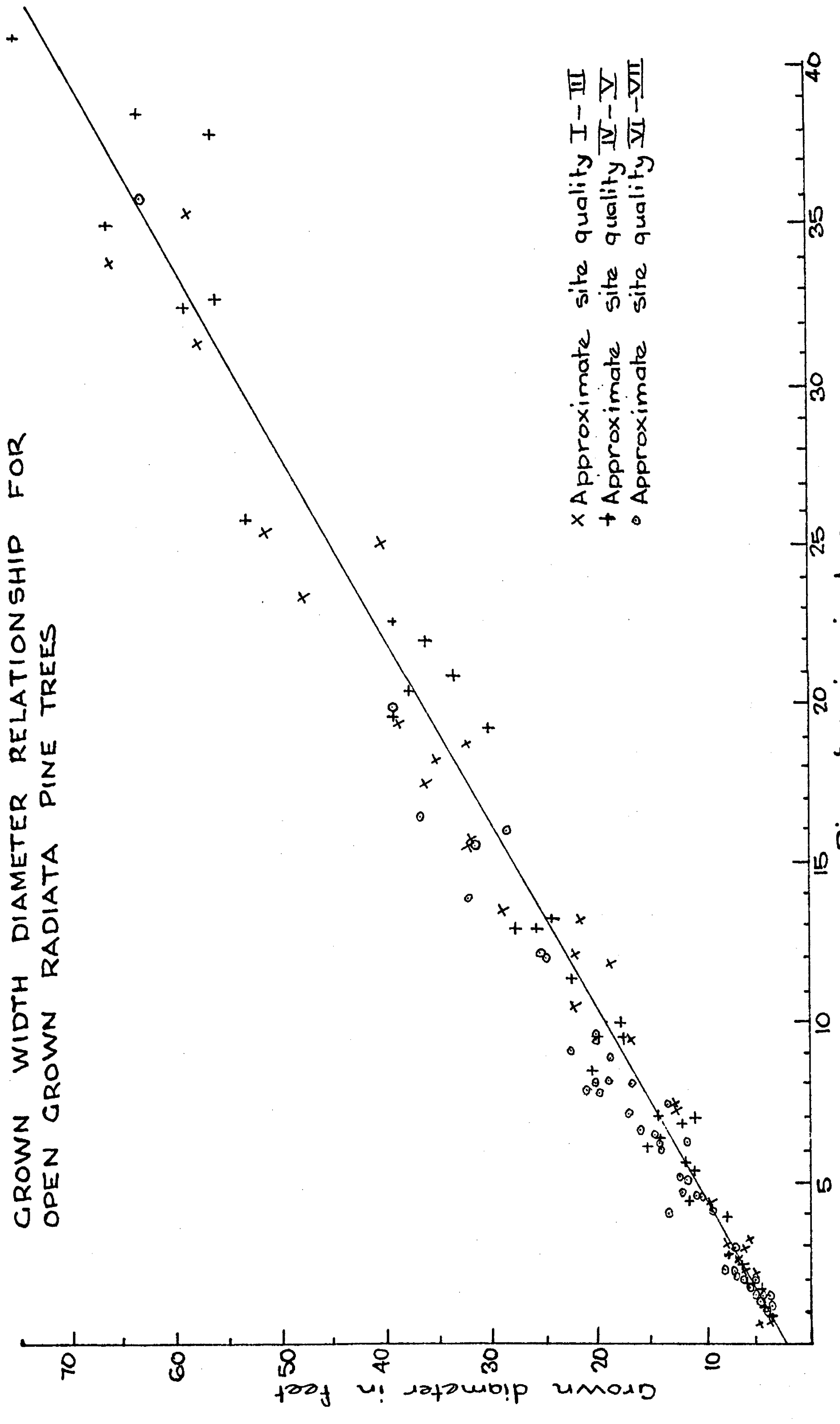
Crown competition factor

In 1961 Krajicek, Brinkman and Gingrich described an index of density, the Crown Competition Factor, based on the observation that for open grown trees the crown width is linearly related to tree diameter, and the assumption that the number of trees whose open grown crown area exactly covers one acre represents a constant level of stand density regardless of diameter.

Using the definition of open grown trees adopted by Krajicek (Krajicek et al, 1961) ten open grown radiata pine trees were measured for diameter (breast height over bark) and the projection of the crown on the ground was mapped. The edge of the crown was ragged but it was approximately circular, although the tree bole was rarely in the centre of the circle, being displaced towards the west and north west, the direction of the prevailing winds. The displacement was greatest in trees on exposed sites, least on trees that were relatively sheltered. It was concluded that the mean of two measurements of crown width at right angles to one another would give a satisfactory approximation of crown width, calculation of crown area from a crown map being considered too time consuming.

A total of 109 open grown radiata pine trees were then measured for diameter and crown width, and an estimate was made of the potential site quality of the site. This estimate of site potential used current unpublished Woods and Forests Department assessment techniques for unplanted land. The data were subdivided into three groups based on the estimated site potential and the graph, figure IV.4, of crown width against tree diameter was plotted. Linear regressions, using crown width as the dependent variable and tree diameter as the independent variable, were calculated for each of the three data groups, and then for combinations of them. A covariance analsis in the manner

FIGURE IV. 4


CROWN WIDTH DIAMETER RELATIONSHIP FOR
OPEN GROWN RADIATA PINE TREES
x Approximate site quality I - III
+ Approximate site quality IV - V
o Approximate site quality VI - VII
Crown diameter in feet
Diameter in inches
10
20
30
40
50
60
70
5
15
25
35

outlined by Freese (1964) indicated no significant difference between the three groups of data, although from figure IV.4 it appears that the trees on sites of poor site potential may have broader crowns for the same diameter.

As the relationship derived from this data, if accepted, will be used to develop volume and increment models for thinnings on sites ranging from SQ I to SQ V it was decided arbitrarily to exclude the 44 trees on the poorer sites and use only the relationship calculated for the 65 trees on sites estimated as having a site potential between SQ I and SQ V. Equation VII.12 resulted, the analysis of variance being shown in Table IV.3.

$$CW = 2.371 + 1.700\ D \quad \text{.......................... Equation VII.12}$$

where CW = Crown width in feet.

From this equation the "Maximum Crown Area" can be calculated, expressing in percent of an acre, the area that would be occupied by the crown of an open grown tree of specified diameter. From this the Crown Competition Factor CCF was calculated in a manner similar to that outlined by Krajicek (Krajicek et al, 1961).

$$CCF = \frac{1}{Ar}\left[0.01039\ N_{Ar} + 0.014531\sum_{1}^{N_{Ar}} D + 0.005208\sum_{1}^{N_{Ar}} (D^2)\right] \ \ldots$$

$$\text{..... Equation IV.13}$$

TABLE IV.3

REGRESSION STATISTICS EQUATION IV.12

$CW = 2.371 + 1.700\ D$

where

CW = Crown width in feet

Source of variation	D.F.	S.S.	M.S.
Regression	1	23140.1	23140.1
Residual	63	599.3	9.513
Total	64	23739.4	

$F_{1/63} = 2432$ i.e. significant at $p = .001$

where CCF = Crown Competition Factor

Ar = Area of the stand in acres

N_{Ar} = Number of trees in the stand

The Crown Competition Factor, as calculated, is based on a statistically satisfactory regression but there are a number of problems associated with its use.

The basic data for open grown trees are relatively imprecise. It is difficult, especially on the larger trees, to satisfactorily measure tree diameter at breast height because the heavy lower whorls that occur in open grown trees cause severe fluting and swelling. It is also impossible to be absolutely certain that the tree is in fact open grown and has not been subjected to competition for a period of time early in its life. The definition of an open grown tree used tends to minimize the risk of measuring a tree which has suffered past competition, but it is still a possibility. The measurement of crown width was shown on the ten selected trees to be a difficult task due to the ragged shape of the crown, and the fact that exposure has affected the crown of most trees only emphasises this difficulty.

Apart from measurement difficulties it is doubtful whether the original assumption, that the number of trees whose open grown crown area exactly covers one acre represents a constant level of density, is tenable. In practice tree crowns increase laterally with age, so that until some external competition affects them the crowns are roughly circular in shape. The competition may be either above or below ground, the latter being more likely to occur first as tree roots commonly occur further from the tree bole than the length of the largest lateral branch. Even open grown trees of radiata pine suffer competition in that their roots are competing for water and nutrients with the pasture or herbaceous plants occuring under them. It is evident that there is some competition before the stage is reached where tree crowns could just cover the whole area, and while this competition may be relatively slight it does cast doubts on the applicability of the technique.

Thus, although a Crown Competition Factor has been developed for radiata pine it is unlikely to be satisfactory in practice and will not be tested in the models.

Tree Area Ratio

Chisman and Schumacher (1940) postulated that the ground area that a tree occupies is related to its diameter.

$$1 = b_0 + b_1 \sum_1^N D + b_2 \sum_1^N (D^2)$$

They then tested the relationship on 133 fully stocked sample plots of loblolly pine calculating the value of the constants by the method of least squares. The index of stand density adopted was to substitute the plot data into the equation derived, and multiply the answer by 100 to obtain the percentage stocking relevant to the average of the fully stocked plots.

The use of the Tree Area Ratio has the disadvantages of Reineke's Stand Density Index or basal area and there is the difficulty of defining fully stocked stands. Because of this the measure was not considered further.

INDICES USING NUMBER OF TREES AND HEIGHT

In all the indices of density in this category the measure of height refers to the tallest part of the stand although the measures vary considerably in the choice of trees that contribute to the measure. It will be shown in Chapter V that the South Australian measure, predominant height, is as satisfactory as any others and it is assumed that the different measures of stand height are interchangeable.

Hart's Index

One of the earliest indices of density was developed by Hart in 1928 (Lewis, 1959; Crowe, 1967) who used the ratio of spacing to height expressed as a percentage. Although Becking (Lewis, 1959) assumed that the trees in the stand are spaced evenly at the apices of equilateral triangles, most authors have assumed that a square spacing is more suitable.

As in South Australia the predominant height is not significantly affected by the low thinning that is generally carried out (vide chapter V), any index of density based on height and number of trees per acre is unlikely to be affected by previous thinning treatment, a factor considered desirable by some authors (Crowe, 1967; Hummel, 1953), leading them to use this index in preference to those indices including mean diameter.

Hummels Height/Spacing ratio

This is similar to Hart's index but, for simplicity, the stocking per unit area is related to the stocking per unit area which is equivalent to 20% of the top height of the stand (Hummel, 1953; Hummel et al, 1959).

The only disadvantage of this index compared with Hart's index is that it assumes a base value which is arbitrarily determined.

INDICES USING NUMBER OF TREES, DIAMETER AND HEIGHT

Volume

Logically, the best measure of the level of competition would be the total production of dry weight and the best index of this is volume. However, what is required is the best index of stand density for use in volume and increment predicting equations for the thinnings subpopulation. As it is obviously impractical to measure the volume of a stand and use this as an estimator of the volume of the thinnings sub-population, this index has not been considered further.

Bole Area

The most commonly quoted index involving the three parameters is bole area, first proposed by Lexen (1943). In essence bole area is the area of cambial surface on the boles of the trees in a stand. Lexen, however did not measure this direct but calculated a bole area index from the product of number of trees per acre, mean stand diameter and mean stand height.

The measure of mean height is difficult to define but it would appear logical to use Lorey's mean height which is the mean height of the trees in the stand weighted in proportion to their basal area. In practice it is common to use a measure of stand top height instead of mean height (Hummel et al, 1959). Lexen found that for ponderosa pine, bole area was linearly related to mean diameter and mean height, but it is not possible to confirm if this linearity also holds for radiata pine.

The measure is the most likely of the indices using diameter, height and number of trees and should be tested in the model, using predominant height instead of mean stand height.

Schumacher and Coile's stocking percent

Schumacher and Coile (1960) working with southern pines introduced an index of stand density based on basal area per acre, dominant height and age, which refers to the ground area a stand occupies. It is considered to have no advantage over the measures using number of trees and diameter.

SUMMARY

A number of indices of stand density have been considered, some of them have been shown to have disadvantages and their use is considered to be undesirable, others are soundly based. It is difficult to determine which of the soundly based indices is likely to be best, so it is proposed that a number of indices be evaluated in the volume and increment models to determine which is most appropriate.

Five indices are considered to be worth evaluating in the models. These are as follows:

(1) Number of trees per acre.

(2) Standing basal area per acre.

(3) Hart's index.

(4) Lexen's bole area index.

(5) The ratio of standing basal area to the theoretical maximum basal area that the site can sustain, as calculated from equation IV.4.

Of these five indices the latter four are expected to be better than number of trees per acre, and are likely to be approximately equal in their effect in the models. As basal area appears from the literature to be the most commonly used index and is a logical index, it was decided to develop the volume and increment models using basal area as the index of stand density and then, after the models are developed, test the other indices by substitution in the models.

PART 2 VOLUME AND INCREMENT MODELS FOR RADIATA PINE THINNINGS

V. SELECTION OF VARIABLES TO BE USED

VOLUME

DIAMETER

RELATIVE TREE SIZE

HEIGHT

SITE POTENTIAL

Discussion of site potential measures

Definition of site potential measures

THINNING

Thinning type

Thinning intensity

Thinning interval

AGE

STAND DENSITY

V. SELECTION OF VARIABLES TO BE USED

It is desirable that the variables used in the models should be capable of being measured precisely, without bias, and at low cost. The variables should also be biologically sound, in that the use of a statistically significant variable that is not soundly based may lead to the development of models that are innaccurate if used in practice.

In practice the efficiency of forest inventory is fixed by the quantity of labour available, labour costs being the greatest component in inventory costs. To achieve maximum efficiency a balance has to be established between making more measurements on a plot and measuring more plots. In South Australian practice (described in chapter II) such a balance is believed to have been attained, although it cannot be proven by objective criteria at this stage.

Models developed in this study will provide estimates of the precision of volume and increment estimated from plot measurements. Further studies using different measurement techniques and different sampling intensities can be compared with these models and an objective choice made as to how best to deploy the limiting labour resource to achieve the most accurate estimate of the volume available from a forest in the next five year Working Plan period.

The following information is available for each inventory plot:

(1) Dimensions of the plot.

(2) Number of rows in the plot, and the number of extraction rows removed from the plot. There is generally one extraction row, the number of rows in the plot generally being the same as the number of rows from one extraction row to the next.

(3) Age of the plot.

(4) Site quality rating of the plot as mapped at age 9½.

(5) Predominant height of the plot.

(6) Diameter of each tree in the plot.

(7) Thinning history of the plot. This is used together with the thinning range of Lewis (1963) to determine the stocking to be left after the proposed thinning.

(8) Thinnings elect; a record of which trees have been selected for removal in the proposed thinning.

This information can be used to estimate the following variables.

VOLUME

The dependent variable to be used in this study is the volume of a tree to be removed in thinning, measured underbark to a four inch (10 cm) top diameter underbark in cubic feet.

DIAMETER

The diameter (overbark) D of the tree at breast height is measured by girth tape giving a direct conversion to diameter and is recorded to the nearest 0.05" (0.1 cm) for trees less than 12" (30 cm) and the nearest 0.1" (0.2 cm) for trees greater than 12" (30 cm). Breast height is defined as 4'3" (1.3 m) above ground, taken on the high side of the tree if the plot is located on a slope, and is marked as a single mark if the point is unaffected by swelling due to branches. Two marks are used if the point 4'3" above ground is affected by branches or other malformation, diameter being taken as the mean of the diameter at each mark.

RELATIVE TREE SIZE

A measure of the relative size of the tree would indicate the relative level of supression of the tree and should therefore be of significance in the increment model because the level of supression is likely to influence the proportion of the stand increment that accrues on an individual tree. As the level of supression will also affect the crown structure of the tree and hence tree form, it may be of significance in the volume model as well.

A measure suggested by Clutter (Pers. comm.) is the ratio of tree diameter to the mean diameter of the stand. Mean diameter is defined as the diameter corresponding to the arithmetic mean basal area (i.e. quadratic mean diameter).

HEIGHT

Stand height is one of the most commonly used parameters in the construction of growth and yield models. Crowe (1967) points out that if stand height is to be used to construct a growth or yield model it is essential to differentiate between mean height and top height, and to select the measure best suited for the model that is being constructed.

Mean height can be defined in a number of ways (Crowe, 1967; Jerram, 1939) but is generally affected by the type and intensity of the thinning to be carried out (Braathe, 1957; Spurr, 1952).

In any model constructed by regression analysis it is desirable that the independent variables used are not correlated and that changing the value of one independent variable does not cause a change in another. It is thus advisable that the measure of height used should be relatively unaffected by the type and intensity of thinning (Braathe, 1957; Crowe, 1967; Spurr, 1952).

In South Australian practice the stands are generally thinned from below so that all measures of mean height are likely to be affected by the thinning. It is therefore desirable that the height measure adopted be a measure of upper stand height, or top height.

Of the measures of upper stand height Naslund's top height and Wiese's top height, (Crowe 1967), are based on the mean tree and are thus likely to be affected by thinning and should therefore not be used. Other measures of top height are based on the mean height of the largest trees per unit area, or the mean height of the tallest trees per unit area. Although it is easier to identify the largest trees, it has always been considered more desirable in South Australia to measure the tallest trees, because the largest trees tend to have a high proportion of malformations which affect their height.

The measure of top height used in South Australia is called predominant height H and is defined as the mean height of the tallest 30 trees per acre with the restriction that the number of trees chosen from each quarter of the approximately one-fifth acre plot are to be the same, or as close as possible. As stocking at clear felling is generally above 45 trees per acre, and thinnings are generally from below, it is likely that the measure is independent of thinning.

Grut (1970) working with radiata pine in South Africa found that top height was affected by thinning. It was therefore decided to investigate whether predominant height is affected by thinning by investigating those permanent sample plots that might most reasonably be expected to show a difference.

Three broad categories of thinning might be expected to affect predominant height.

(1) First thinnings, which commonly involve the removal of some dominant trees such as "wolf" trees and large malformed trees.

(2) Heavy thinnings to a low stocking such that the thinning may be a crown thinning rather than a low thinning.

(3) Late thinnings to a stocking of approximately 30 trees per acre, for in these late thinnings spacing is the most common reason for removal, and the stocking after thinning is only slightly higher than the number of trees used in the estimation of predominant height.

Tables V.1, V.2, V.3 show plots that have been thinned in the past in a manner that would be considered to fall into one of these categories. The data was chosen by subjective selection of those plots known to have received a thinning that might fit one of the three categories described.

Although the predominant height after thinning is always less than or equal to the predominant height before thinning, approximately half the observations in each category differ by less than 0.2 feet. It therefore appears that a bias of approximately one foot may sometimes occur. But as these categories were chosen as examples of the extreme magnitude of the effect, it is considered that the bias introduced for an

TABLE V.1

FIRST THINNINGS

Plot	Stocking (trees per acre)		Predominant Height (feet)		
	Before Thinning	After Thinning	Before Thinning	After Thinning	Difference
389	867	369	65.5	65.5	0.0
177A	911	351	49.6	48.3	1.3
177C	837	355	49.3	49.1	0.2
176B	895	350	49.8	49.8	0.0
387	790	300	66.1	65.1	1.0
397	759	250	62.0	62.0	0.0
176A	876	249	47.1	46.9	0.2
177E	882	250	49.9	49.4	0.5
177J	896	250	48.8	48.8	0.0
399	769	178	76.6	75.2	0.4
395	708	151	56.7	56.2	0.5
398	790	157	62.1	62.0	0.1
177B	919	152	48.7	47.7	1.0
176C	878	151	48.2	47.5	0.7
177D	827	148	49.7	49.4	0.3
177H	846	149	49.6	49.1	0.5
176G	770	148	48.8	48.3	0.5
176J	747	152	46.7	46.1	0.6
158	639	155	65.8	65.7	0.1
				Mean	0.4

TABLE V.2

HEAVY THINNINGS TO LOW STOCKINGS

Plot	Stocking (trees per acre)		Predominant Height (feet)		
	Before Thinning	After Thinning	Before Thinning	After Thinning	Difference
396	243	148	73.5	73.5	0.0
394	168	130	72.4	72.4	0.0
156	252	145	84.4	84.3	0.1
439	173	113	80.8	80.8	0.0
150	249	110	91.4	90.6	0.8
153	180	105	94.8	94.8	0.0
398	157	100	83.2	83.2	0.0
395	151	92	75.1	75.1	0.0
158	155	84	85.4	84.8	0.6
394	130	82	95.5	95.3	0.2
				Mean	0.2

TABLE V.3

LATE THINNINGS TO A STOCKING OF APPROXIMATELY 30 TREES PER ACRE

Plot	Stocking (trees per acre)		Predominant Height (feet)		
	Before Thinning	After Thinning	Before Thinning	After Thinning	Difference
53	142	80	119.1	119.1	0.0
75	125	77	133.3	133.3	0.0
57	82	57	138.5	137.7	0.8
129	85	52	135.7	135.0	0.7
42	139	52	127.4	127.4	0.0
41	92	51	131.2	131.2	0.0
125	101	51	131.1	130.6	0.5
54	101	51	115.5	115.3	0.2
402	73	46	134.6	134.6	0.0
404	94	45	135.5	134.5	1.0
138	67	39	137.6	136.4	1.2
				Mean	0.4

average thinning is of little consequence and can be ignored in the development of the models.

SITE POTENTIAL

Discussion of site potential measures

At the turn of the century the standard system of estimating site potential in Germany was based on volume at age 100 (Mader, 1963), and this was accepted by many foresters, although in practice it was found to be difficult to apply. From this point onwards there developed three different systems for estimating site potential; volume growth, height growth and site type.

One of the foremost proponents of volume growth as an index of site was Bates, who in 1918 stated,

> "the only criterion of site quality is the current annual cubic foot increment of a fully stocked stand of the species under consideration .... I argue that while height growth is a criterion of one of the most important qualities of the site, it does not sum up all the qualities which a forester must be interested in, and which he attempts to express in the term Site Quality".

The use of volume growth as an index of site potential has the advantage of being directly related to what the forester is primarily concerned with, timber production. Providing that volume growth can be accurately assessed it is the ideal index of site potential. However, volume growth is costly and difficult to measure and because of this it has not been widely used.

The most commonly used index of site potential is site index or stand top height at a given age. The way in which it is commonly used in prediction models makes the assumption that height growth is closely related to volume growth. Generally, the advantages of using a readily measured index outweighs the disadvantages inherent in the basic assumption. Stand top height-age curves can generally be prepared with more accuracy than volume-age curves because it is possible to measure a larger pool of basic data and because estimates of volume generally include a larger sampling error than the estimates of stand top height.

The third technique of assessment of site potential is from environmental factors. This technique was expounded by Cajander (1926, 1949) and was later used to assess suitability of native forest for reafforestation with exotic conifers in New Zealand, (Ure, 1950) and to predict volume growth (Havel, 1967). Rennie (1963) has summarised the extensive literature in this field. The technique suffers from the disadvantage that correlations between volume production and environmental parameters are generally not high, probably because the basic interactions between environmental factors are not well understood.

Definition of site potential measures

In South Australia all unplanted land is assessed for suitability for afforestation with radiata pine or maritime pine P.Pinaster, Soland. Tree and ground vegetation, soils and topography, are compared with other sites that have already been planted and an estimate made of likely volume productivity. At present it has not been possible to develop an accurate method (Lewis and Harding, 1963) but it is possible to obtain an approximate assessment.

At age 9½ all radiata pine plantations in the south east of South Australia owned by the Woods and Forests Department, are assessed for site quality. The method of assessment is described in chapter II.

This assessment is based on volume production rather than a measure of stand height. For the purposes of this thesis, site quality SQ is defined as the total volume production to four inches top diameter, underbark, measured in cubic feet per acre at age thirty.

Age thirty was chosen as the base age because it was considered to be approximately in the middle of the range of age over which the model will probably be used. This age has been used in the past as the base age for predominant height in other South Australian publications by Keeves (1966) and Lewis (1967). It is approximately two-thirds of the rotation age, which is in line with common practice (Carron, 1968).

The more widely used measure of site potential is site index, which has been included for comparison purposes and is here defined as the predominant height of the stand (mean height of the 30 tallest trees per acre), measured in feet, at age 30.

It is considered that site quality SQ is a better index of site potential than site index SI, so the models will be developed using SQ, and after the models are developed SI will be substituted for SQ to see if in fact the choice of the measure is correct.

THINNING REGIME

The description of a thinning regime can be separated into three parts (Lewis, 1959).

(1) Thinning type; indicating which categories of trees are to be removed either on a size or crown classification basis.

(2) Thinning intensity; indicating how many trees are to be removed.

(3) Thinning interval; indicating at what stages in the development of a stand these removals are to be made.

<u>Thinning type</u>

Qualitatively, thinning type ranges from a low thinning in which a percentage of the smaller trees or poorer trees are removed, to a crown thinning, or thinning from above in which a percentage of the largest or best trees are removed.

The most used quantitative definition of thinning type is the ratio of the mean diameter of trees thinned to the mean diameter of the stand before thinning (Lewis, 1959; Braathe, 1957; Joergensen, 1957). Vezina (1963) characterises a low thinning as having a ratio less than 0.7, a severe low to light crown thinning a ratio of between 0.9 and 1.0 and a selection thinning a ratio greater than 1.0. A similar index uses the mean diameter of the stand after thinning instead of before thinning but this is considered inferior as the use of the mean diameter of stand before thinning gives a better representation of the characteristics of the stand that produces the thinning.

Another measure that has been used is Ullens Index (Lewis, 1959; Braathe, 1957; Vezina, 1963) which is defined as the ratio between the percentage number of trees removed and the percentage volume removed, but this index is not widely used as it is difficult to apply in practice. It is necessary to measure the volume of thinnings and main crop before felling the thinnings if it is desired to thin to Ullens Index. This is considerably more difficult than measuring the diameter of all trees in a plot.

The ratio of the mean diameter of the thinnings to the mean diameter of the stand before thinning is considered the best measure of thinning type. Current South Australian practice is to carry out a low thinning which commonly varies, using the above definition, from 0.75 to 0.95 with the average about 0.85. There is thus not a wide range of thinning type evident in practice.

Thinning Intensity

The intensity of thinning is essentially an attempt to measure the change in stand density due to thinning. Thinning is generally specified in practice in terms of either residual basal area (Gentle et al, 1962; Robinson, 1968) or as residual number of trees (Lewis, 1963).

The most logical measure of thinning intensity is the proportion of the forest cut (Buckman, 1962), either as basal area or number of trees, but it will only be valid if stand density before thinning is included in the model so that the percentage can be related to an absolute level.

Although there is little to choose between the two measures for use in the volume model, the percentage of basal area cut should not be used in the increment model. This is because between inventory and time of thinning the percentage of basal area cut is likely to change, but the percentage of number of trees per unit area cut is not likely to change. Therefore the percentage of the number of trees per unit area removed in thinning is preferred as the measure of thinning intensity for testing in the models.

Thinning Interval

This is defined as the interval in years between thinnings. A compromise must be reached between the possible value loss due to uneven ring widths in stands left for long periods between thinnings, and the high cost of extracting small volumes in thinning.

In South Australia the optimum is considered to be five years for SQ I and SQ II, six years for SQ III, and seven years for SQ IV and SQ V. In thinning research plots there are few plots with different thinning intervals, and in field operations the 5, 6, and 7 year intervals are adhered to if at all possible. There is currently little fluctuation in practice, and as available plot data does not fluctuate widely either it is unlikely that this parameter will be of significance in this model.

AGE

Buckman (1962) considers age to be the most important independent variable in growth and yield studies. It has the advantage of being capable of precise measurement, and is generally available for any even aged stand.

All plantations in South Australia are established in winter with one year old seedlings. The age of the plantation is taken as the difference between the current date and the year of planting, ignoring the period in the nursery.

All permanent sample plot data is collected in the three month period following the end of May, with the work program starting in the same locality each year and progressing logically so that measurements in each plot are made on approximately the same date each year.

This ensures that as far as possible increment figures obtained refer to exact multiples of one year, and the seasonal fluctuations that occur in the growth of radiata pine (Pawsey, 1964) are largely eliminated. Seasonal fluctuations in growth are probably related to fluctuations in rainfall and other climatic factors but these variations cannot be accurately measured for each plot and have therefore been ignored.

STAND DENSITY

The indices of stand density have been discussed at length in Part 1. The conclusions are summarized briefly here so that the chapter is complete.

Basal area is the index of stand density considered most significant for this study although other indices worthy of trial are number of trees per acre, Hart's height spacing ratio (1928), Lexen's bole area index (1943), and the ratio of the standing basal area to the maximum basal area as calculated from equation IV.4.

VI. DATA PREPARATION

CULLING THE DATA

DATA EXTRACTION

DISTRIBUTION OF THE DATA

VI. DATA PREPARATION

The data to be used in the development of the models had to be extracted from measurements of permanent sample plots. These data are currently held in manilla folders, one for field use and one for office use. The data required for this study are the individual tree data for trees removed in silvicultural thinning, and one plot may provide information for up to six thinnings which may have been made up to thirty years ago.

The total available data is voluminous, the office files alone occupying some 16 filing cabinet drawers. These data will ultimately be stored on magnetic tape which will make data extraction for a project such as this a simpler task. However the proposed computer system is a large one needing complex editing and update procedures and will not be implemented for some time. The data therefore had to be extracted manually after careful culling had reduced the plots to be extracted to an adequate well balanced number of plots.

CULLING THE DATA

As the aim of the study is to provide models for use in inventory calculations, it is desirable that the data used to develop the models should have a similar range as the data collected at inventory. In fact it would be ideal if the data used to develop the model covered a wider range so that extrapolation of the model in practice is reduced to a minimum.

Stands due to receive their first thinning have their yields estimated by interpolation within the unpublished thinned stand yield tables prepared by Lewis. First thinnings should therefore be excluded from the data to be extracted as the model will not be used to estimate their volume.

If a stand is thinned according to the thinning range of Lewis (1963) then the total volume removed from second and subsequent thinnings will range from approximately 60,000 super feet per acre if the stand is SQ V to approximately 130,000 super feet per acre if the stand is SQ I. If the stand is SQ VII the corresponding yield is approximately 25,000 super feet per acre. However there is considerably more area of SQ IV and SQ V than of SQ I and II so that there is more volume removed from SQ V areas than from SQ I or SQ II areas. The proportion of each site quality stata in each age class varies frcm forest to forest so it is difficult to determine the relative importance of each site quality strata in terms of total volume available from second and subsequent thinning operations. It is therefore desirable that all site quality strata be approximately equally represented in the data so that the model will be equally precise over the range of site quality.

Thus, as a preliminary to data extraction, the sample plot registers were inspected and a list of plots prepared that eliminated plots in the categories listed below.

(1) Plots that were unthinned at time of last volume measurement.

(2) Plots that had received only one thinning.

(3) Plots which were not planted, but established by natural regeneration.

(4) Plots that were not planted with radiata pine.

All remaining plots were then stratified into site quality classes based on the total volume production to four inches top diameter underbark using the unpublished provisional yield table for radiata pine developed by Lewis. An inspection then revealed that all SQ VII plots had been eliminated as no SQ VII plot had at that time been second thinned. All but one SQ VI plot was eliminated for the same reason. The exclusion of all SQ VI and SQ VII plots is unlikely to be a serious disadvantage in practice because few areas of such low site potential have received, or are scheduled for, a second thinning in the next six or seven years. In the future more low site quality stands will receive a second or subsequent thinning, but by that time permanent sample plot data will have been collected enabling an examination of the problem. Because higher site quality areas grow faster and are thinned more frequently, the number of second or subsequent thinnings was considerably higher in the higher site quality plots, than the lower.

Sample plots thinned soon after the first plots were established in 1935 were generally more lightly thinned than is currently commercial logging practice. The available data therefore includes more plots that are lightly thinned than plots that are heavily thinned so that it was therefore considered desirable to stratify the available data into stand density or competition levels. The plots were subdivided into three broad density categories based on the stocking after thinning.

(1) Above the optimum thinning range (Lewis, 1963).

(2) Within the optimum thinning range.

(3) Below the optimum thinning range.

Many of the permanent sample plots have received more than two thinnings and therefore have a number of thinnings available for consideration. The term plot thinnings is therefore used to separate these different thinnings which can be regarded as distinct entities occuring in different years.

The data in the last category, below the optimum thinning range, were in the minority. The number of plot thinnings selected for analysis included as many as possible thinned to below the optimum thinning range, and an approximately equal number of plots from the category above the optimum thinning range. These latter were selected at random. The remainder were chosen at random from the plot thinnings within the optimum thinning range.

The selection was further constrained by the desire to have approximately equal numbers from each site quality class. The minimum class was the SQ V class so all 29 plot thinnings in this class were chosen. Similarly all the plot thinnings below the optimum thinning range were chosen in the four higher site quality classes.

A random selection of the other classifications was made difficult by the desire to have if possible, equal numbers of second, third, fourth and fifth thinnings. This was not possible because relatively few plots have received fourth and fifth thinnings. However, a random selection was made from those cells where the number of plot thinnings desired was less than the number available.

Table VI.1 shows the breakdown of plot thinnings in the broad after thinning density classes, and table VI.2 the breakdown by operation. Table VI.3 demonstrates the breakdown of the plot thinnings by predominant height classes. From these three tables it can be seen that the data is fairly evenly distributed between site quality class, predominant height class, density class and operation. All but the last are dynamic and the classification only demonstrates the broad distribution over the continuous range. The number of plot thinnings which were of low site quality, and the number of plot thinnings which were late multiple thinnings were maximised within this range.

Having decided which plot thinnings to use it was desirable to cull the available tree data so that the only necessary data be extracted.

TABLE VI.1

NUMBER OF PLOT THINNINGS BY SITE QUALITY
AND DENSITY

	Site Quality					Total
	I	II	III	IV	V	
Above Optimum Thinning Range	5	5	4	5	5	24
Within Optimum Thinning Range	19	25	22	21	13	100
Below Optimum Thinning Range	6	6	3	7	11	33
Total	30	36	29	33	29	157

A permanent sample plot may be represented by a number of plot thin nings, each one representing a different thinning operation carried out in different years.

TABLE VI.2

NUMBER OF PLOT THINNINGS BY SITE QUALITY
AND THINNING OPERATION

Operation	Site Quality					Total
	I	II	III	IV	V	
Second Thinning	13	13	12	15	24	77
Third Thinning	13	11	11	12	5	52
Fourth Thinning	3	7	5	4	0	19
Fifth Thinning	1	4	1	2	0	8
Sixth Thinning	0	1	0	0	0	1
Total	30	36	29	33	29	157

A permanent sample plot may be represented by a number of plot thinnings, each one representing a different thinning operation carried out in different years.

TABLE VI.3

NUMBER OF PLOT THINNINGS BY SITE QUALITY
AND PREDOMINANT HEIGHT

H	Site Quality					Total
	I	II	III	IV	V	
≤ 70	0	0	0	2	5	7
70½ - 80	4	0	2	1	1	8
80½ - 90	7	7	9	5	9	37
90½ - 100	5	7	3	7	6	28
100½ - 110	8	6	5	10	7	36
110½ - 120	3	4	6	4	1	18
≥ 120½	3	12	4	4	0	23
Total	30	36	29	33	29	157

A permanent sample plot may be represented by a number of plot thinnings, each one representing a different thinning operation carried out in different years.

In practice inventory takes place the year before the first year of the five year working plan, and it is thus necessary to predict volume increment over a five year period. In some rare cases inventory is carried out two years before the start of the working plan period, and in some plans a sixth year of the plan is tentatively prepared. These cases are rare but as they do occur it was decided to extract data up to and including six years before thinning but not data over six years before thinning. The time interval for increment, the increment period, is denoted throughout this study as x, measured in years.

Although other data are available it is necessary to extract data comparable with that measured or derived for inventory plots. This can be done for each year between one and six prior to thinning on some plots, but generally only for two of the six possible years. Data at time of thinning can also be extracted.

DATA EXTRACTION

Because of the relative inaccessibility of the large volume of available data it was necessary to develop an efficient extraction technique. The basic data were extracted by hand, coded into 80 column punch cards and then edited and reformatted by three computer programs into card image records on magnetic tape which could be used as the basic input into the model development phase of the study.

The stand data were kept separate from the tree data and were coded onto plot cards, a number of which formed a plot record (see Appendix 1 for card and record layouts). A plot code was recorded, being 1 for all the plots included in the basic sample and 2 for the plots coded up ready for use as independent test data.

The tree data were coded on a tree card which was reformatted into a tree record, (Appendix 1). A tree code was used which enabled the data chosen for particular analyses to be restricted to certain specific subpopulations.

In current practice a tree greater than 4.2 inches dbhob (10.5 cm) is classed as a normal tree (A tree) if its volume to four inches underbark is not affected by a fork or malformation below thirty feet (10 m). This is designed to keep separate grossly malformed trees (B trees) where the volume to 4 inches top diameter is grossly affected by malformation. All existing South Australian volume and increment models used in inventory are based on the normal A tree subpopulation, it being assumed that the malformed trees have similar characteristics. It is proposed to develop a model for the non malformed trees and later, as a subsequent study, examine the malformed tree data collected and develop if necessary new models. As few malformed trees are left after first thinning, malformed trees are not a major problem in practice.

The normal trees were further subdivided so that a varying number of randomly selected trees could be chosen from each plot. The number chosen were one, five, ten and all available trees. As all plots are approximately uniform in size there are considerably more trees removed per plot in second thinning than in the later thinnings. To prevent bias being introduced because of this it was decided to choose ten random trees per plot thinning as the basic set of data. If, after the models had been developed, analysis showed serial correlation to be a significant problem then the data could be reduced by stages to one randomly selected tree per plot, a data set that theoretically should show no serial correlation.

The plot and tree records were sorted and then input into the third program which prepared the final record for use in model development; the layout of the record being described in Appendix 1.

A single data card was used to enable the basic plot and tree records to be manipulated to produce the data output required.

DISTRIBUTION OF THE DATA

The later thinnings remove relatively few trees compared with the second thinning. To avoid the problem of over-representation of earlier thinnings, and to try to avoid the problem of auto-correlation, the initial data base was chosen using ten random trees from each plot for those plots where more than ten normal trees were removed in thinning.

Eight sets of data were prepared, the first set being for the trees at time of thinning. For this set, there were where possible ten random trees from each plot thinning. Table VI.4 shows the distribution of the 1418 trees in this data set

TABLE VI.4

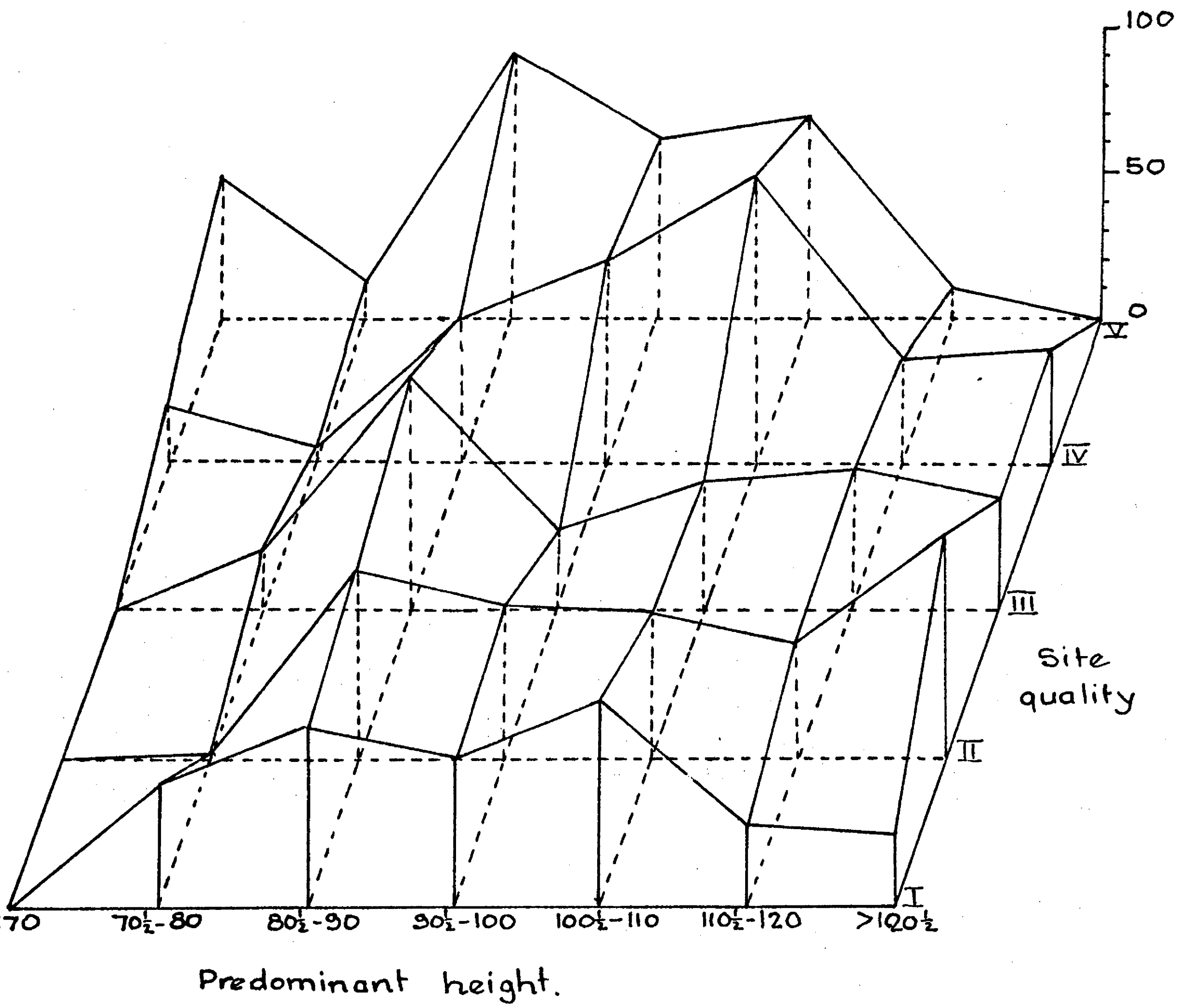
NUMBER OF TREES BY SITE QUALITY
AND PREDOMINANT HEIGHT

H	Site Quality					Total
	I	II	III	IV	V	
≤ 70	0	0	0	18	48	66
70½ - 80	40	0	18	6	10	74
80½ - 90	60	66	81	47	90	344
90½ -100	50	57	28	69	61	265
100½ -110	70	49	46	98	70	233
110½ -120	27	40	49	34	10	160
≥ 120½	20	78	39	39	0	176
Total	267	290	261	311	289	1418

From each plot thinning ten normal trees were selected at random. If less than ten trees were removed in thinning then all normal trees were included.

NUMBER OF TREES IN EACH STRATA

Ten randomly selected trees per plot at each thining



which is graphically depicted in Figure VI.1. This table is equivalent to table VI.3 for 157 plots. The difference between 157 plots multiplied by 10 trees per plot and 1,418 being due to a few plots having less than 10 normal trees removed in thinning. The distribution was not as uniform as would be desirable but it was the best available. It was hoped that the large total number of trees in the set would counterbalance any possible bias due to the data not being distributed uniformly. These data were used to develop the volume models.

Six subsets of data were prepared for trees measured in each of the six years prior to thinning. Ten random trees were chosen where possible from each plot thinning. These data are tabulated in tables VI.5 to VI.10. These six subsets of the data were amalgamated to provide the basic data for increment model development, table VI.11.

It is desirable that if possible the combined data be used because the individual subsets are not well distributed. As in current South Australian practice thinning interval is generally fixed for each site quality class it follows that measurement intervals are also relatively fixed for each site quality class, changing between site quality classes because the thinning interval changes. This is shown in Table VI.12 where it can be seen that SQ IV plots, are generally measured two, four and six years before thinning, SQ II plots two, three and five years before thinning wheras SQ I plots are rarely measured six years before thinning because the thinning interval is generally five years. The choice of one representative year is difficult as all years show some maldistribution. The combined set,

TABLE VI.5

NUMBER OF TREES
MEASURED ONE YEAR BEFORE THINNING
BY SITE QUALITY AND PREDOMINANT HEIGHT

H	Site Quality					Total
	I	II	III	IV	V	
$\leq$ 70	0	0	0	18	38	56
70½ - 80	20	0	8	0	0	28
80½ - 90	51	10	10	10	0	81
90½ - 100	10	0	0	0	0	10
100½ - 110	26	0	0	0	0	26
110½ - 120	0	0	0	0	0	0
$\geq$ 120½	0	0	0	0	0	0
Total	107	10	18	28	38	201

From each plot thinning ten normal trees were selected at random. If less than ten trees were removed in thinning then all normal trees were included.

TABLE VI.6

NUMBER OF TREES
MEASURED TWO YEARS BEFORE THINNING
BY SITE QUALITY AND PREDOMINANT HEIGHT

H	Site Quality					Total
	I	II	III	IV	V	
≦ 70	0	0	0	18	48	66
70½ - 80	40	0	18	6	0	64
80½ - 90	41	48	56	37	20	202
90½ - 100	40	16	0	59	50	165
100½ - 110	60	34	10	38	20	162
110½ - 120	27	0	20	24	0	71
≧120½	3	0	0	39	0	42
Total	211	98	104	221	138	772

From each plot thinning ten normal trees were selected at random. If less than ten trees were removed in thinning then all normal trees were included.

TABLE VI.7

NUMBER OF TREES
MEASURED THREE YEARS BEFORE THINNING
BY SITE QUALITY AND PREDOMINANT HEIGHT

H	Site Quality					Total
	I	II	III	IV	V	
$\leq$ 70	0	0	0	0	0	0
70½ - 80	10	0	8	0	0	18
80½ - 90	38	28	25	10	10	111
90½ - 100	20	22	10	10	0	62
100½ - 110	70	5	36	40	10	161
110½ - 120	10	30	12	10	0	62
$\geq$ 120½	10	18	19	0	0	47
Total	158	103	110	70	20	461

From each plot thinning ten normal trees were selected at random. If less than ten trees were removed in thinning then all normal trees were included.

TABLE VI.8

NUMBER OF TREES
MEASURED FOUR YEARS BEFORE THINNING
BY SITE QUALITY AND PREDOMINANT HEIGHT

H	Site Quality					Total
	I	II	III	IV	V	
≦ 70	0	0	0	18	48	66
70½ - 80	48	0	18	6	10	74
80½ - 90	22	20	56	27	60	185
90½ - 100	30	8	0	39	40	117
100½ - 110	70	10	10	30	30	150
110½ - 120	0	0	7	17	10	34
≧ 120½	0	17	0	39	0	56
Total	162	55	91	176	198	682

From each plot thinning ten normal trees were selected at random. If less than ten trees were removed in thinning then all normal trees were included.

TABLE VI.9

NUMBER OF TREES
MEASURED FIVE YEARS BEFORE THINNING
BY SITE QUALITY AND PREDOMINANT HEIGHT

H	Site Quality					Total
	I	II	III	IV	V	
≤ 70	0	0	0	0	0	0
70½ - 80	0	0	0	0	0	0
80½ - 90	38	56	0	20	0	114
90½ - 100	30	38	0	20	21	109
100½ - 110	70	15	0	30	10	125
110½ - 120	27	30	10	10	0	77
≥120½	20	15	20	0	0	55
Total	185	154	30	80	31	480

From each plot thinning ten normal trees were selected at random. If less than ten trees were removed in thinning then all normal trees were included.

TABLE VI.10

NUMBER OF TREES
MEASURED SIX YEARS BEFORE THINNING
BY SITE QUALITY AND PREDOMINANT HEIGHT

H	Site Quality					Total
	I	II	III	IV	V	
$\leq$ 70	0	0	0	0	0	0
70½ - 80	0	0	18	0	0	18
80½ - 90	0	20	61	20	10	111
90½ - 100	0	0	18	29	0	47
100½ - 110	0	14	46	38	10	108
110½ - 120	10	0	19	24	0	53
$\geq$ 120½	0	63	19	20	0	102
Total	10	97	181	131	20	439

From each plot thinning ten normal trees were selected at random. If less than ten trees were removed in thinning then all normal trees were included.

TABLE VI.11

NUMBER OF TREES
MEASURED IN ALL YEARS BEFORE THINNING
BY SITE QUALITY AND PREDOMINANT HEIGHT

H	Site Quality					Total
	I	II	III	IV	V	
$\leq$ 70	0	0	0	54	134	188
70½ - 80	110	0	70	12	10	202
80½ - 90	190	182	208	124	100	804
90½ - 100	130	84	28	157	111	510
100½ - 110	296	78	102	176	80	732
110½ - 120	74	60	68	85	10	297
$\geq$120½	33	113	58	98	0	302
Total	833	517	534	706	445	3035

From each plot thinning ten normal trees were selected at random. If less than ten trees were removed in thinning then all normal trees were included.

TABLE VI.12

NUMBER OF TREES
MEASURED IN EACH YEAR BEFORE THINNING
BY SITE QUALITY

Number of years before thinning when plot measured	Site Quality					Total
	I	II	III	IV	V	
1	107	10	18	28	38	201
2	211	98	104	221	138	772
3	158	103	110	70	20	461
4	162	55	91	176	198	682
5	185	154	30	80	31	480
6	10	97	181	131	20	439
Total	833	517	534	706	445	3035

From each plot thinning ten normal trees were selected at random. If less than ten trees were removed in thinning then all normal trees were included.

table VI.11, is fairly well distributed and provides a sound base for the development of the increment model.

VII. MODEL FORMULATION

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VII. MODEL FORMULATION

The aim of this thesis is to develop models for radiata pine which will predict volume and short term increment on the thinnings elect subpopulation. The models should use only those parameters currently available from inventory plot measurements. The objective of this chapter is to formulate a number of alternative volume and increment models for subsequent evaluation.

VOLUME MODELS

Choice of the dependent variable

In the literature, tree volume equations have been developed using either volume or the logarithm of volume as the dependent variable (Spurr, 1952). The use of the logarithmic transformation (Schumacher and Hall, 1933) is a means of converting certain non-linear equations into a linear form suitable for linear regression analysis. However in making this transformation the assumption is made that the error term in the original equation is multiplicative rather than additive. This assumption is difficult to sustain. It would appear more desirable to postulate a non-linear model with an additive error term and estimate the coefficients by non-linear techniques rather than use the logarithmic transformation.

As the study is to be restricted to the use of linear regression analysis techniques, volume is preferred as the dependent variable.

The combined variable equation

Of all the tree volume equations using diameter and height to predict tree volume the simplest and most commonly used model is the combined variable equation, generally attributed to Spurr (1952) but better attributed to Naslund (1947), equation VII.1.

$$V = b_0 + b_1 D^2H \qquad \text{Equation VII.1}$$

In this equation the constant b_0 will be referred to later as the combined variable constant, and the constant b_1, the combined variable coefficient.

This equation has probably been used more frequently than any other equation, and is the basis on which many other authors have developed equations. In fact, in the literature reviewed, no two way tree volume equation was examined that did not include the combined variable D^2Ht or D^2H as an independent variable.

When this equation, or others derived from it, are used in practice then the measure of height used is generally tree height, but there is no valid reason why the measure of height cannot be a measure of upper or mean stand height. A measure of upper stand height, predominant height, is available from the data and will be used in the study.

If tree diameter and tree height are used in equation VII.1 to estimate total stem volume then the combined variable constant b_0 would logically be zero. However as the dependent variable is volume to a four inch top diameter, and the height measure used is a measure of upper stand height, it is logical to include a negative constant.

Models using diameter and height

Although the simple combined variable equation has been used by many research workers, others have felt it desirable to build up a model containing as many as ten terms. These equations generally include some or all of the first and second order interactions between diameter and height, but few authors explain their choice of combinations.

Gerrard (1966) considered four important questions concerning the functional form of the volume equation:-

"1. Is the Combined Variable Formula of Spurr (1952: p 94) adequate to account for all the systematic variation found in total cubic volume data?

2. Should curves of volume over height within dbh classes exhibit curvilinearity?

3. Are second degree curves of volume over dbh within height classes sufficient, or are sigmoid curves preferable?

4. Would a constant term of zero represent an undue constraint upon the regression?"

From a consideration of these questions he formulated five models of increasing complexity starting with the simple combined variable equation. His second equation used H in combination with D and D^2 to fit a model in which the curve of volume over height in diameter classes was linear, equation VII.5.

$$V = b_0 + b_1 D + b_2 H + b_3 DH + b_4 D^2 + b_5 D^2H \quad \text{..Equation VII.5}$$

The third equation included H^2 and DH^2 and the fourth, D^2H^2 completing the series of interactions. Finally D^3 and H^3 were added, the cubic terms acting as a proxy for sigmoid curves. Equation VII.6 is similar to Gerrard's final equation except that the only fourth power combination D^2H^2 has been rejected as any tendency to a sigmoidal function should be shown by third and lesser power combinations.

$$V = b_0 + b_1 D + b_2 H + b_3 DH + b_4 D^2 + b_5 D^2H + b_6 H^2 + b_7 DH^2 + b_8 D^3 + b_9 H^3 \quad \text{........................ Equation VII.6}$$

An alternative to Gerrard's model development is to consider the effect of the use of predominant height instead of tree height and the use of volume to four inches top diameter. In the combined variable equation a simple constant reduction to diameter may be sufficient to allow for the use of volume to a four inch top diameter. If predominant height is a proxy for tree height then equation VII.2 results.

$$V = b_0 + b_1 (D-b_2)^2 H \qquad \text{or}$$

$$V = b_0 + b_1 H + b_2 DH + b_3 D^2H \quad \ldots\ldots\ldots\ldots\ldots \quad \text{Equation VII.2}$$

The effect of the use of predominant height is difficult to postulate but if a simple constant reduction to predominant height is incorporated in equation VII.2 then equation VII.5 results.

$$V = b_0 + b_1 (D-b_2) \quad (H-b_3)$$

$$V = b_0 + b_1 D^2H - 2b_1b_2 DH + b_1b_2 H - b_1b_3 D^2 + 2b_1b_2b_3 D$$
$$- b_1b_2{}^2b_3 \qquad \text{or}$$

$$V = b_0 + b_1 D + b_2 H + b_3 DH + b_4 D^2 + b_5 D^2H \ldots \text{Equation VII.5}$$

If the correction applied to diameter used to develop equation VII.5 is replaced by a correction to the basal area this results in equation VII.3.

$$V = b_0 + b_1 (D^2-b_2) \quad (H-b_3)$$

$$V = b_0 + b_1 D^2H - b_1b_2 H - b_1b_3 D^2 + b_1b_2b_3 \text{ or}$$

$$V = b_0 + b_1 H + b_2 D^2 + b_3 D^2H \quad \ldots\ldots\ldots\ldots\ldots \quad \text{Equation VII.3}$$

This equation has been called the Australian equation (Spurr, 1952) as Stoate (1945) was one of the first to propose its use. It was tested by Spurr (1952) on limited data and proved to be better than the combined variable equation for some plots.

In South Australia volume equations have previously been developed using the volume-basal area line approach. Keeves

(1961) using South Australian data for radiata pine demonstrated that for each permanent sample plot the tree volume is linearly related to tree basal area, with the constant being known as the volume line constant and the coefficient, the volume line coefficient. He then developed equations to predict the volume line coefficient and constant from predominant height, the coefficient being related to H and the constant to H and H^2. If these equations are combined then equation VII.4 results.

$$V = b_0 + b_1 H + b_2 D^2 + b_3 H^2 + b_4 D^2H \quad \text{........ Equation VII.4}$$

The use of the square of height was considered by Spurr (1952) to be unsound biologically, although it was used by Naslund (1941, 1947). If the equation is changed by eliminating this parameter then it is the same as the Australian equation as used by Henry (1960), equation VII.3. Keeves (1961) found that the relationship between the volume line constant and predominant height was linear for thinned plots but the addition of a quadratic term was very highly significant in the case of unthinned plots. As the comparison between thinned and unthinned plots showed that there was no significant difference between them, he combined the data and calculated the regression using H and H^2. If the linear relationship for thinned stands is accepted, this would reduce the volume equation by the one term not found in the Australian equation.

Six models have been formulated around the combined variable equation. These models cover a range of alternative hypotheses and all should be fitted to the data and evaluated.

Models using other variables

In addition to diameter and height other variables may have an effect on volume. The most obvious of these parameters is tree form. The most effective measure of form would be a measure of taper up the tree (Lewis and McIntyre, 1963), but this is impractical to measure on inventory plots.

The combined variable coefficient in equation VII.1 embodies the effect of form and is similar to the "form factor" (Spurr, 1952; Jerram, 1939). The use of the combined variable constant in equation VII.1 is not inconsistent with this as the constant can be regarded as a proxy for the effect of the use of predominant height and volume to a four inch top diameter.

There are a number of parameters that may affect the form of a tree and of these the one most likely to be significant is stand density. Baskerville (1962) cites Canadian and Japanese evidence that indicates that with increasing levels of competition the point of maximum cambial activity moves up the tree and from this infers that form factor increases with increasing density.

However if trees of equal diameter, from stands of different stand density are considered, then generally, the more dense the stand the more supressed the tree will be. From the accepted curvilinear form of the tree height - diameter relationship (Spurr, 1952; Carron, 1968) it follows that the more supressed a tree the lower is its diameter relative to the other trees in the stand and the greater the difference between tree height and stand top height. Therefore the tree in the stand of higher density will, in general, have a lower tree height than the tree in the stand of lower density. Thus as the stand density increases, the position of the point of maximum cambial activity will be higher relative to tree height (Baskerville, 1962), but may be lower relative to predominant height. The effect of stand density on the form factor may therefore be positive or negative depending on the magnitude of the two opposing effects.

Despite confusion as to the direction of the effect stand density should be tested in the model. In Part 1 of this thesis the indices of stand density were discussed and it was concluded that a number of indices are worth evaluating in the model.

(1) Number of trees per acre.
(2) Basal area per acre.
(3) Hart's height-spacing ratio.
(4) Lexen's bole area index.
(5) Ratio of basal area to theoretical maximum basal area.

These indices of stand density are likely to have a similar effect in the model so it is logical to evaluate the model using one index alone. After the model has been developed, and the form of the model fixed, the other indices can then be substituted in the model.

This will enable a practical assessment of each of the indices to be made. Basal area appears to be the most promising index and will be tested in the first instance.

Apart from stand density, site potential may have an effect on tree form. In the unpublished yield table for thinned stands of radiata pine of Lewis, a stand based form factor can be calculated as the total production volume to four inches top diameter per acre divided by the product of basal area per acre and predominant height. This stand based form factor increases with increasing site potential, although at later ages the increase is less marked. From this it is inferred that tree form factor as used in this model may increase with increasing site potential, and that the inclusion of a quadratic term may also be significant.

It is expected that site quality SQ, will be more effective measure than site index SI but both should be tested in the model. Initially however it will be desirable to test the model using SQ and after the volume model is developed re-estimate the coefficients and their significance using SI as a replacement for SQ.

A volume model can be formulated from these variables, equation VII.7

$$V = b_0 + b_1 D^2H + b_2 D^2HB + b_3 D^2HSQ \quad \ldots\ldots\ldots \quad \text{Equation VII.7}$$

In view of the likely effect of site quality on form at later ages, age should also be included in the model. It is likely that form and hence volume will be inversely proportional to age if diameter and height are constant (Nelson et al, 1961).

Other authors (Gruschow and Evans, 1959; Mesavage, 1961), however, found that age was as satisfactory as the reciprocal. As age is the simpler form it will be used initially and if it is found to be significant the reciprocal will be evaluated.

Thinning type may also affect the form of the tree for if the thinning is from below then trees with lower than average form factor will generally be removed in preference to those of higher form factor. Thinning type will probably be significant in the model with a positive coefficient.

It is evident from a consideration of the effect of stand density that a change in relative tree size R may effect tree form, the lower the value of R the lower the form factor. However thinning type and relative tree size R may have a combined effect. As the smallest diameter classes, lower R, are generally completely removed in thinning, these trees will all be removed irrespective of thinning type. As R increases the trees chosen for removal in thinning are only a proportion of the trees in a particular diameter class and if the thinning is a low thinning the trees chosen for removal are more likely to have a lower than average tree form factor. The effect of the thinning type variable TT and relative tree size variable R may therefore be multiplicative rather than additive. The combined term should be tested in the model, but it is not considered likely to be significant.

Thinning intensity should also be considered, for the higher the thinning intensity N% the higher will be the average form factor of trees to be removed in a given diameter class. However, as the effect of thinning intensity is also reflected in parameters such as stand density, tree rank and thinning type, it is not considered likely to be significant, although it should be tested.

Apart from the effect of tree and stand variables on form, they may well act as proxies for the effect of the use of predominant height and volume to four inches top diameter. The combined variable constant in equation VII.1 may change with age, for the older the stand the less will be the distance between four inches top diameter and the tip and therefore the unmerchantable volume will be less.

Similarly relative tree size R may be significant as the smaller trees are more likely to have ineffective volume.

Other parameters that are worth evaluating are stand density B, site potential SQ, thinning type TT and thinning intensity N%.

Apart from form variables including stand density and site potential, equation VII.7, it is difficult to decide which of the other variables are most likely to be significant. However three equations VII.8 to VII.10 are formulated with increasing numbers of variables in each, such that in equation VII.10 all the variables are included.

$$V = b_0 + b_1 D^2H + b_2 D^2HB + b_3 D^2HSQ + b_4 D^2HTT + b_5 A + b_6 R$$

.... Equation VII.8

$$V = b_0 + b_1 D^2H + b_2 D^2HB + b_3 D^2HSQ + b_4 D^2HTT + b_5 D^2HN\% + b_6 D^2HA + b_7 D^2HR + b_8 A + b_9 R$$

.... Equation VII.9

$$V = b_0 + b_1 D^2H + b_2 D^2HB + b_3 D^2HSQ + b_4 D^2HSQ^2 + b_5 D^2HTT + b_6 D^2HN\% + b_7 D^2HA + b_8 D^2HR + b_9 D^2HTTR + b_{10} A + b_{11} R + b_{12} B + b_{13} SQ + b_{14} TT + b_{15} N\%$$

.... Equation VII.10

Combined models

The two alternative lines of model development can be combined as they are not incompatible. In formulating combined models it is necessary to use simple equations so that the resulting equation is not too large. Equations VII.2 and VII.3 are logical three variable equations developed using predominant height and tree diameter and combinations of them as independent variables. Equation VII.7 is a logical equation using other stand parameters. Two combined equations VII.11 and VII.12 can then be formulated from these equations.

$$V = b_0 + b_1 D^2H + b_2 D^2 + b_3 H + b_4 D^2HB + b_5 D^2HSQ$$

.... Equation VII.11

$$V = b_0 + b_1 D^2H + b_2 DH + b_3 H + b_4 D^2HB + b_5 D^2HSQ$$

.... Equation VII.12

The combined models can be compared with equations VII.5 and VII.8 to determine which is the better line of model development, or whether the combination is best.

Summary

Twelve volume models have been formulated and these are tabulated in table VII.1. They represent two major alternative forms, one the two way volume equations using diameter, height and combinations of these two parameters as the independent variables, and the other alternative form based on the effect of other tree and stand parameters on the simple combined variable equation. Two equations are formulated incorporating both lines of development. It is evident that once these models are fitted and evaluated it may be found desirable to try other alternative combinations but until the twelve models are evaluated it is difficult to determine which combinations to choose.

INCREMENT MODELS

The increment model will be used to predict the volume that the tree will put on between inventory and the time of thinning up to six years later, and necessitates the summation of tree increment over this period.

The working plan (vide chapter II) details logging operations for each year of the five year plan, and the data available for model development are measured at yearly intervals. It is therefore convenient to consider tree increment in the context of these models as composed of two multiplicative components, increment period and annual increment.

TABLE VII.1

VOLUME

MODELS TO BE EVALUATED

Equation	Independent variables																							No. of variables
	D^2H	D	H	DH	D^2	H^2	D^3	H^3	DH^2	D^2HB	D^2HSQ	D^2HSQ^2	D^2HTT	$D^2HN\%$	D^2HA	D^2HR	D^2HTTR	A	R	B	SQ	TT	N%	
VII.1	x																							1
VII.2	x		x	x																				3
VII.3	x		x		x																			3
VII.4	x		x		x	x																		4
VII.5	x	x	x	x	x																			5
VII.6	x	x	x	x	x	x	x	x	x															9
VII.7	x									x	x													3
VII.8	x									x	x		x					x	x					6
VII.9	x									x	x		x	x	x	x		x	x					9
VII.10	x									x	x	x	x	x	x	x	x	x	x	x	x	x	x	15
VII.11	x		x		x					x	x													5
VII.12	x		x	x						x	x													5

Choice of the dependent variable

It is desirable that the dependent variable chosen for the increment model should be consistent with the dependent variable chosen for the volume model.

In most previous work, increment has either been estimated in absolute terms or as a percentage of the estimated volume standing at the start of the increment period (Petrini, 1957; Kuusela and Kilkii, 1969) but increment percent appears to offer few advantages over increment.

The dependent variable is therefore the difference between the estimated tree volume standing at time of thinning and the estimated tree volume standing at inventory 1-6 years (x) prior to thinning. The volumes are to be estimated from the volume model after it has been proven to be statistically and biologically acceptable.

Increment period

Immediately after a thinning the stand does not fully occupy the site and the stand will not put on as much growth as a stand that has not been thinned. This lag in growth due to thinning will be apparent in the growth rates of all individual trees in the stand. The extent of this lag is likely to be greater in areas of lower site potential where growth rates are slower, and will be greater after a heavy thinning. Evidence from permanent sample plots indicates that unless the thinning is extremely heavy this lag is either not apparent or is of only one year duration.

In practice the model will generally be used to predict increment from one to five years ahead with the average prediction period being three years. Thinning interval is generally not less than five years for SQ I and SQ II, six years for SQ III and seven years for SQ IV and SQ V. Thus if the lag is taken as one year then this will have very little effect in SQ III, SQ IV and SQ V, as inventory will be at least one year after thinning. Only stands of SQ I and SQ II due to be thinned in the last year of the five year plan period will be affected. In a few cases a six year projection period is used but these are relatively rare. Thus if the lag is one year then it will affect one-fifth of the SQ I and SQ II areas and not affect the SQ III, SQ IV and SQ V areas. As SQ I and SQ II areas combined represent between 10 and 15% of the area currently thinned or likely to be thinned, within the next five years it is evident that the practical effect of the lag is minimal as less than 2-3% of the area may be affected.

After the stand fully reoccupies the site, growth will continue at a steady rate following the sigmoidal volume - age curve. However as the increment period is only a small proportion of the rotation length a simple linear approximation is likely to adequately represent the effect of increment period during this phase.

If the thinning interval is long and the stand is at a high level of stand density, then as the stand approaches the time of next thinning the less vigorous trees, likely to be the next thinnings, will begin to be suppressed and their increment relative to the main crop of the stand will be reduced. This is not to say that stand increment will tend to fall off, only the increment on the thinnings elect subpopulation.

In practice nearly all stands are thinned at intervals within one year of that considered desirable, so that excessive thinning intervals are avoided. The optimum thinning range of Lewis (1963) has been used to prescribe the last thinning and in many cases the last two thinnings so that stands are rarely left to become overstocked. Permanent sample plot evidence indicates that annual basal increment on the thinnings elect subpopulation was depressed only if the thinning interval was at least two years longer than current practice, and then only in those plots that have been deliberately kept at stand density levels higher than is current practice.

It is evident that a simple linear function of time may be an adequate approximation, although the lag and the competition effect indicate that a sigmoidal relationship may be more desirable theoretically. Obviously the effect of the increment period x must be conditioned so that when it is zero there is no increment.

It was therefore decided to develop an increment model in which all independent variables in the annual increment component were multiplied by x, the increment period, and then test the assumption of a simple linear effect with respect to time by the use of dummy variables as defined in table VII.2.

In effect the use of dummy variables (Johnston, 1960) in place of the increment period variable enables separate regressions to be fitted to the data for each different increment period. An analysis of variance can then be used to test whether the coefficients of a particular variable, such as site potential, are the same over the range of increment period for all values of site potential. This technique makes no assumptions about the effect of time, but an examination of the coefficients of each variable will enable conclusions to be drawn about this effect.

Annual increment

Of all the variables that are considered likely to affect tree increment one of the most important is likely to be site potential. This variable has been used by many writers (Gruschow and Evans, 1959; Nelson et al, 1963; Wenger et al, 1958; Dahms, 1964; Deetleefs, 1954) to estimate stand increment and it is logical that it should also be an important parameter in any tree increment prediction equation. Generally site potential is included as a linear term but Nelson and Brender (1963) found the quadratic form to be significant. Whether site index SI or site quality SQ should be used as the site potential measure in evaluating the increment models will depend on which is the more significant in the volume model, but SQ is likely to be better.

TABLE VII.2

DEFINITION OF DUMMY VARIABLES

x	Dummy variables					
	x_1	x_2	x_3	x_4	x_5	x_6
1	1	0	0	0	0	0
2	0	1	0	0	0	0
3	0	0	1	0	0	0
4	0	0	0	1	0	0
5	0	0	0	0	1	0
6	0	0	0	0	0	1

x = Increment period in years.

The quadratic is considered unlikely to be of significance, but it should be evaluated. As increment should increase with increasing site potential the coefficient of SQ should be positive.

The interaction with relative tree size RSQ is likely to be more effective in the model than SQ because the inclusion of R will apportion the total stand increment between the individual trees in proportion to their size.

Age is also likely to be an effective variable for in the unpublished yield table of Lewis, periodic annual increment declines with age in a curvilinear manner that over the range of the data is approximated by the reciprocal of age. However the increment model will be used over a restricted range of ages and so age may well be as significant as the reciprocal.

As with site potential, the interactions RA and R/A are likely to be more efficient that A and 1/A as the interaction will ensure that each tree receives a proportion of the stand increment according to its relative tree size. As RA and R/A are considered likely to be equally efficient the simpler form RA will be tested in the first instance.

If site potential and age are constant then changes in stand density will affect tree increment, for with increasing stand density there are generally more trees to share the available increment on the site. Tree increment will therefore be inversely proportional to stand density. The index of stand density to be evaluated initially in the increment model should be the same as that found to be the most efficient in the volume model, and this is likely to be basal area B. Again the interaction with relative tree size is likely to be more effective than stand density alone, which leads to the formulation of four alternative forms of the density variable of which R/B is likely to be more effective than 1/B, RB or B. The coefficient of R/B should be positive.

Thinning may also affect tree increment for if thinning type TT or thinning intensity N% increase then it is more likely that trees with good increment potential will be included in the thinnings elect subpopulation. The effect of thinning will possibly be masked by the interactions RSQ, RA and R/B, but they should be tested in the model.

Apart from the variables considered so far the estimated volume of the tree at time of inventory x years before thinning may be a significant variable in the increment model, as increment may be proportional to the volume on which it is produced.

Of the variables considered the variables including site potential, age and stand density are considered likely to be more efficient than the thinning variables and the estimated volume at time of thinning.

Summary

Combining the increment period x and annual increment variables provides the increment function. Only after the increment model is developed using this simple time function will it be desirable to test the effect of substituting dummy variables to evaluate the effect of increment period. Annual increment can be formulated at three levels of complexity, so the following models should be evaluated initially, all being conditioned to pass through the origin.

Equation VII.13 includes the three interactions RSQ, RA and R/B which are considered likely to be the most effective.

$$\text{Increment} = b_1 \text{ xRSQ} + b_2 \text{ xRA} + b_3 \frac{\text{xR}}{\text{B}} \quad \ldots\ldots\ldots \text{ Equation VII.13}$$

Equation VII.14 incorporates the thinning variables.

$$\text{Increment} = b_1 \text{ xRSQ} + b_2 \text{ xRA} + b_3 \frac{\text{xR}}{\text{B}} + b_4 \text{ xTT} + b_5 \text{ xN\%} \quad \ldots\ldots \text{ Equation VII.14}$$

Equation VII.15 incorporates the estimated volume of the tree at time of thinning.

$$\text{Increment} = b_1 \text{ xRSQ} + b_2 \text{ xRA} + b_3 \frac{\text{xR}}{\text{B}} + b_4 \text{ xTT} + b_5 \text{ xN\%} + b_6 \text{ x}\hat{\text{V}} \quad \ldots\ldots \text{ Equation VII.15}$$

These equations are summarised in table VII.3.

TABLE VII.3

INCREMENT

MODELS TO BE EVALUATED

Equation	Independent variables						No. of variables
	xRSQ	xRA	xR/B	xTT	xN%	$x\hat{V}$	
VII.13	x	x	x				3
VII.14	x	x	x	x	x		5
VII.15	x	x	x	x	x	x	6

Where $\hat{V}$ is the volume of the tree at inventory as estimated from the volume model.

VIII. MODEL EVALUATION AND DEVELOPMENT

VIII. MODEL EVALUATION AND DEVELOPMENT

WEIGHTING FUNCTION FOR THE VOLUME MODELS

In chapter III it was noted that one of the conditions underlying linear regression analysis is that the variance is homogeneous. Before commencing to evaluate the models it is therefore necessary to analyse the variance of the dependent variable, this being a proxy for the variance of the residuals that will result when a model is fitted to the data. If the variance is not homogeneous then a suitable weighting function must be developed.

To facilitate the analysis of the variance the data was partitioned into cells, each cell covering a one inch range of diameter and a ten foot range of predominant height. The variance was then calculated for each cell that had greater than two observations in it. These variances are summarised in table VIII.1. It is evident that cell variance range widely, from 0.34 to 113.97, but generally increase with increasing height and diameter. Bartlett's test (Bartlett, 1937) was carried out in the data and gave evidence of significant heterogeneity at a probability level of $p = .01$. It is therefore necessary to review the possible variance models so that a suitable weighting function for this study can be developed.

TABLE VIII.1

CLASS VARIANCE

VOLUME

Tree diameter range in inches	Predominant height range in feet																	
	50½ - 60		60½ - 70		70½ - 80		80½ - 90		90½ - 100		100½ - 110		110½ - 120		120½ -130		130½ - 140	
	No	Var	No	Var	No	Var	No	Var	No	Var	No	Var	No	Var	No	Var	No	Var
6.0 - 6.95			10	0.60	10	0.53	46	1.54	10	0.34	4	1.15						
7.0 - 7.95	4	1.43	17	0.45	18	1.54	86	1.66	51	1.73	44	1.74						
8.0 - 8.95	4	0.93	11	0.53	23	2.22	104	2.51	76	3.28	65	2.70	7	3.38				
9.0 - 9.95			10	0.90	12	4.51	58	3.74	70	3.11	56	5.32	42	4.38				
10.0 -10.95			7	2.34	7	7.25	33	4.06	23	10.77	66	5.97	43	9.28	5	8.08		
11.0 -11.95							13	7.24	19	8.69	32	15.75	28	9.97	24	6.71		
12.0 -12.95									9	5.69	30	9.25	17	15.57	18	14.08		
13.0 -13.95											14	9.91	11	6.89	22	29.78		
14.0 -14.95											13	15.90	8	19.64	35	35.04		
15.0 -15.95											5	24.40			22	29.44		
16.0 -16.95															14	42.32	8	26.37
17.0 -17.95															5	37.22		
18.0 -18.95															6	20.60		
19.0 -19.95																		
20.0 -20.95															4	113.97		

Data at time of thinning partitioned into one inch diameter and ten foot predominant height classes.
Cells with less than or equal to 2 trees excluded.

Cunia (1962) using the combined variable equation, developed the hypothesis that variance is linearly related to $(D^2H)^2$. Cunia found that the simple function D^2H was not as satisfactory as $(D^2H)^2$. Frayer (1966) considered that for tree volume estimates a constant should be added. If Frayer is correct then it is better to test the two equations below, VIII.1 and VIII.2, rather than Cunia's simplified form.

$$\text{Variance} = b_0 + b_1 D^2H \quad \text{.....................} \quad \text{Equation VIII.1}$$

$$\text{Variance} = b_0 + b_1 D^2H + b_2 (D^2H)^2 \quad \text{..........} \quad \text{Equation VIII.2}$$

Inspection of the data in table VIII.1 suggests that the variance is roughly proportional to diameter and height, equation VIII.3.

$$\text{Variance} = b_0 + b_1 D + b_2 H \quad \text{................} \quad \text{Equation VIII.3}$$

Gerrard (1966) found that the exponential form was the best and used the logarithm of variance as the dependent variable and D and H as the independent variables, equation VIII.4.

$$\log(\text{Variance}) = b_0 + b_1 D + b_2 H \quad \text{...........} \quad \text{Equation VIII.4}$$

However inspection of the cell variances for the data suggested that D^2H may be a satisfactory independent variable, either in the simple linear form or the quadratic form, equations VIII.5 and VIII.6.

$\log(\text{Variance}) = b_0 + b_1 D^2H$ .... Equation VIII.5

$\log(\text{Variance}) = b_0 + b_1 D^2H + b_2 (D^2H)^2$ ..... Equation VIII.6

Finally the logarithmic equation VIII.7 appears promising.

$\log(\text{Variance}) = b_0 + b_1 \log(D) + b_2 \log(H)$

..... Equation VIII.7

These seven models were then fitted to the 53 observations in table VIII.1. The equations are summarised in table VIII.2 and the analysis of variance statistics for the regressions are summarised in table VIII.3.

Of the three equations using variance as the independent variable all have a negative constant which results in negative estimates of variance for some trees in the data. This is obviously erroneous as variance must be positive, so these three models were rejected for that reason alone.

All four models having the logarithm of variance as the dependent variable appear sensible. The addition of $(D^2H)^2$ is significant when added to D^2H. The model providing the best fit was equation VIII.7 with log (D) and log (H) as the independent variables, and this was selected for use in the development of the volume model. This is equation VIII.7, depicted in figure VIII.1.

$\log(\text{Variance}) = -13.9364 + 3.3735 \log(D) + 1.6674 \log(H)$

.... Equation VIII.7

TABLE VIII.2

VARIANCE ESTIMATION MODELS

VOLUME

Equation		
VIII.1	Variance	$= -7.8499 + 1.3564\ 10^{-3}\ D^2H$
VIII.2	Variance	$= -2.4541 - 1.0889\ 10^{-4}\ D^2H + 3.2016\ 10^{-8}\ (D^2H)^2$
VIII.3	Variance	$= -34.9622 + 4.3338\ D - 1.2944\ 10^{-2}\ H$
VIII.4	log (Variance)	$= -3.3552 + 2.7555\ 10^{-1}\ D + 1.9770\ 10^{-2}\ H$
VIII.5	log (Variance)	$= -0.1996 + 1.0109\ 10^{-4}\ D^2H$
VIII.6	log (Variance)	$= -0.5850 + 2.1267\ 10^{-4}\ D^2H - 2.4379\ 10^{-9}\ (D^2H)^2$
VIII.7	log (Variance)	$= -13.9364 + 3.3735 \log (D) + 1.6674 \log (H)$

TABLE VIII.3

VARIANCE ESTIMATION MODELS

VOLUME

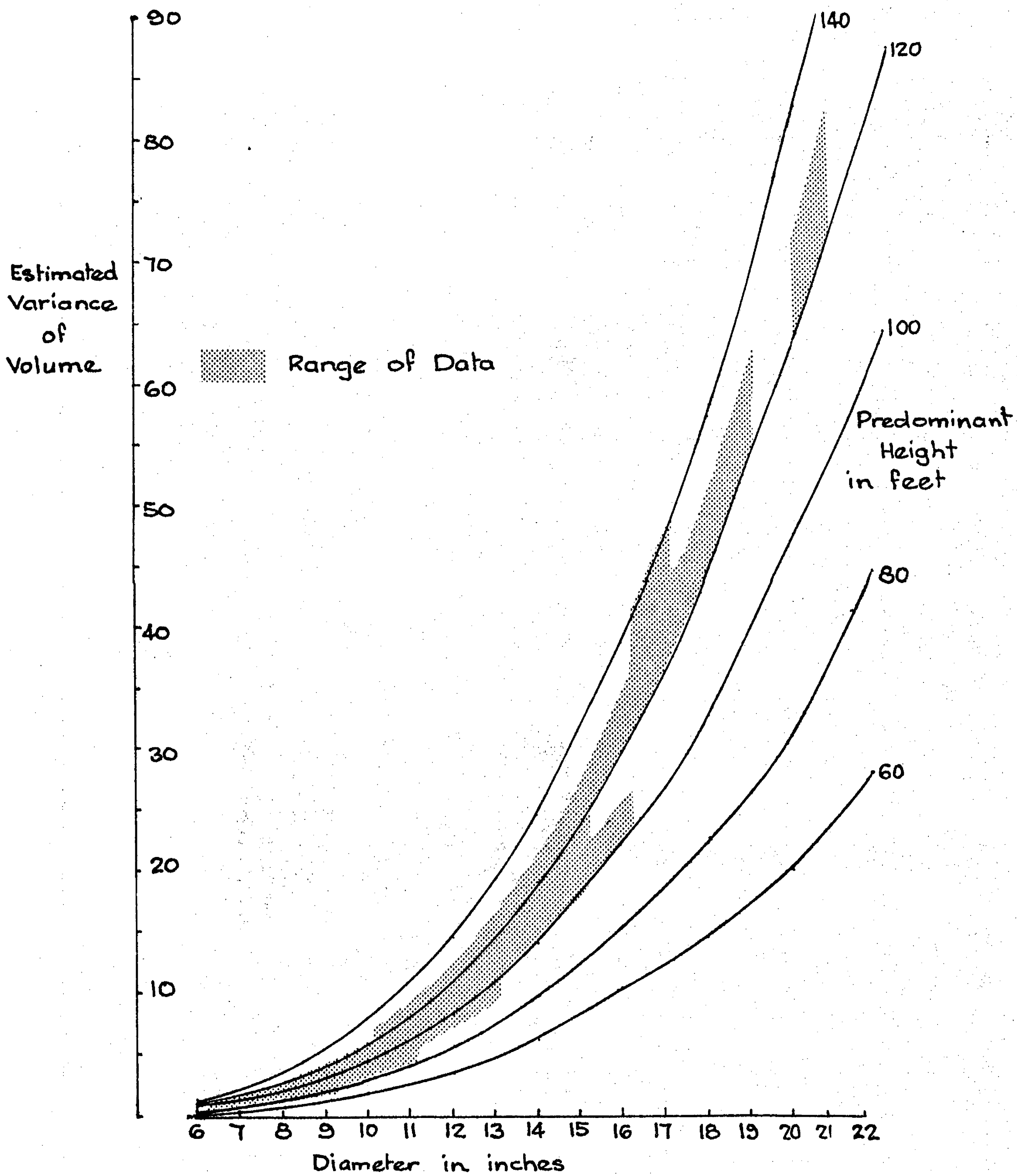
SUMMARY OF REGRESSION STATISTICS

Dependent variable		Degrees of freedom	Total sums of squares
Variance		52	16408.2
log (Variance)		52	91.865

Equation	Dependent variable	Independent variables	Degrees of freedom	Sum squares due to regression
VIII.1	Variance	D^2H	1	12277.9
VIII.2	Variance	D^2H, $(D^2H)^2$	2	13904.4
VIII.3	Variance	D, H	2	10534.4
VIII.4	log (Variance)	D, H	2	79.244
VIII.5	log (Variance)	D^2H	1	68.186
VIII.6	log (Variance)	D^2H, $(D^2H)^2$	2	77.617
VIII.7	log (Variance)	log (D), log (H)	2	82.109

FIGURE VIII. 1

VARIANCE OF VOLUME ESTIMATED FROM DIAMETER AND PREDOMINANT HEIGHT



From this function the weight to be used can be derived as the reciprocal of the square root of the estimated variance if the observations are to be multiplied by the weighting function, or as the reciprocal of the estimated variance if weighted regression analysis is to be used. Both techniques are valid and give the same results (Furnival, 1961).

VOLUME MODELS

In chapter VII twelve volume models were formulated for testing, along two alternative lines of development. One line of development using tree diameter, predominant height and combinations of them as the independent variables, and the other based on Naslund's combined variable equation using other tree and stand variables. Two equations were formulated using a combination of the two approaches. These twelve equations (table VII.1) were then fitted to the data, the regression statistics being summarised in table VIII.4. The correlation coefficients between the independent variables for each of the two alternative lines of model development are tabulated in Appendix 2, tables A2.1 and A2.2.

The simple combined variable equation of Naslund, equation VII.1, is in itself a relatively precise estimator of the volume of each tree to be removed in thinning as evidenced by the value of the multiple coefficient of determination R^2 of .94.

Table VIII.4 shows that the Australian equation, equation VII.3, is marginally better than equation VII.2, and both are

TABLE VIII.4

REGRESSION STATISTICS

VOLUME MODELS

	D.F.	S.S.
Total	1417	68564.6

Equation	Regression		Comparison with other equations	
	D.F.	S.S.	Equation	Significance
VII.1	1	64405.0		
VII.2	3	65315.8	VII.1	YES
VII.3	3	65340.7	VII.1	YES
VII.4	4	65377.8	VII.3	YES
VII.5	5	65346.1	VII.3	NO
VII.6	9	65433.4	VII.3	YES
VII.7	3	65464.5	VII.1	YES
VII.8	6	65707.6	VII.7	YES
VII.9	9	65775.5	VII.8	YES
VII.10	15	65809.8	VII.9	YES
VII.11	5	65772.1	VII.7	YES
VII.12	5	65748.3	VII.7	YES

significantly better than the simple combined variable equation, equation VII.1. The addition of the variable H^2 to equation VII.3 is significant (equation VII.4) but the addition of D and DH, equation VII.5 are not significant. Equation VII.6 including nine variables is significantly better than equation VII.3.

These results support Keeves (1961) model based on the volume - basal area line. They also indicate that Gerrard's hypothesis, that the graph of volume against height within diameter classes may be curvilinear, is likely to be correct as the difference between equation VII.6 which is significantly better than equation VII.3, and equation VII.5 which is not, are all variables including quadratic and cubic terms. However this line of development was not pursued as none of the models using tree diameter, predominant height and combinations of them explain as much of the variation as equation VII.7, based on Naslund's equation and other tree and stand parameters, using only three independent variables.

The addition of other tree and stand parameters to equation VII.7 shows in table VIII.4 that each equation from VII.8 to VII.10 is significantly better than the previous one. This result is anomalous as experience suggests that more than seven independent variables are seldom worthwhile (Grosenbaugh, 1967).

The combined approach, equations VII.11 and VII.12 are significantly better than equation VII.7 with equation VII.11 being marginally the better. As this equation is better than equation VII.8 which has one more variable, and equation VII.9 is not significantly better than it, it was selected as the most appropriate of the models formulated in chapter VII.

The line of model development based on Naslund's combined variable equation and other tree and stand parameters explains considerably more of the variation than the models using tree diameter, predominant height and combinations of them. The results of equations VII.7 to VII.10 are however anomalous and indicate that although they include significant variables the relative importance of each variable is not as formulated in chapter VII.

Using equation VII.11 as a base model twelve equations were formulated, equations VIII.8 to VIII.19 in which each of the variables in equation VII.10 that are not in equation VII.11 are added singly to the base model. These are summarised in table VIII.5 and the regression statistics are summarised in Appendix 2, table A2.3.

Of the twelve variables tested six are significant and six are not. Age, thinning type and thinning intensity are all significant when included either as a measure of form or as a proxy for the combined variable constant. Relative tree size is not significant regardless of how it is incorporated in the model. Stand density B and site quality SQ are not significant when used as a proxy for the combined variable constant. Although site quality is significant when used as a form variable, the quadratic term is not.

TABLE VIII.5

VOLUME

MODELS EVALUATED

Equation	Independent variables																	No. of variables
	D^2H	H	D^2	D^2HB	D^2HSQ	D^2HSQ^2	D^2HTT	$D^2HN\%$	D^2HA	D^2HR	D^2HTTR	A	R	B	SQ	TT	N%	
VIII.8	x	x	x	x	x	x												6
VIII.9	x	x	x	x	x		x											6
VIII.10	x	x	x	x	x			x										6
VIII.11	x	x	x	x	x				x									6
VIII.12	x	x	x	x	x					x								6
VIII.13	x	x	x	x	x						x							6
VIII.14	x	x	x	x	x							x						6
VIII.15	x	x	x	x	x								x					6
VIII.16	x	x	x	x	x									x				6
VIII.17	x	x	x	x	x										x			6
VIII.18	x	x	x	x	x											x		6
VIII.19	x	x	x	x	x												x	6
VIII.20	x	x	x	x	x		x		x									7
VIII.21	x	x	x	x	x			x	x									7
VIII.22	x	x	x	x	x				x			x						7
VIII.23	x	x	x	x	x				x							x		7
VIII.24	x	x	x	x	x				x								x	7

Age is better than either of the thinning variables and is better incorporated in combination with D^2H as an estimator of form. This equation, VIII.11, provides the best base for further development. To this equation were added singly each of the five variables that were significant when added to equation VII.11; and these were fitted to the data as equations VIII. 20 to VIII.24 table VIII.5 with the regression statistics summarised in Appendix 2, table A2.4.

Of the variables tested only thinning intensity was significant and then only when used as a proxy for the combined variable constant. Equation VIII.24 appears to be a satisfactory model but to confirm this a series of analyses of variance were carried out.

To test that each variable in equation VIII.24 is significant, seven equations were calculated, each of six independent variables, each variable being excluded in turn. Analyses of variance comparing these equations with equation VIII.24 showed that the addition of each variable is significant when all the others are included. This test is equivalent to confirming that each coefficient is significantly different from zero by a t test.

To confirm that no other variable should be included in the volume model a series of equations of eight independent variables were calculated in which each of the variables considered in chapter VII as worth evaluating were added singly to equation VII.24. Analyses of variance show that none of the other 16 variables are significant thereby confirming that equation VIII.24 is a satisfactory model. The correlation coefficients between the independent variables in equation VIII.24 are tabulated in Appendix 2, table A2.5.

$$V = -1.161 - 1.7739\ 10^{-2}\ H + 7.3651\ 10^{-2}\ D^2 + 8.7154\ 10^{-4}\ D^2H - 2.2024\ 10^{-6}\ D^2HB + 4.0774\ 10^{-8}\ D^2HSQ + 1.1455\ 10^{-5}\ D^2HA + 1.9502\ 10^{-2}\ N\% \quad \text{Equation VIII.24}$$

In chapter IV five indices of stand density were selected as being logical indices to be tested in the model. In chapters V and VI two alternative measures of site potential and two alternative variables based on age were proposed. Equation VIII. 24 was then changed by substituting each of the alternative variables in turn, the sums of the squared deviations due to the regressions being summarised in table VIII.6, the equations being equations VIII.25 - VIII.30.

The table demonstrates that SQ is marginally better than SI. This result is strictly only valid for this particular model but it does support the South Australian contention that stratification of the forest into volume productivity classes is more effective than stratification based solely on some convenient measure of upper stand height (Keeves, 1970).

Of the indices of stand density basal area per acre was marginally better than Lexen's bole area index and the ratio of standing basal area to the theoretical maximum basal area as derived in chapter IV. Number of trees per acre and Hart's height spacing ratio were considerably poorer. The sign of

TABLE VIII.6

COMPARISON OF ALTERNATIVE FORMS OF SITE POTENTIAL, STAND DENSITY AND AGE

BASED ON EQUATION VIII.24

	D.F.	S.S.
Total	1417	68564.6

Equation		Regression	
		D.F.	S.S.
VIII.24	SQ used as the measure of site potential	7	65857.9
VIII.25	SI used as the measure of site potential	7	65823.3
VIII.24	B used as the index of stand density	7	65857.9
VIII.26	N used as the index of stand density	7	65683.1
VIII.27	Hart's index used as the index of stand density	7	65667.1
VIII.28	Lexen's bole area index used as the index of stand density	7	65821.4
VIII.29	B/Maximum B used as the index of stand density	7	65848.6
VIII.24	A used as the parameter of age	7	65857.9
VIII.30	1/A used as the parameter of age	7	65842.0

the coefficients for the stand density variable were all consistent indicating that regardless of the choice of index, the use of predominant height instead of tree height masks the effect of stand density on tree form. Although only marginally better, the use of basal area as the index of stand density for the volume model is justified. However this result may not be applicable in other management models.

Age proved to be slightly better than the reciprocal and is therefore preferred.

<u>Testing the volume model</u>

Before statistical tests are conducted on any model it is advisable to inspect the coefficients of the model to see if the signs are logical.

In equation VIII.24 the coefficient of the major independent variable D^2H is positive, which is as expected. The coefficient if D^2HB is negative indicating that, as hypothesized in chapter VII, the use of predominant height instead of tree height is greater than the effect of density on tree form. With increasing site potential and age, form factor increases, which is biologically sound. The three variables H, D^2 and N% together with the constant are included because tree volume to a four inch top diameter is used as the dependent variable and because predominant height is used, rather than tree height. When thinning intensity is low only trees with poor form and low vigour are removed so that it is logical that the coefficient of N% be positive. The sign of the coefficient of H is negative, as postulated in the development of equation VII.3, but the sign of D^2 is positive.

This result appears to be anomalous and casts doubt on the original assumptions in equation VII.3. However the effect of the use of volume to a four inch top diameter instead of total stem volume could explain this apparent discrepancy. As trees get older and larger there is less stem volume above the arbitrary top diameter limit, that is, the height to the arbitrary top diameter limit approaches tree height. Thus the volume of this unmerchantable portion is invesely proportional to tree size a proxy for which is tree basal area.

Total stem volume $-V = b_0 - b_1 D^2$

where b_0 b_1 are absolute values of constants.

From the combined variable equation, if total stem volume is the dependent variable and tree diameter and tree height the independent variables then there should be no constant term, as in the following equation.

Total stem volume $= b_2 D^2 Ht$

If these two equations are combined, Ht being replaced by $(H-b_3)$, D^2 by (D^2-b_4) as in the derivation of equation VII.3, then the following equation results.

$$V = -b_0 + b_1 (D^2-b_4) + b_2 (D^2-b_4) (H-b_3)$$

$$V = (-b_0-b_1b_4 + b_2b_3b_4) + (b_1-b_2b_3) D^2 - b_2b_4 H + b_2 D^2H$$

where b_0, b_1, b_2, b_3, b_4 are absolute values of constants.

From this it is evident that although the coefficient of D^2H must always be positive and the coefficient of H always negative. Depending on the values of the constants, the constant term in the regression model and the coefficients of D^2 may be either positive or negative.

The signs of the coefficients of the variables in equation VIII.24 can therefore be satisfactorily explained on logical grounds, so the model was then tested to ensure that basic assumptions underlying linear regression analysis were not violated.

The first test applied was the test for homogeneity of the variance of the residuals that resulted when the model was fitted to the data. The 1418 observations were divided into twenty approximately equal groups and Bartlett's test (1937) and Cochran's test (1941) applied. Both were not significant at a probability level of $p = .01$, the value of Chi-square for Bartlett's test being 21.1, the value of the statistic for Cochran's test 0.033.

The residuals were then tested for normality; neither the moment statistic of kurtosis or the moment statistic of skewness (Sokal and Rohlf, 1969) being significant at a probability level of $p = .01$, the statistics being -0.03 and 0.25 respectively.

The Durbin-Watson d statistic was also calculated and tested as described in chapter III, the value of d being 1.91 which was not significant at a probability level of p = .01.

Two sets of test data were then extracted. There were 590 trees not included in the ten randomly selected trees from each plot and 390 trees in plots that were not included in the basic data pool. Independent t tests were carried out (Freese, 1967; Sokal and Rohlf, 1969) testing the mean error of each of these populations based on equation VIII.24. Values less than .001 resulted in each case and it was therefore concluded that the model can be used to estimate volume at time of thinning.

Before the model could be used to determine the dependent variable for the increment models it was necessary to ensure that the volume model is satisfactory as an estimator for trees to be thinned when measured at inventory up to six years before the thinning. A data set of 217 observations was extracted, consisting of trees measured for volume a number of years prior to thinning. A t test was carried out to compare the mean residual that resulted when equation VIII.24 was fitted to these data. A value of .015 resulted which is not significant at a probability level of p = .01. The model can therefore be safely used to estimate volume at some time prior to thinning, as well as at thinning.

WEIGHTING FUNCTION FOR THE INCREMENT MODEL

As with the volume models it is necessary to analyse the variance of the dependent variable to determine whether a weighting function will be required or not. The dependent variable is the difference between the estimated volume at time of thinning and the estimated volume when inventory is carried out 1 to 6 years (x) prior to thinning.

Increment is dependent on the length of the increment period so the data was partitioned on x as well as on D and H. Each cell covered a single year increment period, two inch diameter range and twenty foot predominant height range. The variance was then calculated for each cell with more than two observations in it, these variances being summarised in table VII.7. Cell variances range widely, from 0.07 to 56.16, generally increasing with increasing diameter and increment period but reducing with increasing height. For trees of similar size, increment will be lower at later ages and in this context predominant height acts as a proxy for age.

It is probably desirable that the weighting functions used for the volume model and the increment model be similar in form. There are two ways that the increment period can logically be introduced into the variance estimating function. The effect of the increment period is probably multiplicative rather than additive as it is logical to assume that the variance of annual increment will be constant for a given diameter and predominant height. log (D) and log (H) could be multiplied by the increment period, which infers a multiplicative effect of increment period on the logarithm of variance, equation VIII.31. Alternatively, log (x) can be added to log (D) and log (H) to make three inde-

TABLE VIII.7

CLASS VARIANCE

INCREMENT

Predominant height range in feet	Tree diameter range in inches	Increment period x					
		1	2	3	4	5	6
40½- 60	6.0 - 7.95	0.11	0.24		0.53		
	8.0 - 9.95	0.10	0.19		0.14		
	10.0 -11.95						
	12.0 -13.95						
	14.0 -15.95						
	16.0 -17.95						
	18.0 -19.95						
	20.0 -21.95						
60½- 80	6.0 - 7.95	0.14	0.38	0.27	0.89		0.71
	8.0 - 9.95	0.47	1.38	2.53	3.61		3.26
	10.0 -11.95	0.44	1.67	3.17	4.71		6.21
	12.0 -13.95						
	14.0 -15.95						
	16.0 -17.95						
	18.0 -19.95						
	20.0 -21.95						
80½-100	6.0 - 7.95	0.07	0.14	0.46	0.49	1.21	0.74
	8.0 - 9.95	0.18	0.57	1.28	1.89	3.15	2.90
	10.0 -11.95	0.32	1.61	3.78	3.32	7.36	7.12
	12.0 -13.95	0.76	5.72	5.01	2.85	9.94	
	14.0 -15.95						
	16.0 -17.95						
	18.0 -19.95						
	20.0 -21.95						
100½-120	6.0 - 7.95		0.07	0.18	0.26		0.37
	8.0 - 9.95		0.33	0.68	1.12	1.79	1.32
	10.0 -11.95	0.44	1.53	3.14	5.43	7.67	3.31
	12.0 -13.95	0.74	3.46	4.50	10.00	13.21	5.41
	14.0 -15.95		5.78	7.46	18.71	27.67	1.83
	16.0 -17.95						
	18.0 -19.95						
	20.0 -21.95						
120½-140	6.0 - 7.95						
	8.0 - 9.95						
	10.0 -11.95		0.20	0.52	0.92	0.24	2.00
	12.0 -13.95		0.80	1.88	2.39	5.26	10.03
	14.0 -15.95		0.32	1.40	1.55	4.66	6.44
	16.0 -17.95		4.13	3.95		15.53	18.29
	18.0 -19.95						28.60
	20.0 -21.95						56.16

Data measured 1-6 years prior to thinning (x), partitioned into two inch diameter, twenty foot predominant height, and one year increment period classes. Cells with less than or equal to two trees excluded.

pendent variables, inferring a multiplicative effect of increment period on variance, equation VIII.32. A third model, equation VIII.33, infers that increment period has no effect on variance and that the form of the variance estimating function is the same for volume and for increment.

$$\log(\text{Variance}) = b_0 + b_1 \log(D) + b_2 \log(H) + b_3 \log(x)$$

.... Equation VIII.31

$$\log(\text{Variance}) = b_0 + b_1\, x \log(D) + b_2\, x \log(H)$$

.... Equation VIII.32

$$\log(\text{Variance}) = b_0 + b_1 \log(D) + b_2 \log(H)$$

.... Equation VIII.33

These three models were then fitted to the 91 observations in table VIII.7. The equations derived are summarised in table VIII.8, and the analysis of variance statistics for these regressions are summarised in table VIII.9.

All have positive coefficients for log (D) and negative for log (H), as suggested by the inspection of the data. The addition of log (x) to log (D) and log (H) was significant at a probability level of $p = .05$, and this three variable equation was significantly better than the two variable equation using x log (D) and x log (H) as independent variables, suggesting that the effect of increment period is multiplicative on variance rather than the logarithm of variance. The following equation, VIII.31, was selected for use in development of the increment model, and is depicted in figure VIII.2.

TABLE VIII.8

VARIANCE ESTIMATION MODELS

INCREMENT

Equation	
VIII.31	$\log(\text{Variance}) = -1.9102 + 4.5908 \log(D) - 2.2484 \log(H) + 1.5212 \log(x)$
VIII.32	$\log(\text{Variance}) = -1.3537 + 0.95054 \times \log(D) - 0.38707 \times \log(H)$
VIII.33	$\log(\text{Variance}) = -3.5892 + 4.5037 \log(D) - 1.7068 \log(H)$

TABLE VIII.9

VARIANCE ESTIMATION MODELS

INCREMENT

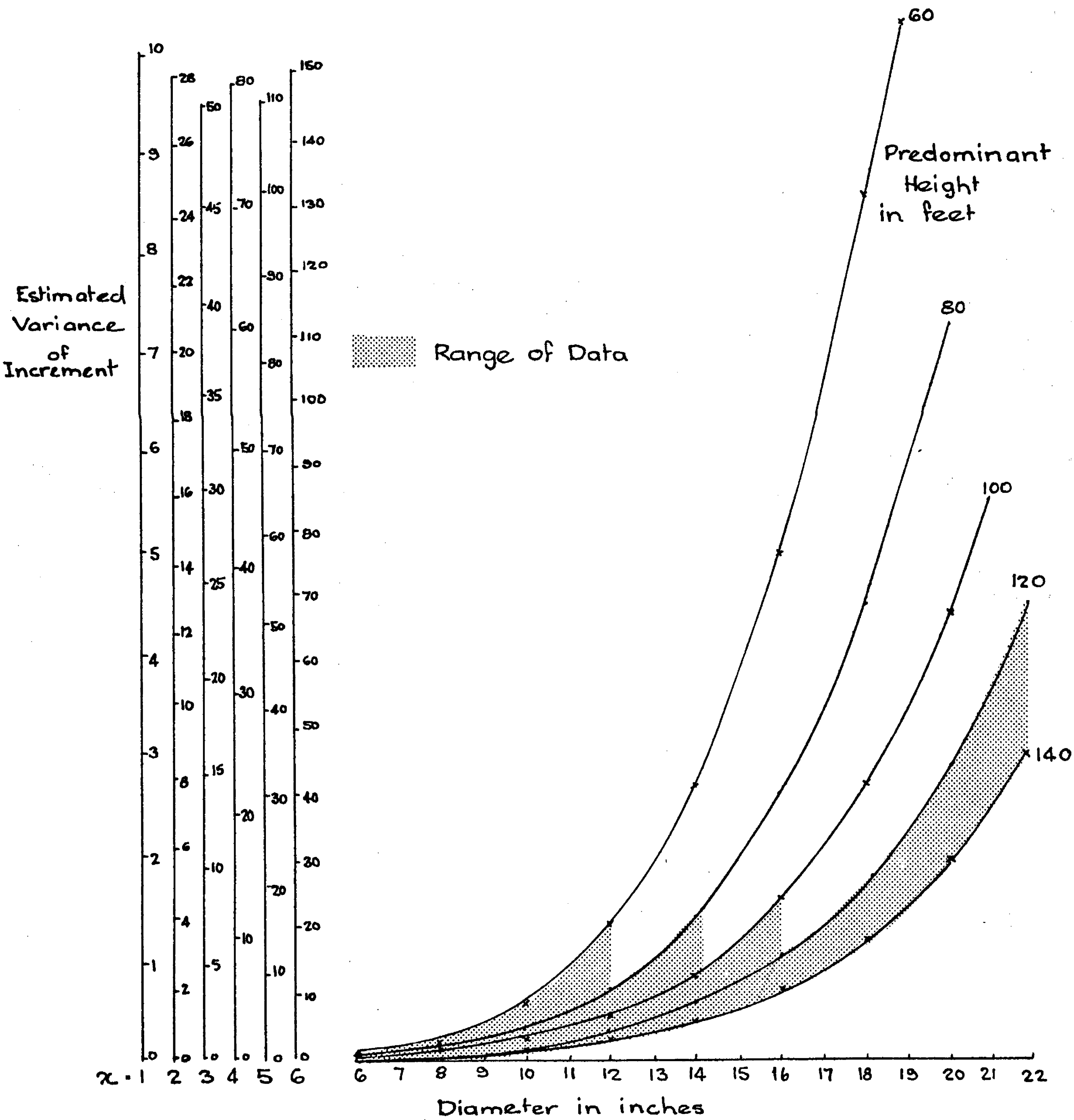
SUMMARY OF REGRESSION STATISTICS

Dependent variable		Degrees of freedom	Total sums of squares
log (Variance)		90	213.93

Equation	Dependent variable	Independent variables	Degrees of freedom	Sum squares due to regression
VIII.31	log (Variance)	log (D), log (H), log (x)	3	158.52
VIII.32	log (Variance)	x log (D), x log (H)	2	138.56
VIII.33	log (Variance)	log (D), log (H)	2	84.38

FIGURE VIII.2

VARIANCE OF INCREMENT ESTIMATED FROM DIAMETER AND PREDOMINANT HEIGHT FOR EACH INCREMENT PERIOD



$$\log(\text{Variance}) = -1.9102 + 4.5908 \log(D) - 2.2484 \log(H) + 1.5212 \log(x) \quad \text{......... Equation VIII.31}$$

The weighting function to be used in the development of the increment model can be derived in a similar manner to that described for the volume model.

INCREMENT MODELS

The three increment equations formulated in chapter VII, equations VII.13, VII.14 and VII.15, table VII.3, were then fitted to the data. The regression statistics are summarised in table VIII.10 with the correlation coefficients between the independent variables summarised in Appendix 2, table A2.6.

Equation VII.13, which includes three independent variables, xRSQ, xRA and xR/B is a relatively precise estimating function with a multiple coefficient of determination R^2 of .73. The addition of two independent variables TT and N% to equation VII.13 to form equation VII.14 is significant as also is the addition of $x\hat{V}$ to equation VII.14, fitted as equation VII.15.

$$\text{Increment} = 3.7038\ 10^{-5}\ \text{xRSQ} - 2.2765\ 10^{-2}\ \text{xRA} + 86.695\ \frac{\text{xR}}{\text{B}} - 0.61993\ \text{xTT} + 5.4327\ 10^{-3}\ \text{xN\%} + 4.9783\ 10^{-2}\ x\hat{V} \quad \text{......................... Equation VII.15}$$

To test that each variable in equation VII.15 is significant six equations were calculated in which each variable was excluded in turn. Analyses of variance showed that each variable was significant when all the others are included. This

TABLE VIII.10

REGRESSION STATISTICS

INCREMENT MODELS

	D.F.	S.S.
Total	3035	16763.4

Equation	Regression		Comparison with other equations		
	D.F.	S.S.	Equation	Variables added	Significance
VII.13	3	12289.9			
VII.14	5	13492.3	VII.13	xTT, xN%	YES
VII.15	6	14291.9	VII.14	$x\hat{V}$	YES

is equivalent to a t test confirming that each coefficient is significantly different from zero.

In chapter VII a number of alternative variables were proposed including site potential, age and stand density. Eight equations VIII.34 to VIII.41 were then formulated in which each variable in turn was replaced by the alternative forms. These are summarised in Appendix 2, table A2.7, with the regression statistics summarised in table VIII.11.

From this table it can be seen that the best variables are those forms used in equation VII.15, relative tree size being included in all three variables. Age was better than its reciprocal which although not entirely logical (vide chapter VII) was not unexpected. The reciprocal of stand density was better than the simple linear form as expected.

In developing the volume model the various indices of stand density and site potential were evaluated and basal area per acre B and site quality SQ were adopted although only marginally better than other alternative forms. Equation VII.15 was modified by substituting SI for SQ, equation VIII.42, and by substituting the various indices of stand density, equations VIII.43 to VIII.46. The regression statistics are summarised in table VIII.12 which shows that the forms used in the volume model are marginally better in the increment models also. Of the indices of stand density, number of trees per acre, Lexen's bole area index and the ratio of basal area to the theoretical maximum basal area are almost as effective as basal area, but Hart's

TABLE VIII.11

COMPARISON OF ALTERNATIVE VARIABLES OF SITE POTENTIAL

AGE AND STAND DENSITY

BASED ON EQUATION VII.15

	D.F.	S.S.
Total	3035	16763.4

Equation		Regression	
		D.F.	S.S.
VII .15	xRSQ used as the variable of site potential	6	14291.9
VIII.34	xSQ used as the variable of site potential	6	14261.7
VIII.35	xSQ^2 used as the variable of site potential	6	14260.3
VII .15	xRA used as the variable of age	6	14291.9
VIII.36	xR/A used as the variable of age	6	14197.2
VIII.37	xA used as the variable of age	6	14263.2
VIII.38	x/A used as the variable of age	6	14204.4
VII .15	xR/B used as the variable of stand density	6	14291.9
VIII.39	xRB used as the variable of stand density	6	13884.0
VIII.40	X/B used as the variable of stand density	6	14199.7
VIII.41	xB used as the variable of stand density	6	13896.8

TABLE VIII.12

COMPARISON OF ALTERNATIVE FORMS OF SITE POTENTIAL AND STAND DENSITY

BASED ON EQUATION VII.15

	D.F.	S.S.
Total	3035	16763.4

Equation		Regression	
		D.F.	S.S.
VII .15	SQ used as the measure of site potential	6	14291.9
VIII.42	SI used as the measure of site potential	6	14290.0
VII .15	B used as the index of stand density	6	14291.9
VIII.43	N used as the index of stand density	6	14280.9
VIII.44	Hart's index used as the index of stand density	6	13518.4
VIII.45	Lexen's bole area index used as the index of stand density	6	14262.7
VIII.46	B/Maximum B used and the index of stand density	6	14281.8

index is considerably poorer. Site quality and site index are very similar in their effect on the model.

Equation VII.15 was therefore accepted as the best increment model assuming that the time element is a linear one.

To test this assumption a 36 independent variable equation VIII.47 was calculated in which the increment period element x was replaced by the dummy variables as described in chapter VII. Thus xTT was replaced by six variables x_1TT, x_2TT, x_3TT, x_4TT, x_5TT, x_6TT, and the other five variables were each replaced in a similar manner.

The regression statistics for equstion VII.47 are summarised in table VIII.13 and show that the equation is not significantly better than equation VII.15. It was concluded that the simple linear time function is satisfactory and that equation VIII.15 is an acceptable model.

Testing the increment model

In equation VII.15 the signs of the coefficients are such that increment increases with increasing tree size, relative tree size, site potential and stand density, and decreases with increasing age, all of which are biologically sound. The effect of thinning intensity is also logical, as increasing thinning intensity increases the likelihood of thinning trees with greater increment potential. The coefficient of thinning type cannot be assessed in the same way, as increasing thinning type

TABLE VIII.13

REGRESSION STATISTICS

INCREMENT MODELS

	D.F.	S.S.
Total	3035	16763.4

Equation	Regression		Comparison with other equations		
	D.F.	S.S.	Equation	Variables added	Significance
VII .15	6	14291.9			
VIII.47	36	14326.4	VII.15	Dummy variables instead of x	NO

increases the average value of relative tree size for the component trees of the thinning elect subpopulation, so that thinning type cannot change without changing the average relative tree size for the elect thinnings in the stand although if thinning type is constant relative tree size can change. Thus all the coefficients that can readily be corroborated biologically are corroborated and the increment model appears to be sound.

In the same way that the volume model was tested, statistical tests were then applied to ensure that the basic assumptions underlying linear regression analysis were not violated.

The 3035 observations were divided into twenty approximately equal groups and Bartlett's test (1937) and Cochran's test (1941) applied. Both were not significant at a probability level of p = .05, the value of Chi-square for Bartlett's test, after correction, being 30.09, the value of the statistic for Cochran's test 0.069.

Although the tests are no significant the relatively high values of the statistics infer that the weighting function is not as efficient as it might have been. This is not surprising as the weighting function is based on variables that are not used in the final increment model although they are inherent in the use of volume as measured at inventory x years before thinning. Although it is likely that a better weighting function could be developed, the one developed is adequate and statistically satisfactory.

The residuals were then tested for normality, neither the moment statistic of kurtosis nor the moment statistic of skewness (Sokal and Rohlf, 1969) being significant at a probability level of p = .05, the statistics being 1.92 and 0.60 respectively. These are consistent with the tests for homogeneity of the variance in that the distribution tends to be leptokurtic if the variance is heterogeneous. A value of 1.96 for the moment statistic of kurtosis would have indicated significant leptokurtosis.

The Durbin-Watson (1950, 1951) d statistic described in chapter III was then calculated, the value of d being 1.85. This was significant using the statistic of Theil and Nagar (1961). To reduce serial correlation to an acceptable non significant level it was necessary to reduce the number of serially correlated observations from each plot, re-estimate the regression coefficients and re-evaluate the d statistic.

The data described in chapter VI was reduced so that only five random trees were selected from each plot thinning and the coefficients of equation VII.15 were re-estimated. The d statistic was recalculated for this regression and was still significant at 1.89, indicating that a further culling of the data was necessary.

The data was then further reduced so that five random trees were selected from each plot thinning, measurements being extracted for only one of the six possible years of measurement prior to thinning. Also where a plot was represented by more than two plot thinnings the number of plot thinnings (each representing a different thinning operation carried out in a different year) was restricted to two. This reduced the number of plot thinnings from 157 to 147 and the number of observations to 731. The coefficients of equation VII.15 were re-estimated and are shown below in equation VIII.48, regression statistics being summarised in Appendix 2, table A2.8. For this equation the Durbin-Watson d statistic was 1.93 which is not significant using the statistic of Theil and Nagar.

$$\text{Increment} = 4.9077\ 10^{-5}\ \text{xRSQ} - 1.4804\ 10^{-2}\ \text{xRA} + 71.578\ \frac{\text{xR}}{\text{B}} - 0.80638\ \text{xTT} + 7.9239\ 10^{-3}\ \text{xN\%} + 4.4680\ 10^{-2}\ \text{x}\hat{\text{V}}$$

.... Equation VIII.48

To test the model independent test data were then extracted. One random measurement was selected from each of the 390 trees used to test the volume model. A t test (Freese, 1967; Sokal and Rohlf, 1969) was carried out comparing the residuals resulting when the increment model, equation VIII.48, was fitted to both sets of data. A value of 1.85 resulted which is not significant. It was concluded that the model was satisfactory.

As the model developed, equation VIII.48, was biologically sound, was based on sound statistical analysis, met the assumptions of linear regression analysis and satisfactorily tested on independent data, it was accepted as the increment model.

METRIC MODELS

In South Australia, as with the rest of Australia, future forest inventory will be carried out using metric measurements. To facilitate the calculation of these metric inventory data the relevant equations have been converted to metric units.

The wieghting function for the volume model is derived from equation VIII.7m which estimates variance.

$$\log(\text{Variance}) = -20.5610 + 3.3735 \log(Dm) + 1.6674 \log(Hm)$$

.... Equation VIII.7m

The volume equation is equation VIII.24m

$$Vm = -3.2876\ 10^{-2} - 1.6480\ 10^{-3}\ Hm + 3.2326\ 10^{-4}\ Dm^2 + 1.2550\ 10^{-5}\ Dm^2Hm - 1.2015\ 10^{-6}\ Dm^2HmBm + 8.3909\ 10^{-9}\ Dm^2HmSQm + 1.6495\ 10^{-7}\ Dm^2HmA + 5.5223\ 10^{-4}\ N\%$$

.... Equation VIII.24m

The weighting function for the increment model is derived from equation VIII.31m which estimates variance.

$$\log(\text{Variance}) = -18.2380 + 4.5908 \log(Dm) - 2.2484 \log(Hm) + 1.5212 \log(x)$$

.... Equation VIII.31m

The increment model is equation VIII.48m.

$$\text{Increment } m = 1.9861\ 10^{-5}\ xRSQm - 4.1920\ 10^{-4}\ xRA + 0.46530\ \frac{xR}{Bm} - 2.2834\ 10^{-2}\ xTT + 2.2438\ 10^{-4}\ xN\% + 4.4680\ 10^{-2}\ x\hat{V}m \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad \text{Equation VIII.48m}$$

IX. CONCLUSIONS

IX. CONCLUSIONS

In South Australian radiata pine forests inventory precedes thinning by from one to six years. Tree volume and increment models have been developed which will enable estimates to be made of the volume at time of inventory of trees selected for removal in thinning, and the increment that will be added to these trees between inventory and the time of thinning.

The models were estimated by linear regression analysis and appear to be soundly based. The volume model has a multiple coefficient of determination of 0.96 and the mean volume of 19.26 cubic feet has an estimated standard deviation of 2.92 cubic feet. The increment model has a multiple coefficient of determination of 0.85 and the mean increment of 1.67 cubic feet has an estimated standard deviation of 0.92 cubic feet.

Extensive statistical tests were carried out to ensure that the assumptions underlying linear regression analysis were met. Independent test data were used to test the models, both of which were satisfactory.

The study demonstrates that satisfactory models can be developed using a combination of tree and stand parameters.

Basal area was accepted as the index of stand density being marginally better than the ratio of basal area to the maximum basal area that the site can sustain, an index derived from an investigation of mortality and stand dynamics during an examination of Reineke's stand density index. Lexen's bole area index (1943), Hart's height-spacing ratio (1928) and number of trees per acre were less efficient.

Further development of the simple model of stand dynamics presented in chapter IV may lead to the development of a better index than the ratio used, and this index may be better than basal area in both the volume and increment models.

Site quality SQ, a measure of site potential based on volume production, was marginally better than site index SI, a measure based on stand top height.

Age was shown to be better than its reciprocal in both the volume and increment models.

Thinning intensity N% was significant in both volume and increment models but thinning type TT was significant only in the increment model. That thinning type was not significant in the volume model was not unexpected as previous evidence (Keeves, 1970) indicated a similar result, attributing the result to the relatively narrow range of thinning type in the data. In the data used in this study the average value of thinning type was 0.86 with a standard deviation of 0.05, indicating that the range of the data is relatively restricted.

The data used to develop the models were selected from trees not seriously malformed, and it is possible that the models will not satisfactorily estimate the volume and increment of malformed trees. As part of a future study the applicability of the models developed to the estimation of malformed trees will be investigated. However few malformed trees remain in the stand after first thinning, only 2.5% of the tree data available for this study were seriously malformed and 4.0% had minor malformations. Until the problem is investigated the models developed are unlikely to cause serious errors when used in practice.

In current inventory procedures the volume on areas to be clear felled is estimated by the use of a predominant height tarif relationship developed by Keeves (1961). Increment is estimated from the unpublished yield table prepared by Lewis. These relationships are stand based rather than tree based so it is desirable that the models developed in this study be extended to predict volume and increment on the main crop, as well as on the thinnings elect subpopulation, so that all inventory calculations will be consistent.

In summary, the study has been successful in that volume and increment models have been satisfactorily developed.

The volume model is equation VIII.24 below:

$$V = -1.161 - 1.7739\ 10^{-2}\ H + 7.3651\ 10^{-2}\ D^2 + 8.7154\ 10^{-4}\ D^2H - 2.2024\ 10^{-6}\ D^2HB + 4.0774\ 10^{-8}\ D^2HSQ + 1.1455\ 10^{-5}\ D^2HA + 1.9502\ 10^{-2}\ N\% \quad \ldots\ldots\ldots\ldots \quad \text{Equation VIII.24}$$

The increment model is equation VIII.48 below:

$$\text{Increment} = 4.9077\ 10^{-5}\ \text{xRSQ} - 1.4804\ 10^{-2}\ \text{xRA} + 71.578\ \frac{\text{xR}}{\text{B}} - 0.80638\ \text{xTT} + 7.9239\ 10^{-3}\ \text{xN\%} + 4.4680\ 10^{-2}\ \text{x}\hat{\text{V}}$$

.... Equation VIII.48

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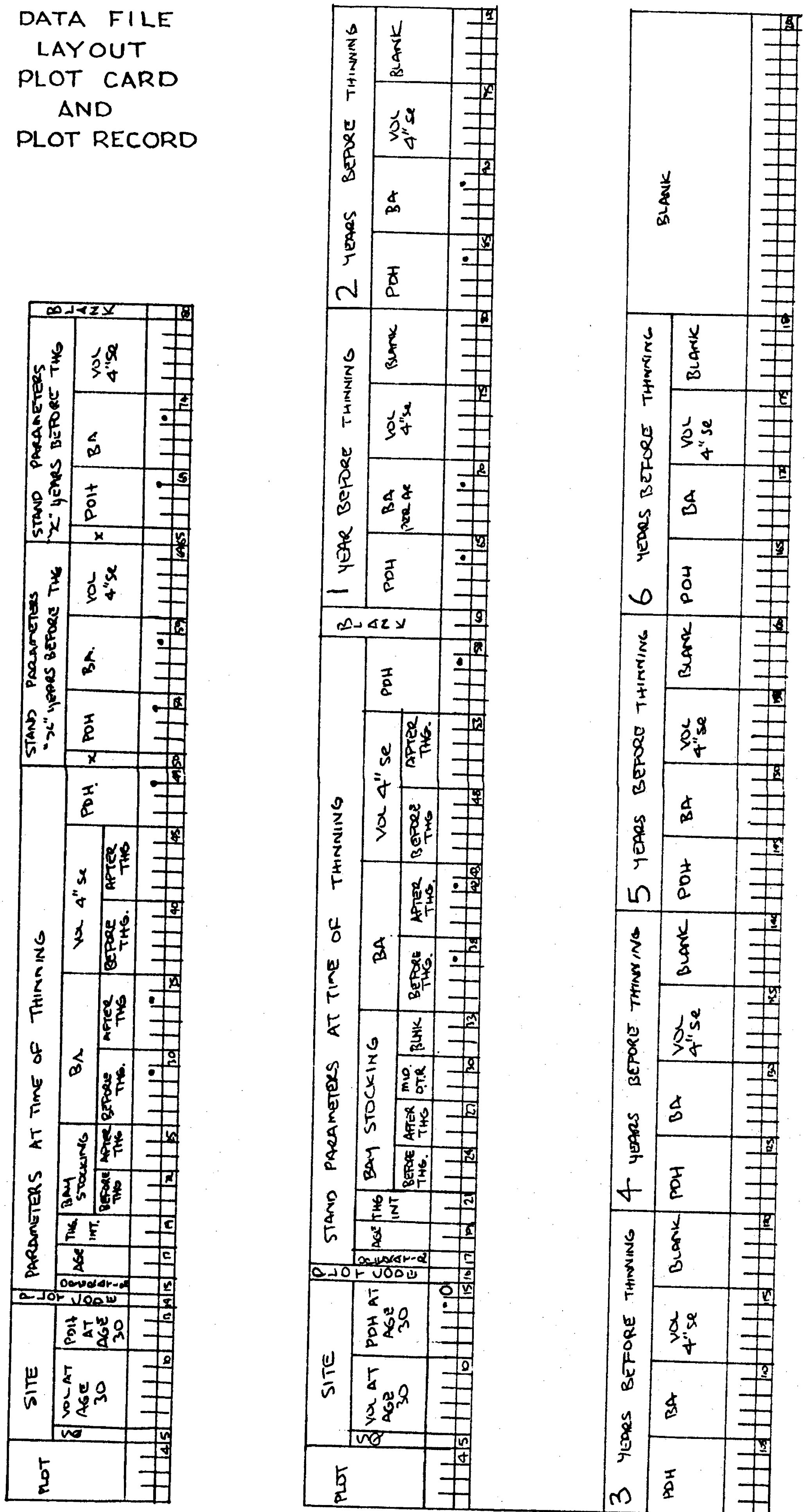
APPENDIX 1

PUNCH CARD AND MAGNETIC TAPE FILE LAYOUT FOR DATA

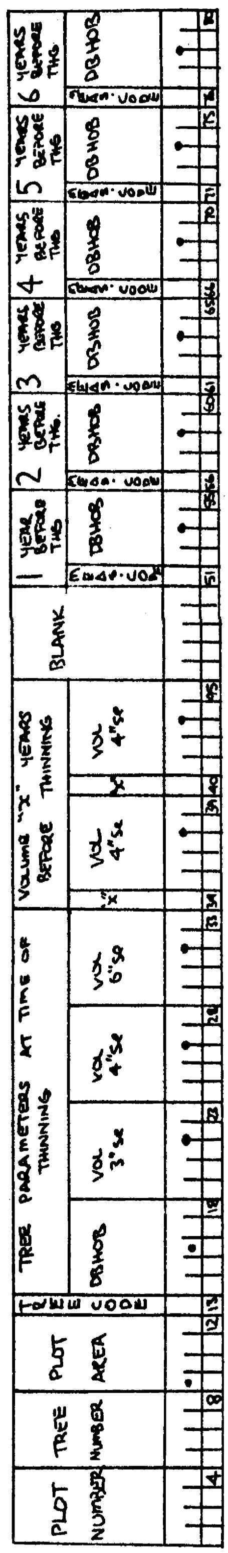
DATA FILE LAYOUT
PLOT CARD
AND
PLOT RECORD



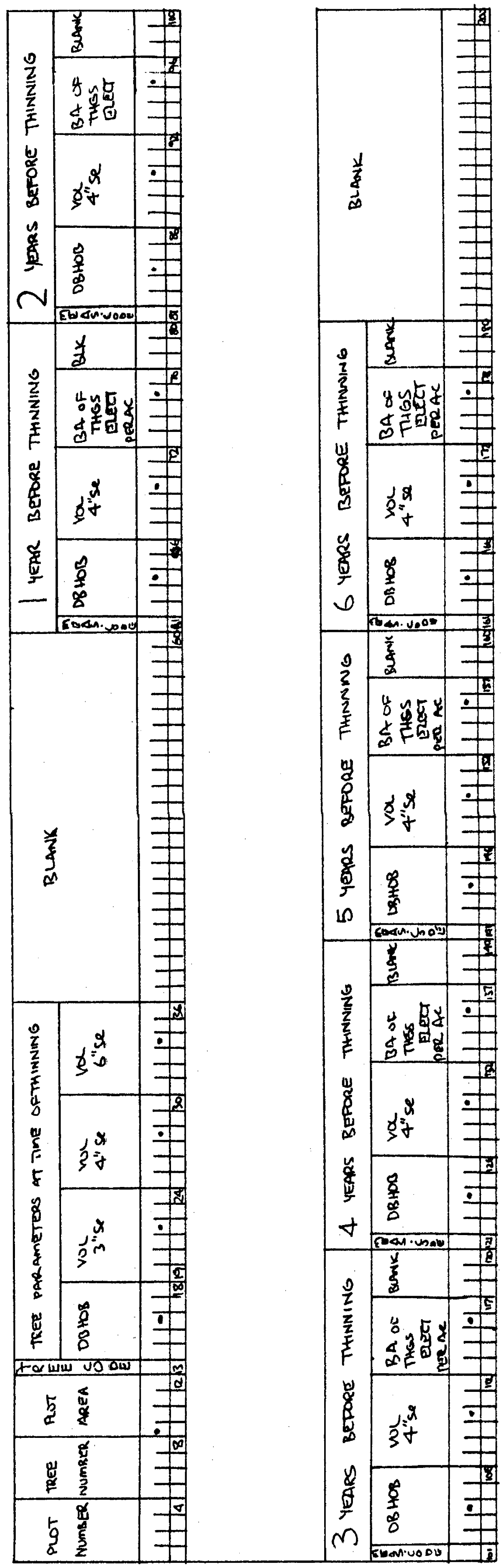
PLOT CARD

PLOT RECORD

DATA FILE
LAYOUT
TREE CARD
AND
TREE RECORD

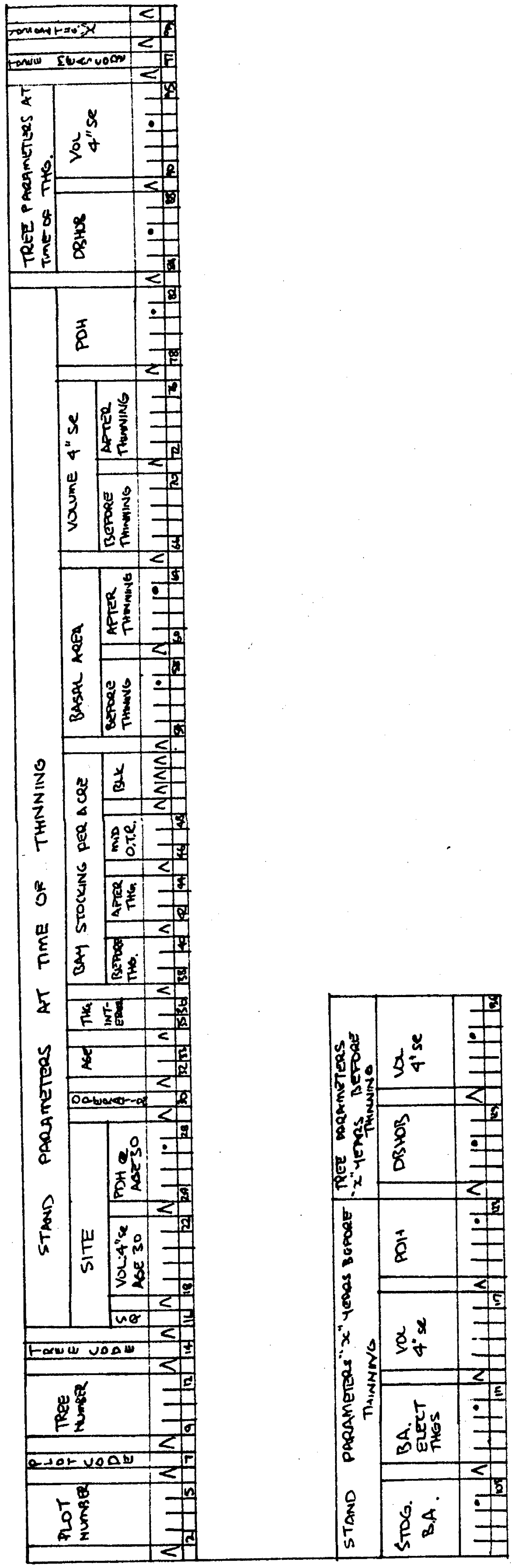


TREE CARD



TREE RECORD.

DATA FILE
LAYOUT
MAIN RECORD



MAIN RECORD

APPENDIX 2

REGRESSION STATISTICS

FOR

VOLUME AND INCREMENT MODELS

TABLE A2.1

CORRELATION COEFFICIENTS

BETWEEN

INDEPENDENT VARIABLES IN EQUATIONS VII.1 - VII.6

	D	H	DH	D^2	H^2	D^3	H^3	D^2H	DH^2
D	1.00	0.49	0.90	0.99	0.52	0.94	0.57	0.93	0.81
H		1.00	0.81	0.49	0.99	0.48	0.98	0.67	0.87
DH			1.00	0.90	0.84	0.88	0.85	0.97	0.98
D^2				1.00	0.53	0.98	0.56	0.96	0.83
H^2					1.00	0.51	0.99	0.71	0.90
D^3						1.00	0.55	0.97	0.82
H^3							1.00	0.74	0.92
D^2H								1.00	0.94
DH^2									1.00

TABLE A2.2

CORRELATION COEFFICIENTS
BETWEEN
INDEPENDENT VARIABLES IN EQUATIONS VII.7 - VII.10

	D^2H	D^2HB	D^2HSQ	D^2HSQ^2	D^2HTT	$D^2HN\%$	D^2HTTR	D^2HA	D^2HR	A	R	B	SQ	TT	N%
D^2H	1.00	0.95	0.91	0.75	0.99	0.87	0.99	0.95	0.98	0.62	0.35	0.36	0.09	0.07	0.08
D^2HB		1.00	0.90	0.75	0.93	0.80	0.87	0.94	0.90	0.68	0.19	0.58	0.16	-0.08	0.05
D^2HSQ			1.00	0.95	0.90	0.74	0.86	0.81	0.88	0.42	0.27	0.43	0.44	0.01	-0.03
D^2HSQ^2				1.00	0.73	0.58	0.70	0.62	0.72	0.24	0.20	0.41	0.64	-0.02	-0.10
D^2HTT					1.00	0.87	0.98	0.94	0.98	0.58	0.38	0.29	0.07	0.18	0.11
$D^2HN\%$						1.00	0.86	0.85	0.86	0.54	0.34	0.20	-0.03	0.20	0.48
D^2HTTR							1.00	0.89	0.99	0.49	0.52	0.20	0.05	0.23	0.13
D^2HA								1.00	0.90	0.74	0.19	0.41	-0.03	0.01	0.08
D^2HR									1.00	0.53	0.51	0.25	0.06	0.14	0.10
A										1.00	-0.13	0.61	-0.25	-0.32	0.01
R											1.00	-0.31	-0.08	0.39	0.17
B												1.00	0.38	-0.59	-0.30
SQ													1.00	-0.15	-0.31
TT														1.00	0.43
N%															1.00

TABLE A2.3

REGRESSION STATISTICS

VOLUME MODELS

	D.F.	S.S.
Total	1417	68564.6

Equation	Regression		Comparison with other equations		
	D.F.	S.S.	Equation	Variable added	Significance
VII .11	5	65772.1			
VIII. 8	6	65781.3	VII .11	D^2HSQ^2	NO
VIII. 9	6	65798.3	VII .11	D^2HTT	YES
VIII.10	6	65789.0	VII .11	$D^2HN\%$	YES
VIII.11	6	65824.4	VII .11	D^2HA	YES
VIII.12	6	65772.9	VII .11	D^2HR	NO
VIII.13	6	65778.5	VII .11	D^2HTTR	NO
VIII.14	6	65802.9	VII .11	A	YES
VIII.15	6	65772.6	VII .11	R	NO
VIII.16	6	65772.8	VII .11	B	NO
VIII.17	6	65784.9	VII .11	SQ	NO
VIII.18	6	65790.2	VII .11	TT	YES
VIII.19	6	65794.7	VII .11	N%	YES

TABLE A2.4

REGRESSION STATISTICS

VOLUME MODELS

	D.F.	S.S.
Total	1417	68564.6

Equation	Regression		Comparison with other equations		
	D.F.	S.S.	Equation	Variable added	Significance
VIII.20	7	65831.2	VIII.11	D^2HTT	NO
VIII.21	7	65829.9	VIII.11	$D^2HN\%$	NO
VIII.22	7	65827.8	VIII.11	A	NO
VIII.23	7	65836.9	VIII.11	TT	NO
VIII.24	7	65857.9	VIII.11	N%	YES

TABLE A2.5

CORRELATION COEFFICIENTS

BETWEEN

INDEPENDENT VARIABLES IN ACCEPTED VOLUME MODEL

	H	D^2	D^2H	D^2HB	D^2HSQ	D^2HA	N%
H	1.00	0.49	0.67	0.76	0.64	0.70	-0.09
D^2		1.00	0.96	0.86	0.86	0.87	0.11
D^2H			1.00	0.95	0.91	0.95	0.08
D^2HB				1.00	0.90	0.94	0.05
D^2HSQ					1.00	0.81	-0.03
D^2HA						1.00	0.08
N%							1.00

TABLE A2.6

CORRELATION COEFFICIENTS

BETWEEN

INDEPENDENT VARIABLES IN INCREMENT EQUATIONS

	xRSQ	xSQ	xSQ^2	xRA	xR/A	xA	x/A	xB	xRB	xR/B	x/B	xTT	xN%	$x\hat{V}$
xRSQ	1.00	0.97	0.91	0.68	0.75	0.65	0.73	0.77	0.82	0.68	0.67	0.88	0.73	0.60
xSQ		1.00	0.94	0.67	0.71	0.67	0.73	0.82	0.82	0.62	0.65	0.88	0.74	0.56
xSQ^2			1.00	0.45	0.56	0.46	0.60	0.67	0.66	0.43	0.46	0.68	0.54	0.48
xRA				1.00	0.35	0.98	0.34	0.88	0.92	0.48	0.48	0.81	0.71	0.82
xR/A					1.00	0.31	0.98	0.40	0.46	0.92	0.90	0.78	0.68	0.20
xA						1.00	0.33	0.91	0.90	0.43	0.46	0.80	0.71	0.78
x/A							1.00	0.44	0.46	0.88	0.91	0.80	0.70	0.17
xB								1.00	0.97	0.38	0.42	0.82	0.70	0.67
xRB									1.00	0.45	0.46	0.84	0.71	0.73
xR/B										1.00	0.99	0.80	0.71	0.31
x/B											1.00	0.83	0.74	0.28
xTT												1.00	0.86	0.56
xN%													1.00	0.53
$x\hat{V}$														1.00

TABLE A2.7

INCREMENT

MODELS EVALUATED

Equation	Independent variables														No. of variables
	xRSQ	xSQ	xSQ^2	xRA	xR/A	xA	x/A	xR/B	xRB	x/B	xB	xTT	xN%	$x\hat{V}$	
VIII.34		x		x				x				x	x	x	6
VIII.35			x	x				x				x	x	x	6
VIII.36	x				x			x				x	x	x	6
VIII.37	x					x		x				x	x	x	6
VIII.38	x						x	x				x	x	x	6
VIII.39	x			x					x			x	x	x	6
VIII.40	x			x						x		x	x	x	6
VIII.41	x			x							x	x	x	x	6

TABLE A2.8

REGRESSION STATISTICS EQUATION VIII.48

$$\text{Increment} = 4.9077\ 10^{-5}\ \text{xRSQ} - 1.4804\ 10^{-2}\ \text{xRA} + 71.578\ \frac{\text{xR}}{\text{B}} - 0.80638\ \text{xTT} + 7.9239\ 10^{-3}\ \text{xN\%} + 4.4680\ 10^{-2}\ \text{x}\hat{\text{V}}$$

Source of Variation	D.F.	S.S.	M.S.
Regression	6	4770.4	795.1
Residuals	725	919.4	1.268
Total:	731	5689.8	

$F_{6/725} = 627$ i.e. significant

APPENDIX 3

NOTATION

Definition	Abbreviation	Description
The number of years before thinning when the inventory plot was measured.	x	
The volume of the tree to a four inch (10 cm) top diameter, underbark,		Tree volume
in cubic feet.	V	
in cubic metres.	Vm	
The diameter at breast height are overbark,		Tree diameter
in inches.	D	
in centimetres.	Dm	
The ratio of the tree diameter to the quadratic mean diameter of the stand, (before thinning if measured at time of thinning).	R	Relative tree size
Age in years.	A	Age
The mean height of the tallest 30 trees per acre, with the restriction that the number of trees chosen from each quarter of the approximately one-fifth acre plot is as close as possible to the same,		Predominant height
in feet.	H	
in metres.	Hm	
The height of the tree,		Tree height
in feet.	Ht	
in metres.	Htm	
The total production volume to 4" top diameter, underbark, at age 30,		Site Quality
in cubic feet per acre.	SQ	
in cubic metres per hectare.	SQm	
The Predominant height of the stand at age 30,		Site index
in feet.	SI	
in metres.	SIm	
The standing basal area,		Basal area
in square feet per acre.	B	
in square metres per hectare.	Bm	
Number of trees,		Number of trees
per acre.	N	
per hectare.	Nm	
The ratio of the quadratic mean diameter of thinnings to be removed (or thinnings elect) to be quadratic mean diameter of the stand before thinning.	TT	Thinning type
The ratio of the quadratic mean diameter of thinnings or thinning elect to the quadratic mean diameter of the stand, (before thinning if measured at time of thinning), expressed as a percentage.	N%	Thinning intensity
	Log	Natural logarithm
Constants and coefficients used in models.	b_o b_1 b_2 b_3 b_4 .....b_n	Constants