FITTING DISTRIBUTIONS TO DIAMETER AND HEIGHT DATA FROM EVEN-AGED STANDS OF *PINUS RADIATA*

by

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Unless it is stated otherwise in the text to the contrary, the material contained in this thesis is a product of my own work.

CONTENTS

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Summary

The data used in this study were obtained from forty even-aged yield plots of *P. radiata* from the Kowen plantation in the Northeast corner of the Australian Capital Territory. The trees were planted between 1927 and 1944 and measurements of diameter and height were made on all plots on three or more occasions between 1955 and 1971 inclusive.

In this dissertation the six univariate distributions, normal, lognormal, gamma, beta, Weibull and Johnson's S_B, are fitted to the 149 measurement data by plot data sets. As a means of comparing the relative fits of these distributions, sums of the ranks of the maximized log-likelihood values, for the six distributions on 149 data sets, were obtained. The Kolmogorov-Smirnov goodness of fit statistic was also calculated for these distributions and the sums of the ranks of these obtained. There was very little agreement between these two methods of assessing the relative adequacy of the distributions of interest, for either the diameter or the height data. Hence it appeared that, for this data, there was little difference overall between any of the six distributions.

Some of the reasons for the difference between the results of fitting these distributions to the Kowen data, and earlier reported work on similar data, is discussed. The Kowen plots were thinned at various times between 1955 and 1971 and this appeared to have a lasting effect on the diameter and height distributions. The number of trees measured in these plots is relatively quite small, and this had a marked effect on the parameter estimation, particularly for the three parameter distributions.

Bivariate S_B (S_{BB} distribution) distributions and bivariate normal distributions were fitted. The S_{BB} seemed to be largely inappropriate for this data. In view of the confused situation regarding the best univariate distribution to use, it was decided, in the interests of simplicity, to choose the normal distribution to represent the univariate data. The bivariate normal could be used, with caution, to represent the diameter and height data simultaneously.

1. Introduction

Considerable work has been done on fitting distributions to diameter data in both even-age and mixed-age stands of trees. See for example Bailey and Dell (1973), Bailey (1974), and Schreuder and Swank (1974). There are several references to earlier work in these papers. Bailey and Dell, and Bailey discuss in detail the fitting of two and three parameter Weibull distributions to diameter data. Bailey states that diameter is generally well correlated with other variables of interest such as tree volume. He asserts that the quantification of the distribution of diameter, and its relationship to site, stand composition, age and density is often valuable for both economic and biological purposes.

Schreuder and Swank used the two parameter Weibull distribution to summarize diameter, basal area, surface area, biomass and crown profile data for several different ages of white and loblolly pine plantations. They found that the data for these variables were easily summarised by this one distribution in a theoretically consistent fashion. This was evidently not possible with the normal and gamma distributions, and the lognormal gave less satisfactory results. They then assert that the distribution function should prove useful in modelling tree stands since only the parameter values need to be changed over time for the above variables. The change in these parameter values, they say, may be a good way to characterize and interpret changes in stands over time.

More recently, Hafley and Schreuder (1977), have fitted six distributions to both diameter and height data for even-aged stands of shortleaf pine, longleaf pine and loblolly pine. The distributions they chose were the normal, lognormal, gamma, beta, Weibull and Johnson's S_B. They conclude that overall the Johnson's S_B distribution gave the best performance in terms of the log likelihood criterion.

In another paper Schreuder and Hafley (1977), extend their results from the univariate work to an examination in detail of the bivariate Johnson S_{BB} distribution in its application to diameter/height data. The rationale for endeavouring to fit bivariate distributions to diameter/height data from forest stands is explained quite succinctly in the Introduction to Schreuder and Swank's paper:

> "An important problem in forestry is the prediction of stand yields in volume on the basis of stand age, productive capacity of the site, and stand density. Forest managers are interested in studying the effect of such stand management practices as thinning and fertilization on volume in pulpwood, sawtimber, and veneer. The volume in each of these products is heavily dependent on tree diameter and height distributions. For example, a given stand volume consisting of small trees may be entirely pulpwood; whereas, the same volume in a few large trees may be primarily sawtimber. Hence, there is considerable interest in the successful fitting of a bivariate statistical distribution to describe diameter-height frequency data. Current

practice is to fit a marginal distribution to the diameter frequency data and then use an empirical height-diameter relationship to estimate average height per diameter class and thence volume. Although this latter approach is satisfactory in many ways, it ignores the fact that height can vary considerably for a given diameter. This is especially important in older, commercially more important stands. Hence, the possibility of generating a bivariate distribution of diameters and heights for a stand would often be of interest to managers".

2. The Data

The data were obtained from forty* even-aged yield plots of *P. radiata* from the Kowen plantation in the Northeast corner of the Australian Capital Territory (A.C.T.). The trees were planted between 1927 and 1944 and measurements of diameter and height were made on all plots in 1955/56, 1962 and 1971, with additional measurements made on some plots in 1958 and 1967. In Table 1 the plot code, the year of planting, the measurement dates and the number of trees are given.

Measurements (in cm) of diameter at breast height over bark (DBHOB) were made using a girth tape; breast height is 1.3m (4.25 feet) above ground level. Total height (in m) of the trees was measured by Haga Altimeter, with two readings being taken from different positions. In the case of leaning trees, the vertical component was taken as the tree height. The measurements of tree diameter are quite accurate. The height readings, however, have quite large measurement errors associated with them. There were 8056 pairs of measurements made on tree height and of these 253 (3.14%), the difference between the two values was greater than 1.5m. In 27 measurements (0.34%) this difference between the two heights was greater than 3m. As expected most of the more disparate readings occurred with plots containing older and taller trees. One obviously erroneous measurement of tree height was corrected in the data set.

* The last few sets of data were lost from plot 40 and all of the plot 41 data, during a transfer between computers.

Thinning was carried out progressively on the plots over time, however the first thinning was not carried out on a plot until the trees were 15 years of age. The thinning that was carried out in 1956 was quite light. This can be seen by looking at the tree numbers per plot given in Table 1. Usually the trees that were thinned, on each occasion, were those with the smallest diameter. In some instances thinning could remove up to 50% of the trees from a plot. The time when the plots were thinned relative to the time of measurement of the plots may have a profound effect on the nature of the distribution of the diameter and height values. If several years had elapsed between the time of the last thinning and the measurement of the diameter and height then the distributions of these variables might be expected to have stabilised again for that plot. Generally speaking the plots were thinned after the measurements were made so that thinning should not have a profound affect on the diameter and height distributions.

There is one occurrence in the data set where a plot was measured on two consecutive days, before and after thinning. The plot involved is 350601, which was measured on 13/5/71 when it had 68 trees and also on 14/5/71 when it had 19 trees. Histrograms for the distribution of diameter and height before and after thinning are given in Figures 1 to 4. The change is more noticeable for the diameter data (Figures 1 and 2), where there is a negatively skewed distribution before thinning and a positively skewed distribution after thinning. It is also apparent that it is not only the smallest diameter trees that have been thinned on this plot. However, in general it is predominantly the smallest diameter trees that are removed when thinning a plot.

In a number of sets of data there appeared to be an abnormally large number of height measurements where both readings were the same. This siutation is illustrated in Figure 5, which is a histogram of the proportion of trees on a plot for which both the height readings are the same. The plots and measurements dates involved are listed in Table 2. There are two possible explanations for this; either there was only one height measurement taken in the field, or there were two readings taken, but only the mean value recorded. In both instances the recorded height was coded twice on the data sheets prior to the entry of the data into the computer. Hence the height data for these data sets will have greater variability than that for the other data sets.

All subsequent data analyses have used the mean of the two tree height measurements as the observed value of tree height.

In order to obtain some insight into the distribution of the data, histograms were plotted for a few of the data sets. Figures 6 and 7 are histograms for the diameter and height values for the first plot, 31A501 planted in 1941 and measured 23/1/56. Histograms for another plot, 31A601 planted in 1941 and measured 22/1/58 are given for diameter and height variables in Figures 8 and 9 respectively. None of the Figures appear to represent a normally distributed set of data, although, save for some slight bimodality, the data in Figure 8 seem closest to normality. From these figures we can also see that we have to deal with data that can be either positively or negatively skewed.

To illustrate the relationship between the diameter and height measurements, a bivariate plot for the first set of data (31A501, measured 23/1/56) is given in Figure 10. Note that there appears to be a tree here with a rather low value for its diameter. This may reflect breakage which is common in trees with double leaders (forks of comparable size).

3. The β_1-β_2 Plane

Before proceeding to fit univariate distributions to the diameter (DBHOB) and height data, we should look in some detail at the sample moments for this data and endeavour to assess the suitability or otherwise of these distributions. The distributions that we will look at are the normal, lognormal, gamma, beta, Weibull and Johnston's S_B.

Let us define μ_r as the r^{th} sample moment about the mean of the distribution of a random variable x with density function f(x). That is

$$\mu_r = \int_{-\infty}^{\infty} (x-E(x))^r f(x)dx \ .$$

In addition to the mean μ_1 and the variance μ_2, we have two coefficients which are functions of lower order moments. The first is the skewness coefficient $\sqrt{\beta_1}$ which is defined as $\mu_3/(\mu_2)^{3/2}$, and the second is the kurtosis coefficient β_2 defined as μ_4/μ_2^2. Skewness measures the departure of the distribution from symmetry about the mean, with negative values of $\sqrt{\beta_1}$ indicating a distribution with a long tail to the left, and positive values indicating a distribution with a long tail to the right. On the other hand kurtosis is considered to be a measure of the peakedness of a distribution with larger values of β_2 indicating a more peaked distribution.

The moment estimators of $\sqrt{\beta_1}$ and β_2, for a sample of size n, are

$$\sqrt{b_1} = \frac{\sqrt{n}\ \Sigma_i (X_i - \overline{X})^3}{\{\Sigma_i (X_i - \overline{X})^2\}^{3/2}}$$

and

$$b_2 = \frac{n\Sigma_i(X_i - \bar{X})^4}{\{\Sigma_i(X_i - \bar{X})^2\}^2} .$$

The variances are $\frac{6n(n-1)}{(n-2)(n+1)(n+3)}$ and $\frac{24n(n-1)^2}{(n-2)(n-3)(n+3)(n+5)}$ for $\sqrt{b_1}$ and b_2 respectively, Fisher (1970).

Values of the sample moments and the values of $\sqrt{b_1}$ and b_2 with their standard errors, for the diameter and height data from Kowen are given in Tables 3 and 4 respectively. In these Tables the plot codes used in Table 1 do not appear. However the plots are in the same order, with the results for all measurement dates on a plot given consecutively. That is the first three lines in Tables 3 and 4 are the results for the three measurement dates on plot 31A501, and so forth. There are a couple of points to notice in these Tables. Firstly for the height data, almost all (84%) the values of $\sqrt{b_1}$ are negative, whereas for the diameter data, both negative and positive values occur almost equally often (44% to 56%). Secondly, it is fairly obvious that for the majority of the 149 data sets for both variables the value of the skewness coefficient would not be significantly different from zero.

In order to demonstrate the range of the skewness and kurtosis coefficients covered by the various statistical distributions, authors have often used a graph of the β_1-β_2 plane; for example Johnson and Kotz (1970a) and Hafley and Schreuder (1977). The sample values of b_1 and b_2 for the diameter data have been plotted in Figure 11. In this diagram I have indicated the possible values of β_1 and β_2 that the six distributions we are interested in can take.

There is an impossible region on this graph where it is mathematically impossible for certain combinations of β_1 and β_2 to occur. The normal distribution is represented by a point in this plane, whereas the lognormal, Weibull, and gamma distributions are all represented by lines, thus illustrating their greater flexibility over the normal in their ability to adequately represent data. The beta distribution is even more flexible since it is represented in the β_1 - β_2 plane by the area between the impossible region line and the gamma distribution line. The S_B distribution is one of three distributions proposed by Johnson (1949a). These distributions are transformations of a standard normal variate, and together the three of them span the β_1 - β_2 plane. The S_B distribution spans the area between the impossible region line and the lognormal line; the S_L distribution is the three parameter lognormal distribution; and the S_U distribution covers the remainder of the β_1 - β_2 plane. Therefore the S_B distribution is a little more flexible than the beta in the range of values of skewness and kurtosis that it can represent.

It is worth noting at this stage that the gamma and lognormal distributions can only adequately represent positively skewed data. From Figure 11 we are not able to discern which points represent negatively skewed data and which represent positively skewed data.

Figure 12 contains the corresponding results for the height data.

In both Figures 11 and 12 there are some data sets represented by points lying above the lognormal line. That is these points are from sets of data where the skewness is relatively small compared to the kurtosis. This situation does not appear to occur with the data of Hafley and Schreuder. Table 5 enumerates the plots and measurement dates where this has occurred for our data set and we can see that there are more points for the height data than for the diameter data. One possible explanation for this is that there may be a second population of small trees that are managing to survive below the main canopy. Data from these data sets may be more adequately represented by an S_U distribution, although this was not attempted since the number of data sets involved is relatively small. Furthermore many of the points, particularly for the height data, are actually quite close to the lognormal line.

4. Maximum Likelihood Estimation

To enable comparisons to be made between the fit of each of the distributions of interest to the data, we will fit each of them using maximum likelihood. We can compare values of the maximized likelihood function for these distributions and assess their relative fits. Since the calculation of the likelihood function itself is quite difficult, we will concentrate as is usual on the log of the likelihood, which has a maximum at the same parameter values as the likelihood.

(a) Normal distribution

The probability density function (p.d.f.) for a normal distribution with mean μ and variance σ^2 is

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\{(x-\mu)/\sigma\}^2}$$

$$-\infty < x,\ \mu < \infty,\ \sigma^2 > 0.$$

The likelihood function for a random sample of size n is

$$L = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{X_i-\mu}{\sigma}\right)^2}$$

Hence the log of the likelihood is

$$\ell = -\frac{n}{2} \ln 2\pi - n \ln\sigma - \frac{1}{2\sigma^2} \sum_{i=1}^{n} (X_i-\mu)^2$$

where $\ln$ is the natural logarithm.

Differentiating ℓ with respect to the parameters μ and σ^2, and equating the resultant two equations to zero, we can solve to obtain the well known maximum likelihood estimates for the normal distribution,

$$\hat{\mu} = \frac{\sum_{i=1}^{n} X_i}{n}$$

$$\hat{\sigma}^2 = \frac{1}{n} \sum_{i=1}^{n} (X_i - \hat{\mu})^2$$

(b) Lognormal distribution

For a three parameter lognormal distribution with location parameter θ we have the p.d.f.

$$f(x) = \frac{1}{(x-\theta)\sqrt{2\pi}\,\sigma} \, e^{-\frac{1}{2\sigma^2}\{\ell n(x-\theta)-\mu\}^2} ,$$

$$x > \theta, -\infty<\mu<\infty, \sigma^2>0 .$$

If θ were known (usually it is zero), we then have a two parameter lognormal distribution. That is, if $z_i = \ell n(x_i - \theta)$ then we obtain the density function

$$f(z) = \frac{1}{e^z\sqrt{2\pi}\,\sigma} \, e^{-\frac{1}{2\sigma^2}(z-\mu)^2}$$

Hence the likelihood for a random sample of size n is

$$L = \prod_{i=1}^{n} \frac{1}{e^{Z_i}\sqrt{2\pi}\,\sigma} \, e^{-\frac{1}{2\sigma^2}(Z_i-\mu)^2} ,$$

and we obtain the M.L.E.'s of the parameters of

$$\hat{\mu} = \frac{1}{n} \sum_{i=1}^{n} Z_i$$

$$\hat{\sigma}^2 = \frac{1}{n} \sum_{i=1}^{n} (Z_i - \hat{\mu})^2 .$$

If θ were unknown we have the likelihood

$$L = \prod_{i=1}^{n} \frac{1}{(X_i - \theta)\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2\sigma^2}[\ln(X_i - \theta) - \mu]^2}$$

From the log of this likelihood we can obtain the equations for estimating the parameters μ and σ^2 of

$$\mu = \hat{\mu}(\theta) = \frac{1}{n} \sum_{i=1}^{n} [\ln(X_i - \theta)]$$

$$\sigma^2 = \hat{\sigma}^2(\theta) = \frac{1}{n} \sum_{i=1}^{n} \{\ln(X_i - \theta) - \hat{\mu}(\theta)\}^2$$

Johnson and Kotz (1970a) suggest that these could be solved by taking a sequence of values of θ, calculate the maximized log-likelihood corresponding to each, then try to estimate numerically the value $\hat{\theta}$ of θ which maximizes the maximized log-likelihood. They refer to a paper by Hill (1963) who has shown that this procedure runs into trouble since the maximized log-likelihood tends to infinity as θ tends to min $(X_1, X_2, \ldots, X_n)$. The M.L.E. of θ should really be $<$ min$(X_1 X_2, \ldots, X_n)$. Hill shows that we should solve the above two equations subject to the constraint that

$$\sum_{i=1}^{n} \ell n[\frac{X_i-\theta}{G(\theta)}] \ (X_i-\theta)^{-1} = -\sum_{i=1}^{n} (X_i-\theta)^{-1} \ \hat{\sigma}^2 \ (\theta)$$

$$\text{where } G(\theta) = \ell n[\prod_{i=1}^{n} (X_i-\theta)]^{\frac{1}{n}} .$$

This reduces to

$$\frac{1}{\hat{\sigma}^2(\theta)} \sum_{i=1}^{n} (X_i-\theta)^{-1}[\ell n(X_i-\theta)-\mu(\theta)] + \sum_{i=1}^{n} (X_i-\theta)^{-1} = 0$$

$$\text{where } \mu(\theta) = \frac{1}{n}\sum_{i=1}^{n} \ell n(X_i-\theta) .$$

(c) <u>Gamma distribution</u>

The three parameter gamma distribution, with location parameter θ has p.d.f.

$$f(x) = (x-\theta)^{\alpha-1} \frac{e^{-(x-\theta)/\beta}}{\beta^{\alpha}\Gamma(\alpha)} , \qquad \alpha, \beta > 0, \quad x > \theta ,$$

and where $\Gamma(\alpha)$ is the gamma function.

When θ is 0 (the most usual case) we obtain

$$f(x) = \frac{x^{\alpha-1}e^{-x/\beta}}{\beta^{\alpha}\Gamma(\alpha)}$$

and this has likelihood

$$L = \prod_{i=1}^{n} \frac{X_i^{\alpha-1} e^{-X_i/\beta}}{\beta^{\alpha}\Gamma(\alpha)}$$

The log likelihood is then

$$\ell = -\frac{1}{\beta}\sum_{i=1}^{n} X_i - n\ \ell n\ \Gamma(n) + (\alpha-1)\sum_{i=1}^{n} \ell n X_i - \alpha n \ell n \beta \quad ,$$

and we can obtain the two equations (in the two unknown parameters)

$$\frac{1}{n}\sum_{i=1}^{n} \ell n X_i = \ell n\hat{\beta} + \psi(\hat{\alpha})$$

$$\hat{\alpha}\hat{\beta} = \frac{1}{n}\sum_{i=1}^{n} X_i$$

where $\psi(\cdot)$ is the digamma function, viz. $\psi(a)=\frac{\Gamma'(a)}{\Gamma(a)}$.

Eliminating $\hat{\beta}$ we obtain

$$\frac{1}{n}\sum_{i=1}^{n} \ell n X_i - \ell n\ \overline{X} = \psi(\hat{\alpha}) - \ell n\ \hat{\alpha} \quad ,$$

and since

$$\ell n\ \overline{X} - \frac{1}{n}\sum_{i-1}^{n} \ell n X_i = \ell n \overline{X} - \ell n \prod_{i=1}^{n}(X_i)^{\frac{1}{n}} \quad ,$$

we have

$$\ell n\ \left[\frac{\text{arithmetic mean } (X_1,\ldots,X_n)}{\text{geometric mean}(X_1,\ldots,X_n)}\right] = \ell n\hat{\alpha} - \psi(\hat{\alpha}) \quad .$$

Several approximations have been suggested for the value of $\hat{\alpha}$ (Johnson and Kotz, 1970a). One such approximation is due to Thom (1968) and is as follows:

$$\text{if } Y = \ln \bar{X} - \frac{1}{n} \sum_{i=1}^{n} \ln X_i \quad ,$$

then an approximation for $\hat{\alpha}$ is

$$\hat{\alpha} \doteq \frac{1}{4Y} \left(1 + \sqrt{1 + \frac{4}{3}Y}\right) \quad .$$

If θ is unknown then we would have to obtain the M.L.E.'s of the three unknown parameters from the following equations obtained from the log-likelihood.

$$\sum_{i=1}^{n} \ln(X_i - \hat{\theta}) - n \ln \hat{\beta} - n\,\psi(\hat{\alpha}) = 0$$

$$\sum_{i=1}^{n} (X_i - \hat{\theta}) - n\,\hat{\alpha}\hat{\beta} = 0$$

$$- \sum_{i=1}^{n} (X_i - \hat{\theta})^{-1} + n/\{\hat{\beta}(\hat{\alpha}-1)\} = 0$$

These can be solved iteratively.

(d) Beta distribution

When the range parameters a and b are unknown, we have the p.d.f. for a beta random variable of

$$f(x) = \frac{1}{B(p,q)} \frac{(x-a)^{p-1}(b-x)^{q-1}}{(b-a)^{p+q-1}}, \qquad \begin{array}{l} a \leqslant x \leqslant b \\ p,\ q > 0, \end{array}$$

where B(p,q) is the beta function.

For a random sample of size n the likelihood function is

$$L = \prod_{i=1}^{n} \frac{1}{B(p,q)} \frac{(X_i-a)^{p-1}(b-X_i)^{q-1}}{(b-a)^{p+q-1}}$$

and hence the log-likelihood is

$$\ell = -n\,\ell n[B(p,q)] + (p-1)\sum_{i=1}^{n} n(X_i-a)$$

$$+ (q-1)\sum_{i=1}^{n} \ell n(b-X_i) - n(p+q-1)\,\ell n(b-a) \quad .$$

If the values of the range parameters a, and b are known then we obtain the likelihood equations for the parameters p and q:

$$\psi(\hat{p}) - \psi(\hat{p}+\hat{q}) = \frac{1}{n}\sum_{i=1}^{n} \ell n\left(\frac{X_i-a}{b-a}\right)$$

$$\psi(\hat{q}) - \psi(\hat{p}+\hat{q}) = \frac{1}{n}\sum_{i=1}^{n} \ell n\left(\frac{b-X_i}{b-a}\right)$$

where ψ is the digamma function.

Johnson and Kotz (1970b) state that we can solve these equations iteratively for $\hat{p}$ and $\hat{q}$. They further state, that if a and b are unknown, then the M.L.E.'s of all four parameters can be obtained by using a succession of trial values of a and b, until a pair (a,b) for which the maximized log-likelihood is as great as possible, is attained.

(e) Weibull distribution

The p.d.f. for a three parameter Weibull distribution is

$$f(x) = \frac{c(x-\theta)^{c-1}}{b^c} e^{-[(x-\theta)/b]^c}$$

where θ is the location parameter, $x \geqslant \theta \geqslant 0$,

b is the scale parameter, $b > 0$,

and c is the shape parameter, $c > 0$.

The likelihood for a random sample from the two parameter Weibull distribution with $\theta=0$ is

$$L = \prod_{i=1}^{n} \frac{cX_i^{\,c-1}}{b^c} e^{-[X_i/b]^c}$$

Bailey and Dell (1973) say that the M.L.E.'s of the parameters can be obtained by iteratively solving

$$[\sum_{i=1}^{n} X_i^{\hat{c}} \ln X_i]/[\sum_{i=1}^{n} X_i^{\hat{c}}] - \frac{1}{\hat{c}} = \frac{1}{n} \sum_{i=1}^{n} \ln X_i$$

for $\hat{c}$ and then solving

$$\hat{b} = [\frac{1}{n} \sum_{i=1}^{n} X_i^{\hat{c}}]^{\frac{1}{\hat{c}}} \qquad \text{for } \hat{b} .$$

Details of the estimating equations for the three parameter case are given in Harter and Moore (1965). They outline an iterative technique for solving these equations.

The log-likelihood for the three parameter model is

$$\ell = n \ln c - nc \ln b + \sum_{i=1}^{n} (k-1)\ln(X_i-\theta) - \sum_{i=1}^{n} \left(\frac{X_i-\theta}{b}\right)^c .$$

(f) Johnson's S_B distribution

Johnson (1949 a) proposed three distributions, which together span the β_1-β_2 plane. As mentioned earlier the one we are interested in is the S_B distribution.

Let $$z = \gamma + \delta \ln \left(\frac{x-\xi}{\xi-\lambda-x}\right)$$

where z is a unit normal variable and $\xi < x < \xi + \lambda$. Using the change of variable technique we can obtain the p.d.f. for the variable x.

That is $$f(x) = \frac{1}{\sqrt{2\pi}} \frac{\delta\lambda}{(x-\xi)(\xi+\lambda-x)} e^{-\frac{1}{2}[\gamma+\delta\ln(\frac{x-\xi}{\xi+\lambda-x})]^2}$$

where the variable x is said to have and S_B distribution. When the range parameters ξ and λ are both known, then Johnson has shown that closed form estimates of γ and δ can be obtained.

$$\text{viz.} \quad \hat{\gamma} = -\frac{\bar{y}}{s} , \quad \hat{\delta} = \frac{1}{s} \quad \text{where } \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i ,$$

$$s^2 = \frac{1}{n}\sum_{i=1}^{n} (y_i-\bar{y})^2 \quad \text{and}$$

$$y_i = \ln \frac{x_i-\xi}{\xi+\lambda-x_i} .$$

The likelihood function for a random sample of size n from an S_B distribution is

$$L = \prod_{i=1}^{n} \frac{\delta\lambda}{\sqrt{2\pi}(X_i-\xi)(\xi+\lambda-X_i)} e^{-\frac{1}{2}[\gamma+\delta \ \ln(\frac{X_i-\xi}{\xi+\lambda-X_i})]^2}$$

The log-likelihood is

$$\ell = n \ \ln \delta\lambda - \frac{n}{2} \ln 2\pi - \sum_{i=1}^{n} \ln(X_i-\xi)$$

$$- \sum_{i=1}^{n} \ln(\xi+\lambda-X_i) - \frac{1}{2} \sum_{i=1}^{n} [\gamma+\delta \ \ln(\frac{X_i-\xi}{\xi+\lambda-X_i})]^2$$

(g) General method of estimation

For each of the distributions above, where closed form maximum likelihood estimates of the parameters do not exist, a specific iterative method can be used to obtain these estimates. However, rather than write a program for each of the iterative methods outlined above, it was decided to reduce the amount of programming required and utilize a general minimization routine. The minimization technique used is the simplex method of Nelder and Mead (1965). This has been programmed by Shaw (1971, unpublished) in a Fortran subroutine called MINIM. The maximum likelihood estimates are the parameter estimates which minimize "minus the log likelihood function". Simplex is not the fastest minimization technique available (and it would be unlikely to be as fast as the specific iterative techniques mentioned earlier, but it does have the advantage that partial derivatives of the function being minimized, with respect to the parameters, do not have to be provided.

If requested, MINIM will fit a quadratic surface in the region of the minimum. The coefficients of this polynomial form the Hessian matrix of the second derivatives of the function being minimized. It is well known that if the function being minimized is minus a log likelihood function, then the Hessian matrix is the information matrix and its inverse is the variance-covariance matrix. Hence approximate standard errors (S.E.'s) for the parameter estimates can be obtained from MINIM, if desired.

5. Kolmogorov-Smirnov Test for Goodness of Fit

In order to assess the adequacy of fit of the six distributions to the diameter and height data, we also used a Kolmogorov-Smirnov test, Massey (1951) in addition to the value of the maximized likelihood.

Suppose we have a population which is thought to have some specific cumulative frequency distribution function $F_0(x)$. That is, the value of $F_0(x)$ is the proportion of individuals in the population having values less than or equal to a particular value x. For a random sample of N observations we would expect its cumulative step function to be quite close to the specified distribution function $F_0(x)$.

Now $F_0(x)$ is the population cumulative distribution function, and if $S_N(x)$ is the observed cumulative step function of a sample ($S_N(x) = \frac{n}{N}$, where n is the number of observations less than or equal to x), then the sampling distribution of $d = \text{maximum}|F_0(x) - S_N(x)|$ is known, and is independent of $F_0(x)$, if $F_0(x)$ is continuous.

Massey tabulated critical points for the distribution of d for various sample sizes.

The population cumulative distribution functions for the six distributions of interest were computed using the following methods.

(i) Normal We have $\Phi(u) = P_r(U \leqslant u)$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{u} e^{-\frac{1}{2}t^2} dt \quad ,$$

where U has a standard normal distribution. For a normal distribution X with mean μ and variance σ^2, we have $P_r(X \leqslant x) = P_r(U \leqslant \frac{X-\mu}{\sigma})$. An approximation due to Hastings is

$$\Phi(u) \doteqdot 1 - \tfrac{1}{2}(1+a_1u + a_2u^2 + a_3u^3+a_4u^4)^{-4}$$

where a_1=.196854, a_2=.115194, a_3=.000344, a_4=.019527 and $u \geqslant 0$.

(ii) <u>Lognormal</u> We can obtain the cumulative distribution from that for the normal, using the transformation $U=(\log x - \mu)/\sigma$.

(iii) <u>Gamma</u> Johnson and Kotz (1970a) give the following approximation, due to Wilson and Hilferty (1931), for the χ^2 distribution;

$$P_r(\chi^2_\gamma < x) \doteqdot \Phi[\{(x/\gamma)^{\frac{1}{3}} - 1 - \tfrac{2}{9}\gamma^{-1}\}\sqrt{9\gamma/2}\,] \quad .$$

We can obtain the result for the general gamma distribution, using the transformation $y = \frac{x-\theta}{\beta}$. That is $x = \theta + \beta y$. Note that $\gamma=2\alpha$ and that $\theta=0$ for the two parameter gamma distribution. The approximation is very good if $\gamma>30$ (ie $\alpha>15$), Abramowitz and Stegun (1965). Only 6 of the 298 estimated values of α are less than 15, and of these the smallest two are just under 10. Hence it seemed quite reasonable for us to use the approximation.

(iv) Beta Johnson and Kotz (1970b) suggested an approximation to the incomplete beta function ratio. However this approximation breaks down completely if either or both of p and q are less than 0.5. This does occur in a few data sets and so it was necessary to use a more exact procedure, I used the IMSL routine MDBETA which is a Fortran program based on the Collected Algorithms from A.C.M., algorithm 179, Ludwig (1963).

(v) Weibull The cumulative distribution function is $F_x(x)=1-\exp[-\{(x-\theta)/\alpha\}^c]$, Johnson and Kotz (1970a).

(vi) Johnson's S_B We can obtain the cumulative distribution function from the normal using the transformation $U=\gamma+\delta\log\left(\frac{x-\xi}{\xi+\lambda-x}\right)$, where $\xi=0$ for the diameter data and 1.3 for the height.

6. Results of Maximum Likelihood Estimation and the Test for Goodness of Fit

The minimum value for DBHOB was set at 0 cm for all data sets. Further the minimum value for height was set at 1.3 m; that is the height at which the diameter measurement was made. These seem reasonable lower bounds as we are trying to fit distributions to a number of data sets on different aged trees. Hence we are estimating two parameters for normal, lognormal, gamma and Weibull, and three parameters for the beta and Johnson's S_B distributions. Therefore estimates of the parameters of the normal and the lognormal and the values of their log-likelihoods could be obtained directly, and for the gamma the Thom approximation could be used. For the other three distributions the parameter estimates and the values of the maximized log-likelihood were obtained using MINIM.

Most of the computations carried out in this section were performed on a PDP 11/34 computer.

(a) The Diameter Data

Initially the second range parameter (upper limit for the range of the data values), for the beta and S_B distributions was set to be maximum (observed diameter on the plot) + 5 cm. The values of the maximized log-likelihood for all six distributions and all 149 sets of data were calculated. For the S_B distribution we obtained the M.L. estimates directly (see earlier section). In order to obtain insight into the overall performance of each of the distributions the following procedure was used. For each data set the six log-likelihood values were ranked (largest to smallest) and the sum of these ranks calculated for each distribution. The values obtained were: normal, 434; lognormal, 587; gamma, 472; beta, 429; Weibull, 644; S_B, 563. Hence we conclude that overall the beta distribution was the best performer, the normal the next best and so on.

The Kolmogorov-Smirnov (K-S) statistic d was calculated for all distributions and all plots. Values of d were ranked (smallest to largest) and the sums of these ranks obtained for each distribution. The values were: normal, 406; lognomal, 457; gamma, 413; beta, 556; Weibull, 624; S_B, 673. From these summed ranks we could conclude that the best performer was the normal distribution, the gamma the next best, and so on.

If we now estimate the second range parameter, for the beta and S_B distributions, we encounter difficulties with several data sets. Not suprisingly the problems occur where there is positive skewness in the data. The log-likelihood surface has a long ridge where it is close to its maximum value for a wide range of values of the range parameter. In order to overcome this problem the values of the range parameter were constrained to lie between the maximum observed diameter + 0.0001 cm, and an upper limit derived from the relationship,

$$\text{maximum diameter} = 10.0 + 62.353(1-\exp(-.098(\text{age}-7.546))), \quad (6.1$$

Since there was no relationship that we could find in the literature, relating maximum tree diameter on a plot to tree age for *p. radiata*, it was necessary to obtain a suitable relationship from the data we had available. This was done as follows. In Figure 13, the maximum observed tree diameter for each of the 149 data sets was plotted against tree age. Since we were interested in setting an upper bound for diameter on plots of a certain age, then eight points, arrowed in this figure, were chosen and a Mischerlich curve was fitted. This seemed to be a reasonable model for this data with no apparent systematic trend in the residuals from the fitted model. In order to be satisfied that we were setting a reasonable upper bound for the iteration procedure to work within, an addition of 10 cm was made to the value of the maximum diameter obtained from the Mischerlich model. This upper bound is plotted on Figure 13.

For the beta distribution, the estimated value of the second range parameter, b, was almost identical to the value obtained for maximum diameter from the above relationship (6.1) in 74 of the 149 data sets. In all but two of these 74 data sets, the data had positive skewness. For the two with negative skewness, (data sets 87 and 88), the estimated value of skewness was just less than zero. Altogether there were 83 sets with positive skewness, but for some of these the value of the skewness parameter was barely greater than zero, (see Table 3).

The iterative procedure in MINIM failed to converge satisfactorily for 23 of the sets of data. All of these were data sets where the estimated second range parameter from the iterative procedure, was the maximum diameter for trees of that particular age on that plot, (obtained from the equation 6.1 given above). Recall that the plots consist of equal aged trees. It would seem that for these 23 data sets, there is a long ridge in the likelihood surface, where a large change in the magnitude of the second range parameter will have very little effect on the value of the log-likelihood. This is quite a common problem encountered when fitting nonlinear models to data. Also it is quite likely, that for some of the other data sets where the second range parameter estimate was the same as the maximum plot diameter, that there is a ridge in the log-likelihood surface. However the iterative procedure in MINIM did converge in our situation where we were constraining the value of the second range parameter. Fortunately for the diameter data and the beta distribution there are not to many instances where MINIM failed to converge satisfactorily.

Another serious problem is one that arises when the quadratic surface near the minimum (maximum of the log-likelihood function), does not approximate the actual surface. When this occurs we obtain estimated standard errors for the parameter estimates that are unreasonably large. This was quite a serious problem for the beta distribution, since in 40 of the data sets the estimated standard error for one or more of the parameters was considered unreasonably large (i.e. greater than 10.0). In many of these 40 data sets all three parameters had large standard errors associated with them.

Now let us look at the results for the S_B distribution. There were 71 data sets where the estimated value of the second range parameter, λ, was almost identical to the value obtained from the equation for maximum diameter. In all but three of these 71 data sets the data had positive skewness. Once again for those with negative skewness (data sets 87, 88 and 120), the actual value of the estimated skewness was just less than zero. MINIM failed to converge satisfactorily for 31 of the sets of data, all but three of which were from plots where the estimated second range parameter was the maximum diameter for trees of that particular age on that plot. For the S_B distribution there were 26 data sets where the estimated errors, for one or more of the parameters, were considered to be rather large (i.e. greater than 10.0). The second range parameter accounted for 25 of these 26 instances.

The values of the maximized log-likelihood for the six distributions we are comparing are given in Table 6. The log-likelihood values for the beta and S_B distributions are those for the three parameter distributions we have just been discussing. These log-likelihood values were ranked for all 149 sets of data and the sums of these ranks found for the six distributions. The values obtained are: normal, 555; lognormal, 627; gamma, 517; beta, 379; Weibull, 691; S_B, 360.

We also calculated the value of the K-S statistic d for the three parameter beta and S_B distributions. Values of d, for all six distributions are given in Table 7, together with the critical value for the 5% significance level. The critical values of d were obtained from Table 1 of Massey (1951). For five data sets the calculated value of d exceeded the tabulated value for at least one the the six distributions. The beta and S_B were not in this category.

The six K-S values of d were ranked for all 149 data sets, and the sums of these ranks for the six distributions calculated. The values obtained were: normal, 499; lognormal, 471; gamma, 428; beta, 523; Weibull, 701; S_B, 507.

(b) The Height Data

The value of the second range parameter for fitting the beta S_B distribution to the height data was initially set to the maximum(height) + 5 m. Values of the maximized log-likelihood were obtained for the six distributions of interest for the 149 sets of data. The six log-likelihood values for the distributions have been ranked for all data sets and the sums of these rankings calculated, The values obtained were: normal, 534; lognormal, 769; gamma, 644; beta, 385; Weibull, 429; S_B, 368. We can see from these rankings that overall the best performer appears to be the S_B distribution, closely followed by the beta. As expected the lognormal and gamma distributions, which are unable to adequately fit negatively skewed data, were the worst overall performers.

As before, for the diameter data, we also calculated the values of the K-S statistic d, ranked them for each data set, and calculated the sum of these ranks. The values obtained were: normal 451; lognormal, 578; gamma, 510; beta, 459; Weibull, 633; S_B, 498. Hence the best performer appeared to be the normal distribution, closely followed by the beta. The worst performers were the Weibull and lognormal distributions.

If we now estimate the second range parameter, for the beta and S_B distributions, we encounter a similar difficulty to that mentioned earlier for the diameter data. For the height data we constrained our second range parameter to lie between the maximum observed height + 0.0001 m and an upper limit derived from the relationship maximum height = 4.57 x age $^{0.7}$, which is due to Allison *et. al.* (1979). This equation enables the maximum height attainable for a tree in a stand of given age, to be calculated.

The above relationship was obtained from data from New Zealand plantations of *P. radiata*. It is well known that the relationship of height to age is dependent on site quality. Furthermore it would be very unlikely that the Kowen site in the A.C.T. was of the same quality as the site in New Zealand, from which the above relationship was derived. Hence it may not hold for Australian data, or for the A.C.T. data in particular. However that does not really present any problems for us here, since we are only seeking a reasonable upper bound for the second range parameter to use in our iteration procedure with MINIM. In Figure 14 the New Zealand relationship is plotted on the same graph as the maximum tree height observed on the plots. From this figure we can see that by using this relationship that we might be overestimating the maximum attainable height for trees in the Kowen plots.

In fitting the beta distribution the estimated value of the second range parameter, b, was identical to that obtained from the above relationship in 12 of the 149 data sets. (For a further two data sets it was very close.) These 12 data sets all had positive skewness, although altogether there were 24 data sets with positive skewness (some barely so). For 21 data sets the iterative procedure in MINIM failed to converge satisfactorily. Of these, 8 were data sets in common with those whose estimate of the second range parameter was the maximum for trees of that particular age on that plot. It would seem that, for these 8 data sets, there is a long ridge in the log-likelihood surface where a large change in the magnitude of the second range parameter will have very little effect on the value of the log-likelihood. Fortunately for the height data and the beta distribution there are not really very many plots that cause this problem.

As mentioned earlier another problem is the one that arises when the quadratic surface near the minimum (maximum of the log-likelihood function), does not approximate the actual surface, and thence we obtain estimated standard errors for the parameters that are unreasonably large. This was quite a serious problem for the beta distribution, since for 42 data sets the estimated standard error for one or more of the parameters was unreasonably large, (i.e. greater than 10.0). In many of these data sets all three parameters had large standard errors associated with them.

The situation for the S_B distribution was a little different. There were 19 data sets where the second range parameter, λ, was set almost identical to the value obtained from the New Zealand relationship, (one other value was quite close). Once again all 19 data sets in question had positive skewness. For 8 data sets MINIM failed to converge satisfactorily. All of these 8 were in common with the 19 above. There were only 15 plots where the estimated standard errors for the parameters of the S_B

distribution were considered to be unreasonably large (i.e. greater than 10.0), markedly fewer than for the beta distribution. For the S_B distribution the second range parameter accounted for 13 of these 15 instances.

The values of the maximized log-likelihood for the six distributions we are interested in are given in Table 8. The log-likelihood values for the beta and S_B distributions are those for the three parameter distributions we have been discussing above. As before we ranked these log-likelihood values for all 149 sets of data and found the sum of these ranks for the six distributions. The values obtained are: normal, 567; lognormal, 780; gamma, 658; beta, 287; Weibull, 543; S_B, 294.

We also calculated the value of the K-S statistic d for the three parameter beta and S_B distributions. Values of d for all six distributions are given in Table 9, together with the critical value for the 5% significance level. As mentioned before the critical values of d were obtained from Table 1 of Massey (1951). For 10 data sets the calculated value of d exceeded the tabulated value for at least one of the six distributions. All distributions, except the S_B, were in this category.

The six K-S values of d were ranked for all 149 data sets, and the sums of these ranks for the six distributions calculated. The values obtained were: normal, 436; lognormal, 541; gamma, 475; beta, 580; Weibull, 612; S_B, 485.

We can see (Table 10) that, for both height and diameter data, there is not a great deal of agreement between the sums of the rankings obtained for the log-likelihood values and those obtained for the values of the K-S statistic d.

Table 10 Comparison of Rankings

(3 parameter beta and S_B distributions)

		Normal	Lognormal	Gamma	Beta	Weibull	S_B
DIAMETER	Log-Likelihood	555^{4}	627^{5}	517^{3}	379^{2}	691^{6}	360^{1}
	K-S d statistic	499^{3}	471^{2}	428^{1}	523^{5}	701^{6}	507^{4}
HEIGHT	Log-Likelihood	567^{4}	780^{6}	658^{5}	287^{1}	543^{3}	294^{2}
	K-S d statistic	436^{1}	541^{4}	475^{2}	580^{5}	612^{6}	485^{3}

This is rather disappointing since we are not now in a position to be confident about any particular distribution being better than any other, as a suitable overall distribution to represent diameter and height data.

There are several reasons for these conflicting results. For each data set, if we look closely at the relative magnitude of the six figures in the tables of the maximized log-likelihood and K-S statistic d, we can see that in many instances there is very little difference amongst them. This is the case for both the diameter height data.

Relatively small changes in the values of the log-likelihood, or in the values of d, for one or more distributions, could lead to quite a marked change in the ranks, the sums of these ranks, and thus our interpretation of what is the best distribution.

The number of trees measured in all data sets is really quite low. In less than 10% of the data sets we have more than 100 trees measured for diameter and height. Furthermore 27 data sets (i.e. 18.1%) had measurements made on fewer than 30 trees. Hafley and Schreuder (1977) had more than 100 trees in all their data sets, and for four of their 21 sets of data more than 500 trees were measured. Since the number of trees measured in our data sets is lower than that in Hafley and Schreuder, then the distributions of the diameter and height measurements we have are likely to be less well defined.

For a data set where values of maximized log-likelihood are nearly the same for all six distributions, we could say that all six fit the data equally well. However it might be more correct to say (in the light of a relatively small number of observations), that all six distributions fit almost equally poorly.

7. Estimation of bivariate distributions

In seeking to fit bivariate distributions to diameter and height data we must bear some points in mind. Firstly we would like to choose a bivariate distribution such that its marginal distributions were univariate distributions of the same distributional form. Secondly it would be desirable that the distribution be relatively easy to fit in terms of parameter estimation, and thirdly, we would like the distribution to be reasonably easy to work with.

We will look briefly at two distributions that meet these criteria.

(a) The bivariate normal distribution

The probability density function for a bivariate normal distributuion in two variables X_1, X_2 is

$$f(x_1 x_2) = \frac{1}{2\pi\sqrt{1-\rho^2}} \exp -\left[\frac{1}{2(1-\rho^2)}\left\{\left(\frac{x_1-\mu_1}{\sigma_1}\right)^2 - 2\frac{(x_1-\mu_1)(x_2-\mu_2)}{\sigma_1\sigma_2} + \left(\frac{x_2-\mu_2}{\sigma_2}\right)^2\right\}\right]$$

where $E(X_i) = \mu_i$, $Var(X_i) = \sigma_i^2$, $(i=1,2)$, and the correlation between X_1 and X_2 is ρ.

It is well known that the maximum likelihood estimators for the five parameters are :

$$\hat{\mu}_i = \sum_{j=1}^{n} X_{ij} , \quad \sigma_i^2 = \frac{1}{n} \sum_{j=1}^{n} (X_{ij}-\bar{X}_i)^2 \quad \text{with } i=1,2,$$

and

$$\hat{\rho} = \frac{\sum_{j=1}^{n} (X_{ij}-\bar{X}_1)\ (X_{2j}-\bar{X}_2)}{\{\sum_{j=1}^{n} (X_{ij}-\bar{X}_1)^2 \sum_{j=1}^{n} (X_{2j}-\bar{X}_2)^2\}^{\frac{1}{2}}}$$

(b) <u>The bivariate S_B distribution, the S_{BB} distribution.</u>

The S_{BB} distribution, Johnson (1949b), has probability density function

$$f(z_1,z_2) = \frac{1}{2\pi\sqrt{1-\rho^2}} \exp -[\frac{1}{2(1-\rho^2)}\{z_1^2-{}^2\rho z_1 z_2 + z_1^2\}]$$

where z_1 and z_2 are standard normal variates defined by

$$z_1 = \gamma_1 + \delta_1 \ \ell n \ (\frac{D-\xi_1}{\xi_1+\lambda_1-D})$$

and

$$z_2 = \gamma_2 + \delta_2 \ \ell n \ (\frac{H-\xi_2}{\xi_2+\lambda-H})$$

with D=diameter and H=height.

In our case ξ_1 and ξ_2 are the smallest values of diameter and height, and λ_1 and λ_2 are the range of values for diameter and height respectively. For the S_{BB} distribution, both marginals are S_B distributions.

The method for fitting the S_{BB} distribution is quite straightforward, Johnson 1949b. Briefly, we obtain estimates of the parameters for the marginal S_B distributions, transform the observation pairs to standard normal variates and then calculate the estimate of the correlation between these two transformed variates.

Schreuder and Hafley (1977) discuss the fitting of the $_{BB}$ distribution to forestry diameter and height data. One of the properties of interest for the S_{BB} distribution, they point out, is the regression relation between height and diameter. Since the usual mean regression is complicated, they suggest using the median regression which takes the relatively simple form

$$y_2 = \theta y_1^{\phi}\{(1-y_1)^{\phi} + \theta y_1^{\phi}\}^{-1}$$

where

$$y_2 = (H-\xi_2)/\lambda_2 \quad ,$$

$$y_1 = (D-\xi_1)/\lambda_1 \quad ,$$

$$\theta = \exp\left(\frac{\rho\gamma_1 - \gamma_2}{\delta_2}\right) \quad ,$$

and $$\phi = \rho\frac{\delta_1}{\delta_2}$$

Since we are regressing tree height on tree diameter, it is reasonable to assume that ρ is positive. Hence ϕ will be positive. As Schreuder and Hafley point out there is a problem when $0<\phi<1$, since the first derivative of y_2 with respect to y_1 does not exist at the extremes, $y_1=0$ and $y_1=1$. They conclude that under these circumstances the median regression relationship may not be of practical use.

8. Results for the S_{BB} distribution

Estimated parameter values for the three parameter S_B distribution, for both diameter and height data, are given in Table 11. After transforming the diameter and height observations to standard normal variates the correlation coefficient, ρ, between them was calculated. We now have estimates for all seven parameters of the bivariate S_{BB} distribution. Values of ρ, together with the derived parameters ϕ and θ, are also given in Table 11.

There is one data set where the correlation parameter between transformed diameter and height variates, is negative (data set 44). This is quite unexpected at first sight. However, this data set is quite small, containing 16 trees, and thus would have been heavily thinned by the time these measurements were made in 1967.

We can also see, from Table 11, that in only 19 data sets is the estimated value of ϕ greater than one. In Figure 15, the diameter and height data from data set 14 (where $\phi>1$), transformed to a 0 to 1 scale, (using parameters of the marginal S_B distributions) is given, together with the fitted S_{BB} median regression curve. Data from another data set (number 24), where $\phi<1$, has been transformed in a similar manner and the transformed observations, and the fitted S_{BB} median regression curve, given in Figure 16.

The difference between the slopes of the two S_{BB} median regression curves is immediately apparent. It is fairly evident that the shape of the regression curve to be preferred is that in Figure 15, which corresponds to $\phi > 1$. Schreuder and Hafley (1977) found that the estimated value of ϕ was between 0 and 1 in six out of their 21 data sets. As this was less than one third of their data sets, they decided to overcome the problem by increasing the value of λ_1, by diameter range class increments, until the value of ϕ was greater than one. This seems to be a rather artificial way of forcing ϕ to be greater than one, but if it were only applied to relatively few plots then would not cause too much concern. Since for most of our data the estimated value of ϕ is between 0 and 1 (130 out of 149), we decided that it was inappropriate to use the method of adjusting λ_1 to make $\phi>1$. Hence the S_{BB} distribution does not appear to be a very appropriate bivariate distribution to use with the Kowen data.

9. Discussion

The results that we have obtained are quite different from the more clearcut results obtained for univariate distributions, by Hafley and Schreuder (1977), and for the S_{BB} distribution, by Schreuder and Hafley (1977). It is interesting to ponder some likely reasons for these differences.

As mentioned earlier there are relatively few observations in the data sets from the Kowen plantation, compared to the number of observations in the data sets of Hafley and Schreuder. With small numbers of observations we are not able to obtain particularly good fits of distributions to the data, particularly in the tail areas of the distributions. Not only will the fits be poor, but there will often be difficulty in estimating parameters which are related to the tails. This was certainly the case when we were estimating the second range parameter for the three parameter beta and S_B distribution, particularly for positively skewed data.

Since we have encountered problems in fitting univariate distributions, that could be attributed to the low numbers of observations, then we would expect to encounter more serious problems when fitting bivariate distributions.

When discussing the data earlier on, mention was made of the thinning that was carried out on all plots. At that time it was thought that thinning might not have a too profound a long term effect. This does not appear to be the case. Hafley and Schreuder make no mention of thinning in their paper, so we must assume that none was carried out. The thinning on the Kowen plots seems to be having a long term effect on all moments of the tree diameter and height distributions. In particular let us briefly look at the values of the skewness squared and kurtosis parameters that we obtained compared to the values obtained by Hafley and Schreuder. For the diameter data we have 4 data sets where $b_1>2$ and 3 data sets where $b_2>6$, whereas Hafley and Schreuder have 5 data sets where $b_1>2$ and 3 where $b_2>6$. Also for the height data we have 2 data sets where $b_1>1$ and 2 where $b_2>4$ and likewise for Hafley and Schreuder's data. They have 21 data sets and we have 149 and hence, proportionately, we have far fewer data sets with relatively high skewness or kurtosis values. They were working with three different species of pines, pure stands of either shortleaf pine, longleaf pine or loblolly pine, but this should not substantially change the way the trees react to competition.

Since the results obtained from fitting univariate distributions was quite confusing, with no distribution being consistently better than the others, it is not surprising that the data are not very well represented by bivariate S_{BB} distributions.

In the light of our efforts in fitting univariate distributions to tree diameter and height data, it would be rather difficult to suggest, with any degree of precision, a desirable number of measurements to be made in any future work in this area. Quite clearly, the thinning that was carried out on the plots in the Kowen plantation, has permanently affected the distributions of both tree height and diameter. Under these circumstances (that is with thinning), it would be very difficult to suggest a suitable minimum number of trees for measurement. However, if no thinning were carried out, then we might expect reasonable results with samples in excess of 50 trees, although results would be much better if more than 100 trees could be measured.

If we were to fit bivariate distributions of diameter and height, to data from plots with no thinning, then a minimum of 100 trees should be measured. Schreuder and Hafley (1977) are also concerned about sample size when fitting bivariate distributions. Their sample sizes ranged from 105 to 728 for the 21 data sets that they looked at. They attribute the large differences they get between the observed and fitted frequencies, in the tails of the bivariate S_{BB} distribution, to the sample sizes they have, stating:

> "... the sample sizes, although relatively large, are still small for adequately fitting tail probabilities of bivariate distributions".

Since one of the principal aims of statistics is to present data in a succinct form, then we should choose the simplest distribution which provides an adequate fit to the data. The normal distribution is the obvious choice. Parameter estimates obtained from fitting the normal distribution to the diameter and height is given in Table 12. As mentioned earlier the fitting of bivariate distributions to data sets the size of the ones we have from Kowen is not really very satisfactory. The fitting of the bivariate normal, or any other bivariate distribution, to this data would not really be recommended. We do, however, give the correlation coefficient between diameter and height, in Table 12, so that all parameters of the bivariate normal appear in this table.

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Table 1

Plot No.	Year Planted	Measurement Dates and No. of Trees									
31A501	1941	23.1.56	95			21.3.62	85			23. 8.71	57
31A601	1941	28.1.55	101	22.1.58	79	13.3.62	76			23. 8.71	56
32A601	1941	19.1.56	97	21.1.58	75	21.2.62	57			25. 8.71	36
33A601	1941	2.2.55	115	22.1.58	79	28.3.62	69			6.11.71	43
270701	1940	25.1.56	98			15.2.62	56			27. 7.71	26
270801	1939	28.1.55	98	21.1.58	71	15.2.62	59			23. 7.71	33
28A601	1940	26.1.55	77	20.1.58	61	16.2.62	46			28. 7.71	35
290701	1940	28.1.55	88	21.1.58	66	21.3.62	65			20. 8.71	44
30B601	1943	24.1.56	109			14.3.62	101			15.12.71	92
30B701	1943	24.1.56	135			21.2.62	117			10.12.71	109
01A501	1927	26.1.55	43	16.1.58	40	13.2.62	40			8. 7.71	25
01A601	1927	17.1.55	33	16.1.58	32	12.2.62	32	1.11.67	16	8. 7.71	16
01A701	1927	18.1.55	39	16.1.58	36	13.2.62	36	1.11.67	22	9. 7.71	22
02A701	1928	20.1.55	38	17.1.58	34	14.2.62	30			12. 7.71	19
030601	1928	18.1.55	51	17.1.58	43	14.2.62	35			13. 7.71	35
04A501	1928	18.1.55	38	17.1.58	37					14. 7.71	25
04A601	1928	18.1.55	35	16.1.58	34	13.2.62	29			13. 7.71	20
04B701	1928	18.1.55	48	15.1.58	43	14.2.62	26	1.11.67	13	14. 7.71	13
070501	1935	19.1.55	38	20.1.58	34	15.2.62	34			15. 7.71	20
070601	1935	14.1.55	50	20.1.58	47	1.2.62	47			16. 7.71	19
070701	1935	26.1.55	39			22.3.62	23			16. 7.71	12

Plot No.	Year Planted	Measurement Dates and No. of Trees									
080701	1935	19.1.55	52	17.1.58	46	14.2.62	46			19. 7.71	14
080801	1935	19.1.55	57	20.1.58	48	28.3.62	33			20. 7.71	17
340501	1941	25.1.56	65			28.3.62	78			7.12.71	45
340601	1941	24.1.56	135			24.1.62	105			9.12.71	55
350601	1943	15.1.56	139			15.2.62	139	13.5.71*	68	14. 5.71	19
350701	1943	24.1.56	95			12.2.62	87			6. 6.71	95
360701	1944	27.1.56	125			14.2.62	91			15. 6.71	90
370601	1944	29.1.56	104			28.3.62	120			9. 6.71	98
180801	1937	14.1.55	50	1.58	50	12.2.62	50			11. 5.71	38
180802	1937	11.1.55	45	15.1.58	38	12.2.62	38			3. 5.71	24
210701	1938	20.1.55	71	17.1.58	40	14.2.62	40			25. 6.71	21
220601	1938	9.1.55	56	17.1.58	35	13.3.62	29			27. 6.71	19
230601	1938	25.1.55	74	20.1.58	58	16.2.62	54			29. 6.71	35
240501	1938	25.1.55	59	17.1.58	41	16.2.62	39			30. 6.71	23
240601	1938	20.1.55	59	15.1.58	43	15.2.62	35			30. 6.71	23
250701	1938	1.2.55	58	21.1.58	48	15.2.62	36			6. 7.71	18
26A601	1940	26.1.55	91	21.1.58	73	21.2.62	74			22. 7.71	33
090601	1935	27.1.55	52	21.1.58	45	16.2.62	45			20. 7.71	22
140601	1936	14.1.55	61								

* This measurement was made just prior to the plot being thinned.

Figure 1

Plot 350601 measured 13/5/71; the diameter data before thinning.

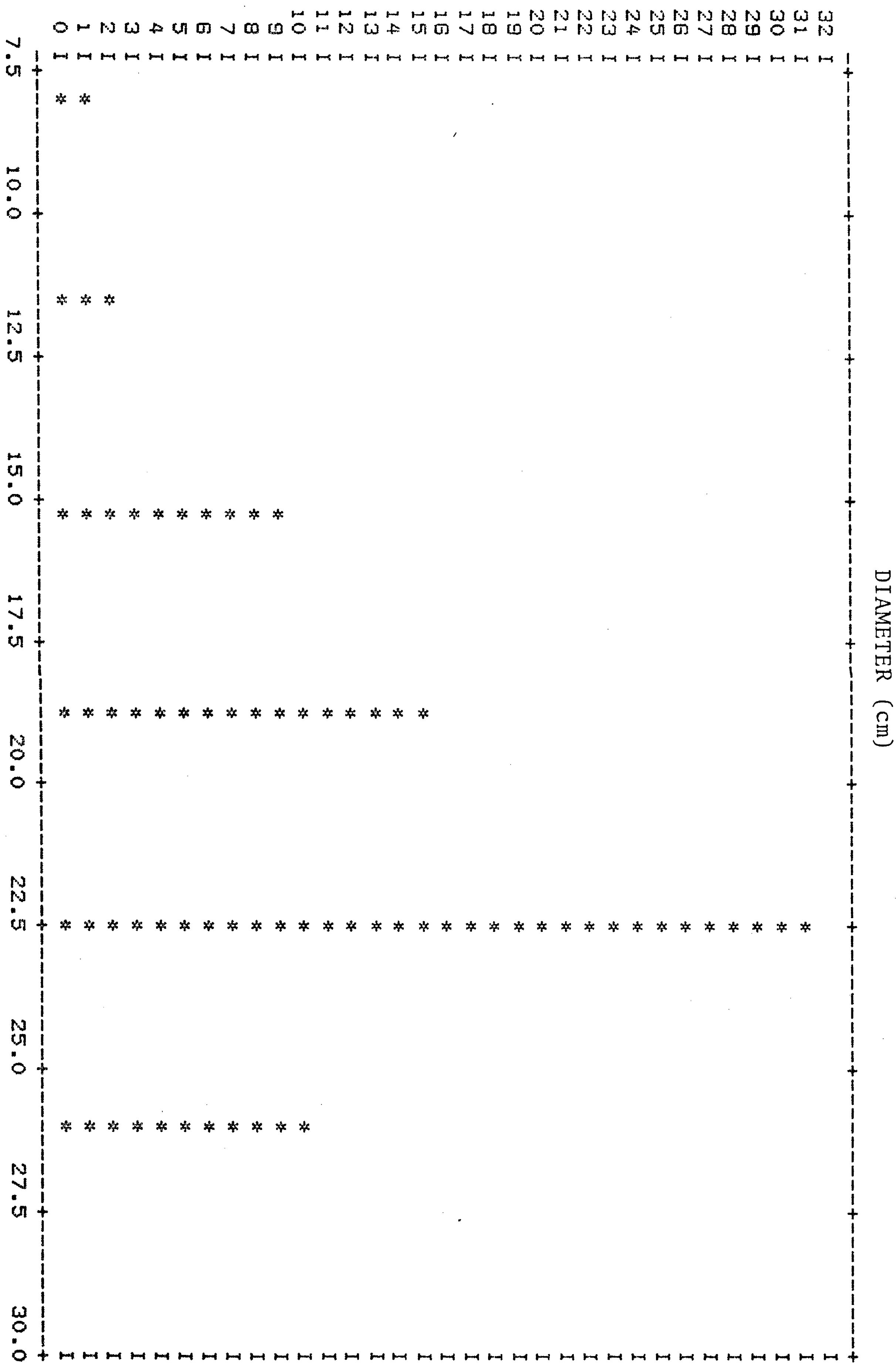

Figure 2

Plot 350601 measured 14/5/71; the diameter data after thinning.

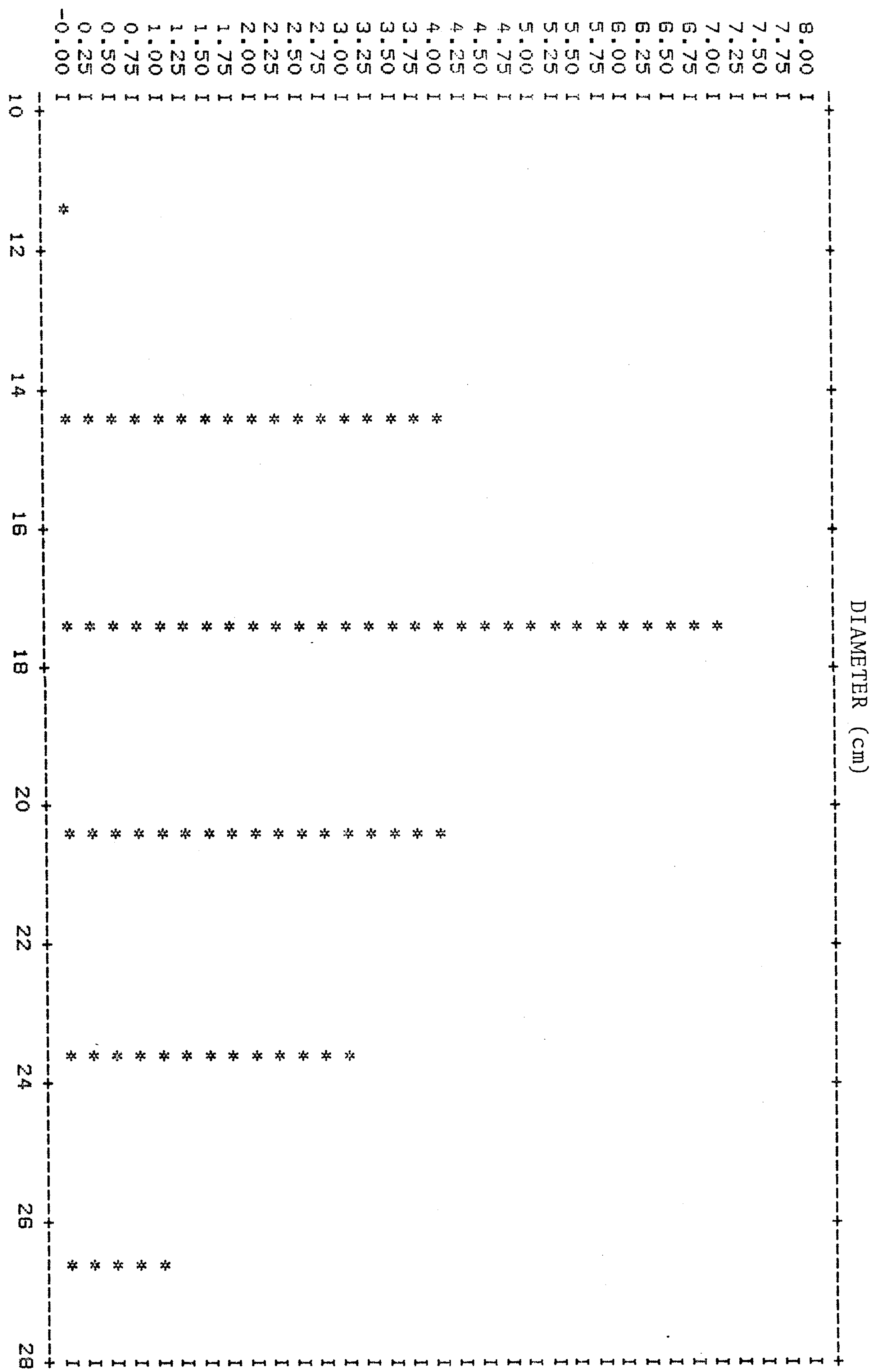

Figure 3

Plot 350601 measured 13/5/71; the height data before thinning.

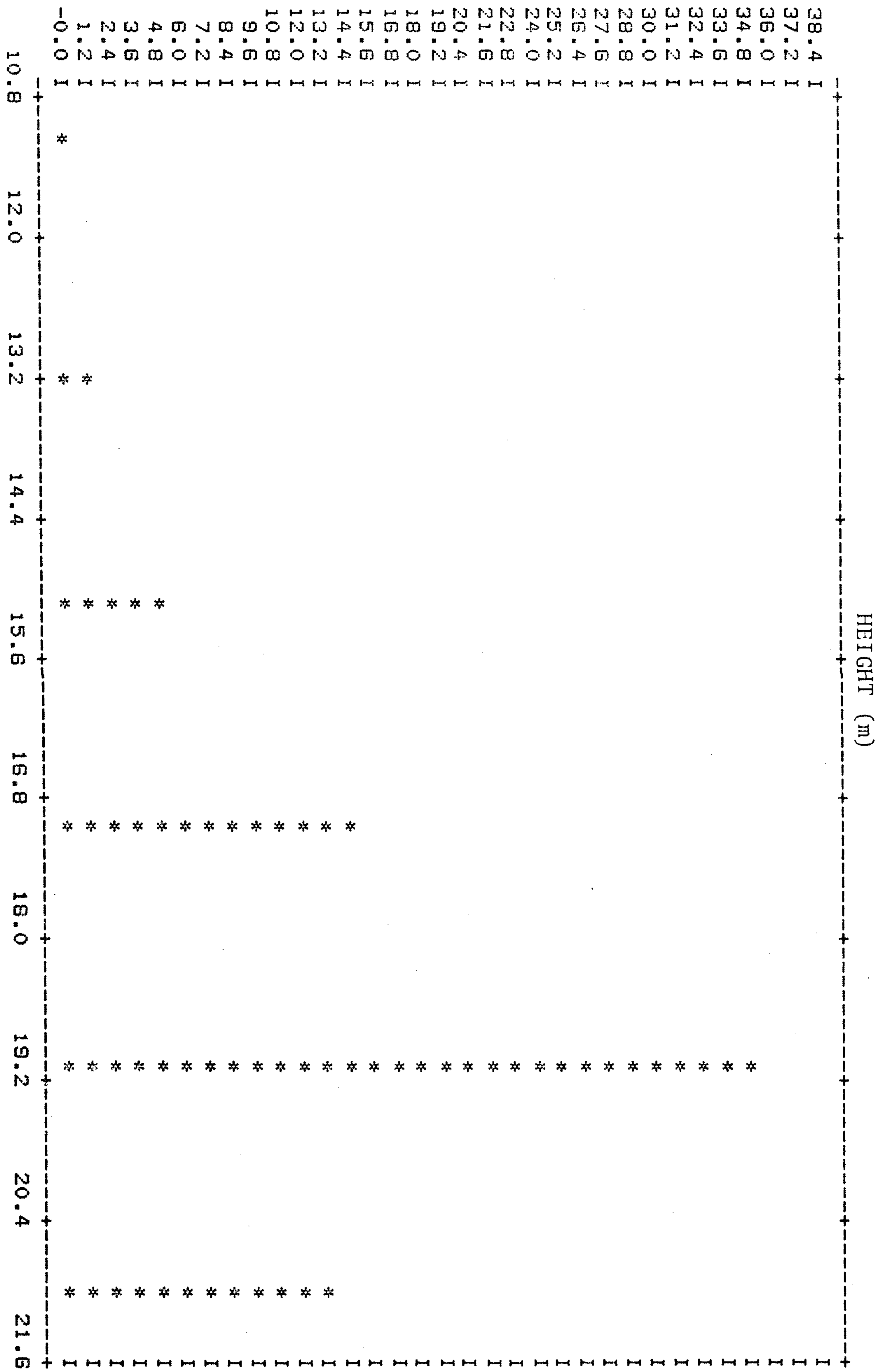

Figure 4

Plot 350601 measured 14/5/71; the height data after thinning.

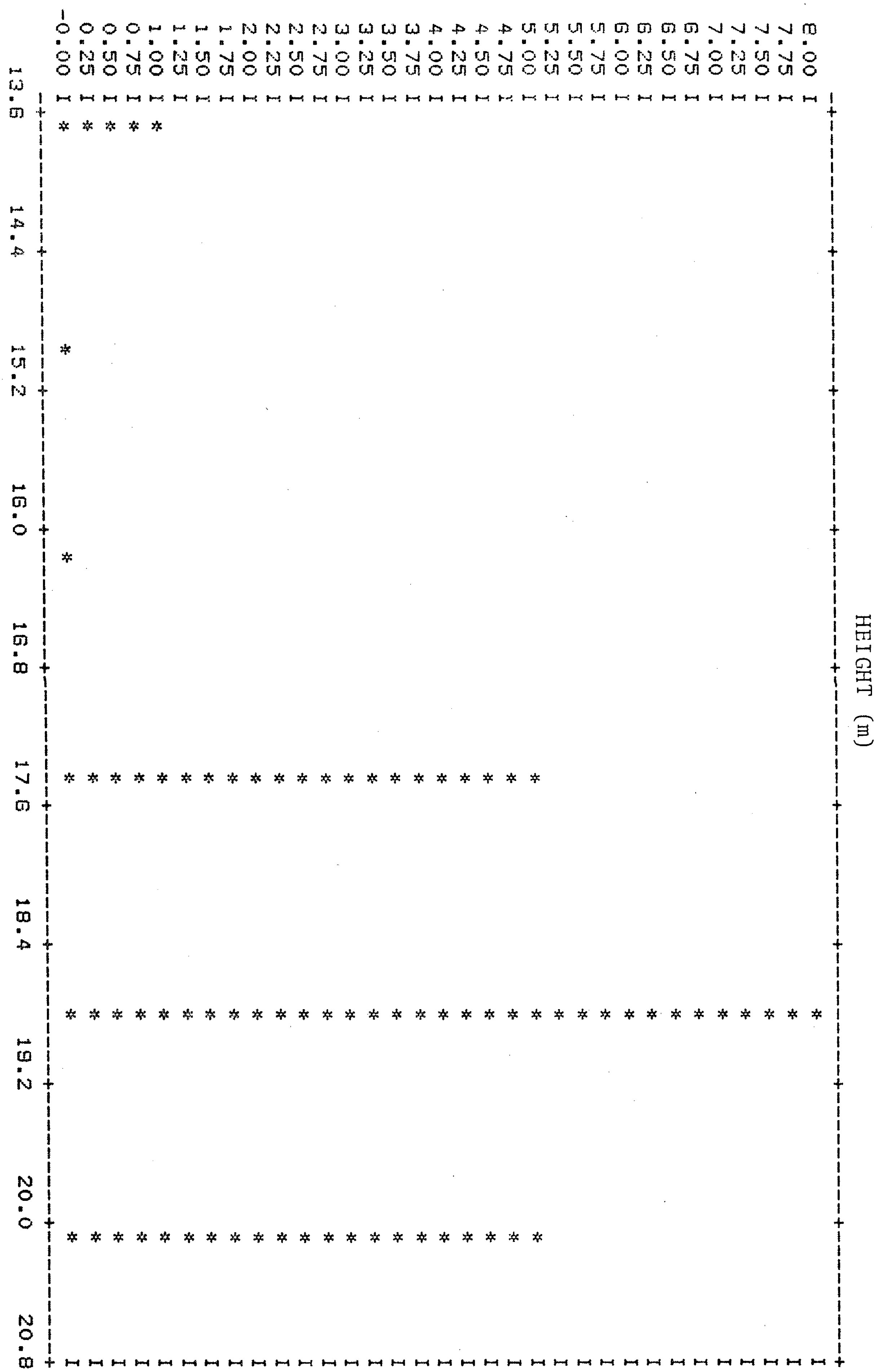

Figure 5

Proportion of trees on a plot for which both height readings are the same.

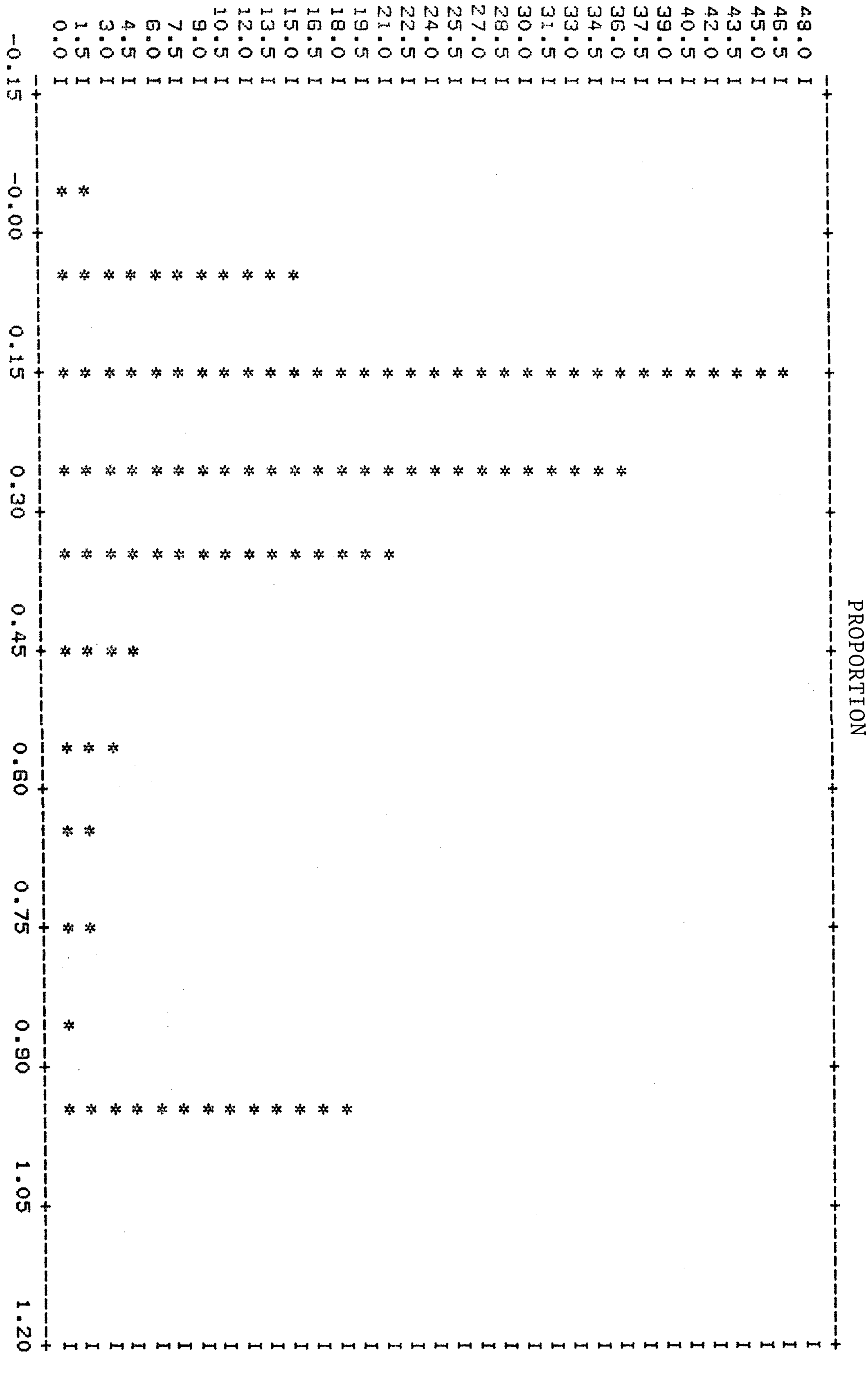

Table 2

(a) Plots in which both height readings for all trees are the same.

Plot No.	Plot Code	Measurement Date	No. of Trees
9	32A601	21. 1.58	75
14	33A601	28. 3.62	69
24	28A601	20. 1.58	61
44	01A601	1.11.67	16
51	02A701	20. 1.55	38
52	02A701	17. 1.58	34
91	340501	28. 3.62	78
96	350601	15. 1.56	139
97	350601	15. 2.62	139
99	350601	14. 5.71	19
109	180801	14. 1.55	50
123	220601	14. 3.62	29
130	240501	17. 1.58	41
142	26A601	21. 1.58	73

(b) Plots in which both height readings for all but one tree are the same.

Plot No.	Plot Code	Measurement Date	No. of Trees
98	350601	13. 5.71	68
107	370601	28. 3.62	120

(c) Plots in which both height readings for all but two trees are the same.

Plot No.	Plot Code	Measurement Date	No. of Trees
2	31A501	21. 3.62	85
35	30B701	21. 2.62	117

Figure 6

Histogram for diameter data from plot 31A501 measured 23/1/56.

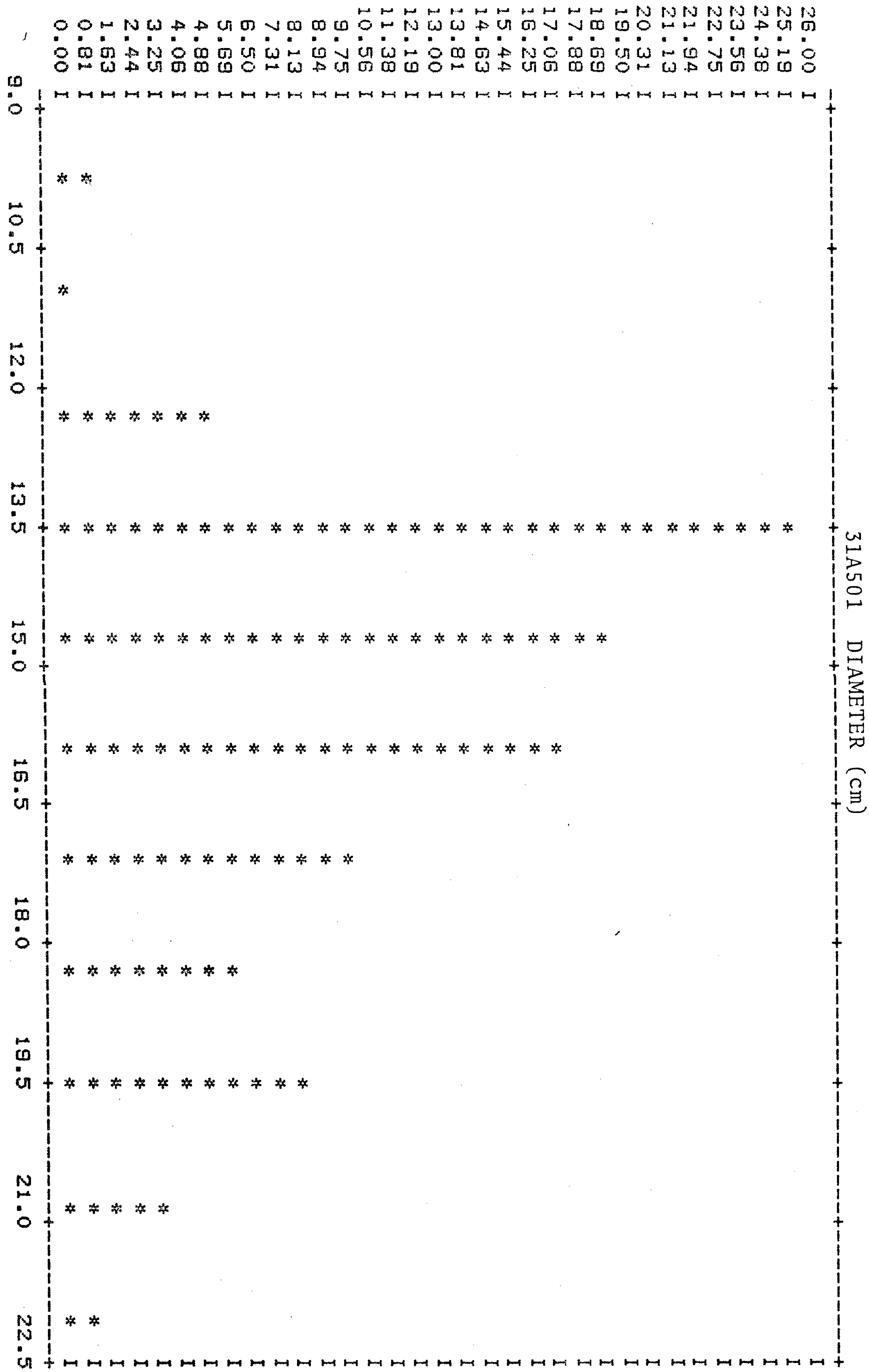

Figure 7

Histogram for height data from plot 31A501 measured 23/1/56.

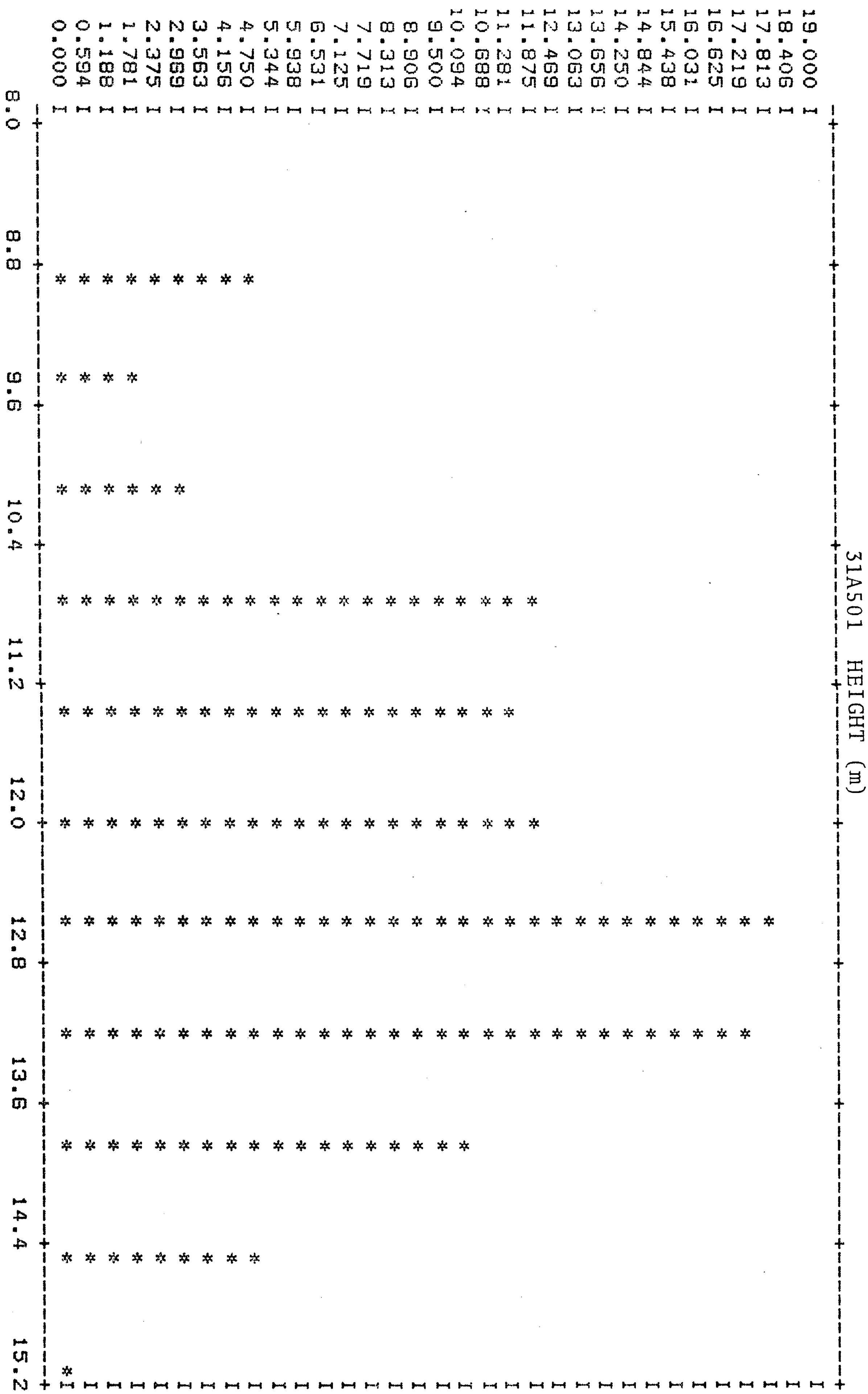

Figure 8

Histogram for diameter data from plot 31A601 measured 22/1/58.

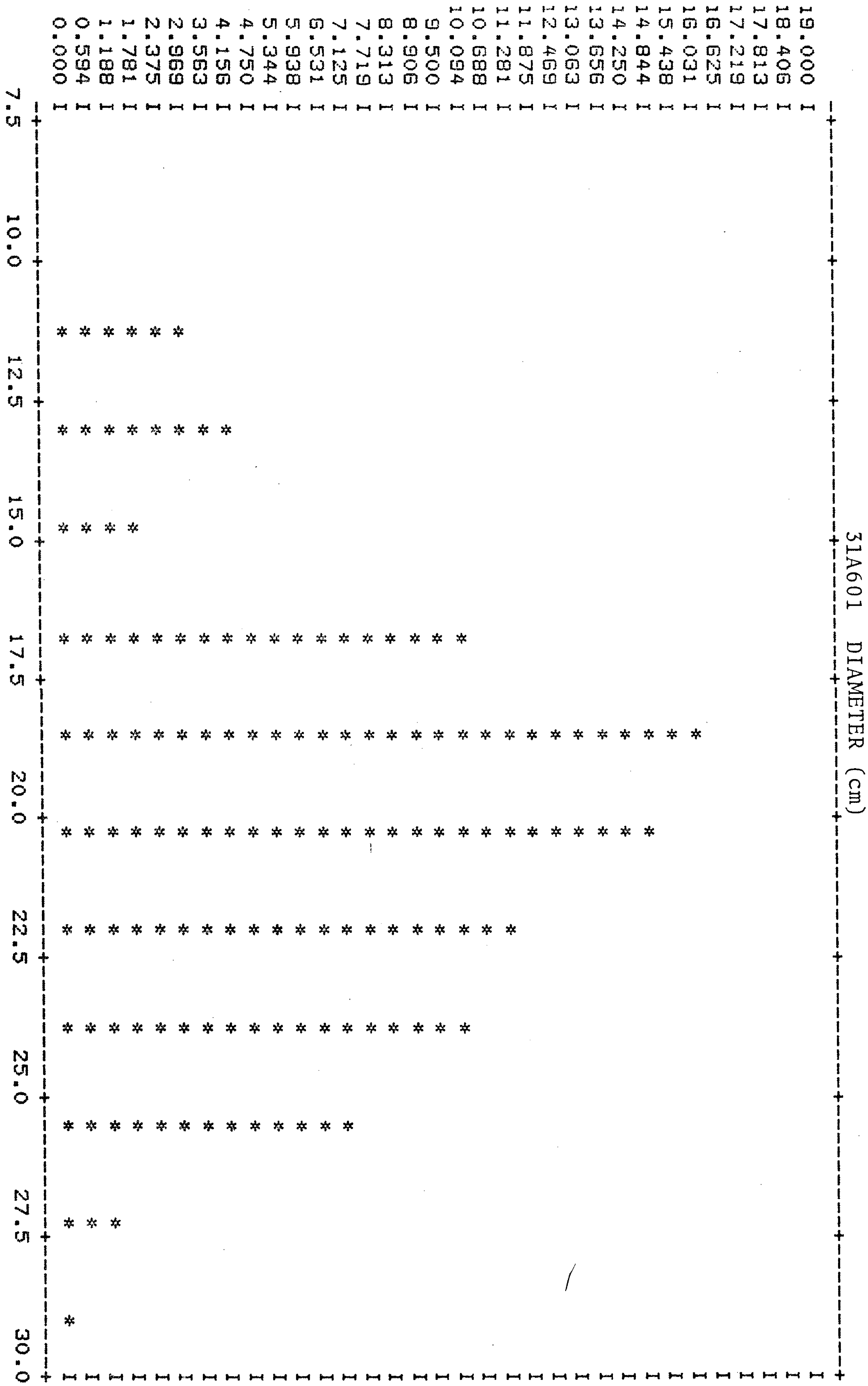

Figure 9

Histogram for height data from plot 31A601 measured 22/1/58.

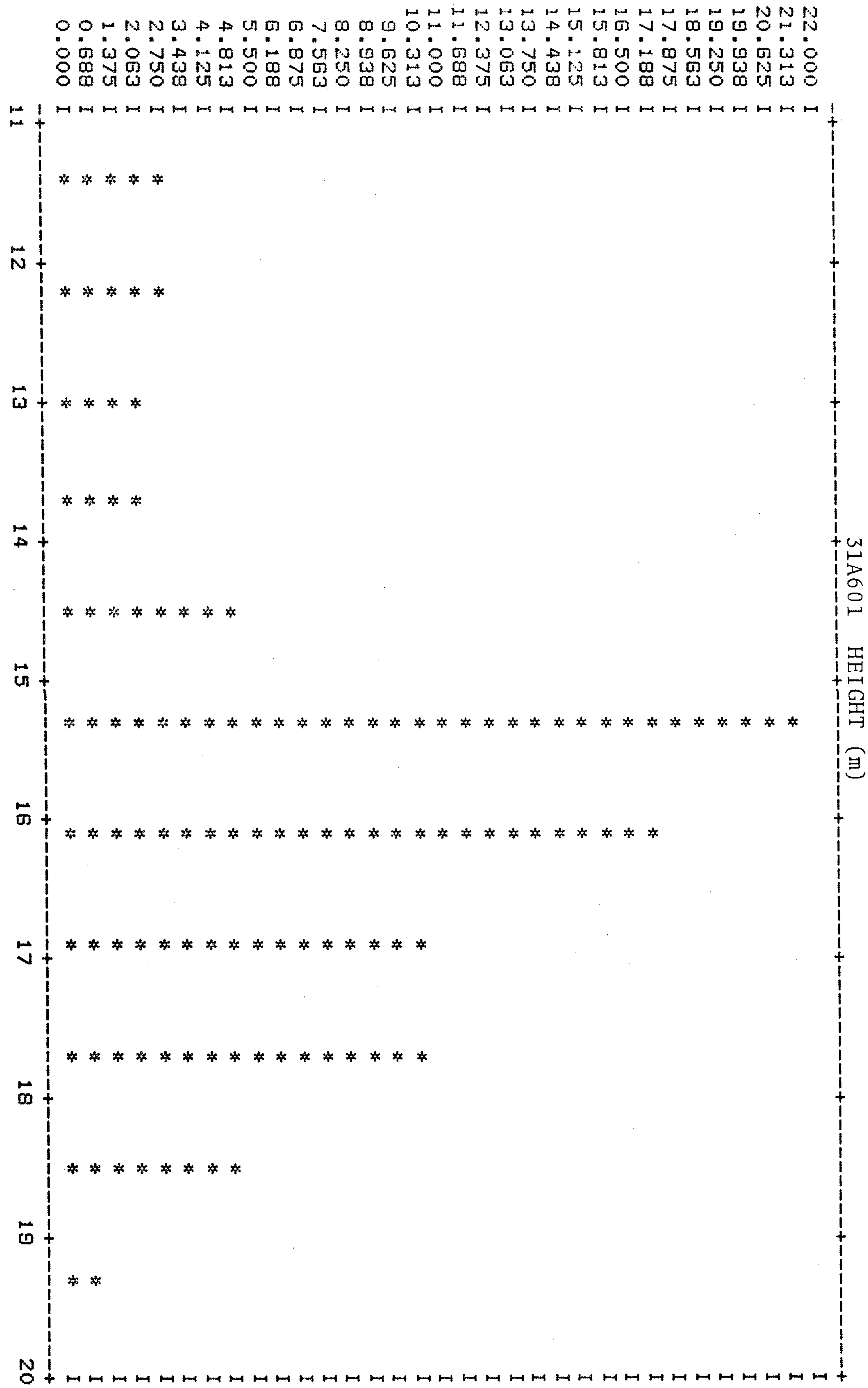

Figure 10

Bivariate plot of diameter vs. height for plot 31A501 measured 23/1/56.

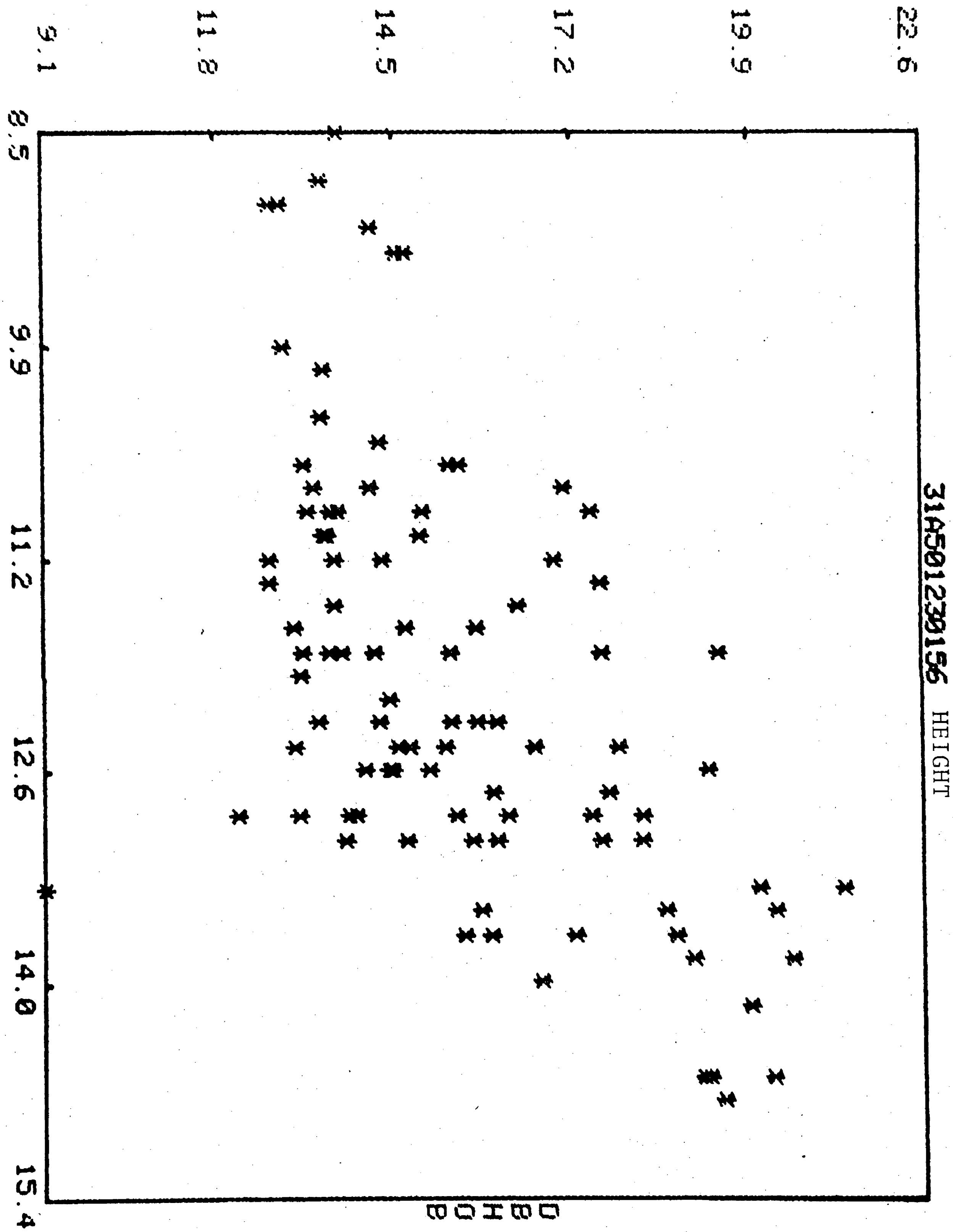

Table 3

DIAMETER AT BREAST HEIGHT (cm)

Plot no.	min. value	max. value	first moment	second moment	third moment	fourth moment	skew-ness	kurt-osis	s.e. skew.	s.e. kurt.
1	9.195	21.387	15.524	5.503	6.523	83.235	0.505	2.749	0.247	0.490
2	11.430	26.924	20.072	12.147	-0.202	358.485	-0.005	2.430	0.261	0.517
3	16.383	35.306	26.080	15.347	0.485	608.210	0.008	2.582	0.316	0.623
4	8.636	24.384	16.138	12.758	-3.475	376.129	-0.076	2.311	0.240	0.476
5	10.414	28.194	19.789	14.174	-14.747	557.119	-0.276	2.773	0.271	0.535
6	12.954	33.020	23.438	21.650	-9.641	1142.911	-0.096	2.438	0.276	0.545
7	14.859	41.783	28.948	37.086	2.742	3052.260	0.012	2.219	0.319	0.628
8	11.328	26.492	18.234	7.377	-0.184	179.613	-0.009	3.300	0.245	0.485
9	12.446	28.956	20.496	8.138	8.725	243.083	0.376	3.671	0.277	0.548
10	16.002	36.068	24.843	15.874	22.522	799.820	0.356	3.174	0.316	0.623
11	18.923	44.323	28.792	45.989	96.168	4542.209	0.308	2.148	0.393	0.768
12	8.382	24.638	16.236	11.074	-13.067	373.241	-0.355	3.043	0.226	0.447
13	14.224	28.448	20.307	8.261	11.615	212.770	0.489	3.118	0.271	0.535
14	14.224	33.528	23.913	16.655	11.647	834.309	0.171	3.008	0.289	0.570
15	20.625	42.139	30.952	30.157	47.019	2324.537	0.284	2.556	0.361	0.709
16	8.839	28.854	20.138	15.917	-18.486	738.873	-0.291	2.916	0.244	0.483
17	19.558	35.306	27.486	15.672	0.361	512.197	0.006	2.085	0.319	0.628
18	20.650	44.958	36.667	42.819	-175.983	4397.791	-0.628	2.399	0.456	0.887
19	9.906	28.194	20.566	13.109	-23.966	588.106	-0.505	3.422	0.244	0.483
20	16.764	32.512	23.629	10.393	13.519	324.050	0.403	3.000	0.285	0.563
21	19.050	39.624	27.337	19.072	42.495	1112.643	0.510	3.059	0.311	0.613
22	19.177	51.918	34.854	64.835	19.863	10329.021	0.038	2.457	0.409	0.798
23	9.144	26.670	17.912	17.499	1.576	698.428	0.022	2.281	0.274	0.541
24	11.430	29.464	21.469	22.436	-22.208	1138.224	-0.209	2.261	0.308	0.604
25	13.970	36.576	26.438	28.092	-49.055	1933.951	-0.329	2.451	0.350	0.688
26	14.605	46.355	33.931	42.995	-223.024	7494.921	-0.791	4.054	0.398	0.778
27	10.414	25.908	17.595	9.570	-6.008	263.625	-0.203	2.878	0.257	0.508
28	11.938	29.718	20.374	11.196	-10.177	441.269	-0.272	3.521	0.295	0.582
29	13.716	34.544	23.407	15.558	-5.547	828.829	-0.090	3.424	0.297	0.586
30	13.665	40.183	27.935	29.778	-21.601	2686.688	-0.133	3.030	0.357	0.702
31	7.112	26.111	14.957	12.244	10.061	458.846	0.235	3.061	0.231	0.459
32	9.652	38.354	19.307	25.260	52.768	2457.783	0.416	3.852	0.240	0.476
33	9.347	47.549	23.021	48.394	118.149	8142.667	0.351	3.477	0.251	0.498
34	8.128	25.298	16.088	14.926	-6.814	535.245	-0.118	2.402	0.209	0.414
35	8.865	33.020	20.395	25.621	-13.054	1703.254	-0.101	2.595	0.224	0.444
36	8.865	41.275	24.005	52.213	-18.280	6555.240	-0.048	2.405	0.231	0.459
37	21.082	36.576	27.013	11.306	15.077	404.346	0.397	3.163	0.361	0.709
38	22.352	38.608	29.235	12.664	10.164	483.758	0.226	3.016	0.374	0.733
39	24.384	41.910	32.468	15.345	-0.110	669.807	-0.002	2.845	0.374	0.733
40	28.042	47.981	39.645	20.637	-40.658	1336.569	-0.434	3.138	0.464	0.902
41	27.178	44.704	32.543	14.642	46.830	856.189	0.836	3.994	0.409	0.798
42	29.718	47.498	35.139	15.728	54.458	969.285	0.873	3.918	0.414	0.809
43	31.750	51.562	38.941	18.953	57.711	1277.532	0.699	3.556	0.414	0.809
44	41.148	56.642	46.133	18.572	80.481	1134.639	1.006	3.290	0.564	1.091
45	43.790	60.274	49.508	21.009	95.978	1446.901	0.997	3.278	0.564	1.091
46	24.384	36.830	30.851	11.702	-4.737	260.200	-0.118	1.900	0.378	0.741
47	25.908	40.132	33.323	13.214	-10.823	378.995	-0.225	2.170	0.393	0.768
48	27.686	44.958	36.280	18.145	-16.994	755.332	-0.220	2.294	0.393	0.768
49	34.798	50.292	42.787	13.095	-17.549	550.097	-0.370	3.208	0.491	0.953

Table 3 (continued)

50	36.297	53.162	45.715	15.023	-33.987	762.613	-0.584	3.379	0.491	0.953
51	25.908	42.926	33.448	16.041	20.112	589.493	0.313	2.291	0.383	0.750
52	27.940	46.482	36.098	17.699	27.793	823.095	0.373	2.627	0.403	0.788
53	32.258	49.784	40.479	21.267	30.430	1005.710	0.310	2.224	0.427	0.833
54	37.541	56.642	48.443	19.583	-35.947	1195.916	-0.415	3.119	0.524	1.014
55	20.828	31.496	25.246	6.151	7.155	105.614	0.469	2.792	0.333	0.656
56	23.114	33.528	27.689	7.576	6.717	137.967	0.322	2.404	0.361	0.709
57	25.146	37.592	31.387	9.513	5.997	210.690	0.204	2.328	0.398	0.778
58	27.559	45.161	36.539	16.649	10.955	672.608	0.161	2.426	0.398	0.778
59	19.304	31.496	23.789	7.838	16.823	207.635	0.767	3.380	0.383	0.750
60	20.574	33.782	25.750	8.744	20.680	280.311	0.800	3.666	0.388	0.759
61	26.670	46.025	34.698	20.016	79.592	1509.601	0.889	3.768	0.464	0.902
62	22.352	36.322	28.303	14.563	15.409	503.485	0.277	2.374	0.398	0.778
63	24.130	39.878	31.078	18.007	14.393	732.711	0.188	2.260	0.403	0.788
64	26.924	44.958	35.805	21.955	9.754	1077.387	0.095	2.235	0.434	0.845
65	30.632	54.864	44.379	41.526	-144.849	4298.489	-0.541	2.493	0.512	0.992
66	24.638	37.846	30.501	12.889	14.189	339.969	0.307	2.046	0.343	0.674
67	25.908	40.132	33.002	15.563	16.899	466.674	0.275	1.927	0.361	0.709
68	32.258	44.704	38.276	16.818	3.859	477.760	0.056	1.689	0.456	0.887
69	40.640	52.832	48.338	11.537	-19.688	374.891	-0.502	2.816	0.616	1.191
70	44.069	57.785	52.732	15.599	-32.077	637.914	-0.521	2.622	0.616	1.191
71	19.812	31.242	24.585	5.956	4.135	105.539	0.284	2.975	0.383	0.750
72	22.860	35.814	28.493	8.332	2.879	187.081	0.120	2.695	0.403	0.788
73	26.416	40.640	33.326	14.246	-5.763	462.499	-0.107	2.279	0.403	0.788
74	37.287	49.022	42.158	10.527	17.499	259.658	0.512	2.343	0.512	0.992
75	15.240	30.988	22.266	11.328	7.362	346.410	0.193	2.700	0.337	0.662
76	18.796	32.258	24.727	12.308	3.557	327.718	0.082	2.163	0.347	0.681
77	19.812	35.306	27.594	17.335	-0.700	650.537	-0.010	2.165	0.347	0.681
78	30.658	43.358	36.834	11.861	14.432	359.136	0.353	2.553	0.524	1.014
79	21.844	38.100	29.633	20.520	28.994	830.778	0.312	1.973	0.378	0.741
80	27.432	47.498	39.006	33.413	-93.346	2598.786	-0.483	2.328	0.481	0.935
81	44.882	59.512	52.610	16.065	-21.445	663.870	-0.333	2.572	0.637	1.232
82	19.050	36.830	27.338	16.088	10.152	709.051	0.157	2.740	0.330	0.650
83	19.812	39.624	30.259	21.457	11.004	1237.029	0.111	2.687	0.350	0.688
84	20.828	45.720	34.273	35.791	0.245	3221.565	0.001	2.515	0.350	0.688
85	37.643	57.125	46.887	34.995	28.861	2207.859	0.139	1.803	0.597	1.154
86	19.558	37.338	28.181	14.258	-5.446	581.719	-0.101	2.862	0.316	0.623
87	20.828	41.148	31.020	16.249	-2.463	821.953	-0.038	3.113	0.343	0.674
88	24.384	46.736	35.691	22.194	-2.452	1513.023	-0.023	3.072	0.409	0.798
89	32.004	58.242	46.327	55.443	-96.596	6800.291	-0.234	2.212	0.550	1.063
90	7.620	26.060	15.945	10.185	29.068	461.339	0.894	4.447	0.297	0.586
91	9.398	32.004	19.415	23.490	21.278	1596.145	0.187	2.893	0.272	0.538
92	9.881	40.538	26.529	39.379	-129.746	5667.362	-0.525	3.655	0.354	0.695
93	7.595	22.403	14.900	10.320	-11.816	272.869	-0.356	2.562	0.209	0.414
94	7.620	28.194	19.169	19.625	-53.591	1091.563	-0.616	2.834	0.236	0.467
95	10.312	35.128	25.650	40.703	-279.288	5318.742	-1.076	3.210	0.322	0.634
96	8.204	19.202	13.979	5.727	-3.695	83.921	-0.270	2.559	0.206	0.408
97	8.636	24.130	16.422	10.314	-7.455	274.588	-0.225	2.581	0.206	0.408
98	9.779	27.788	20.614	14.142	-38.204	686.697	-0.718	3.434	0.291	0.574
99	12.979	28.016	19.154	14.489	26.280	594.205	0.477	2.830	0.524	1.014
100	12.725	20.930	15.433	3.786	4.960	42.328	0.673	2.953	0.247	0.490
101	12.700	26.162	18.367	8.069	9.700	190.484	0.423	2.926	0.258	0.511
102	8.103	33.376	21.269	22.573	-23.055	1762.335	-0.215	3.459	0.247	0.490
103	7.849	21.742	15.582	8.114	-8.751	174.414	-0.379	2.649	0.217	0.430
104	10.160	28.448	19.854	16.577	-41.113	832.599	-0.609	3.030	0.253	0.500

Table 3 (continued)

105	10.084	36.271	24.134	33.853	-96.349	3595.990	-0.489	3.138	0.254	0.503
106	6.604	20.574	15.025	6.317	-8.559	173.932	-0.539	4.359	0.237	0.469
107	4.826	25.654	17.073	19.014	-58.354	1157.098	-0.704	3.200	0.221	0.438
108	4.928	31.623	20.415	34.933	-154.380	3613.429	-0.748	2.961	0.244	0.483
109	13.462	30.480	22.037	14.869	-16.994	556.262	-0.296	2.516	0.337	0.662
110	13.716	33.274	24.110	20.054	-27.625	1029.149	-0.308	2.559	0.337	0.662
111	13.970	37.338	26.645	32.315	-46.775	2614.530	-0.255	2.504	0.337	0.662
112	13.945	45.822	31.006	76.217	-188.818	12533.322	-0.284	2.158	0.383	0.750
113	14.224	35.560	24.164	24.085	17.545	1774.623	0.148	3.059	0.354	0.695
114	19.558	38.862	27.833	20.613	69.767	1386.621	0.745	3.263	0.383	0.750
115	21.336	44.196	31.476	28.242	93.298	2503.065	0.622	3.138	0.383	0.750
116	28.219	50.089	38.386	28.401	82.704	2416.570	0.546	2.996	0.472	0.918
117	8.128	28.448	20.839	15.130	-42.790	811.560	-0.727	3.545	0.285	0.563
118	20.320	32.766	25.756	8.025	4.619	174.595	0.203	2.711	0.374	0.733
119	21.844	39.116	29.978	17.021	12.687	757.456	0.181	2.615	0.374	0.733
120	28.956	50.800	40.122	30.423	-7.199	2112.055	-0.043	2.282	0.501	0.972
121	10.922	37.592	21.957	40.147	30.710	3583.919	0.121	2.224	0.319	0.628
122	18.542	42.926	28.687	26.160	37.624	2128.708	0.281	3.111	0.398	0.778
123	24.130	50.800	34.894	31.241	65.680	3442.951	0.376	3.528	0.434	0.845
124	36.271	52.959	45.005	23.374	9.167	1192.168	0.081	2.182	0.524	1.014
125	8.382	30.480	17.739	21.658	13.998	1300.213	0.139	2.772	0.279	0.552
126	9.398	34.036	21.314	19.731	11.619	1425.992	0.133	3.663	0.314	0.618
127	12.446	37.592	24.492	24.627	13.381	1997.041	0.109	3.293	0.325	0.639
128	18.034	44.044	29.788	40.626	-9.098	4668.123	-0.035	2.828	0.398	0.778
129	6.909	29.464	19.206	23.789	-35.136	1568.131	-0.303	2.771	0.311	0.613
130	14.986	33.274	23.591	20.306	2.895	962.646	0.032	2.335	0.369	0.724
131	17.780	37.592	27.777	24.938	-3.720	1496.418	-0.030	2.406	0.378	0.741
132	18.390	44.755	33.932	53.501	-168.248	6752.789	-0.430	2.359	0.481	0.935
133	9.652	29.972	19.515	16.291	12.214	806.474	0.186	3.039	0.311	0.613
134	10.414	33.528	23.344	19.633	-15.341	1368.142	-0.176	3.549	0.361	0.709
135	19.558	38.100	28.441	20.573	24.202	1088.467	0.259	2.572	0.398	0.778
136	26.340	46.736	36.553	26.277	11.135	1546.704	0.083	2.240	0.481	0.935
137	11.684	32.766	21.892	24.735	-1.478	1566.690	-0.012	2.561	0.314	0.618
138	15.494	36.068	25.723	25.373	14.705	1665.021	0.115	2.586	0.343	0.674
139	20.574	44.196	31.115	32.098	77.236	2575.103	0.425	2.499	0.393	0.768
140	32.055	56.109	42.973	37.129	83.088	3736.875	0.367	2.711	0.536	1.038
141	7.620	22.606	16.195	9.720	-7.435	289.725	-0.245	3.066	0.253	0.500
142	12.954	25.908	19.450	7.949	4.650	177.229	0.208	2.805	0.281	0.555
143	13.716	30.734	22.452	12.357	6.383	433.953	0.147	2.842	0.279	0.552
144	14.402	38.913	28.139	40.326	-105.783	3720.114	-0.413	2.288	0.409	0.798
145	16.256	31.496	22.743	11.492	8.173	362.630	0.210	2.746	0.330	0.650
146	18.542	33.782	26.021	10.568	7.972	322.276	0.232	2.886	0.354	0.695
147	21.844	37.338	29.752	13.203	15.153	466.430	0.316	2.676	0.354	0.695
148	28.880	48.514	37.696	20.760	8.515	1303.986	0.090	3.026	0.491	0.953
149	9.398	28.956	18.305	23.993	23.500	1319.268	0.200	2.292	0.306	0.604

Table 4

TREE HEIGHT (m)

plot no.	min. value	max. value	first moment	second moment	third moment	fourth moment	skew-ness	kurt-osis	s.e. skew.	s.e. kurt.
1	8.534	14.783	12.091	1.948	-1.268	10.890	-0.466	2.869	0.247	0.490
2	10.973	23.165	17.962	4.962	-8.992	95.405	-0.814	3.875	0.261	0.517
3	17.069	25.603	22.438	3.257	-2.490	31.524	-0.424	2.971	0.316	0.623
4	7.010	15.697	12.310	2.874	-3.212	30.101	-0.659	3.645	0.240	0.476
5	10.973	18.898	15.786	3.039	-3.871	32.801	-0.731	3.551	0.271	0.535
6	14.326	24.232	20.446	4.470	-6.549	66.347	-0.693	3.320	0.276	0.545
7	20.117	31.394	26.090	7.475	-8.676	133.915	-0.425	2.397	0.319	0.628
8	9.296	15.697	12.723	2.444	-0.108	13.495	-0.028	2.260	0.245	0.485
9	11.582	18.593	14.569	2.641	1.123	16.227	0.262	2.327	0.277	0.548
10	13.106	23.012	18.852	4.322	-3.345	55.263	-0.372	2.959	0.316	0.623
11	16.764	30.328	23.410	13.916	0.611	357.949	0.012	1.848	0.393	0.768
12	7.925	15.392	11.850	2.787	-2.733	21.461	-0.587	2.763	0.226	0.447
13	10.973	18.288	14.978	1.980	-1.039	13.519	-0.373	3.449	0.271	0.535
14	12.192	22.250	18.341	4.131	-6.887	63.449	-0.820	3.719	0.289	0.570
15	18.288	27.737	24.178	4.387	-4.604	57.808	-0.501	3.004	0.361	0.709
16	10.363	18.136	14.565	3.186	-1.601	25.703	-0.282	2.533	0.244	0.483
17	17.069	24.232	20.947	2.704	-0.957	18.098	-0.215	2.475	0.319	0.628
18	23.470	30.632	27.227	3.927	-2.259	30.123	-0.290	1.954	0.456	0.887
19	12.192	19.660	16.209	2.198	-1.340	14.425	-0.411	2.986	0.244	0.483
20	15.545	22.708	19.748	2.532	-1.673	18.171	-0.415	2.835	0.285	0.563
21	18.288	27.127	23.725	3.486	-5.830	49.744	-0.896	4.094	0.311	0.613
22	18.288	33.376	29.570	10.167	-52.747	611.363	-1.627	5.915	0.409	0.798
23	8.839	16.002	12.730	1.854	-0.298	10.895	-0.118	3.171	0.274	0.541
24	12.192	21.031	15.925	2.789	1.872	30.576	0.402	3.931	0.306	0.604
25	14.326	23.622	20.173	4.402	-7.337	73.853	-0.794	3.811	0.350	0.688
26	17.374	29.261	25.573	7.918	-31.086	305.682	-1.395	4.876	0.398	0.778
27	10.820	16.764	14.471	1.739	-1.622	9.508	-0.707	3.143	0.257	0.508
28	13.411	20.422	17.725	2.049	-1.060	12.898	-0.362	3.073	0.295	0.582
29	16.612	25.908	21.641	2.801	-1.860	28.006	-0.397	3.571	0.297	0.586
30	17.983	28.956	25.804	4.930	-17.467	152.171	-1.596	6.261	0.357	0.702
31	8.687	15.240	11.918	1.847	-0.534	9.491	-0.213	2.781	0.231	0.459
32	10.668	20.726	17.007	4.893	-8.752	70.283	-0.809	2.935	0.240	0.476
33	13.106	26.670	21.560	10.372	-21.330	268.505	-0.639	2.496	0.251	0.498
34	8.077	15.392	12.506	2.544	-1.277	15.640	-0.315	2.417	0.209	0.414
35	9.906	21.946	16.880	6.096	-7.317	101.061	-0.486	2.720	0.224	0.444
36	10.363	27.432	20.661	13.674	-29.885	548.049	-0.591	2.931	0.231	0.459
37	19.507	24.689	22.523	1.317	-0.305	5.240	-0.202	3.019	0.361	0.709
38	21.031	27.584	24.540	2.321	-0.839	14.816	-0.237	2.751	0.374	0.733
39	22.708	29.261	25.653	2.399	1.253	14.667	0.337	2.549	0.374	0.733
40	24.994	31.090	28.188	2.820	-0.041	15.413	-0.009	1.939	0.464	0.902
41	22.860	27.889	25.442	1.443	-0.232	5.441	-0.134	2.614	0.409	0.798
42	25.603	31.394	27.851	1.611	0.738	8.086	0.361	3.114	0.414	0.809
43	26.060	33.680	31.132	2.330	-4.684	27.856	-1.317	5.130	0.414	0.809
44	30.175	36.271	34.138	1.812	-2.973	17.387	-1.219	5.298	0.564	1.091
45	31.242	35.966	34.309	1.326	-1.421	6.630	-0.930	3.768	0.564	1.091
46	20.879	29.413	26.197	2.466	-2.948	28.597	-0.735	4.701	0.378	0.741
47	26.213	32.614	29.041	2.152	1.526	13.171	0.484	2.845	0.393	0.768
48	29.566	33.223	32.063	0.745	-0.451	1.765	-0.702	3.181	0.393	0.768
49	32.004	36.576	34.290	1.166	-0.265	4.268	-0.210	3.142	0.491	0.953

Table 4 (continued)

50	32.004	38.405	35.606	2.621	-1.180	18.884	-0.278	2.748	0.491	0.953
51	27.127	30.785	29.060	1.053	0.220	2.192	0.204	1.978	0.383	0.750
52	27.432	32.918	30.310	1.487	-0.715	7.476	-0.394	3.379	0.403	0.788
53	30.937	35.052	32.588	0.917	0.395	2.398	0.450	2.853	0.427	0.833
54	33.223	37.795	35.132	1.082	0.630	4.407	0.560	3.768	0.524	1.014
55	16.916	23.470	19.952	2.615	0.485	16.280	0.115	2.381	0.333	0.656
56	17.831	24.841	22.073	2.648	-3.252	20.204	-0.756	2.887	0.361	0.709
57	20.117	25.146	23.269	1.645	-1.467	8.045	-0.695	2.971	0.398	0.778
58	22.403	28.194	25.795	2.063	-1.643	12.272	-0.554	2.884	0.398	0.778
59	16.459	21.488	18.958	1.131	0.690	4.311	0.573	3.368	0.383	0.750
60	17.678	23.165	20.603	1.618	0.497	7.503	0.241	2.866	0.388	0.759
61	22.403	28.042	25.634	1.499	-0.465	7.414	-0.253	3.298	0.464	0.902
62	16.612	25.603	20.574	4.404	2.286	47.846	0.247	2.467	0.398	0.778
63	18.898	25.603	22.385	3.452	0.427	24.882	0.067	2.088	0.403	0.788
64	21.946	28.194	25.277	3.293	-0.787	18.307	-0.132	1.688	0.434	0.845
65	23.470	31.547	28.156	3.948	-3.930	43.380	-0.501	2.783	0.512	0.992
66	22.555	30.785	26.756	3.172	0.041	30.424	0.007	3.023	0.343	0.674
67	23.622	32.918	28.910	3.673	-1.693	44.149	-0.240	3.272	0.361	0.709
68	28.194	35.357	32.315	2.927	-1.771	22.161	-0.354	2.586	0.456	0.887
69	31.242	36.424	34.478	3.073	-2.638	16.223	-0.490	1.717	0.616	1.191
70	33.071	38.710	36.377	3.735	-3.476	26.265	-0.482	1.882	0.616	1.191
71	13.106	17.526	15.669	1.322	-0.669	3.928	-0.440	2.246	0.383	0.750
72	14.630	19.964	17.817	1.771	-0.953	7.283	-0.404	2.322	0.403	0.788
73	18.593	23.470	21.565	1.652	-0.720	5.878	-0.339	2.154	0.403	0.788
74	23.470	28.346	25.817	1.569	-0.234	6.257	-0.119	2.543	0.512	0.992
75	13.259	18.288	15.789	1.405	0.076	4.971	0.045	2.519	0.337	0.662
76	15.545	21.031	18.210	1.620	0.791	6.788	0.384	2.587	0.347	0.681
77	16.612	24.079	20.856	2.551	-0.661	19.534	-0.162	3.002	0.347	0.681
78	22.860	28.804	25.619	2.982	0.223	16.987	0.043	1.910	0.524	1.014
79	16.154	24.384	20.304	3.693	-1.294	35.038	-0.182	2.569	0.378	0.741
80	24.994	31.852	29.122	4.084	-6.316	44.712	-0.765	2.681	0.481	0.935
81	33.223	38.405	35.509	2.772	2.403	14.703	0.521	1.914	0.637	1.232
82	15.088	23.774	20.627	3.582	-5.396	43.871	-0.796	3.419	0.330	0.650
83	17.678	26.518	23.539	3.413	-5.091	45.970	-0.807	3.947	0.350	0.688
84	20.422	31.699	28.075	4.965	-12.015	120.976	-1.086	4.907	0.350	0.688
85	32.004	35.814	33.767	1.890	0.321	4.834	0.123	1.353	0.597	1.154
86	15.240	26.365	21.026	4.409	-2.972	64.771	-0.321	3.332	0.316	0.623
87	16.154	27.737	24.279	4.643	-12.513	117.511	-1.251	5.450	0.343	0.674
88	23.470	31.547	28.314	3.405	-5.162	42.732	-0.822	3.686	0.409	0.798
89	31.090	39.929	35.966	5.405	-5.789	74.386	-0.461	2.546	0.550	1.063
90	8.839	14.021	11.840	1.341	-0.232	4.574	-0.149	2.545	0.297	0.586
91	11.887	20.422	16.569	3.838	-4.880	42.326	-0.649	2.874	0.272	0.538
92	13.411	25.451	21.732	5.975	-26.371	240.114	-1.806	6.726	0.354	0.695
93	8.534	15.545	12.378	2.015	-0.610	10.421	-0.213	2.566	0.209	0.414
94	9.144	21.946	17.596	5.208	-11.847	115.751	-0.997	4.267	0.236	0.467
95	14.021	26.213	22.408	8.249	-30.099	257.409	-1.270	3.783	0.322	0.634
96	9.754	14.630	12.497	1.097	-0.260	3.083	-0.226	2.560	0.206	0.408
97	10.973	20.422	15.641	2.631	-2.226	25.005	-0.522	3.613	0.206	0.408
98	12.192	21.946	18.651	2.985	-5.961	45.388	-1.156	5.096	0.291	0.574
99	14.326	20.726	18.481	1.968	-3.147	18.415	-1.140	4.756	0.524	1.014
100	9.906	14.630	12.503	0.795	-0.275	1.979	-0.389	3.135	0.247	0.490
101	13.868	22.250	16.925	2.226	1.708	20.940	0.514	4.225	0.258	0.511
102	12.649	24.841	19.875	4.968	-8.996	110.976	-0.812	4.496	0.247	0.490
103	9.449	16.154	13.227	1.409	-0.776	6.255	-0.464	3.153	0.217	0.430
104	12.192	21.946	18.663	3.298	-5.613	45.737	-0.937	4.206	0.253	0.500

Table 4 (continued)

105	14.630	27.280	22.674	7.347	-17.722	194.675	-0.890	3.606	0.254	0.503
106	8.077	14.173	11.710	1.399	-0.803	7.002	-0.485	3.580	0.237	0.469
107	7.010	19.202	14.794	4.536	-8.606	85.172	-0.891	4.139	0.221	0.438
108	7.315	22.708	17.105	7.890	-23.592	279.309	-1.065	4.487	0.244	0.483
109	14.935	22.860	19.513	3.268	-4.517	39.352	-0.765	3.684	0.337	0.662
110	16.154	25.298	21.580	4.631	-9.418	75.793	-0.945	3.534	0.337	0.662
111	17.374	29.566	26.222	6.844	-24.765	236.422	-1.383	5.048	0.337	0.662
112	18.440	34.595	29.610	17.170	-87.077	1072.275	-1.224	3.637	0.383	0.750
113	16.154	23.470	19.893	2.720	-0.939	19.271	-0.209	2.606	0.354	0.695
114	19.660	26.822	23.819	2.857	-1.196	20.084	-0.248	2.461	0.383	0.750
115	23.317	30.937	27.733	4.107	-2.496	36.145	-0.300	2.143	0.383	0.750
116	28.346	37.490	33.498	4.589	-5.095	61.390	-0.518	2.915	0.472	0.918
117	9.754	18.136	14.506	3.589	-1.667	34.748	-0.245	2.697	0.285	0.563
118	14.021	22.250	18.437	4.419	-1.309	40.663	-0.141	2.082	0.374	0.733
119	16.916	26.213	22.609	4.992	-4.365	64.993	-0.391	2.608	0.374	0.733
120	21.031	31.547	29.050	5.511	-24.505	212.988	-1.894	7.013	0.501	0.972
121	4.267	18.288	13.749	9.264	-21.376	279.201	-0.758	3.254	0.319	0.628
122	15.088	22.250	18.689	3.702	-0.400	30.388	-0.056	2.218	0.398	0.778
123	19.507	26.518	23.449	3.242	-2.118	27.343	-0.363	2.602	0.434	0.845
124	27.127	34.442	31.194	3.677	-2.239	34.363	-0.318	2.541	0.524	1.014
125	8.992	17.831	14.264	3.687	-2.783	36.793	-0.393	2.707	0.279	0.552
126	11.735	19.812	16.764	2.636	-2.765	25.450	-0.646	3.663	0.314	0.618
127	13.411	23.774	20.303	4.388	-7.896	78.963	-0.859	4.102	0.325	0.639
128	14.630	26.060	22.237	6.901	-11.224	153.071	-0.619	3.215	0.398	0.778
129	6.553	16.002	12.626	4.095	-8.551	65.209	-1.032	3.888	0.311	0.613
130	10.058	19.202	15.887	4.469	-10.026	74.339	-1.061	3.722	0.369	0.724
131	12.802	23.927	19.601	5.449	-12.051	118.024	-0.948	3.975	0.378	0.741
132	13.411	28.956	23.986	11.435	-51.294	678.910	-1.327	5.192	0.481	0.935
133	7.163	16.307	12.988	3.298	-4.556	39.922	-0.761	3.670	0.311	0.613
134	11.735	19.355	16.179	2.539	-1.650	21.184	-0.408	3.287	0.361	0.709
135	16.002	23.317	19.908	2.601	-0.741	17.050	-0.177	2.519	0.398	0.778
136	21.946	29.108	25.510	4.008	2.705	36.964	0.337	2.301	0.481	0.935
137	9.601	17.221	14.286	2.495	-1.355	18.357	-0.344	2.948	0.314	0.618
138	11.582	21.641	18.050	3.842	-4.702	56.288	-0.624	3.814	0.343	0.674
139	18.745	27.432	23.406	3.871	-0.140	41.695	-0.018	2.782	0.393	0.768
140	27.889	32.156	30.023	1.375	0.592	4.179	0.367	2.209	0.536	1.038
141	7.620	15.392	12.438	1.813	-1.456	13.780	-0.597	4.192	0.253	0.500
142	13.106	17.678	15.357	1.529	0.141	5.259	0.075	2.250	0.281	0.555
143	13.868	26.518	19.114	3.759	2.945	67.427	0.404	4.772	0.279	0.552
144	17.069	26.060	22.306	6.317	-6.582	87.285	-0.415	2.187	0.409	0.798
145	12.192	20.117	16.735	2.976	-0.901	24.242	-0.175	2.738	0.330	0.650
146	16.764	23.012	20.181	2.151	-0.361	11.672	-0.114	2.523	0.354	0.695
147	17.678	25.908	23.283	2.607	-3.670	29.410	-0.872	4.326	0.354	0.695
148	25.146	30.785	28.333	1.811	-0.776	9.750	-0.318	2.971	0.491	0.953
149	10.668	19.355	15.387	3.796	-1.998	34.114	-0.270	2.367	0.306	0.604

Figure 11

The β_1 - β_2 plane with estimates of b_1 and b_2 for the diameter data for all data sets.

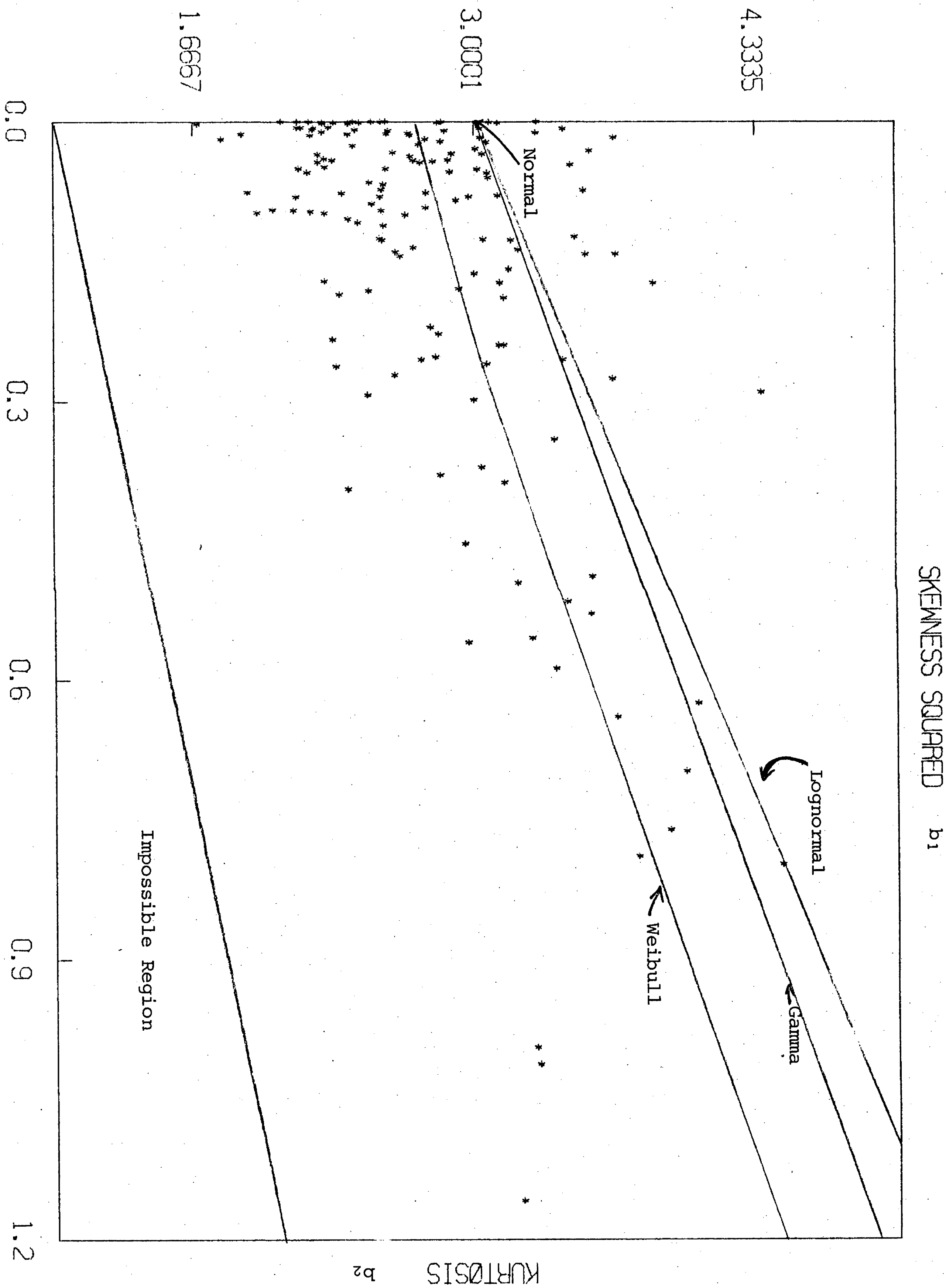

Figure 12

The β_1 - β_2 plane with estimates of b_1 and b_2 for the height data for all data sets.

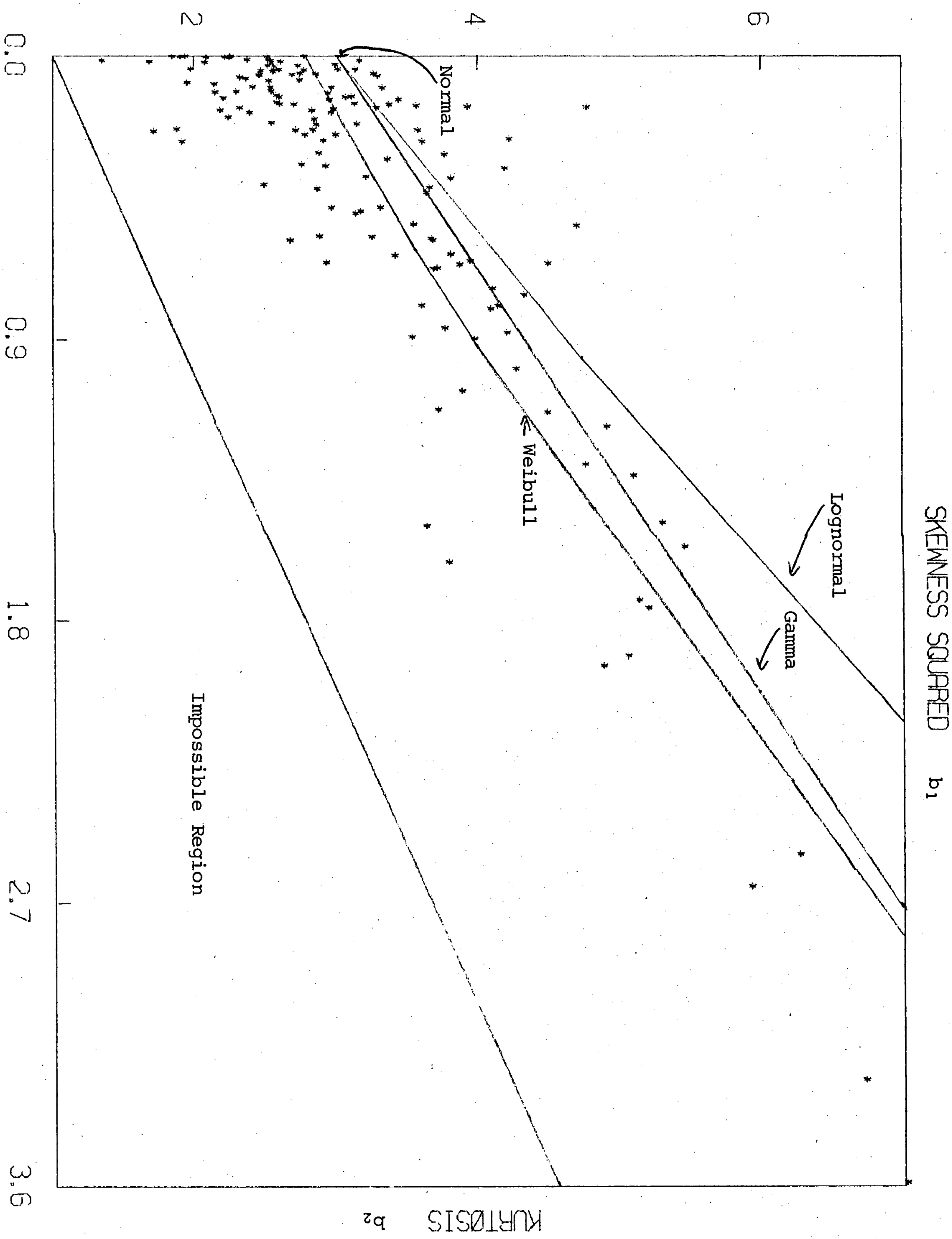

Table 5

Points above lognormal line in the $\beta_1 - \beta_2$ plane

Diameter Plot Number	Diameter Plot Code	Diameter Measurement Date	Diameter No. of Trees	Height Plot Number	Height Plot Code	Height Measurement Date	Height No. of Trees
8	32A601	19. 1.56	97	13	33A601	22. 1.58	79
9	32A601	21. 1.58	75	23	28A601	26. 1.55	77
19	270801	28. 1.55	98	24	28A601	20. 1.58	61
28	290701	21. 1.58	66	29	290701	21. 3.62	65
29	290701	21. 3.62	65	46	01A701	18. 1.55	39
33	30B601	15.12.71	92	49	01A701	1.11.67	22
87	080801	20. 1.58	48	52	02A701	17. 1.58	34
88	080801	28. 3.62	33	54	02A701	12. 7.71	19
92	340501	7.12.71	45	61	04A501	14. 7.71	25
102	350701	6. 6.71	95	67	04B701	15. 1.58	43
123	220601	13. 3.62	29	86	080801	19. 1.55	57
126	230601	20. 1.58	58	97	350601	15. 2.62	139
127	230601	16. 2.62	54	101	350701	12. 2.62	87
134	240601	15. 1.58	43	102	350701	6. 6.71	95
148	090601	20. 7.71	22	106	370601	29. 1.56	104
				138	250701	21. 1.58	48
				141	250701	26. 1.55	91
				143	26A601	21. 2.62	74

Figure 13

Plot of the maximum observed tree diameter against tree age, for all data sets. The curve for maximum diameter used in the iteration procedure is also plotted.

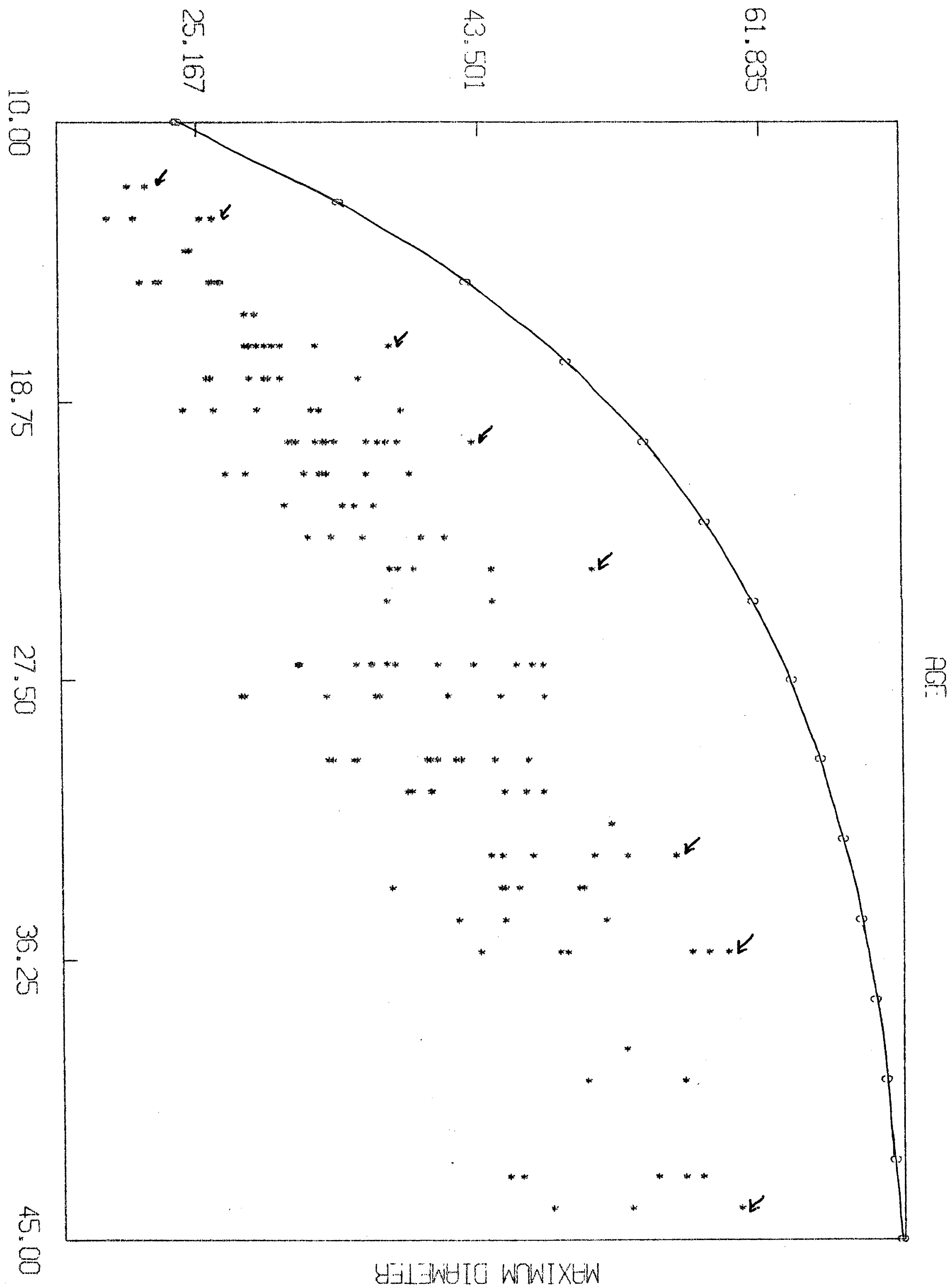

Table 6

Values of the maximized log-likelihood for the normal(N), lognormal(LN), gamma(G), beta(B), Weibull(W), and S_B for the diameter data.

Plot Code	No.	N	LN	G	B	W	S_B
31A50141230156	95	-215.801	-213.557	-213.959	-214.817	-221.306	-214.432
31A50141210362	85	-226.736	-228.256	-227.352	-226.236	-227.568	-226.177
31A50141230871	57	-158.711	-159.499	-159.027	-158.572	-159.741	-158.543
31A60141280155	101	-271.895	-275.158	-273.359	-270.656	-271.503	-270.505
31A60141220158	79	-216.827	-221.195	-219.211	-215.860	-216.079	-215.858
31A60141140362	76	-224.689	-227.165	-225.870	-223.855	-224.626	-223.830
31A60141230871	56	-180.631	-181.748	-181.014	-180.010	-180.756	-179.926
32A60141190156	97	-234.558	-236.632	-235.521	-234.754	-237.394	-234.846
32A60141210158	75	-185.039	-184.633	-184.456	-184.957	-189.993	-184.917
32A60141210262	57	-159.673	-159.163	-159.086	-159.528	-162.544	-159.463
32A60141250871	36	-119.993	-119.256	-119.281	-119.447	-120.426	-119.254
33A60141020255	115	-301.445	-309.836	-306.135	-300.912	-300.057	-300.977
33A60141220158	79	-195.503	-193.859	-194.160	-195.081	-200.779	-194.891
33A60141280362	69	-194.945	-195.542	-194.979	-194.834	-197.129	-194.819
33A60141061271	43	-134.252	-133.915	-133.828	-134.041	-135.615	-133.981
27070140250156	98	-274.659	-281.025	-278.120	-273.716	-273.787	-273.765
27070140150262	56	-156.513	-156.915	-156.639	-156.088	-157.122	-156.075
27070140270771	26	-85.733	-87.655	-86.892	-78.857	-84.343	-81.312
27080139280155	98	-265.148	-273.144	-269.773	-263.792	-263.234	-263.819
27080139210158	71	-183.856	-182.786	-182.937	-183.641	-188.215	-183.542
27080139150262	59	-170.691	-169.279	-169.520	-170.512	-174.173	-170.428
27080139230771	33	-115.661	-116.479	-115.939	-115.410	-115.758	-115.359
28A60140260155	77	-219.451	-221.306	-220.074	-218.561	-219.441	-218.406
28A60140200158	61	-181.430	-184.366	-182.944	-179.236	-180.626	-179.555
28A60140160262	46	-141.987	-144.643	-143.462	-140.587	-141.105	-140.610
28A60140280771	35	-115.482	-120.222	-118.291	-114.569	-114.240	-114.573
29070140280155	88	-224.246	-227.744	-226.130	-224.047	-224.335	-224.060
29070140210158	66	-173.362	-176.700	-175.223	-173.567	-173.803	-173.671
29070140210362	65	-181.429	-183.815	-182.636	-181.680	-182.653	-181.786
29070140200871	44	-137.096	-139.327	-138.222	-137.063	-137.301	-137.104
30B60143240156	109	-291.190	-292.486	-291.046	-290.801	-293.078	-290.736
30B60143140362	101	-306.389	-306.517	-305.308	-305.863	-308.422	-305.733
30B60143151271	92	-308.994	-310.526	-308.555	-308.345	-309.371	-308.373
30B70143240156	135	-374.018	-380.051	-376.879	-372.441	-373.039	-372.247
30B70143210262	117	-355.756	-362.163	-358.771	-355.005	-355.224	-354.971
30B70143101271	109	-370.229	-376.834	-372.990	-368.736	-369.259	-368.565
01A50127260155	43	-113.160	-112.557	-112.655	-112.993	-116.166	-112.916
01A50127160158	40	-107.533	-107.395	-107.347	-107.463	-109.717	-107.435
01A50127130262	40	-111.373	-111.770	-111.544	-111.353	-112.609	-111.348
01A50127080771	25	-73.312	-74.222	-73.858	-72.869	-72.979	-72.881
01A60127170155	33	-91.109	-89.865	-90.207	-91.312	-94.895	-91.370
01A60127160158	32	-89.493	-88.243	-88.601	-89.785	-93.296	-89.922
01A60127120262	32	-92.478	-91.532	-91.783	-92.909	-95.780	-93.136
01A60127011167	16	-46.076	-45.402	-45.610	-46.659	-48.151	-46.986
01A60127080771	16	-47.063	-46.402	-46.605	-47.906	-49.126	-48.395
01A70127180155	39	-103.303	-103.682	-103.505	-102.540	-103.268	-102.771
01A70127160158	36	-97.545	-98.151	-97.896	-96.860	-97.283	-96.941
01A70127130262	36	-103.253	-103.930	-103.639	-102.789	-103.123	-102.800
01A70127011167	22	-59.511	-59.988	-59.803	-59.315	-59.618	-59.308
01A70127090771	22	-61.022	-61.708	-61.452	-60.518	-60.535	-60.493
02A70128200155	38	-106.648	-106.143	-106.244	-106.606	-108.549	-106.609
02A70128170158	34	-97.094	-96.590	-96.696	-97.173	-99.206	-97.232
02A70128140262	30	-88.425	-88.032	-88.119	-88.508	-89.936	-88.592
02A70128120771	19	-55.219	-55.714	-55.520	-54.909	-55.052	-54.915
03060128180155	51	-118.688	-117.793	-118.023	-118.419	-122.772	-118.299

Table 6 (continued)

Plot Code	No.	N	LN	G	B	W	S_B
03060128170158	43	-104.551	-104.050	-104.162	-104.385	-107.095	-104.321
03060128140262	35	-89.084	-88.884	-88.904	-89.004	-90.685	-88.974
03060128130771	35	-98.879	-98.786	-98.756	-98.830	-100.266	-98.817
04A50128180155	38	-93.040	-91.675	-92.059	-92.630	-96.895	-92.436
04A50128170158	37	-92.616	-91.296	-91.658	-92.279	-96.633	-92.114
04A50128140771	25	-72.930	-71.841	-72.138	-72.985	-75.561	-73.001
04A60128180155	35	-96.537	-96.142	-96.195	-96.372	-98.052	-96.308
04A60128160158	34	-97.387	-97.191	-97.180	-97.250	-98.519	-97.202
04A60128130262	29	-85.940	-85.962	-85.891	-85.848	-86.674	-85.824
04A60128130771	20	-65.642	-66.646	-66.254	-64.467	-64.799	-64.520
04B70128180155	48	-129.463	-128.780	-128.935	-129.275	-131.640	-129.208
04B70128150158	43	-120.030	-119.469	-119.590	-119.868	-121.721	-119.819
04B70128140262	26	-73.584	-73.551	-73.533	-73.111	-73.976	-73.437
04B70128011167	13	-34.342	-34.615	-34.513	-26.139	-33.925	-33.801
04B70128140771	13	-36.303	-36.598	-36.490	-31.443	-35.779	-35.621
07050135190155	38	-87.824	-87.537	-87.574	-87.766	-90.454	-87.738
07050135200158	34	-84.286	-84.276	-84.229	-84.253	-85.918	-84.242
07050135150262	34	-93.404	-93.792	-93.607	-93.184	-93.737	-93.186
07050135150771	20	-51.919	-51.567	-51.672	-52.055	-53.614	-52.137
07060135140155	50	-131.629	-131.562	-131.416	-131.439	-133.435	-131.378
07060135200158	47	-125.681	-125.748	-125.612	-125.480	-126.651	-125.415
07060135010262	47	-133.730	-134.160	-133.886	-133.409	-134.211	-133.373
07060135160771	19	-50.456	-50.228	-50.280	-50.471	-51.713	-50.481
07070135260155	39	-114.256	-113.547	-113.690	-114.109	-115.579	-114.099
07070135220362	23	-72.988	-74.040	-73.624	-70.801	-72.094	-71.323
07070135160771	12	-33.687	-33.877	-33.805	-33.482	-33.608	-33.558
08070135190155	52	-146.014	-146.129	-145.915	-145.955	-147.811	-145.945
08070135170158	46	-135.791	-136.123	-135.835	-135.721	-137.054	-135.707
08070135140262	46	-147.558	-148.417	-147.906	-147.348	-148.079	-147.315
08070135190771	14	-44.752	-44.670	-44.675	-44.725	-45.076	-44.800
08080135190155	57	-156.613	-157.724	-157.187	-156.537	-157.621	-156.535
08080135200158	48	-135.022	-135.823	-135.406	-135.055	-136.406	-135.075
08080135280362	33	-97.972	-98.500	-98.219	-98.006	-98.911	-98.042
08080135200771	17	-58.252	-58.759	-58.533	-57.711	-58.039	-57.822
34050141250156	65	-167.661	-165.070	-165.322	-166.900	-172.148	-166.559
34050141280362	78	-233.783	-235.512	-234.078	-233.434	-234.581	-233.449
34050141071271	45	-146.500	-152.976	-150.135	-146.589	-145.906	-146.770
34060141240156	135	-349.108	-358.268	-354.210	-346.673	-346.485	-346.488
34060141240162	105	-305.271	-317.474	-312.364	-301.086	-301.725	-300.718
34060141091271	55	-179.965	-190.050	-186.165	-172.760	-176.588	-172.288
35060143150156	139	-318.522	-324.200	-321.677	-316.536	-317.073	-316.488
35060143150262	139	-359.410	-365.859	-362.854	-357.954	-358.278	-357.872
35060143130571	68	-186.558	-193.605	-190.767	-183.915	-183.770	-183.820
35060143140571	19	-52.357	-51.876	-51.930	-52.043	-53.091	-51.940
35070143240156	95	-198.032	-194.777	-195.674	-197.415	-206.145	-197.040
35070143120262	87	-214.275	-212.701	-212.920	-213.466	-219.005	-213.160
35070143060671	95	-282.846	-291.424	-287.308	-283.573	-283.050	-283.955
36070144270156	125	-308.213	-315.528	-312.376	-305.089	-305.673	-305.021
36070144140262	91	-256.889	-265.985	-262.243	-253.918	-253.788	-253.675
36070144150671	90	-286.195	-296.870	-292.215	-284.716	-284.510	-284.803
37060144290156	104	-243.418	-253.411	-249.169	-243.421	-242.563	-243.638
37060144280362	120	-346.984	-366.990	-358.488	-343.132	-343.723	-343.027
37060144090671	98	-313.174	-332.397	-324.167	-308.785	-311.067	-308.522
18080137140155	50	-138.428	-140.584	-139.642	-137.670	-137.746	-137.630
18080137000158	50	-145.907	-148.394	-147.297	-144.973	-145.162	-144.962

Table 6 (continued)

Plot Code	No.	N	LN	G	B	W	S_B
18080137120262	50	-157.836	-160.572	-159.302	-156.801	-157.197	-156.832
18080137110571	38	-136.258	-139.053	-137.705	-134.208	-135.528	-134.343
18080237110155	45	-135.438	-136.208	-135.624	-135.446	-136.406	-135.462
18080237150158	38	-111.412	-109.754	-110.163	-111.495	-114.187	-111.598
18080237120262	38	-117.395	-116.102	-116.374	-117.549	-119.756	-117.632
18080237030571	24	-74.212	-73.606	-73.738	-74.431	-75.877	-74.559
21070138200155	71	-197.187	-205.547	-202.095	-194.264	-194.348	-194.088
21070138170158	40	-98.409	-98.242	-98.228	-98.352	-100.476	-98.330
21070138140262	40	-113.446	-113.385	-113.292	-113.374	-114.937	-113.352
21070138250671	21	-65.657	-65.902	-65.767	-65.533	-65.889	-65.526
22060138090155	56	-182.852	-183.576	-182.739	-181.952	-182.496	-181.704
22060138170158	35	-106.787	-106.667	-106.533	-107.016	-108.134	-107.159
22060138140362	29	-91.054	-90.791	-90.752	-91.730	-92.694	-92.149
22060138270671	19	-56.900	-56.902	-56.873	-56.890	-57.431	-57.516
23060138250155	74	-218.791	-220.589	-219.182	-218.368	-219.097	-218.330
23060138200158	58	-168.783	-170.988	-169.646	-169.098	-170.267	-169.323
23060138160262	54	-163.127	-164.585	-163.660	-163.214	-164.314	-163.302
23060138290671	35	-114.490	-115.726	-115.050	-114.420	-114.691	-114.415
24050138250155	59	-177.209	-183.052	-180.287	-176.378	-176.530	-176.426
24050138170158	41	-119.900	-120.456	-120.068	-119.634	-120.248	-119.567
24050138160262	39	-118.058	-118.808	-118.378	-117.799	-118.321	-117.763
24050138300671	23	-78.402	-80.037	-79.336	-75.119	-77.742	-76.817
24060138200155	59	-166.041	-166.973	-166.190	-165.923	-167.371	-165.942
24060138150158	43	-125.025	-127.899	-126.515	-125.276	-125.413	-125.396
24060138150262	35	-102.583	-102.333	-102.289	-102.449	-103.848	-102.401
24060138300671	23	-70.226	-70.285	-70.206	-70.142	-70.726	-70.123
25070138010255	58	-175.337	-177.247	-176.132	-174.976	-175.455	-174.915
25070138210158	48	-145.717	-146.268	-145.800	-145.512	-146.490	-145.454
25070138150262	36	-113.520	-112.659	-112.793	-113.449	-114.913	-113.456
25070138060771	18	-58.070	-57.805	-57.842	-58.374	-59.010	-58.613
26A60140260155	91	-232.601	-238.085	-235.545	-232.191	-232.341	-232.272
26A60140210158	73	-179.247	-179.161	-178.944	-178.985	-182.168	-178.913
26A60140210262	74	-198.028	-198.494	-198.029	-197.861	-200.385	-197.843
26A60140220771	33	-107.825	-110.238	-109.185	-105.812	-106.912	-105.879
09060135270155	52	-137.269	-137.129	-137.008	-137.077	-139.292	-137.010
09060135210158	45	-116.903	-116.746	-116.684	-116.810	-119.144	-116.777
09060135160262	45	-121.912	-121.446	-121.504	-121.803	-124.348	-121.760
09060135200771	22	-64.580	-64.695	-64.604	-64.615	-65.528	-64.640
14060136140155	61	-183.477	-183.678	-183.021	-182.605	-183.550	-182.354

Table 7

Values of the Kolmogorov-Smirnov statistic d for the normal(N), lognormal(LN), gamma(G), beta(B), Weibull(W), and S_B, with the critical value for the 5% significance level; diameter data.

Plot Code	No.	N	LN	G	B	W	S_B	5%
31A50141230156	95	0.1188	0.0944	0.1030	0.1115	0.1246	0.1084	0.1395
31A50141210362	85	0.0577	0.0422	0.0369	0.0674	0.0833	0.0633	0.1475
31A50141230871	57	0.0673	0.0574	0.0574	0.0722	0.1017	0.0714	0.1801
31A60141280155	101	0.0631	0.0582	0.0611	0.0562	0.0587	0.0551	0.1353
31A60141220158	79	0.0353	0.0597	0.0493	0.0497	0.0577	0.0503	0.1530
31A60141140362	76	0.0545	0.0611	0.0543	0.0722	0.0761	0.0689	0.1560
31A60141230871	56	0.0653	0.0595	0.0548	0.0706	0.0796	0.0658	0.1817
32A60141190156	97	0.0290	0.0461	0.0396	0.0310	0.0573	0.0312	0.1381
32A60141210158	75	0.0806	0.0537	0.0624	0.0766	0.1084	0.0749	0.1570
32A60141210262	57	0.0796	0.0558	0.0593	0.0758	0.0917	0.0732	0.1801
32A60141250871	36	0.1053	0.0973	0.1010	0.1020	0.0937	0.1004	0.2267
33A60141020255	115	0.0519	0.1001	0.0834	0.0480	0.0402	0.0481	0.1268
33A60141220158	79	0.1218	0.0974	0.1059	0.1177	0.1284	0.1145	0.1530
33A60141280362	69	0.0592	0.0422	0.0467	0.0582	0.0915	0.0572	0.1637
33A60141061271	43	0.0948	0.0611	0.0713	0.0930	0.1197	0.0836	0.2074
27070140250156	98	0.0453	0.0909	0.0752	0.0466	0.0499	0.0470	0.1374
27070140150262	56	0.1016	0.0762	0.0851	0.1135	0.1221	0.1085	0.1817
27070140270771	26	0.1027	0.1287	0.1194	0.1141	0.1293	0.1142	0.2640
27080139280155	98	0.0503	0.0923	0.0780	0.0577	0.0487	0.0580	0.1374
27080139210158	71	0.0799	0.0626	0.0678	0.0797	0.1200	0.0790	0.1614
27080139150262	59	0.0859	0.0573	0.0662	0.0852	0.1192	0.0845	0.1771
27080139230771	33	0.0776	0.0667	0.0510	0.0828	0.0976	0.0843	0.2367
28A60140260155	77	0.0627	0.0469	0.0489	0.0680	0.0761	0.0640	0.1550
28A60140200158	61	0.0455	0.0744	0.0635	0.0872	0.0662	0.0757	0.1741
28A60140160262	46	0.0853	0.1260	0.1127	0.0653	0.0724	0.0659	0.2005
28A60140280771	35	0.0741	0.1284	0.1082	0.0981	0.0775	0.0988	0.2299
29070140280155	88	0.0609	0.0981	0.0857	0.0631	0.0735	0.0643	0.1450
29070140210158	66	0.0704	0.1074	0.0946	0.0726	0.0897	0.0741	0.1674
29070140210362	65	0.0535	0.0632	0.0511	0.0577	0.0822	0.0589	0.1687
29070140200871	44	0.0715	0.0820	0.0722	0.0829	0.0978	0.0834	0.2050
30B60143240156	109	0.0563	0.0681	0.0552	0.0508	0.0753	0.0494	0.1303
30B60143140362	101	0.0578	0.0755	0.0591	0.0514	0.0622	0.0531	0.1353
30B60143151271	92	0.0518	0.0772	0.0610	0.0517	0.0510	0.0556	0.1418
30B70143240156	135	0.0474	0.0759	0.0638	0.0358	0.0331	0.0362	0.1171
30B70143210262	117	0.0473	0.0613	0.0548	0.0428	0.0536	0.0402	0.1257
30B70143101271	109	0.0435	0.0837	0.0661	0.0331	0.0367	0.0319	0.1303
01A50127260155	43	0.1020	0.0826	0.0887	0.1002	0.1430	0.0980	0.2074
01A50127160158	40	0.0860	0.0703	0.0751	0.0844	0.1268	0.0839	0.2150
01A50127130262	40	0.0716	0.0701	0.0702	0.0742	0.0964	0.0741	0.2150
01A50127080771	25	0.1070	0.1296	0.1222	0.0992	0.1061	0.0974	0.2700
01A60127170155	33	0.0926	0.0862	0.0883	0.0810	0.1156	0.0928	0.2367
01A60127160158	32	0.1099	0.1028	0.1055	0.1093	0.1105	0.1088	0.2404
01A60127120262	32	0.1130	0.1044	0.1076	0.1128	0.1308	0.1124	0.2404
01A60127011167	16	0.1521	0.1434	0.1464	0.1570	0.1769	0.1583	0.3280
01A60127080771	16	0.1670	0.1469	0.1535	0.1869	0.2078	0.2049	0.3280
01A70127180155	39	0.0942	0.0801	0.0850	0.1090	0.1107	0.1094	0.2178
01A70127160158	36	0.0885	0.0772	0.0812	0.0937	0.0966	0.0944	0.2267
01A70127130262	36	0.0743	0.0828	0.0758	0.0821	0.0875	0.0823	0.2267
01A70127011167	22	0.1150	0.1124	0.1131	0.1258	0.1451	0.1256	0.2840
01A70127090771	22	0.1256	0.1323	0.1279	0.1429	0.1513	0.1418	0.2840
02A70128200155	38	0.1346	0.1196	0.1249	0.1343	0.1374	0.1327	0.2206
02A70128170158	34	0.1210	0.1089	0.1133	0.1210	0.1138	0.1214	0.2332
02A70128140262	30	0.1162	0.1048	0.1089	0.1179	0.1169	0.1191	0.2400
02A70128120771	19	0.0820	0.0857	0.0842	0.0894	0.0927	0.0876	0.3010
03060128180155	51	0.1617	0.1420	0.1486	0.1576	0.1998	0.1559	0.1904

Table 7 (continued)

Plot Code	No.	N	LN	G	B	W	S_B	5%
03060128170158	43	0.1416	0.1224	0.1288	0.1380	0.1784	0.1349	0.2074
03060128140262	35	0.1432	0.1238	0.1302	0.1413	0.1834	0.1402	0.2299
03060128130771	35	0.1161	0.0940	0.1014	0.1154	0.1510	0.1180	0.2299
04A50128180155	38	0.1038	0.0880	0.0935	0.0994	0.1234	0.0977	0.2206
04A50128170158	37	0.1252	0.1049	0.1113	0.1211	0.1721	0.1191	0.2236
04A50128140771	25	0.1838	0.1572	0.1659	0.1862	0.2159	0.1852	0.2700
04A60128180155	35	0.0713	0.0821	0.0734	0.0701	0.0985	0.0698	0.2299
04A60128160158	34	0.0697	0.0738	0.0727	0.0694	0.0847	0.0690	0.2332
04A60128130262	29	0.0850	0.0673	0.0736	0.0840	0.0944	0.0841	0.2460
04A60128130771	20	0.0909	0.0944	0.0935	0.0710	0.0841	0.0709	0.2940
04870128180155	48	0.1345	0.1146	0.1214	0.1326	0.1545	0.1318	0.1963
04870128150158	43	0.1140	0.0998	0.1048	0.1131	0.1302	0.1100	0.2074
04870128140262	26	0.1144	0.0964	0.1015	0.1781	0.1474	0.1243	0.2640
04870128011167	13	0.1245	0.1103	0.1150	0.3175	0.1813	0.1759	0.3610
04870128140771	13	0.1061	0.1098	0.1066	0.3150	0.1596	0.1604	0.3610
07050135190155	38	0.0859	0.0733	0.0778	0.0822	0.1180	0.0826	0.2206
07050135200158	34	0.0637	0.0779	0.0714	0.0629	0.0648	0.0619	0.2332
07050135150262	34	0.0775	0.0820	0.0742	0.0771	0.0949	0.0770	0.2332
07050135150771	20	0.1302	0.1221	0.1249	0.1313	0.1629	0.1338	0.2940
07060135140155	50	0.0515	0.0477	0.0491	0.0491	0.0871	0.0482	0.1923
07060135200158	47	0.0696	0.0644	0.0664	0.0681	0.0728	0.0673	0.1984
07060135010262	47	0.0758	0.0798	0.0788	0.0707	0.0589	0.0715	0.1984
07060135160771	19	0.1379	0.1195	0.1257	0.1374	0.1728	0.1414	0.3010
07070135260155	39	0.1539	0.1420	0.1464	0.1529	0.1501	0.1540	0.2178
07070135220362	23	0.0923	0.1249	0.1137	0.1384	0.0850	0.1261	0.2790
07070135160771	12	0.1082	0.1119	0.1108	0.1416	0.1573	0.1585	0.3750
08070135190155	52	0.0641	0.0453	0.0502	0.0663	0.1000	0.0652	0.1886
08070135170158	46	0.0693	0.0456	0.0530	0.0730	0.1051	0.0742	0.2005
08070135140262	46	0.0566	0.0460	0.0433	0.0575	0.0770	0.0556	0.2005
08070135190771	14	0.1958	0.1869	0.1901	0.1976	0.1955	0.1992	0.3490
08080135190155	57	0.0526	0.0631	0.0571	0.0544	0.0625	0.0550	0.1801
08080135200158	48	0.0546	0.0591	0.0570	0.0569	0.0710	0.0582	0.1963
08080135280362	33	0.0712	0.0544	0.0562	0.0772	0.1080	0.0795	0.2367
08080135200771	17	0.0691	0.0790	0.0761	0.1081	0.0869	0.0892	0.3180
34050141250156	65	0.1077	0.0828	0.0845	0.1003	0.1382	0.0990	0.1687
34050141280362	78	0.0634	0.0474	0.0386	0.0492	0.0718	0.0471	0.1540
34050141071271	45	0.0616	0.1229	0.0994	0.0694	0.0540	0.0720	0.2027
34060141240156	135	0.0802	0.1116	0.0990	0.0581	0.0571	0.0558	0.1171
34060141240162	105	0.0975	0.1544	0.1354	0.0772	0.0693	0.0724	0.1327
34060141091271	55	0.1905	0.2481	0.2289	0.1426	0.1583	0.1325	0.1834
35060143150156	139	0.0662	0.1021	0.0902	0.0474	0.0561	0.0478	0.1154
35060143150262	139	0.0457	0.0819	0.0704	0.0462	0.0543	0.0469	0.1154
35060143130571	68	0.0987	0.1426	0.1279	0.0777	0.0632	0.0779	0.1649
35060143140571	19	0.1268	0.0885	0.1015	0.1106	0.1467	0.1043	0.3010
35070143240156	95	0.0780	0.0659	0.0698	0.0761	0.1232	0.0734	0.1395
35070143120262	87	0.0866	0.0647	0.0694	0.0777	0.1034	0.0731	0.1458
35070143060671	95	0.0424	0.0892	0.0739	0.0500	0.0617	0.0520	0.1395
36070144270156	125	0.0677	0.0749	0.0734	0.0442	0.0543	0.0462	0.1216
36070144140262	91	0.0591	0.1019	0.0857	0.0599	0.0488	0.0591	0.1426
36070144150671	90	0.0722	0.1349	0.1127	0.0660	0.0562	0.0661	0.1434
37060144290156	104	0.0652	0.1086	0.0926	0.0757	0.0833	0.0783	0.1334
37060144280362	120	0.0786	0.1523	0.1260	0.0743	0.0589	0.0748	0.1242
37060144090671	98	0.1025	0.1662	0.1460	0.0819	0.0978	0.0801	0.1374
18080137140155	50	0.0687	0.0863	0.0751	0.0597	0.0549	0.0594	0.1923
18080137000158	50	0.0567	0.0874	0.0733	0.0519	0.0506	0.0514	0.1923

Table 7 (continued)

Plot Code	No.	N	LN	G	B	W	S_B	5%
18080137120262	50	0.0771	0.1170	0.1043	0.0733	0.0620	0.0693	0.1923
18080137110571	38	0.0919	0.1481	0.1301	0.1006	0.0935	0.0875	0.2206
18080237110155	45	0.0916	0.0905	0.0806	0.0968	0.1164	0.0975	0.2027
18080237150158	38	0.1176	0.0892	0.0970	0.1229	0.1533	0.1108	0.2206
18080237120262	38	0.1188	0.0999	0.1053	0.1271	0.1555	0.1288	0.2206
18080237030571	24	0.2127	0.1851	0.1942	0.2199	0.2419	0.2250	0.2740
21070138200155	71	0.0693	0.1164	0.1005	0.0416	0.0507	0.0369	0.1614
21070138170158	40	0.0861	0.0709	0.0763	0.0826	0.1057	0.0834	0.2150
21070138140262	40	0.1013	0.0742	0.0832	0.1011	0.1342	0.1022	0.2150
21070138250671	21	0.1496	0.1232	0.1323	0.1603	0.1737	0.1583	0.2890
22060138090155	56	0.0818	0.0968	0.0930	0.0819	0.0765	0.0819	0.1817
22060138170158	35	0.0853	0.0859	0.0771	0.0803	0.0685	0.0799	0.2299
22060138140362	29	0.0832	0.0833	0.0825	0.0962	0.1094	0.1015	0.2460
22060138270671	19	0.0813	0.0895	0.0869	0.0895	0.1167	0.1423	0.3010
23060138250155	74	0.0439	0.0687	0.0579	0.0485	0.0488	0.0491	0.1581
23060138200158	58	0.0688	0.0828	0.0761	0.0742	0.0826	0.0773	0.1786
23060138160262	54	0.0557	0.0937	0.0795	0.0657	0.0792	0.0684	0.1851
23060138290671	35	0.0878	0.1285	0.1137	0.0926	0.0832	0.0946	0.2299
24050138250155	59	0.0818	0.0842	0.0681	0.0813	0.0908	0.0797	0.1771
24050138170158	41	0.0430	0.0674	0.0548	0.0415	0.0633	0.0401	0.2124
24050138160262	39	0.0969	0.0905	0.0778	0.1088	0.1255	0.1055	0.2178
24050138300671	23	0.0814	0.1057	0.0895	0.2131	0.0830	0.1298	0.2790
24060138200155	59	0.0684	0.0715	0.0690	0.0714	0.0854	0.0728	0.1771
24060138150158	43	0.0812	0.0811	0.0793	0.0917	0.1005	0.0942	0.2074
24060138150262	35	0.0994	0.0678	0.0783	0.0981	0.1263	0.0969	0.2299
24060138300671	23	0.0837	0.0794	0.0813	0.0819	0.1091	0.0818	0.2790
25070138010255	58	0.0575	0.0694	0.0651	0.0666	0.0704	0.0677	0.1786
25070138210158	48	0.0649	0.0460	0.0512	0.0677	0.0914	0.0681	0.1963
25070138150262	36	0.0999	0.0649	0.0767	0.1034	0.1169	0.1026	0.2267
25070138060771	18	0.1336	0.1106	0.1186	0.1424	0.1403	0.1448	0.3090
26A60140260155	91	0.0483	0.0620	0.0539	0.0606	0.0643	0.0616	0.1426
26A60140210158	73	0.0911	0.0673	0.0748	0.0857	0.1277	0.0830	0.1592
26A60140210262	74	0.0565	0.0402	0.0405	0.0485	0.0793	0.0474	0.1581
26A60140220771	33	0.0682	0.0980	0.0858	0.0554	0.0673	0.0548	0.2367
09060135270155	52	0.0639	0.0451	0.0508	0.0619	0.1000	0.0605	0.1886
09060135210158	45	0.1051	0.0804	0.0886	0.1015	0.1413	0.0985	0.2027
09060135160262	45	0.1481	0.1241	0.1321	0.1466	0.1791	0.1458	0.2027
09060135200771	22	0.0883	0.0691	0.0727	0.0901	0.1134	0.0911	0.2840
14060136140155	61	0.0661	0.0725	0.0611	0.0546	0.0696	0.0518	0.1741

Figure 14

Plot of the maximum observed tree height against tree age, for all data sets. The curve for maximum height used in the iteration procedure is also plotted.

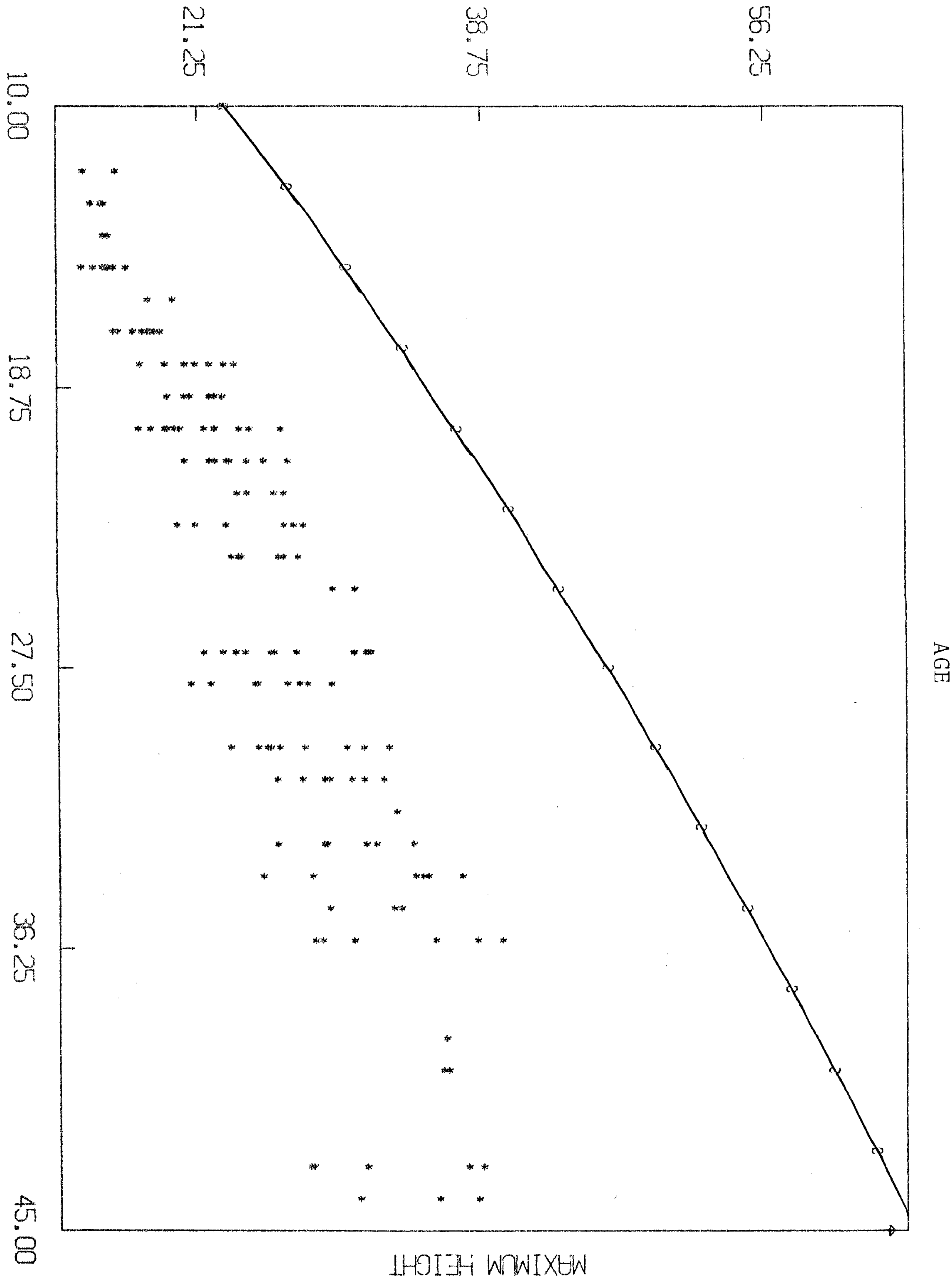

Table 8

Values of the maximized log-likelihood for the normal(N), lognormal(LN), gamma(G), beta(B), Weibull(W), and S_B for the height data.

Plot Code	No.	N	LN	G	B	W	S_B
31A50141230156	95	-166.480	-170.518	-168.914	-163.950	-164.181	-163.984
31A50141210362	85	-188.684	-195.140	-192.666	-186.271	-184.697	-185.955
31A50141230871	57	-114.535	-115.900	-115.371	-112.960	-113.604	-113.188
31A60141280155	101	-196.617	-204.754	-201.487	-193.593	-193.229	-193.554
31A60141220158	79	-156.003	-160.648	-158.879	-152.463	-152.450	-152.497
31A60141140362	76	-164.743	-168.503	-167.078	-160.888	-161.221	-161.001
31A60141230871	56	-135.784	-137.418	-136.783	-133.764	-134.164	-133.832
32A60141190156	97	-180.974	-181.980	-181.402	-180.381	-182.133	-180.364
32A60141210158	75	-142.838	-142.078	-142.189	-142.476	-146.262	-142.303
32A60141210262	57	-122.593	-124.513	-123.725	-121.568	-121.956	-121.623
32A60141250871	36	-98.477	-98.666	-98.495	-97.986	-98.618	-97.984
33A60141020255	115	-222.112	-229.317	-226.487	-217.952	-217.467	-217.626
33A60141220158	79	-139.073	-141.457	-140.484	-138.492	-139.324	-138.481
33A60141280362	69	-146.843	-151.307	-149.621	-143.475	-143.054	-143.294
33A60141061271	43	-92.804	-94.078	-93.592	-91.287	-91.678	-91.452
27070140250156	98	-195.831	-198.702	-197.480	-194.300	-194.976	-194.347
27070140150262	56	-107.314	-108.024	-107.734	-106.784	-107.608	-106.821
27070140270771	26	-54.674	-55.002	-54.879	-53.763	-54.163	-53.931
27080139280155	98	-177.647	-180.397	-179.309	-176.135	-176.591	-176.095
27080139210158	71	-133.720	-135.369	-134.730	-131.884	-132.518	-132.061
27080139150262	59	-120.554	-123.247	-122.270	-117.309	-117.014	-117.187
27080139230771	33	-85.090	-88.941	-87.553	-76.651	-79.822	-77.335
28A60140260155	77	-133.017	-134.560	-133.836	-133.011	-134.839	-133.032
28A60140200158	61	-117.840	-117.277	-117.305	-117.793	-123.053	-117.688
28A60140160262	46	-99.359	-102.059	-101.039	-96.794	-97.007	-96.895
28A60140280771	35	-85.872	-89.353	-88.103	-79.673	-81.392	-80.177
29070140280155	88	-149.218	-153.037	-151.617	-144.312	-144.539	-144.330
29070140210158	66	-117.319	-118.793	-118.203	-116.267	-117.003	-116.359
29070140210362	65	-125.701	-127.229	-126.621	-125.230	-126.273	-125.200
29070140200871	44	-97.531	-101.437	-100.041	-90.304	-91.153	-90.206
30860143240156	109	-188.113	-190.833	-189.645	-187.532	-188.698	-187.512
30860143140362	101	-223.498	-230.695	-227.984	-214.369	-216.240	-214.084
30860143151271	92	-238.139	-244.067	-241.768	-229.737	-233.111	-229.982
30870143240156	135	-254.577	-259.105	-257.193	-250.516	-252.334	-251.047
30870143210262	117	-271.760	-278.438	-275.710	-267.288	-268.120	-267.184
30870143101271	109	-297.207	-306.791	-302.859	-292.147	-293.296	-292.172
01A50127260155	43	-66.942	-67.276	-67.133	-66.770	-68.008	-66.776
01A50127160158	40	-73.596	-74.019	-73.848	-73.299	-74.084	-73.317
01A50127130262	40	-74.254	-73.905	-74.000	-74.191	-77.437	-74.149
01A50127080771	25	-48.431	-48.464	-48.441	-48.362	-48.955	-48.378
01A60127170155	33	-52.873	-53.030	-52.953	-52.774	-53.755	-52.789
01A60127160158	32	-53.039	-52.815	-52.867	-53.074	-56.499	-53.073
01A60127120262	32	-58.941	-60.140	-59.727	-55.394	-54.980	-55.070
01A60127011167	16	-27.457	-27.898	-27.746	-27.632	-26.024	-26.068
01A60127080771	16	-24.963	-25.241	-25.141	-23.573	-23.606	-23.493
01A70127180155	39	-72.943	-74.095	-73.668	-71.931	-72.127	-71.878
01A70127160158	36	-64.873	-64.473	-64.586	-64.961	-68.669	-64.971
01A70127130262	36	-45.780	-46.155	-45.938	-31.990	-43.704	-43.498
01A70127011167	22	-32.901	-32.996	-32.938	-32.835	-33.802	-32.832
01A70127090771	22	-41.818	-41.993	-41.926	-42.250	-42.037	-41.618
02A70128200155	38	-54.894	-54.765	-54.797	-53.112	-57.014	-54.947
02A70128170158	34	-54.994	-55.330	-55.211	-54.655	-55.524	-54.639
02A70128140262	30	-41.265	-41.075	-41.141	-41.330	-44.785	-41.355
02A70128120771	19	-27.704	-27.559	-27.609	-36.976	-30.471	-27.736
03060128180155	51	-96.876	-96.795	-96.770	-96.790	-99.101	-96.757

Table 8 (continued)

Plot Code	No.	N	LN	G	B	W	S_B
03060128170158	43	-81.931	-83.365	-82.852	-78.641	-78.909	-78.640
03060128140262	35	-58.378	-59.168	-58.887	-56.257	-56.273	-55.883
03060128130771	35	-62.335	-62.983	-62.742	-60.750	-61.115	-60.940
04A50128180155	38	-56.265	-55.725	-55.883	-56.129	-60.692	-56.045
04A50128170158	37	-61.404	-61.220	-61.254	-61.342	-64.075	-61.308
04A50128140771	25	-40.537	-40.755	-40.668	-40.435	-41.156	-40.418
04A60128180155	35	-75.605	-75.323	-75.361	-75.490	-77.423	-75.431
04A60128160158	34	-69.307	-69.297	-69.268	-69.246	-70.293	-69.225
04A60128130262	29	-58.429	-58.599	-58.523	-57.417	-58.280	-57.884
04A60128130771	20	-42.111	-42.551	-42.389	-41.439	-41.549	-41.477
04B70128180155	48	-95.815	-95.983	-95.883	-95.829	-98.245	-95.832
04B70128150158	43	-88.989	-89.539	-89.309	-88.818	-89.966	-88.820
04B70128140262	26	-50.855	-51.154	-51.035	-50.257	-50.572	-50.343
04B70128011167	13	-25.744	-25.917	-25.857	-15.596	-24.858	-24.066
04B70128140771	13	-27.012	-27.192	-27.129	-18.105	-26.280	-25.778
07050135190155	38	-59.230	-59.993	-59.713	-57.016	-57.866	-57.465
07050135200158	34	-57.961	-58.612	-58.367	-56.364	-57.002	-56.690
07050135150262	34	-56.779	-57.194	-57.035	-54.155	-55.971	-55.532
07050135150771	20	-32.881	-32.970	-32.938	-32.832	-33.478	-32.835
07060135140155	50	-79.442	-79.531	-79.453	-79.402	-81.366	-79.387
07060135200158	47	-78.023	-77.484	-77.625	-77.889	-81.670	-77.806
07060135010262	47	-88.696	-89.260	-89.012	-88.569	-89.791	-88.574
07060135160771	19	-37.341	-37.335	-37.326	-37.315	-37.866	-37.306
07070135260155	39	-80.813	-81.403	-81.148	-80.551	-81.153	-80.558
07070135220362	23	-48.816	-49.515	-49.268	-45.250	-46.959	-46.324
07070135160771	12	-23.144	-22.997	-23.047	-23.300	-24.179	-23.215
08070135190155	52	-106.959	-109.446	-108.527	-103.260	-103.739	-103.388
08070135170158	46	-93.505	-95.433	-94.723	-90.590	-91.031	-90.813
08070135140262	46	-102.127	-104.720	-103.773	-98.433	-98.515	-98.416
08070135190771	14	-24.322	-24.285	-24.301	-22.938	-24.659	-24.317
08080135190155	57	-123.162	-124.763	-124.104	-122.897	-123.619	-122.890
08080135200158	48	-104.960	-108.601	-107.264	-99.962	-100.216	-99.795
08080135280362	33	-67.040	-68.116	-67.730	-65.293	-65.209	-65.262
08080135200771	17	-38.464	-38.766	-38.652	-37.901	-38.009	-37.940
34050141250156	65	-101.757	-102.762	-102.309	-101.291	-102.330	-101.330
34050141280362	78	-163.126	-167.261	-165.682	-159.359	-159.310	-159.164
34050141071271	45	-104.073	-110.368	-108.106	-96.926	-96.695	-96.115
34060141240156	135	-238.855	-242.051	-240.646	-237.526	-238.852	-237.546
34060141240162	105	-235.627	-246.258	-242.164	-228.676	-228.227	-228.011
34060141091271	55	-136.069	-141.747	-139.703	-124.993	-128.802	-125.403
35060143150156	139	-203.696	-205.860	-204.938	-202.361	-204.259	-202.438
35060143150262	139	-264.457	-270.440	-268.082	-263.267	-263.112	-263.138
35060143130571	68	-133.665	-138.764	-136.879	-129.534	-128.176	-129.026
35060143140571	19	-33.390	-34.457	-34.073	-31.783	-31.691	-31.714
35070143240156	95	-123.874	-125.842	-125.078	-122.763	-123.820	-122.728
35070143120262	87	-158.259	-156.991	-157.250	-158.212	-168.017	-158.040
35070143060671	95	-210.944	-217.805	-215.146	-208.527	-207.406	-208.296
36070144270156	125	-198.776	-202.762	-201.203	-196.601	-196.823	-196.489
36070144140262	91	-183.415	-189.227	-187.055	-177.933	-177.497	-177.724
36070144150671	90	-217.451	-224.047	-221.576	-210.820	-211.227	-210.634
37060144290156	104	-165.011	-169.466	-167.680	-163.334	-163.653	-163.337
37060144280362	120	-261.001	-274.314	-269.045	-255.537	-254.587	-255.015
37060144090671	98	-240.267	-255.498	-249.461	-235.221	-234.031	-234.512
18080137140155	50	-100.554	-102.969	-102.066	-98.217	-98.159	-98.192
18080137000158	50	-109.267	-112.278	-111.180	-105.145	-105.176	-104.914

Table 8 (continued)

Plot Code	No.	N	LN	G	B	W	S_B
18080137120262	50	-119.030	-123.524	-121.900	-109.356	-112.424	-110.057
18080137110571	38	-107.940	-112.071	-110.569	-98.263	-103.192	-98.953
18080237110155	45	-86.363	-86.986	-86.725	-86.050	-86.780	-86.056
18080237150158	38	-73.863	-74.330	-74.141	-73.306	-73.838	-73.392
18080237120262	38	-80.759	-81.278	-81.072	-79.153	-80.092	-79.679
18080237030571	24	-52.338	-52.825	-52.645	-51.658	-51.743	-51.660
21070138200155	71	-146.113	-148.424	-147.401	-145.105	-145.816	-145.202
21070138170158	40	-86.477	-87.033	-86.775	-85.839	-86.438	-85.941
21070138140262	40	-88.914	-90.012	-89.578	-87.094	-88.042	-87.585
21070138250671	21	-47.719	-49.714	-49.014	-36.072	-43.526	-42.187
22060138090155	56	-141.791	-152.691	-147.927	-135.303	-140.139	-136.534
22060138170158	35	-72.566	-72.845	-72.700	-72.392	-73.066	-72.390
22060138140362	29	-58.202	-58.738	-58.531	-57.557	-57.854	-57.631
22060138270671	19	-39.330	-39.565	-39.477	-39.004	-39.242	-39.041
23060138250155	74	-153.275	-156.599	-155.216	-151.109	-151.766	-151.269
23060138200158	58	-110.404	-113.112	-112.072	-108.426	-108.493	-108.450
23060138160262	54	-116.550	-120.002	-118.697	-113.429	-113.537	-113.473
23060138290671	35	-83.466	-85.370	-84.625	-78.474	-82.044	-81.561
24050138250155	59	-125.309	-133.264	-130.188	-119.810	-121.050	-119.959
24050138170158	41	-88.870	-92.883	-91.397	-84.991	-85.250	-84.789
24050138160262	39	-88.398	-91.526	-90.353	-86.185	-85.773	-85.999
24050138300671	23	-60.657	-63.876	-62.671	-58.255	-58.468	-58.240
24060138200155	59	-118.921	-124.282	-122.154	-116.085	-116.176	-116.020
24060138150158	43	-81.044	-82.467	-81.896	-80.437	-80.729	-80.452
24060138150262	35	-66.394	-66.809	-66.629	-66.159	-66.756	-66.170
24060138300671	23	-48.602	-48.347	-48.414	-48.554	-50.051	-48.523
25070138010255	58	-108.815	-110.793	-109.966	-107.525	-108.222	-107.727
25070138210158	48	-100.412	-103.070	-102.019	-98.845	-98.992	-98.902
25070138150262	36	-75.447	-75.668	-75.549	-75.428	-76.727	-75.424
25070138060771	18	-28.410	-28.286	-28.320	-28.671	-29.853	-28.427
26A60140260155	91	-156.196	-161.635	-159.447	-154.726	-154.621	-154.706
26A60140210158	73	-119.076	-119.066	-118.996	-118.962	-121.598	-118.923
26A60140210262	74	-153.993	-153.555	-153.490	-154.200	-161.669	-154.142
26A60140220771	33	-77.239	-78.239	-77.849	-74.557	-76.116	-75.331
09060135270155	52	-102.136	-103.106	-102.674	-101.799	-102.611	-101.824
09060135210158	45	-81.082	-81.433	-81.273	-80.916	-81.971	-80.926
09060135160262	45	-85.415	-87.200	-86.551	-82.222	-82.699	-82.414
09060135200771	22	-37.752	-37.967	-37.883	-37.515	-37.933	-37.528
14060136140155	61	-127.241	-128.962	-128.225	-126.072	-126.546	-126.114

Table 9

Values of the Kolmogorov-Smirnov statistic d for the normal(N), lognormal(LN), gamma(G), beta(B), Weibull(W), and S_B, with the critical value for the 5% significance level; height data.

Plot Code	No.	N	LN	G	B	W	S_B	5%
31A50141230156	95	0.0550	0.0794	0.0718	0.0640	0.0587	0.0618	0.1395
31A50141210362	85	0.0727	0.0953	0.0871	0.0799	0.0761	0.0816	0.1475
31A50141230871	57	0.0395	0.0511	0.0471	0.0764	0.0703	0.0658	0.1801
31A60141280155	101	0.0452	0.0832	0.0706	0.0555	0.0519	0.0576	0.1353
31A60141220158	79	0.0924	0.1197	0.1106	0.0820	0.0686	0.0781	0.1530
31A60141140362	76	0.0679	0.0938	0.0851	0.0607	0.0539	0.0594	0.1560
31A60141230871	56	0.0574	0.0717	0.0666	0.0417	0.0473	0.0412	0.1817
32A60141190156	97	0.0456	0.0419	0.0410	0.0574	0.0746	0.0534	0.1381
32A60141210158	75	0.0928	0.0795	0.0784	0.0881	0.1192	0.0849	0.1570
32A60141210262	57	0.0435	0.0655	0.0575	0.0757	0.0834	0.0739	0.1801
32A60141250871	36	0.1201	0.1230	0.1226	0.1124	0.1093	0.1148	0.2267
33A60141020255	115	0.0860	0.1061	0.1001	0.0667	0.0609	0.0661	0.1268
33A60141220158	79	0.0777	0.0738	0.0748	0.0901	0.1076	0.0905	0.1530
33A60141280362	69	0.0825	0.1108	0.1014	0.0645	0.0546	0.0652	0.1637
33A60141061271	43	0.0793	0.0947	0.0900	0.0886	0.0798	0.0802	0.2074
27070140250156	98	0.0540	0.0735	0.0674	0.0540	0.0528	0.0521	0.1374
27070140150262	56	0.0510	0.0539	0.0494	0.0639	0.0713	0.0635	0.1817
27070140270771	26	0.0886	0.0925	0.0912	0.1080	0.1140	0.1101	0.2640
27080139280155	98	0.0465	0.0635	0.0582	0.0641	0.0767	0.0641	0.1374
27080139210158	71	0.0511	0.0625	0.0588	0.0623	0.0682	0.0631	0.1614
27080139150262	59	0.0892	0.1089	0.1024	0.0811	0.0849	0.0820	0.1771
27080139230771	33	0.1207	0.1520	0.1419	0.1047	0.0749	0.0602	0.2367
28A60140260155	77	0.0615	0.0621	0.0538	0.0685	0.0985	0.0689	0.1550
28A60140200158	61	0.1482	0.1379	0.1409	0.1486	0.1921	0.1472	0.1741
28A60140160262	46	0.0786	0.1050	0.0962	0.0947	0.0943	0.0956	0.2005
28A60140280771	35	0.1452	0.1802	0.1688	0.1496	0.0834	0.1199	0.2299
29070140280155	88	0.0686	0.0799	0.0757	0.0622	0.0502	0.0572	0.1450
29070140210158	66	0.0514	0.0522	0.0476	0.0793	0.0896	0.0777	0.1674
29070140210362	65	0.0737	0.0775	0.0759	0.0766	0.0886	0.0766	0.1687
29070140200871	44	0.1107	0.1375	0.1288	0.0641	0.0422	0.0472	0.2050
30B60143240156	109	0.0444	0.0537	0.0498	0.0452	0.0626	0.0453	0.1303
30B60143140362	101	0.1111	0.1234	0.1196	0.0736	0.0926	0.0737	0.1353
30B60143151271	92	0.1215	0.1409	0.1350	0.0634	0.0956	0.0679	0.1418
30B70143240156	135	0.0484	0.0574	0.0547	0.0529	0.0474	0.0472	0.1171
30B70143210262	117	0.0628	0.0821	0.0763	0.0556	0.0671	0.0584	0.1257
30B70143101271	109	0.0799	0.1213	0.1084	0.0439	0.0451	0.0436	0.1303
01A50127260155	43	0.0790	0.0723	0.0746	0.0924	0.1287	0.0911	0.2074
01A50127160158	40	0.0703	0.0619	0.0648	0.0894	0.1182	0.0863	0.2150
01A50127130262	40	0.0876	0.0752	0.0794	0.0861	0.1310	0.0852	0.2150
01A50127080771	25	0.1259	0.1143	0.1182	0.1452	0.1644	0.1347	0.2700
01A60127170155	33	0.1159	0.1082	0.1106	0.1242	0.1326	0.1216	0.2367
01A60127160158	32	0.0910	0.0867	0.0880	0.0902	0.0913	0.0910	0.2404
01A60127120262	32	0.1267	0.1320	0.1301	0.1290	0.1113	0.1235	0.2404
01A60127011167	16	0.0665	0.0742	0.0713	0.2708	0.1236	0.1216	0.3280
01A60127080771	16	0.0864	0.0909	0.0891	0.2411	0.1437	0.1457	0.3280
01A70127180155	39	0.0987	0.0875	0.0909	0.1299	0.1484	0.1319	0.2178
01A70127160158	36	0.0855	0.0833	0.0840	0.0851	0.1199	0.0857	0.2267
01A70127130262	36	0.0896	0.0937	0.0920	0.3131	0.0935	0.0897	0.2267
01A70127011167	22	0.1041	0.1022	0.1028	0.1143	0.1528	0.1127	0.2840
01A70127090771	22	0.0691	0.0596	0.0630	0.3272	0.1182	0.0910	0.2840
02A70128200155	38	0.2078	0.2024	0.2042	0.2923	0.2301	0.2102	0.2206
02A70128170158	34	0.1212	0.1136	0.1159	0.1408	0.1720	0.1420	0.2332
02A70128140262	30	0.1232	0.1242	0.1241	0.1223	0.1795	0.1238	0.2400
02A70128120771	19	0.1515	0.1451	0.1471	0.3570	0.2052	0.1515	0.3010
03060128180155	51	0.0665	0.0645	0.0647	0.0648	0.0876	0.0640	0.1904

Table 9 (continued)

Plot Code	No.	N	LN	G	B	W	S_B	5%
03060128170158	43	0.1013	0.1153	0.1110	0.0732	0.0746	0.0731	0.2074
03060128140262	35	0.0717	0.0808	0.0777	0.1572	0.1093	0.1131	0.2299
03060128130771	35	0.0839	0.0742	0.0772	0.1443	0.1391	0.1376	0.2299
04A50128180155	38	0.1344	0.1260	0.1288	0.1321	0.1620	0.1312	0.2206
04A50128170158	37	0.1248	0.1134	0.1173	0.1233	0.1761	0.1209	0.2236
04A50128140771	25	0.0598	0.0546	0.0564	0.0665	0.0864	0.0689	0.2700
04A60128180155	35	0.1091	0.1022	0.1047	0.1067	0.1090	0.1061	0.2299
04A60128160158	34	0.1138	0.0964	0.1021	0.1142	0.1585	0.1124	0.2332
04A60128130262	29	0.1594	0.1507	0.1536	0.1780	0.1798	0.1785	0.2460
04A60128130771	20	0.1183	0.1309	0.1270	0.1121	0.1093	0.1074	0.2940
04870128180155	48	0.0915	0.0826	0.0857	0.0949	0.1394	0.0950	0.1963
04870128150158	43	0.0669	0.0668	0.0666	0.0717	0.0935	0.0714	0.2074
04870128140262	26	0.0787	0.0818	0.0807	0.1181	0.1281	0.1146	0.2640
04870128011167	13	0.1337	0.1382	0.1364	0.2727	0.1701	0.1684	0.3610
04870128140771	13	0.1493	0.1503	0.1500	0.2615	0.1452	0.1270	0.3610
07050135190155	38	0.0730	0.0763	0.0751	0.0802	0.0745	0.0717	0.2206
07050135200158	34	0.1000	0.0996	0.0999	0.0939	0.0980	0.0953	0.2332
07050135150262	34	0.1031	0.0960	0.0984	0.2046	0.1233	0.1216	0.2332
07050135150771	20	0.0876	0.0827	0.0844	0.0969	0.1334	0.0994	0.2940
07060135140155	50	0.0905	0.0742	0.0799	0.0893	0.1289	0.0885	0.1923
07060135200158	47	0.1238	0.1101	0.1150	0.1213	0.1492	0.1197	0.1984
07060135010262	47	0.0736	0.0582	0.0631	0.0863	0.1190	0.0860	0.1984
07060135160771	19	0.1294	0.1272	0.1282	0.1292	0.1200	0.1289	0.3010
07070135260155	39	0.0509	0.0566	0.0543	0.0685	0.0934	0.0672	0.2178
07070135220362	23	0.1359	0.1495	0.1454	0.2171	0.1042	0.1013	0.2790
07070135160771	12	0.2391	0.2307	0.2338	0.3574	0.2715	0.2438	0.3750
08070135190155	52	0.0832	0.1065	0.0989	0.0779	0.0708	0.0807	0.1886
08070135170158	46	0.0533	0.0683	0.0631	0.0981	0.0911	0.0960	0.2005
08070135140262	46	0.0687	0.0853	0.0792	0.1149	0.0979	0.1129	0.2005
08070135190771	14	0.2255	0.2192	0.2211	0.2970	0.2611	0.2286	0.3490
08080135190155	57	0.0604	0.0601	0.0577	0.0742	0.0982	0.0754	0.1801
08080135200158	48	0.1234	0.1409	0.1356	0.1042	0.0783	0.1005	0.1963
08080135280362	33	0.1055	0.1204	0.1156	0.0955	0.0918	0.0932	0.2367
08080135200771	17	0.0981	0.0986	0.0983	0.1157	0.1125	0.1106	0.3180
34050141250156	65	0.0714	0.0525	0.0588	0.0828	0.0899	0.0814	0.1687
34050141280362	78	0.1007	0.1185	0.1131	0.0873	0.0760	0.0876	0.1540
34050141071271	45	0.1634	0.2033	0.1906	0.1157	0.0911	0.1043	0.2027
34060141240156	135	0.0517	0.0635	0.0589	0.0567	0.0672	0.0570	0.1171
34060141240162	105	0.0707	0.0994	0.0896	0.0664	0.0482	0.0625	0.1327
34060141091271	55	0.1874	0.2175	0.2081	0.1105	0.1222	0.1096	0.1834
35060143150156	139	0.0574	0.0568	0.0574	0.0720	0.0930	0.0698	0.1154
35060143150262	139	0.0710	0.0830	0.0783	0.0705	0.0696	0.0706	0.1154
35060143130571	68	0.1100	0.1230	0.1181	0.1115	0.0971	0.1092	0.1649
35060143140571	19	0.0742	0.0852	0.0794	0.0916	0.0793	0.0851	0.3010
35070143240156	95	0.0520	0.0675	0.0616	0.0527	0.0773	0.0535	0.1395
35070143120262	87	0.0709	0.0558	0.0611	0.0686	0.1019	0.0670	0.1458
35070143060671	95	0.0633	0.0889	0.0800	0.0546	0.0599	0.0573	0.1395
36070144270156	125	0.0465	0.0661	0.0599	0.0452	0.0468	0.0447	0.1216
36070144140262	91	0.0673	0.0937	0.0848	0.0707	0.0558	0.0684	0.1426
36070144150671	90	0.1049	0.1319	0.1234	0.0648	0.0541	0.0649	0.1434
37060144290156	104	0.0466	0.0443	0.0425	0.0654	0.0753	0.0668	0.1334
37060144280362	120	0.0603	0.1030	0.0890	0.0455	0.0392	0.0494	0.1242
37060144090671	98	0.1040	0.1511	0.1363	0.0827	0.0629	0.0796	0.1374
18080137140155	50	0.0788	0.0955	0.0896	0.0999	0.1011	0.1023	0.1923
18080137000158	50	0.0873	0.1038	0.0988	0.0866	0.0679	0.0874	0.1923

Table 9 (continued)

Plot Code	No.	N	LN	G	B	W	S_B	5%
18080137120262	50	0.1040	0.1338	0.1241	0.0934	0.0742	0.0537	0.1923
18080137110571	38	0.1620	0.1812	0.1757	0.0901	0.1202	0.0756	0.2206
18080237110155	45	0.0463	0.0550	0.0518	0.0532	0.0715	0.0529	0.2027
18080237150158	38	0.0718	0.0576	0.0622	0.0963	0.1045	0.0940	0.2206
18080237120262	38	0.1186	0.1084	0.1118	0.1373	0.1402	0.1384	0.2206
18080237030571	24	0.0729	0.0780	0.0760	0.0748	0.0751	0.0723	0.2740
21070138200155	71	0.0410	0.0450	0.0385	0.0724	0.0785	0.0681	0.1614
21070138170158	40	0.0937	0.1011	0.0988	0.0819	0.0857	0.0805	0.2150
21070138140262	40	0.0594	0.0674	0.0624	0.1282	0.1082	0.1154	0.2150
21070138250671	21	0.1470	0.1705	0.1625	0.2362	0.0820	0.0778	0.2890
22060138090155	56	0.0929	0.1321	0.1214	0.0595	0.0739	0.0396	0.1817
22060138170158	35	0.0596	0.0713	0.0636	0.0563	0.0718	0.0565	0.2299
22060138140362	29	0.1072	0.1104	0.1093	0.1271	0.1335	0.1205	0.2460
22060138270671	19	0.0887	0.0880	0.0885	0.0961	0.1071	0.0905	0.3010
23060138250155	74	0.0501	0.0525	0.0446	0.0717	0.0775	0.0725	0.1581
23060138200158	58	0.0440	0.0507	0.0479	0.0677	0.0691	0.0671	0.1786
23060138160262	54	0.0841	0.1102	0.1015	0.0663	0.0559	0.0655	0.1851
23060138290671	35	0.0908	0.1128	0.1061	0.2490	0.0846	0.0900	0.2299
24050138250155	59	0.0763	0.1279	0.1108	0.0629	0.0581	0.0550	0.1771
24050138170158	41	0.1272	0.1575	0.1483	0.0949	0.0813	0.0876	0.2124
24050138160262	39	0.0997	0.1327	0.1218	0.0898	0.0751	0.0897	0.2178
24050138300671	23	0.1543	0.1740	0.1618	0.2053	0.1559	0.1927	0.2790
24060138200155	59	0.0568	0.0847	0.0741	0.0570	0.0697	0.0615	0.1771
24060138150158	43	0.0480	0.0469	0.0426	0.0668	0.0787	0.0655	0.2074
24060138150262	35	0.0502	0.0537	0.0508	0.0556	0.0750	0.0562	0.2299
24060138300671	23	0.1808	0.1656	0.1709	0.1797	0.2094	0.1786	0.2790
25070138010255	58	0.0800	0.0613	0.0630	0.1212	0.1218	0.1161	0.1786
25070138210158	48	0.0788	0.0965	0.0912	0.0757	0.0816	0.0776	0.1963
25070138150262	36	0.0662	0.0617	0.0630	0.0686	0.1005	0.0688	0.2267
25070138060771	18	0.1667	0.1586	0.1611	0.3371	0.2117	0.1678	0.3090
26A60140260155	91	0.0349	0.0501	0.0439	0.0583	0.0648	0.0589	0.1426
26A60140210158	73	0.0779	0.0676	0.0710	0.0760	0.1028	0.0752	0.1592
26A60140210262	74	0.0630	0.0562	0.0532	0.0615	0.0831	0.0596	0.1581
26A60140220771	33	0.0884	0.1039	0.0989	0.1089	0.0956	0.1074	0.2367
09060135270155	52	0.0964	0.0771	0.0832	0.1186	0.1398	0.1157	0.1886
09060135210158	45	0.0725	0.0635	0.0666	0.0864	0.1170	0.0839	0.2027
09060135160262	45	0.0617	0.0745	0.0698	0.0772	0.0790	0.0816	0.2027
09060135200771	22	0.0666	0.0598	0.0621	0.0765	0.0958	0.0770	0.2840
14060136140155	61	0.0665	0.0860	0.0798	0.0565	0.0649	0.0569	0.1741

Table 11

Estimated parameter values for the three parameter S_B distribution for both diameter and height data, together with the estimates of ρ, ϕ and θ.

	No.	$\hat{\gamma}_1$	$\hat{\delta}_1$	$\hat{\lambda}_1$	$\hat{\gamma}_2$	$\hat{\delta}_2$	$\hat{\lambda}_2$	$\hat{\rho}$	ϕ	θ
1	95	2.316	4.188	42.319	-2.022	2.191	15.261	0.568	1.085	4.587
2	85	-0.581	2.470	36.080	-1.893	2.268	24.143	0.681	0.741	1.935
3	57	-0.311	3.093	49.723	-3.283	2.567	27.266	0.596	0.718	3.342
4	101	-0.470	1.890	28.927	-1.642	1.908	15.900	0.793	0.786	1.945
5	79	-0.913	1.947	32.513	-2.330	2.027	19.345	0.677	0.650	2.328
6	76	-0.715	1.991	40.152	-2.577	2.005	24.798	0.744	0.739	2.772
7	56	-0.461	2.024	52.315	-2.539	2.147	32.810	0.823	0.776	2.735
8	97	0.374	3.451	38.507	-1.133	2.896	19.235	0.630	0.750	1.604
9	75	1.153	4.042	47.653	2.009	4.885	33.207	0.666	0.551	0.776
10	57	0.746	3.392	55.668	-2.162	2.459	25.079	0.692	0.954	2.971
11	36	0.582	2.317	65.445	-0.983	2.280	36.771	0.835	0.848	1.905
12	115	-0.571	1.990	28.600	-1.584	1.918	15.393	0.721	0.749	1.843
13	79	1.217	4.033	47.640	-2.199	3.248	20.730	0.516	0.641	2.388
14	69	0.949	3.272	55.667	-2.373	2.042	22.685	0.642	1.029	4.308
15	43	0.344	2.929	65.427	-3.079	2.350	29.365	0.699	0.871	4.106
16	98	-0.814	1.905	33.608	-1.737	2.390	19.865	0.625	0.498	1.672
17	56	-1.044	2.775	46.581	-3.083	3.474	27.872	0.563	0.450	2.051
18	26	-1.561	0.886	45.570	-3.597	2.325	31.771	0.666	0.254	3.006
19	98	-1.157	1.959	32.335	-2.601	2.923	21.177	0.553	0.370	1.956
20	71	0.742	3.954	52.048	-3.248	2.636	24.036	0.438	0.657	3.878
21	59	0.451	3.279	58.640	-3.433	2.336	27.869	0.731	1.026	5.008
22	33	-0.191	1.956	66.677	-2.694	1.439	33.376	0.832	1.132	5.824
23	77	-0.272	1.891	33.578	-0.883	3.627	20.430	0.706	0.368	1.210
24	61	-0.972	1.479	33.346	1.568	5.014	34.562	0.546	0.161	0.658
25	46	-1.097	1.632	40.686	-2.567	1.957	24.320	0.777	0.648	2.402
26	35	-1.174	1.636	51.367	-2.549	1.479	29.261	0.799	0.884	2.973
27	88	-0.480	2.483	32.198	-2.864	2.103	16.764	0.531	0.627	3.456
28	66	-0.436	2.701	37.788	-3.118	2.990	22.367	0.547	0.494	2.619
29	65	0.034	2.881	47.073	-2.929	3.871	29.996	0.641	0.477	2.143
30	44	-0.471	2.180	50.669	-3.236	1.788	28.956	0.791	0.964	4.960
31	109	0.831	2.395	35.810	-1.372	2.971	17.391	0.783	0.632	1.975
32	101	1.276	2.318	52.059	-2.079	1.697	20.726	0.745	1.018	5.965
33	92	1.216	1.998	63.926	-1.811	1.424	26.670	0.839	1.177	7.302
34	135	-0.337	1.782	29.580	-1.837	1.949	15.814	0.730	0.667	2.261
35	117	-0.177	1.795	38.995	-1.645	1.785	22.166	0.725	0.729	2.340
36	109	-0.054	1.490	47.234	-1.313	1.532	28.203	0.850	0.827	2.287
37	43	1.459	4.611	63.949	-4.760	5.426	30.111	0.408	0.347	2.683
38	40	1.021	4.515	65.779	-3.976	4.392	32.741	0.248	0.255	2.620
39	40	0.035	4.103	65.211	2.035	8.762	55.050	0.202	0.094	0.793
40	25	-2.232	2.558	56.710	-3.449	5.593	41.468	0.101	0.046	1.779
41	33	-0.152	4.087	63.950	-4.451	6.918	36.870	0.396	0.234	1.887
42	32	-0.525	4.044	66.089	-0.999	9.894	50.560	0.424	0.173	1.082
43	32	-1.083	3.685	68.121	-4.964	2.344	33.681	0.397	0.625	6.921
44	16	-2.292	3.346	69.761	-5.745	2.599	36.650	-0.174	-0.224	10.632
45	16	-2.523	2.869	70.601	-5.964	2.400	35.966	0.163	0.195	10.103
46	39	-2.459	2.191	41.418	-4.488	3.349	31.581	0.644	0.422	2.380
47	36	-2.430	2.513	46.431	-1.663	8.492	50.565	0.371	0.110	1.094
48	36	-2.019	2.724	53.966	-6.831	2.641	33.224	0.574	0.592	8.560
49	22	-2.934	3.644	62.186	-8.446	7.441	43.643	0.284	0.139	2.782
50	22	-3.271	2.982	61.396	-5.918	4.784	44.381	0.602	0.375	2.283
51	38	-0.471	3.865	63.080	-4.541	10.656	45.904	0.334	0.121	1.509
52	34	-0.795	3.766	65.447	-6.667	5.206	37.154	0.197	0.143	3.492
53	30	-1.397	3.440	67.685	-4.336	13.427	53.942	0.209	0.053	1.351
54	19	-2.932	2.982	67.017	-1.986	15.367	63.567	0.278	0.054	1.079
55	51	2.486	6.110	63.080	2.572	6.814	45.804	0.494	0.443	0.821

Table 11 (continued)

	No.	$\hat{\gamma}_1$	$\hat{\delta}_1$	$\hat{\lambda}_1$	$\hat{\gamma}_2$	$\hat{\delta}_2$	$\hat{\lambda}_2$	$\hat{\rho}$	ϕ	θ
56	43	1.741	5.751	65.061	-3.550	2.080	24.841	0.509	1.407	8.436
57	35	0.792	5.426	67.652	-4.303	2.137	25.146	0.353	0.896	8.538
58	35	-0.327	4.257	70.421	-4.489	2.505	28.805	0.400	0.679	5.698
59	38	2.694	5.326	63.085	4.826	10.247	45.906	0.524	0.272	0.717
60	37	2.316	5.307	65.442	4.125	9.247	49.419	0.433	0.248	0.713
61	25	0.112	3.850	70.411	-5.288	5.487	33.684	0.597	0.419	2.653
62	35	0.848	4.055	63.087	1.730	5.303	45.906	0.581	0.444	0.792
63	34	0.388	3.802	65.443	0.560	5.913	44.251	0.339	0.218	0.930
64	29	-0.419	3.538	67.682	-3.544	2.190	29.057	0.221	0.357	4.835
65	20	-1.932	1.640	59.296	-3.795	2.639	33.509	0.394	0.245	3.158
66	48	0.292	4.351	63.079	-1.393	6.324	45.906	0.443	0.305	1.272
67	43	-0.066	4.108	65.420	-3.403	4.679	41.055	0.596	0.523	2.052
68	26	-1.683	3.545	62.305	-4.957	3.161	37.691	0.599	0.672	3.487
69	13	-3.901	2.411	58.456	-4.071	1.645	36.424	0.331	0.485	5.426
70	13	-3.674	2.246	63.664	-3.871	1.599	38.710	0.398	0.559	4.513
71	38	0.977	5.448	53.951	-3.475	2.198	17.526	0.337	0.836	5.647
72	34	0.286	5.020	58.635	-3.399	2.124	20.091	0.115	0.272	5.032
73	34	-1.736	3.213	52.984	-3.944	2.045	23.470	0.353	0.554	5.099
74	20	-2.319	4.883	68.514	-4.907	5.895	35.251	0.002	0.002	2.297
75	50	1.375	3.836	53.949	0.708	6.415	30.656	0.637	0.381	1.027
76	47	0.202	3.572	50.865	2.786	7.813	41.030	0.589	0.269	0.711
77	47	-0.825	2.769	48.271	-2.500	4.399	30.710	0.588	0.370	1.581
78	19	-0.746	4.889	68.517	-0.410	6.804	47.221	0.236	0.169	1.035
79	39	-0.591	2.874	53.936	-2.194	3.352	29.015	0.423	0.363	1.786
80	23	-1.894	1.316	49.811	-3.506	1.704	31.852	0.636	0.492	3.857
81	12	-3.603	2.443	65.286	-3.547	7.952	56.147	0.584	0.179	1.199
82	52	-0.087	3.294	53.947	-2.902	1.890	23.856	0.738	1.286	4.487
83	46	-0.205	3.090	58.639	-3.352	2.075	26.994	0.738	1.099	4.676
84	46	-0.460	2.527	63.037	-3.379	2.025	32.231	0.777	0.969	4.445
85	14	-1.916	2.374	68.517	-3.182	9.976	56.087	0.716	0.170	1.199
86	57	-0.663	3.294	51.321	-1.874	3.395	31.210	0.540	0.524	1.563
87	48	-0.421	3.549	58.630	-3.065	1.838	27.737	0.613	1.183	4.605
88	33	-0.874	3.214	63.028	-4.079	2.518	32.636	0.669	0.854	4.006
89	17	-1.560	1.816	67.106	-4.143	2.723	42.557	0.837	0.559	2.835
90	65	1.585	3.081	42.319	-1.993	3.104	16.174	0.624	0.619	2.614
91	78	0.781	2.229	46.537	-2.186	1.950	20.558	0.868	0.992	4.344
92	45	-0.535	1.625	46.031	-2.639	1.767	25.451	0.779	0.717	3.518
93	135	-0.742	1.757	24.948	-1.674	2.685	17.137	0.778	0.509	1.504
94	105	-0.907	1.446	30.008	-2.035	1.804	21.946	0.796	0.638	2.071
95	55	-1.105	1.080	36.290	-2.312	1.491	26.214	0.818	0.592	2.571
96	139	-1.120	2.105	22.414	-2.659	3.258	16.236	0.741	0.479	1.753
97	139	-0.731	2.020	28.087	-1.874	3.092	22.277	0.722	0.472	1.546
98	68	-1.349	1.652	30.287	-2.928	2.248	22.319	0.684	0.503	2.441
99	19	3.039	3.512	63.930	-3.492	2.118	20.726	0.847	1.405	17.543
100	95	1.267	4.505	35.816	-3.316	3.543	15.671	0.482	0.614	3.030
101	87	2.566	4.176	52.045	1.541	5.885	35.896	0.726	0.515	1.056
102	95	-0.253	1.947	40.052	-2.180	2.399	26.316	0.816	0.663	2.276
103	125	-1.210	1.800	23.899	-2.637	2.865	16.798	0.708	0.445	1.862
104	91	-1.060	1.614	30.678	-2.759	2.051	22.187	0.821	0.646	2.511
105	90	-0.683	1.502	40.051	-2.283	1.743	27.651	0.848	0.731	2.659
106	104	-1.063	2.181	24.453	-2.234	2.608	14.954	0.559	0.468	1.876
107	120	-0.734	1.332	27.452	-1.666	1.820	19.202	0.777	0.569	1.826
108	98	-0.612	1.177	33.260	-1.459	1.639	22.708	0.874	0.628	1.757
109	50	-1.068	2.083	35.586	-2.874	2.208	23.447	0.530	0.500	2.844
110	50	-1.063	1.896	38.352	-2.742	1.852	25.299	0.638	0.654	3.048

Table 11 (continued)

	No.	$\hat{\gamma}_1$	$\hat{\delta}_1$	$\hat{\lambda}_1$	$\hat{\gamma}_2$	$\hat{\delta}_2$	$\hat{\lambda}_2$	$\hat{\rho}$	ϕ	θ
111	50	-0.855	1.682	43.309	-2.799	1.539	29.566	0.632	0.691	4.340
112	38	-0.679	1.195	50.049	-2.151	1.272	34.595	0.824	0.774	3.495
113	45	0.166	2.445	49.965	-2.671	3.633	27.622	0.683	0.459	2.152
114	38	-0.019	2.957	55.251	-3.687	3.206	29.827	0.548	0.506	3.148
115	38	-0.181	2.762	61.079	-3.535	2.188	32.047	0.447	0.564	4.850
116	24	-0.834	2.988	67.687	-4.231	2.994	40.295	0.516	0.514	3.559
117	71	-1.332	1.586	30.439	-1.582	2.277	20.002	0.696	0.485	1.333
118	40	0.431	4.709	53.953	-2.011	2.478	24.980	0.571	1.085	2.487
119	40	-0.006	3.569	59.920	-2.665	1.948	27.146	0.770	1.411	3.918
120	21	-1.161	2.857	67.163	-3.378	1.579	31.547	0.759	1.373	4.865
121	56	0.296	1.764	47.660	-1.028	1.194	18.288	0.771	1.139	2.866
122	35	-0.336	2.512	53.946	-1.592	3.457	28.459	0.716	0.521	1.479
123	29	-0.842	2.407	59.920	-3.441	2.833	28.942	0.589	0.500	2.828
124	19	-2.453	1.813	57.710	-4.365	3.364	38.264	0.486	0.262	2.568
125	74	0.671	2.085	41.802	-1.734	1.932	18.528	0.845	0.911	3.290
126	58	0.880	2.658	50.669	-2.663	2.310	20.573	0.744	0.856	4.205
127	54	0.777	2.705	56.833	-2.598	1.966	24.431	0.695	0.956	4.935
128	35	-0.033	2.213	59.152	-2.250	1.642	26.814	0.763	1.028	3.876
129	59	-0.536	1.502	33.072	-1.533	1.606	16.002	0.813	0.761	1.980
130	41	-0.174	2.446	45.641	-2.028	1.635	19.202	0.757	1.133	3.189
131	39	-0.537	2.395	50.182	-2.229	1.943	24.478	0.698	0.860	2.597
132	23	-1.235	1.145	47.404	-1.972	1.497	29.458	0.891	0.682	1.790
133	59	1.036	2.762	47.585	-1.737	1.746	16.308	0.788	1.247	4.315
134	43	-0.404	2.281	43.043	-2.365	2.778	21.390	0.702	0.576	2.115
135	35	0.334	3.233	59.919	-2.769	3.671	27.474	0.685	0.604	2.263
136	23	-0.587	3.188	67.100	1.098	6.532	52.829	0.683	0.334	0.795
137	58	-0.158	2.010	42.231	-2.172	2.246	18.136	0.772	0.691	2.491
138	48	0.248	2.588	53.944	-2.353	2.156	22.658	0.617	0.741	3.198
139	36	-0.208	2.554	59.916	-0.672	5.212	41.562	0.776	0.380	1.103
140	18	-1.411	2.358	67.200	-1.955	11.140	52.829	0.612	0.130	1.103
141	91	-0.731	2.043	27.731	-2.073	2.479	16.108	0.747	0.615	1.852
142	73	1.904	4.172	49.956	1.552	6.344	31.983	0.635	0.418	0.948
143	74	1.288	3.675	54.124	1.058	5.013	39.775	0.502	0.368	0.921
144	33	-1.091	1.291	41.505	-2.317	1.506	26.135	0.751	0.644	2.703
145	52	1.228	3.826	53.925	-1.909	3.102	23.903	0.771	0.951	2.510
146	45	0.954	4.384	58.245	-2.855	4.391	28.815	0.630	0.629	2.197
147	45	0.492	4.282	63.084	-3.671	2.059	25.987	0.593	1.233	6.849
148	22	-0.751	3.646	68.480	-5.620	4.423	34.729	0.676	0.557	3.177
149	61	0.530	2.012	41.802	-1.745	2.251	20.801	0.823	0.736	2.635

<u>Figure 15</u>

S_{BB} median regression curve. Plot 33A601 measured 28/3/62; ϕ>1.

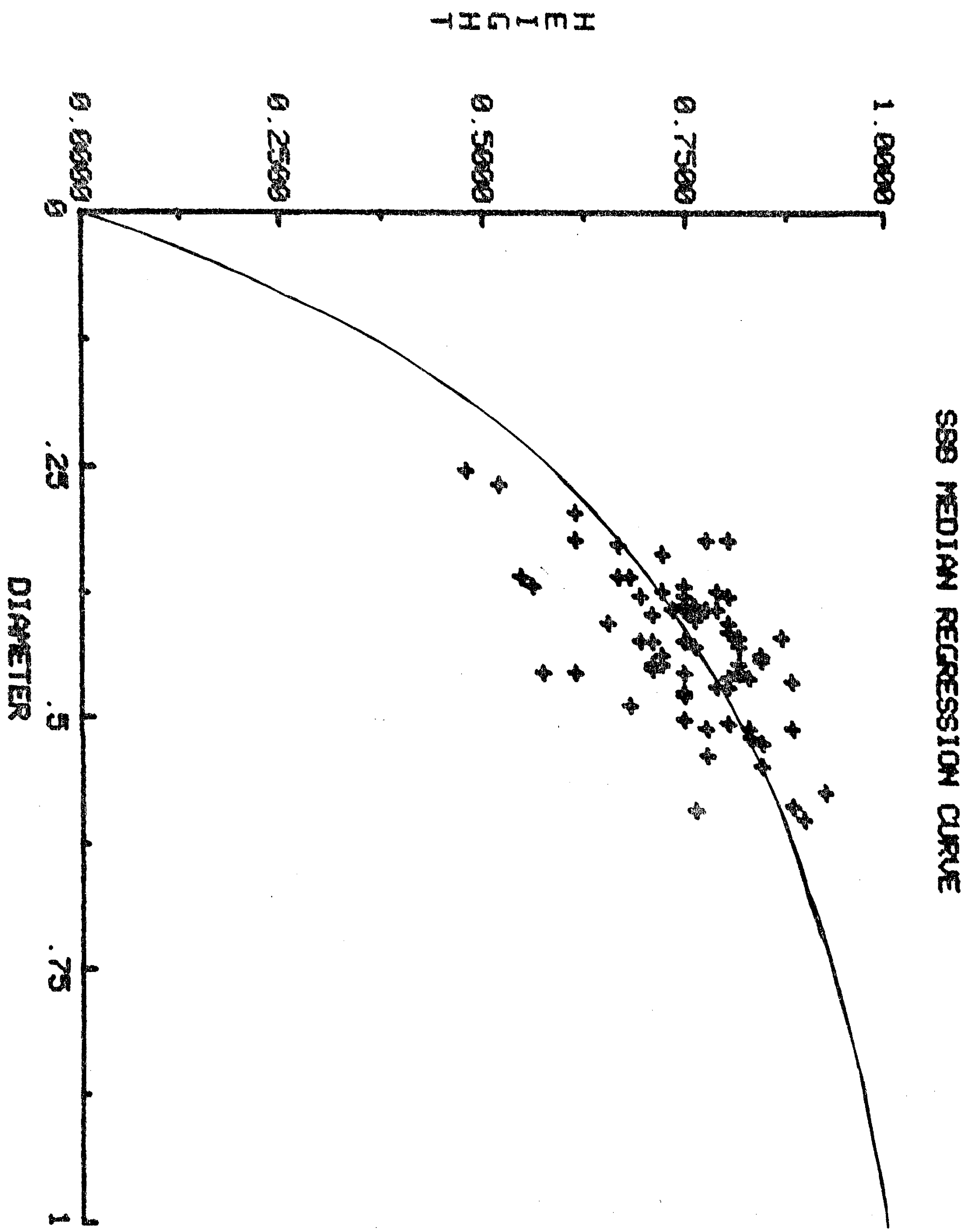

Figure 16

S_{BB} median regression curve. Plot 28A601 measured 20/1/58; $\phi<1$.

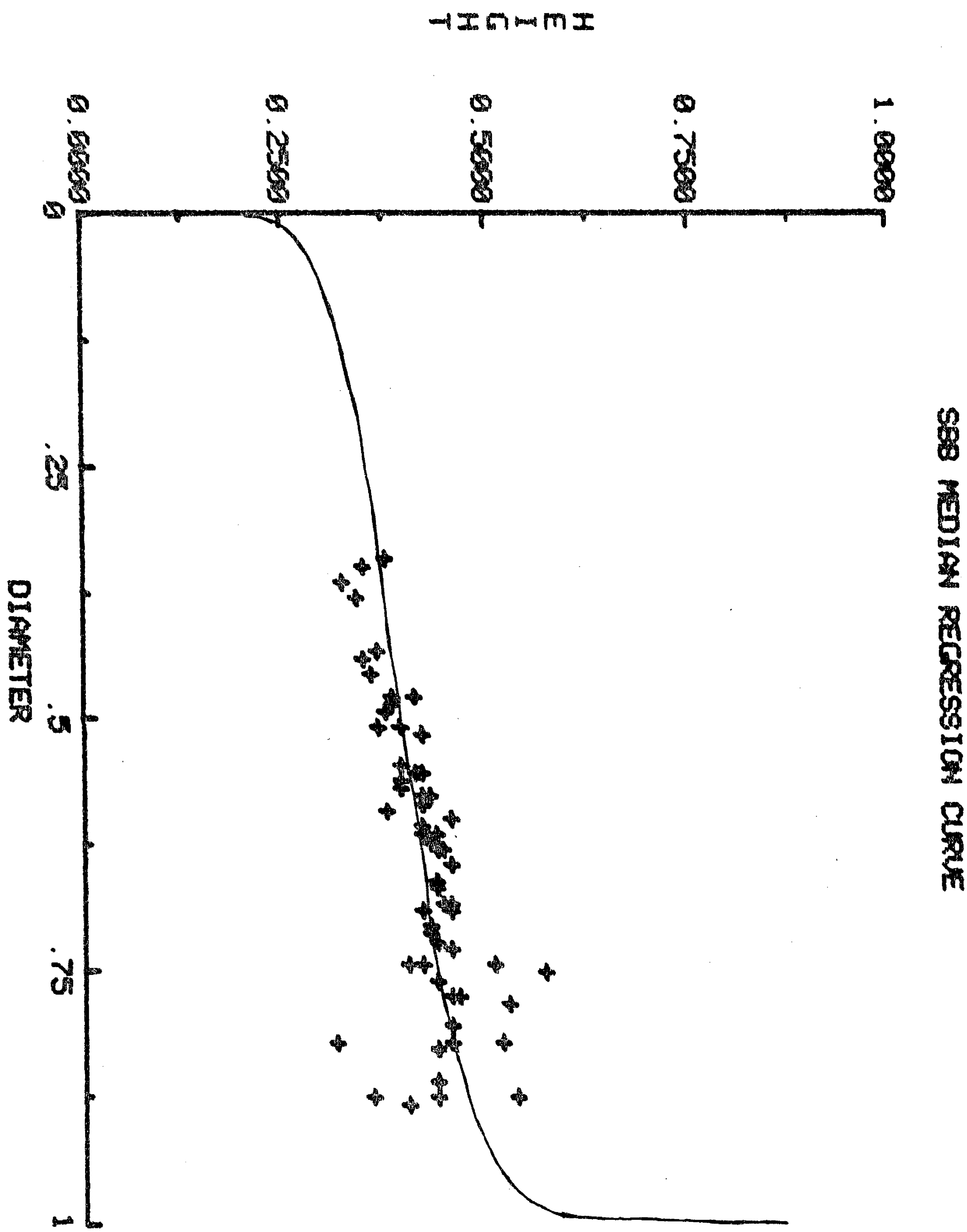

Table 12

Parameter estimates for the bivariate normal distribution.

Plot Code	No.	$\hat{\mu}_1$	$\hat{\sigma}^2_1$	$\hat{\mu}_2$	$\hat{\sigma}^2_2$	$\hat{\rho}$
31A50141230156	95	15.524	5.503	12.091	1.948	0.551
31A50141210362	85	20.072	12.147	17.962	4.962	0.683
31A50141230871	57	26.080	15.347	22.438	3.257	0.607
31A60141280155	101	16.138	12.758	12.310	2.874	0.786
31A60141220158	79	19.789	14.174	15.786	3.039	0.692
31A60141140362	76	23.438	21.650	20.446	4.470	0.751
31A60141230871	56	28.948	37.086	26.090	7.475	0.831
32A60141190156	97	18.234	7.377	12.723	2.444	0.631
32A60141210158	75	20.496	8.138	14.569	2.641	0.669
32A60141210262	57	24.843	15.874	18.852	4.322	0.685
32A60141250871	36	28.792	45.989	23.410	13.916	0.833
33A60141020255	115	16.236	11.074	11.850	2.787	0.725
33A60141220158	79	20.307	8.261	14.978	1.980	0.507
33A60141280362	69	23.913	16.655	18.341	4.131	0.620
33A60141061271	43	30.952	30.157	24.178	4.387	0.690
27070140250156	98	20.138	15.917	14.565	3.186	0.641
27070140150262	56	27.486	15.672	20.947	2.704	0.564
27070140270771	26	36.667	42.819	27.227	3.927	0.657
27080139280155	98	20.566	13.109	16.209	2.198	0.574
27080139210158	71	23.629	10.393	19.748	2.532	0.416
27080139150262	59	27.337	19.072	23.725	3.486	0.696
27080139230771	33	34.854	64.835	29.570	10.167	0.813
28A60140260155	77	17.912	17.499	12.730	1.854	0.719
28A60140200158	61	21.469	22.436	15.925	2.789	0.589
28A60140160262	46	26.438	28.092	20.173	4.402	0.802
28A60140280771	35	33.931	42.995	25.573	7.918	0.812
29070140280155	88	17.595	9.570	14.471	1.739	0.575
29070140210158	66	20.374	11.196	17.725	2.049	0.589
29070140210362	65	23.407	15.558	21.641	2.801	0.657
29070140200871	44	27.935	29.778	25.804	4.930	0.797
30B60143240156	109	14.957	12.244	11.918	1.847	0.778
30B60143140362	101	19.307	25.260	17.007	4.893	0.724
30B60143151271	92	23.021	48.394	21.560	10.372	0.820
30B70143240156	135	16.088	14.926	12.506	2.544	0.745
30B70143210262	117	20.395	25.621	16.880	6.096	0.741
30B70143101271	109	24.005	52.213	20.661	13.674	0.849
01A50127260155	43	27.013	11.306	22.523	1.317	0.402
01A50127160158	40	29.235	12.664	24.540	2.321	0.244
01A50127130262	40	32.468	15.345	25.653	2.399	0.203
01A50127080771	25	39.645	20.637	28.188	2.820	0.121
01A60127170155	33	32.543	14.642	25.442	1.443	0.407
01A60127160158	32	35.139	15.728	27.851	1.611	0.434
01A60127120262	32	38.941	18.953	31.132	2.330	0.455
01A60127011167	16	46.133	18.572	34.138	1.812	-0.093
01A60127080771	16	49.508	21.009	34.309	1.326	0.121
01A70127180155	39	30.851	11.702	26.197	2.466	0.605
01A70127160158	36	33.323	13.214	29.041	2.152	0.327
01A70127130262	36	36.280	18.145	32.063	0.745	0.580
01A70127011167	22	42.787	13.095	34.290	1.166	0.309
01A70127090771	22	45.715	15.023	35.606	2.621	0.634
02A70128200155	38	33.448	16.041	29.060	1.053	0.335
02A70128170158	34	36.098	17.699	30.310	1.487	0.192
02A70128140262	30	40.479	21.267	32.588	0.917	0.200
02A70128120771	19	48.443	19.583	35.132	1.082	0.292
03060128180155	51	25.246	6.151	19.952	2.615	0.500

Table 12 (continued)

Plot Code	No.	$\hat{\mu}_1$	$\hat{\sigma}^2_1$	$\hat{\mu}_2$	$\hat{\sigma}^2_2$	$\hat{\rho}$
03060128170158	43	27.689	7.576	22.073	2.646	0.488
03060128140262	35	31.387	9.513	23.269	1.645	0.363
03060128130771	35	36.539	16.649	25.795	2.063	0.402
04A50128180155	38	23.789	7.838	18.958	1.131	0.521
04A50128170158	37	25.750	8.744	20.603	1.618	0.432
04A50128140771	25	34.698	20.016	25.634	1.499	0.590
04A60128180155	35	28.303	14.563	20.574	4.404	0.583
04A60128160158	34	31.078	18.007	22.385	3.452	0.338
04A60128130262	29	35.805	21.955	25.277	3.293	0.231
04A60128130771	20	44.379	41.526	28.156	3.948	0.358
04B70128180155	48	30.501	12.889	26.756	3.172	0.440
04B70128150158	43	33.002	15.563	28.910	3.673	0.594
04B70128140262	26	38.276	16.818	32.315	2.927	0.593
04B70128011167	13	48.338	11.537	34.478	3.073	0.466
04B70128140771	13	52.732	15.599	36.377	3.735	0.454
07050135190155	38	24.585	5.956	15.669	1.322	0.327
07050135200158	34	28.493	8.332	17.817	1.771	0.106
07050135150262	34	33.326	14.246	21.565	1.652	0.436
07050135150771	20	42.158	10.527	25.817	1.569	0.007
07060135140155	50	22.266	11.328	15.789	1.405	0.633
07060135200158	47	24.727	12.308	18.210	1.620	0.586
07060135010262	47	27.594	17.335	20.856	2.551	0.581
07060135160771	19	36.834	11.861	25.619	2.982	0.235
07070135260155	39	29.633	20.520	20.304	3.693	0.438
07070135220362	23	39.006	33.413	29.122	4.084	0.771
07070135160771	12	52.610	16.065	35.509	2.772	0.540
08070135190155	52	27.338	16.088	20.627	3.582	0.716
08070135170158	46	30.259	21.457	23.539	3.413	0.735
08070135140262	46	34.273	35.791	28.075	4.965	0.779
08070135190771	14	46.887	34.995	33.767	1.890	0.712
08080135190155	57	28.181	14.258	21.026	4.409	0.542
08080135200158	48	31.020	16.249	24.279	4.643	0.594
08080135280362	33	35.691	22.194	28.314	3.405	0.675
08080135200771	17	46.327	55.443	35.966	5.405	0.857
34050141250156	65	15.945	10.185	11.840	1.341	0.614
34050141280362	78	19.415	23.490	16.569	3.838	0.861
34050141071271	45	26.529	39.379	21.732	5.975	0.812
34060141240156	135	14.900	10.320	12.378	2.015	0.789
34060141240162	105	19.169	19.625	17.596	5.208	0.836
34060141091271	55	25.650	40.703	22.408	8.249	0.904
35060143150156	139	13.979	5.727	12.497	1.097	0.754
35060143150262	139	16.422	10.314	15.641	2.631	0.736
35060143130571	68	20.614	14.142	18.651	2.985	0.780
35060143140571	19	19.154	14.489	18.481	1.968	0.789
35070143240156	95	15.433	3.786	12.503	0.795	0.474
35070143120262	87	18.367	8.069	16.925	2.226	0.731
35070143060671	95	21.269	22.573	19.875	4.968	0.825
36070144270156	125	15.582	8.114	13.227	1.409	0.726
36070144140262	91	19.854	16.577	18.663	3.298	0.840
36070144150671	90	24.134	33.853	22.674	7.347	0.871
37060144290156	104	15.025	6.317	11.710	1.399	0.585
37060144280362	120	17.073	19.014	14.794	4.536	0.816
37060144090671	98	20.415	34.933	17.105	7.890	0.876
18080137140155	50	22.037	14.869	19.513	3.268	0.568
18080137000158	50	24.110	20.054	21.580	4.631	0.657

Table 12 (continued)

Plot Code	No.	$\hat{\mu}_1$	$\hat{\sigma}^2_1$	$\hat{\mu}_2$	$\hat{\sigma}^2_2$	$\hat{\rho}$
18080137120262	50	26.645	32.315	26.222	6.844	0.683
18080137110571	38	31.006	76.217	29.610	17.170	0.856
18080237110155	45	24.164	24.085	19.893	2.720	0.686
18080237150158	38	27.833	20.613	23.819	2.857	0.545
18080237120262	38	31.476	28.242	27.733	4.107	0.470
18080237030571	24	38.386	28.401	33.496	4.589	0.544
21070138200155	71	20.839	15.130	14.506	3.589	0.696
21070138170158	40	25.756	8.025	18.437	4.419	0.565
21070138140262	40	29.978	17.021	22.609	4.992	0.769
21070138250671	21	40.122	30.423	29.050	5.511	0.725
22060138090155	56	21.957	40.147	13.749	9.264	0.779
22060138170158	35	28.687	26.160	18.689	3.702	0.713
22060138140362	29	34.894	31.241	23.449	3.242	0.623
22060138270671	19	45.005	23.374	31.194	3.677	0.519
23060138250155	74	17.739	21.658	14.264	3.687	0.852
23060138200158	58	21.314	19.731	16.764	2.636	0.753
23060138160262	54	24.492	24.627	20.303	4.388	0.711
23060138290671	35	29.788	40.626	22.237	6.901	0.791
24050138250155	59	19.206	23.789	12.626	4.095	0.832
24050138170158	41	23.591	20.306	15.887	4.469	0.751
24050138160262	39	27.777	24.938	19.601	5.449	0.688
24050138300671	23	33.932	53.501	23.986	11.435	0.909
24060138200155	59	19.515	16.291	12.988	3.298	0.789
24060138150158	43	23.344	19.633	16.179	2.539	0.722
24060138150262	35	28.441	20.573	19.908	2.601	0.689
24060138300671	23	36.553	26.277	25.510	4.008	0.686
25070138010255	58	21.892	24.735	14.286	2.495	0.789
25070138210158	48	25.723	25.373	18.050	3.842	0.573
25070138150262	36	31.115	32.098	23.406	3.871	0.781
25070138060771	18	42.973	37.129	30.023	1.375	0.584
26A60140260155	91	16.195	9.720	12.438	1.813	0.771
26A60140210158	73	19.450	7.949	15.357	1.529	0.633
26A60140210262	74	22.452	12.357	19.114	3.759	0.503
26A60140220771	33	28.139	40.326	22.306	6.317	0.800
09060135270155	52	22.743	11.492	16.735	2.976	0.772
09060135210158	45	26.021	10.568	20.181	2.151	0.634
09060135160262	45	29.752	13.203	23.283	2.607	0.565
09060135200771	22	37.696	20.760	28.333	1.811	0.661
14060136140155	61	18.305	23.993	15.387	3.796	0.817