6-1-1980

Dynamic Model of the Zone of Aeration

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DYNAMIC MODEL OF THE ZONE OF AERATION

(Part I of Completion Report for Project A-023-ARK)

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Publication No. 70

1980

Research Project Technical Completion Report

OWRT Project No. A-023-ARK

Effects of Changes in Surface Water Regime and/or Land Use on the Vertical Distribution of Water Available for Wetland Vegetation

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The work upon which this report is based was supported in part by funds provided by the United States Department of the Interior, Office of Water Research and Technology, as provided for under Public Law 88-379 as amended.

June 1980

DISCLAIMER

REPORT FORMAT

PART I - DYNAMIC MODEL OF THE ZONE OF AERATION
(Included herein)

PART I (Appendix) - DOCUMENTATION OF COMPUTER PROGRAM
(Included as a separate document and available upon request from the Arkansas Water Resources Research Center, University of Arkansas, Fayetteville, AR 72701.

PART II - PORTABLE ENVIRONMENTAL DATA LOGGER AND SENSORS
(Incuded as a separate document)

Each of parts I and II is complete within itself, and may be distributed separately or as a single report.

PART I

DYNAMIC MODEL OF THE ZONE OF AERATION

by

Robert N. MacCallum

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June, 1980

ABSTRACT

A mathematical model by Green (1), simulating one-dimensional vertical ground-water movement in unsaturated soils of the prairie region of Kansas, has been adapted for use in a wetlands environment typified by the wetlands forest of Eastern Arkansas. The model consists of two second-order, non-linear, partial differential equations and an algorithm for their numerical solution. The original model was extended to include functions for seasonal changes in transpiration and for drainage of excess precipitation. Before the addition of the two functions, the model reliability was limited to one growth season.

With the mathematical model presented in this work it is possible to study interactions between hydrologic changes and the long term vegetative changes. The model potentially is a versatile management tool which could be used to help predict the environmental impact of proposed flood control projects.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

INTRODUCTION

The ten million acre Delta region in Eastern Arkansas was once a gigantic hardwood Forest. For milleniums, the Mississippi and its tributaries have been raising their crests and spreading silt-laden water over portions, and sometimes over all, of the Delta area. The soil that supported most of this wetland forest was highly suitable for agriculture after it had been drained and cleared. A report by the Arkansas Planning Commission showed that 4.6 million acres have been drained, and almost all of the Delta region is affected by some flood control. As a result of these efforts, 8.2 million acres have been cleared for agriculture leaving less than two million acres in forests.

As flood control measures are put into effect in Eastern Arkansas, parts of the remaining wetland forest will be denied their annual flooding. Assuming that vegetation is in dynamic equilibrium with the environment, the vegetative composition of the wetland forest will change to reflect the environmental change of reduced flooding. In order to predict the environmental impact of proposed flood control project; a relationship between hydrologic change and vegetation change should be available.

To develop such a relationship, two models would be needed. The first, a model which would predict vegetative reaction to moisture change, in being developed by R. L. Phipps of the U. S. Geological Survey. The second is a hydrology model which will describe the vertical distribution of water available for vegetative use. This second model is the subject of this report.

A suitable numerical model of unsaturated groundwater flow including effects of evapotranspiration has been developed by Green (1). It is a digital computer algorithm for the solution of the differential equations which describe isothermal, one-dimensional, unsteady-state, simultaneous flow of water and air in a porous medium. These are a set of two second order, non-linear, partial differential equations. Evaporation and transpiration loss functions are included in the equations as source/sink terms. The formulation of these terms has been made using an Ohm's law analogy to water flow in the soil-atmosphere and soil-plant-atmosphere systems. Energy considerations for the evapotranspiration process have not been made. Verification of the model was made using input data from the Arkansas River Valley in Kansas.

Green's model was modified for this study by providing for seasonal changes, by providing for rainwater run-off or drainage, and by use of input data representative of the wetlands of Eastern Arkansas. The partial differential equations were converted to a finite difference form and solved using an iterative implicit procedure. Input data to the model include soil parameters (such as porosity and permeability), plant parameters (such as plant conductivity and root density), atmospheric parameters (such as humidity and mass-transfer coefficients) and model parameters (such as time-step size, grid size and convergence criteria). The model computes soil moisture saturations and evapotranspiration losses as a function of time and position in the soil column.

Ultimately, this model may be combined with the vegetative reaction model of the U. S. Geological Survey to determine optimum use of wetlands by predicting the impact of proposed flood control and drainage projects.

ACHIEVEMENT OF PROJECT OBJECTIVES

A. Objectives

The primary objective of this research project was to develop a dynamic mathematical model of the zone of aeration for wetlands which describes the vertical distribution of water available for use by vegetation.

A secondary objective was to provide data collected from test plots in the wetlands of Eastern Arkansas for verification of the model.

B. Extent of Achievement

The primary objective was achieved with the adaptation of Green's (1) model to a wetlands environment. The model was programmed for the IBM System 360 computer, and theoretical data pertinent to the Arkansas Delta region was developed.

Achievement of the secondary objective was partially achieved in the following manner. Study plots were established in the White River National Wildlife Refuge. These sites were chosen such that historical, ecological, and climatological data were available. The vegetation on these study plots was surveyed and cataloged as to type and size during the early months of the project. However, it became impossible to continue the field work necessary to obtain additional data at the test sites because of repeated heavy rains and floods. These adverse weather conditions began a few months after project initiation and continued for the two-year duration of the project.

In the interim, instrumentation was developed to obtain data at the remote test sites. Because of the demonstrated liklihood that the test sites would be under water for extended periods of time, it was necessary to design an instrumentation and recording package that could operate automatically, and without human interaction, for long periods of time.

Such a package has been designed and constructed. Unfortunately, the original project termination date was reached before test plot data could be obtained and made available to the computer model. The instrument package was subsequently operated very successfully, and test plot data were obtained during an extension of the project.

It is anticipated that this instrument and recording package will find wide application in other studies where data collection in remote areas is required. Thus, this phase of the project is included as a separate document, "Portable Environmental Data Logger and Sensors." This is attached as Part II of this report, but that document is complete within itself and may be distributed with or without Part I.

LITERATURE SURVEY

A literature review of references cited by Green et al (1) confirmed their conclusion that no mathematical models exist which adequately describe the movement of soil moisture in a soil column including the effects of evaporation and transpiration. The leterature showed that Green's approach is valid to solve the fluid-flow equations with source/sink terms to account for evapotranspiration. An Ohm's law analogy was used for the evapotranspiration terms and the resulting equations were solved using a numerical solution and digital computer.

This project will be an application of the well proven mathematical relationships for unsteady, isothermal flow through saturated and unsaturated porous media. Some references supporting these relationships are listed in the bibliography (2, 3, 4, 5). A concise history of research prior to 1969 is given by Freeze (6). Since then investigation has continued to define the mechanism of the groundwater cycle. Many studies have been concerned with the problem of infiltration (7, 8, 9, 10) while others have looked at the interface between the saturated and unsaturated zone (8, 11). An extensive review of recent research is given by Whisler and Bouwer (12).

A brief description of representative literature pertinent to the computer model will be given in three parts: 1) Solution of the fluid flow equations, 2) Transpiration, and 3) Evaporation. Detailed literature reviews related to this work are given by Dabiri (13) and Green (14).

Solution of the Fluid Flow Equations

In the past decade, several studies in water resources and soil science have used numerical techniques and a digital computer to solve various forms of the fluid-flow equations for two-phase isothermal, one-dimensional, unsteady-state flow through a porous medium.

Staple's (15) equation is typical:

$$\frac{\partial S}{\partial t} = \frac{\partial}{\partial z}\left(D \frac{\partial S}{\partial z}\right) - \frac{\partial K}{\partial z} \qquad (1)$$

where,

t = time

z = position

S = volumetric water saturation

K = conductivity

D = function of conductivity and derivative of the capillary pressure-water saturation curve

This equation was solved numerically for a silt loam for which physical property data were available. Equation (1) or similar forms have also been solved by Rugin and Steinhard (16), Hanks et al (17), and others.

Many studies have been made on petroleum reservoirs and simultaneous oil and gas flow have been treated extensively. Studies of water and air flow, especially in the petroleum industry, typically have treated a set of two simultaneous, partial-differential equations. Douglas et al (18) have derived these equations for an immiscible system and solved the equations for two dimensions using an alternating direction, iterative, numerical procedure. Others using this approach are Knapp et al (19), Blair (20), and Coats (21).

Green et al (22) used the petroleum reservoir engineering approach to describe water flow in the presence of an air phase. The solutions were compared to results from a field experiment conducted by the U. S. Geological Survey using neutron logging. The agreement between calculated and observed values was good, within two percent absolute moisture.

Transpiration

Approaches describing the transpiration process have generally been based on a water balance, energy balance, or a combination of the two.

The technique of using an analogy to Ohm's law to deacribe the transpiration process dates back at least to Gradmann (23) who used the agrument that the law is applicable where velocity of flow is proportional to a potential difference. Van Den Honert (24) used the analogy and assumed the transport of water in the transpiration stream to be a steady-state process. His main equation is:

$$\frac{dm}{dt} = \frac{\Psi_1 - \Psi_0}{r_r} = \frac{\Psi_2 - \Psi_1}{r_x} = \frac{\Psi_3 - \Psi_2}{r_1} = \frac{\Psi_4 - \Psi_3}{r_g} \qquad (2)$$

where,

$\frac{dm}{dt}$ = rate of water transport through the system

r_r, r_x, r_1, r_g = resistance in root, xylem, leaf, and atmosphere

Ψ_0, Ψ_1, Ψ_2, Ψ_3, Ψ_4 = water potential on either side of each respective part of the system

Van Den Honert agreed with Gradmann's conclusion that the resistance to water transport, as vapor in the atmosphere, was much greater than the total resistance to liquid flow inside the plant. This leads to the conclusion that the slowest process, e.g., the rate of vapor transport, in the system.

Ray (25) disagreed with Van Den Honert's assumption that diffusion is directly proportional to potential difference. This disagreement was based on a non-linear relationship between Ψ and vapor pressure given by:

$$\Psi = \frac{RT}{V} \ln P/P_0 \qquad (3)$$

where,

Ψ = water potential based on pure water at atmospheric pressure and temperature as a reference

R = universal gas constant

T = temperature

V = molal water volume

Po = water vapor pressure at Ψ = 0

P = water vapor pressure corresponding to Ψ

Slayter and Gardner (26) also assumed that the non-linear relationship between Ψ and vapor concentration is applicable when the transport of water takes place in the form of vapor in the atmosphere. The extended Van Den Honert's plant-atmosphere relationship to include water transport in the soil as well. They considered the case of a non-uniform root system, with a low root impedance compared to the soil impedance. Assuming the system varied only in the vertical direction, and the root zone consisted of n discrete layers of thickness h, they wrote, for the *i*th layer:

$$W_i = Bh \left(\Psi_{root} - \Psi_{bulk,i} - \rho g z_i\right) k_i L_i \qquad (4)$$

and

$$E = Bh \sum_{i=1}^{n} \left(\Psi_{root} - \Psi_{bulk,i} - \rho g Z_i\right) k_i L_i \qquad (5)$$

where,

E = total rate of transpiration

W_i = rate of water uptake per unit cross-section of soil with thickness h

B = a constant which contains the geometrical factors in the flow equation

Ψ_{root}, Ψ_{bulk} = water potential in root and soil, respectively

Z_i = average depth of the ith soil layer

ρ = depth

g = acceleration due to gravity

k = capillary conductivity of the soil

L = length of root per unit volume of soil (root density)

The equation gave good results for the upper root zone, but there was evidence that the impedance to water movement in the smaller roots towards the bottom of the root zone is not negligible compared to the soil impedance as assumed.

Cowan (27) has made an analysis of the dynamic aspects of water transport in the soil-plant-atmosphere system. He presented a solution for the differential equation describing radial soil-moisture flow towards a single plant root which is absorbing water at a periodically varying rate. This is combined with hypothetical plant characteristics to form a model of the hydraulic behavior of a crop. For the rate of transpiration per unit area of the crop, he wrote:

$$E = \frac{\tau_r - \Psi}{Z} \quad \text{; an analogy to Ohm's Law.} \qquad (6)$$

where,

E = transpiration rate

τ_r = soil water potential at the root surface

Ψ = leaf water potential

Z = plant imdedance

The transpiration rate is determined according to soil moisture conditions, e.g.:

(i) For soil moisture greater than the permanent wilting point saturation,

when $\tau_r - AE_t > \Psi_w$

then $E = E_t$

and $\Psi = \tau_r - ZE_t$

where E_t = transpiration rate

Ψ_w = leaf water potential at which stomata close

(ii) For soil moisture less than wilting point saturation,

when $\tau_r = ZE_t < \Psi_w$

then $\Psi = \Psi_w$

and $E = E_w$

where E_w = the "supply function"

The potential rate of transpiration in (i) is controlled by the environmental conditions, e.g., temperature and humidity, whereas transpiration rate in (ii) is controlled by soil moisture content as well as soil and plant conductivities.

Whistler, et al (28) numerically analyzed the steady-state equation for boundary conditions of a fixed transpiration rate at the top of the column and a water table at the bottom.

Philip (29) stated that the mechanism of transpiration must involve the entire soil-plant-atmosphere continuum (SPAC). He proposed a model on the basis of mass and energy transfer in the SPAC, which used two partial differential equations, essentially of the heat conduction or diffusion type, for each of the three domains of the SPAC. He also listed the minimum information that in needed to set up the proposed model, in terms of boundaries, energy sources, initial conditions, conductivities and diffusivities in SPAC.

Evaporation

Evaporation from a wet porous medium to a moving gas stream was first considered many years ago. One early worker, Sherwood, (30, 31, 32) determined that under constant environmental conditions the drying process could be divided into a "constant rate" and one or two "falling rate" periods. When the moisture content is sufficiently high, a considerable amount of moisture leaves the porous medium at an approximately constant rate, which is roughly equal to the rate of evaporation from a continuous water surface under identical environmental conditions. During the constant rate period, drying takes place at the exposed surface by diffusion of vapor through a stationary air film.

Based on experimental data Ceaglske and Hougen (33) developed an emperical equation to estimate the evaporation rate during the constant rate period:

$$Q_e = 0.0059r^{0.11} v^{0.8} K_g (P_{ws} - P_{wa}) \quad (7)$$

where,

Q_e = evaporation rate

r = radius of sand grains in porous media

K_g = mass-transfer coefficient

V = wind velocity

P_{ws} = water vapor pressure at surface temperature

P_{wa} = partial pressure of water vapor in the atmosphere

During the constant rate drying period, water evaporated at the surface is continuously replenished by liquid water flow in the porous media as a result of capillary forces. The water mobility in the porous media is relatively high during this period. As drying proceeds, the liquid water mobility decreases and saturation at the surface diminishes. The water retreats into smaller volumes between the grains of the porous media and the evaporating surface area decreases. Capillary forces become large at the surface due to the reduced water content, and the evaporation rate begins to decrease.

During the initial and first falling rate periods of evaporation, the temperature of the surface remains constant and, as a rule, is equal to the ambient wet-bulb temperature. Philip (34), Gardner and Hillel (35), and Craig (36) have shown experimentally that the soil temperature stays constant during these periods and departure from isothermal model does not occur until the late stage of the drying process.

In the late stages of drying, with the breakdown of a continuous liquid phase in the media, the water mobility decreases sharply, decreasing the evaporation rate. When the evaporative surface recedes into the porour media, the second falling rate period begins. Moisture migrates outward by vapor diffusion by an evaporation-condensation mechanism (37).

Philip, et al. (38) gave the equation for vapor diffusion through porous media as:

$$q_{vap} = -D_{atm} \ \gamma \ \alpha \ \nabla(\rho a) \qquad (8)$$

where,

q_{vap} = vapor flux density

D_{atm} = molecular diffusivity of water vapor in air

a = volumetric air content of the medium

α = tortuosity factor allowing for the extra path lengths = 0.62

ρ = density of water vapor

γ = "mass-flow factor" introduced to allow for the mass flow of vapor arising from the difference in boundary conditions governing the air and vapor components of the diffusion system.

$$\gamma = \frac{P}{P-p} = 1$$

p = partial pressure of water vapor

P = total gas pressure

THEORETICAL DEVELOPMENT

In this section, the partial differential equations describing unsteady-state, two-phase flow in a porous media will be developed. Evaporation and transpiration will be included in the water phase equation as source/sink terms derived from an Ohm's law analogy. The algorithm used to solve the equations numerically will be discussed briefly.

Derivation of the Fluid-Flow Equations

Since flow takes place only in the pore spaces, the modified equation of continuity for flow in unsaturated porous media is:

$$\phi \frac{\partial(S\rho)}{\partial t} = -(\nabla \cdot \rho v_o) \quad (9)$$

of for one dimensional, incompressible flow:

$$\phi \frac{\partial(S\rho)}{\partial t} = -\rho \frac{\partial V_o}{\partial x} \quad (10)$$

Darcy's equation may be written as:

$$V_o = -\frac{k}{\mu}\left(\frac{\partial P}{\partial x} - \rho g \frac{\partial h}{\partial x}\right) \quad (11)$$

Substituting (11) for (10) gives:

$$\frac{\phi\, \partial(S\rho)}{\partial t} = \frac{\partial}{\partial x}\left[\frac{\rho k}{\mu}\left(\frac{\partial P}{\partial x} + \rho g \frac{\partial h}{\partial x}\right)\right] \quad (12)$$

or for the water phase:

$$\frac{\phi\, \partial(S_w \rho_w)}{\partial t} = \frac{\partial}{\partial x}\left[\frac{\rho_w k_{rw} k}{\mu_w}\left(\frac{\partial P_w}{\partial x} + \rho_w g \frac{\partial h}{\partial x}\right)\right] \quad (13)$$

For the above equations:

V_o = overall velocity, L/T

P = Pressure, FL^{-2}

S = Saturation, L^3L^{-3}(pore)

ρ = density, ML^{-3}

h = height above an arbitrary reference plane, L

g = acceleration due to gravity, LT^{-2}

ϕ = porosity, dimensionless

X = space coordinate, L

t = time, T

M = viscosity, $ML^{-1}T^{-1}$

k = aboslute permeability, L^2

subscripts:

W = water phase

rw = relative permeability to water phase

A similar equation applies for the air phase. Also, the relative permeability of each phase in a function of saturation.

For water:

$$k_{rw} = f_1(S_w) \tag{14}$$

The pressures in the two phases are not equal, but related by:

$$P_a - P_w = P_c \tag{15}$$

$$P_c = f_2\ (S_w) \tag{16}$$

Capillary pressure, P_c, is the difference between pressure of the liquid phase beneath the air-liquid interface and the pressure in the gaseous phase above the interface.

These equations were included in a model with algorithms for their numerical solution. The model includes effects of evapotranspiration which is developed in the following section.

Evapotranspiration Terms

To include the effects on the fluid flow equation, a variable source/sink term is included in equation (13):

$$\phi \frac{\partial(S_w \rho w)}{\partial_t} = \frac{\partial}{\partial_x} \left[\frac{\rho_w k_{rw} k}{\mu_w} \left(\frac{\partial P_w}{\partial_x} + \rho_w g \frac{\partial_b}{\partial_x} \right) \right] + Q_w' (X,t) \quad (17)$$

where,

$Q_w' (X,t)$ = rate of removal of water by evapotranspiration per unit volume of soil, $MT^{-1}L^{-3}$

The source/sink term $Q'_w(X,t)$ includes the evaporation rate $Q'_e(X,t)$, and the transpiration rate, $Q'_t (X,t)$. These have been formulated from an analogy to Ohm's law considering driving forces and resistances in parallel/series from point to point in the soil-plant-atmosphere system. For evaporation, the system has two resistances in series, one from the soil to the soil surface and one from the soil surface to the atmosphere. For transpiration, there are three resistance; one set of parallel resistances from the soil to the root system, a second from the root system to the leaf surface, and a third from the leaf system to the atmosphere. These resistances are shown schematically in Figure 1.

<u>Explanation of the Evaporation Formula</u>

According to Sherwood (31,32), the equation for constant rate drying of a wet porous medium is:

$$R_1 = K_{air} V^{0.8} (p_1 - pa) \quad (18)$$

where:

R_1 = rate of evaporation per unit _____, $MT^{-1}L^{-2}$

v = wind velocity parallel to the surface, LT^{-1}

P_1 = water vapor pressure at the porous medium surface, FL^{-2}

P_a = partial pressure of water in the air, FL^{-2}

K_{air} = mass transfer coefficient, $(MT^{-1}L^{-2})(L^{-0.8}T^{0.8})(F^{-1}L^{2})$

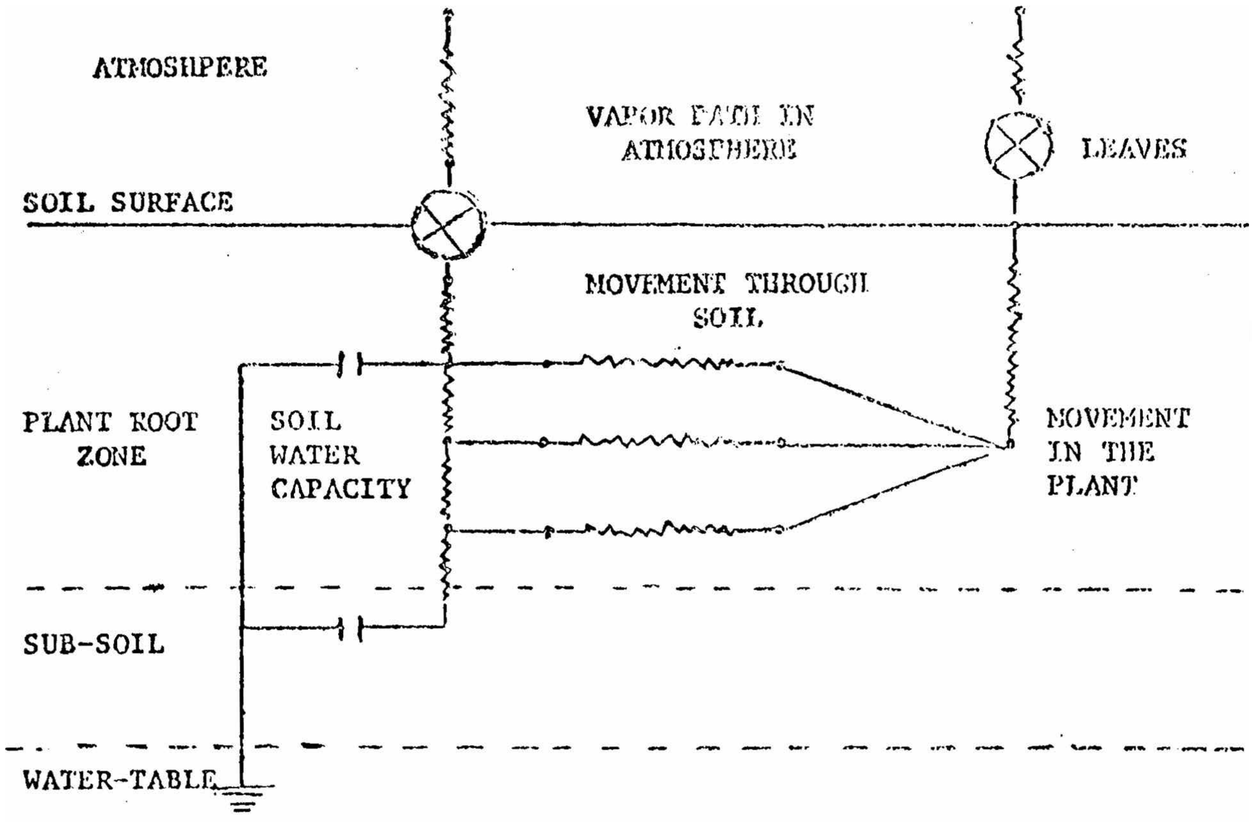


Figure 1. REPRESENTATION OF PATHWAYS OF WATER TRANSPORT IN THE SOIL, PLANT AND ATMOSPHERE. SITES OF PHASE CHANGE, LIQUID TO VAPOR, ARE SHOWN BY ⊗

FROM GREEN, ET AL (1)

During the first falling rate period, liquid flows through the porous medium to the surface to replenish evaporated water at a reduced rate. When the water content at the surface becomes sufficiently small, the evaporative surface retreats into the porous medium. The rate of evaporation in this period is given by (38):

$$R_2 = D_{atm} \quad \phi S_a \ \alpha \ \frac{dc}{dx} \tag{19}$$

Assuming ideal gas properties and a finite difference approximation, eq. 19 becomes:

$$R_2 = \frac{D_{atm} \ \phi \ S_a \ \alpha}{RT} \left(\frac{P_2 - P_1}{\Delta x} \right) \tag{20}$$

where:

R_2 = rate water vapor diffuses through dx in the porous media per unit area, $MT^{-1}L^{-2}$

D_{atm} = water-vapor diffusion coefficient in air, L^2T^{-1}

α = porous-media tortuosity, dimensionless

P_1 = water vapor partial pressure at the porous media surface, FL^{-2}

P_2 = water-vapor pressure at the temperature and existing capillary pressure

R - gas constant, $FL \ ^{o}R^{-1}M^{-1}$

T - soil temperature, ^{o}R

If it is assumed that the vapor diffusional resistances in the porous media and in the air film above the media are in series and, if a succession of steady states is assumed, then

$$R_1 = R_2 = Q_c^{(t)} \tag{21}$$

from eq. (18), (20), and (21),

$$Q_e^{(t)} = \frac{}{\frac{1}{K_{air} V^{0.8}} + \frac{RT\Delta x}{D_{atm} \phi S_a \alpha}} \tag{22}$$

Since P_2 is not usually known, it may be substituted in terms of capillary pressure, using the equation given by Edlepen and Slayter (39).

$$P_c = \frac{-RT}{V} \ln \frac{P_2}{P_s*} \tag{23}$$

where:

v = molal water volume, L^3M^{-1}

P_c = soil capillary pressure at the evaporative surface, FL^{-2}

P_s* = saturation water-vapor pressure at soil temperature, FL^{-2}

Using the definition of relative humidity:

$$\frac{RH}{100} = \frac{Pa}{P*_{air}} \tag{24}$$

where:

RH = percent relative humidity

$P*_{air}$ = saturation water vapor pressure at air temperature, FL^{-2}

And substituting (23) and (24) into (22) given:

$$Q_a(t) = \frac{P*s\ e^{-(Pc^r/RT)} - 0.01\ RH\ P*_{air}}{\left(\frac{1}{Kair\ V^{0.8}}\right) + \left(\frac{R\ TAX}{D_{atm}\ S_a\alpha}\right)} \tag{25}$$

This is the equation used for water evaporating at the soil surface in the model.

Explanation of the Transpiration Formula

All three components of the conducting media: soil, plant, and atmosphere, are important in the transpiration process. For this model, a succession of steady states is assumed. That is, water flow is assumed equal in all three components at any given time. The component with the greatest resistance governs the transpiration rate.

The rate water flows from the soil to the roots, as Whistler (28) described, is:

$$Q_t(t) = \int_{-L}^{0} A(x)\ K_e(x)\ (\Psi_{ws} - \Psi_{wr})\ dx \tag{26}$$

where:

$Qt(t)$ = transpiration rate per unit area $MT^{-1}L^{-2}$

$A(x)$ = root density, i.e., length of roots per volume of soil, L^{-2}

$K_e(x)$ = effective soil conductivity, $MLF^{-1}T^{-1}$, and

$(\Psi_{ws} - \Psi_{wr})$ = difference between the average water potential in the soil and at the root surface, FL^{-2}

Assuming constant Ψ_{wr} along the root system, (26) becomes:

$$Q_t(t) = \int_{-L}^{0} A(x)\ K_e(x)\ \Psi_{ws} d_x - \Psi_{wr} \int_{-L}^{0} A(x) K_e(x)\ dx \tag{27}$$

The water rate through the roots, xylem, and leaf system is given by:

$$Q_t(t) = h_p\ (\Psi_{wr} - \Psi_{wl}) \tag{28}$$

where:

h_p = overall plant conductivity, $MF^{-1}T^{-1}$

Ψ_{wl} = water potential in the stomates, FL^{-2}

This same amount of water must flow from the leaf to the atmosphere as a vapor:

$$Q_t(t) = K_{gl} \ (P_{wl} - Pa) \qquad (29)$$

where:

K_{gl} = mass transfer coefficient $(ML^{-2}T^{-1})(F^{-3}L^{2})$

P_{wl} = water vapor pressure at leaf surface at leaf temperature and existing water tension in the leaf, FL^{-2}

P_a = water vapor partial pressure in the air, FL^{-2}

The overall model has been designed to account for unsteady state flow, but for small time increments Δt, the flow is equal from the soil to the roots, the roots to the leaves, and the leaves to the atmosphere. Therefore:

$$Q_t(t) = \int_{-L}^{o} A(x)K_e(x) \ \Psi_{ws} \ dx - \Psi_{wr}\int_{-L}^{o} A(x)K_e(x)dx = h_p(\Psi_{wr} - \Psi_{wl}) \qquad (30)$$

$$= Kg\ l\ (Pwl - Pa)$$

To relate P_{wl} with Ψ_{wl}, Slayter's (39) equation is used:

$$P_{cl} = \frac{-RT'}{V} \ln \frac{P_{wl}}{P_1^*} \qquad (31)$$

where:

R_{cl} = capillary pressure in the stomata, FL^{-2}

T_1 = leaf temperature, ^{o}R

PI* = saturated water vapor pressure at leaf temperature, FL^{-2}

Since $\frac{P_{w1}}{P_1^*} > 0.95$ as given by Kozlowski (40), equation (31) can be approximated by:

$$P_{c1} = \frac{RT}{V^1} \left(1 - \frac{P_{w1}}{P_1^*}\right) \tag{31}$$

P_{c1} is the difference between air pressure P_{a1}, and water pressure P_{w1}, in the pores of the leaf. So, in the leaf:

$$P_{c1} = P_{a1} - P_{w1} = P_{a1} - \Psi_{w1} + \rho_w gh, \tag{33}$$

where:

h_1 = height of leaves above the datumplane, L.

Substituting P_{c1} into (32) and solving for P_{w1} gives:

$$P_{w1} = p'^* + \frac{(\Psi_{w1} - P_{a1} - \rho_w gh,)\ V\ P^*_1}{RT,} \tag{34}$$

Substituting P_{w1} into equation (30), eliminating Ψ_{wx} and Ψ_{w1}, and solving for $Q_t(t)$ gives:

$$Q_t(t) = \frac{\frac{N}{F} - P_{a1} - \rho_w gh, + \frac{RT}{vp,^*}(P,^* - P_a)}{\left(\frac{1}{F} + \frac{1}{hp} + \frac{RT,}{K_{g1} VP^*_1}\right)} \tag{35}$$

where:

$$F = \int_{-L}^{0} A\ (x)\ K_e\ (x)\ dx$$

$$N = \int_{-L}^{0} A\ (x)\ K_w\ (x)\ \Psi_s\ dx$$

Equation (35) is the transpiration equation used in the model.

DESCRIPTION OF THE COMPUTER MODEL

The model that was developed by Green (1) and used in this project is a digital computer algorithm for the solution of the differential equations which describe isothermal, one-dimensional, unsteady state, simultaneous flow of water and air in a porous medium. The equations are two second order, non-linear, partial differential equations. Evaporation and transpiration loss functions are included in the equations as source/sink terms. The formulation of these terms has been made using an Ohm's law analogy to water flow in the soil-atmosphere and soil-plant-atmosphere systems. Energy considerations for the evapotranspiration process have not been made.

Numerical Solution of the Equation

Methods for analytical solutions to equations (13) and (17) are not feasible so these have been solved by numerical analysis. The region from $-L \leq X \leq 0$ (i.e., the depth of the soil column) was divided into (NX) intervals and the differential terms in the air phase counterpart to equation (13) and in equation (17) were replaced by finite differences. Essentially an iterative implicit procedure was used to calculate pressures and saturation at each time step, converging after meeting a maximum allowable residual mass or pressure. Briefly, the computation by the computer for each time step is:

1) The coefficients of the difference are calculated.
2) The residual mass values are determined.
3) The difference equations are solved by the Gaussian elimination method yielding pressure residuals.
4) New values of pressure are calculated at each grid mode.

5) The iteration parameter is advanced by one and the residuals are recalculated. Convergence is checked by comparing residuals to the maximum allowable residuals.

6) If convergence criteria are not met, steps 3, 4, and 5 are repeated. This cyclical process continues until a preset maximum number of iterations is reached. The program continues after an error message if convergence is not attained.

7) New values of saturation are calculated using the capillary pressure-saturation relationship once convergence has been reached.

8) At the end of each time step a material balance calculation is made.

Description of the Computer Program

A digital computer program was used to perform the calculations discussed above. The bulk of the program is directly concerned with the numerical solution of equations (13) and (17). The model includes precipitation, evaporation, transpiration, effective permeabilities of the water and air phases, capillary pressure as a function of liquid saturation, gravity, and variable atmosphere, soil and fluid properties.

The computer program was written in FORTRAN IV for the GE 635 system and adapted to the IBM 360 system. This model requires 24K of 36 bit-word of core storage. It uses a magnetic tape for restart purposes.

The main program is an executive routine. Subroutines are called to carry out the calculations, data manipulation, etc. in the desired sequence. The four subroutines are shown schematically in Figure 2. The arrows indicate the flow of information.

Subroutine TAB

This is a table-look-up subroutine. Linear interpolation or extrapolation of table values is carried out to determine the dependent variable at the desired value of the independent variable.

Subroutine LNKA

If the program is starting from initial conditions, this subroutine causes all input data to be read and properly stored. If the computer run is a restart from magnetic tape, this subroutine controls the tape-reading and data storage.

Subroutine CAPPR

In the computer program, capillary pressure-saturation data are described using the following equation.

$$S_w = A + BP_c$$

where:

S_w = Water saturation, fractional

P_c = Capillary pressure, psi

A;B = Constants

In LNKA, S_w values are read in at equal increments of P_c beginning at a specified P_c starting point. Subroutine CAPPR generates A and B values for each increment of P_c.

At each value of S_w read in, the capillary pressure curve is approximated by a second degree polynomial. This polynomial is used to determine the slope of the tangent to the curve at the particular S_w

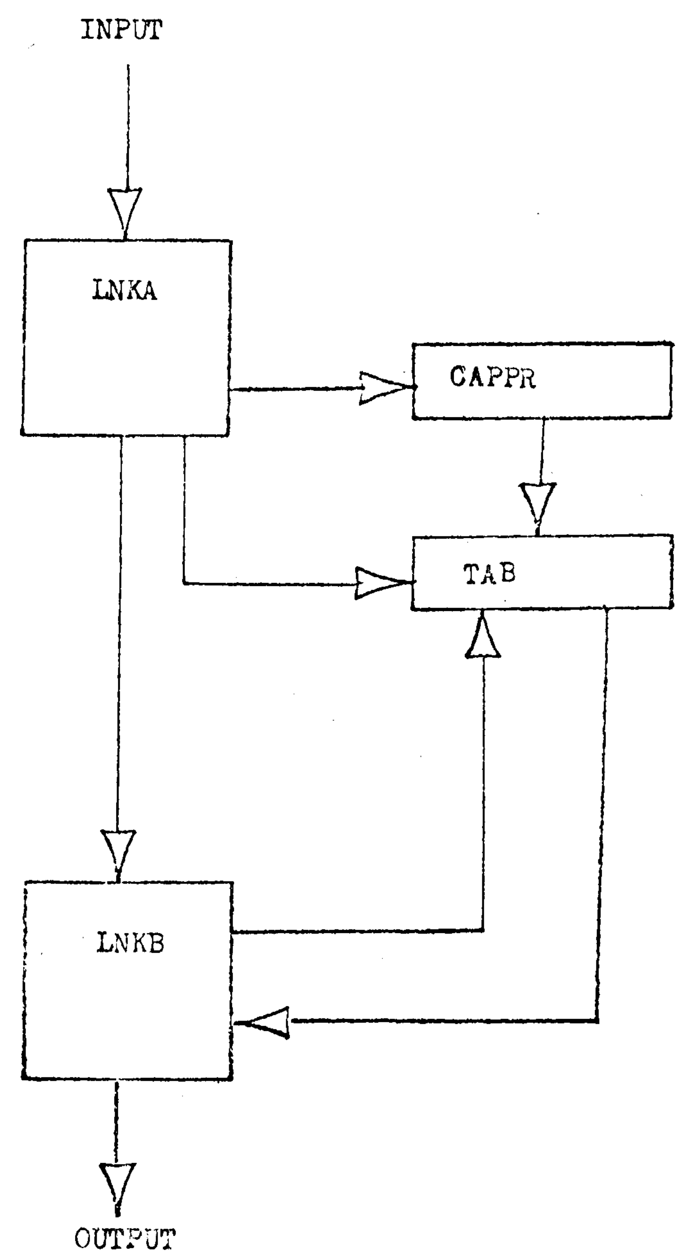


Figure 2. Flow Diagram of Computer Program

value and the intercept of that tangent on the S_w axis. The constants A and B corresponding to S_w are thus determined.

Subroutine LNKB

This subroutine causes the necessary calculations to be performed in order to solve the finite difference equations. Data manipulation is controlled. The subroutine contains a material balance routine and a "check out" procedure.

Complete documentation of the computer program is included as an unattached Appendix to this report, and it is available upon request from the Arkansas Water Resources Research Center, University of Arkansas, Fayetteville, Arkansas 72701.

Input Data

Since Green (1) made an extensive literature search and developed the model, most of this work has been to adapt the computer program to the IBM 360 and to find appropriate theoretical data for Eastern Arkansas Wetlands. Originally it was planned to obtain actual data from the test plots established in the White River National Wildlife Refuge, but prolonged flooding prevented obtaining soil and plant data from the plots. The following lists all data needed, with an explanation of value and source for each individual data item.

Soil Parameters:

i) Soil properties (absolute permeability, porosity),

ii) Capillary pressure-water saturation curve,

iii) Water relative permeability water saturation curve,

iv) Air relative permeability-water saturation curve,

v) Initial water saturation as a function of depth,

vi) Soil temperature versus time.

Plant Parameters:

i) Water saturation in soil at permanent wilting point,

ii) Plant conductivity,

iii) Mass transfer coefficient for evaporation of water from crop community,

iv) Height of leaf system from datum plane,

v) Critical leaf potential,

vi) Root density versus depth,

vii) Percent ground coverage by vegetation.

Fluid Parameters:

i) Air and water viscosity-pressure curves,

ii) Air and water density-pressure curves.

Atmosphere Parameters:

i) Mass transfer coefficient for evaporation of water from bare soil,

ii) Air temperature versus time,

iii) Air relative humidity versus time,

iv) Wind velocity versus time,

v) Precipitation data versus time.

Program Parameters:

i) Time increment,

ii) Rate of increasing time increment,

iii) Maximum time increment,

iv) Space increments,

v) Convergence criteria for maximum allowed

- air phase residual mass,
- water phase residual mass,
- change in air pressure,
- change in water pressure,

vi) Total simulation time.

The data sources and values used in this study are:

Soil Parameters:

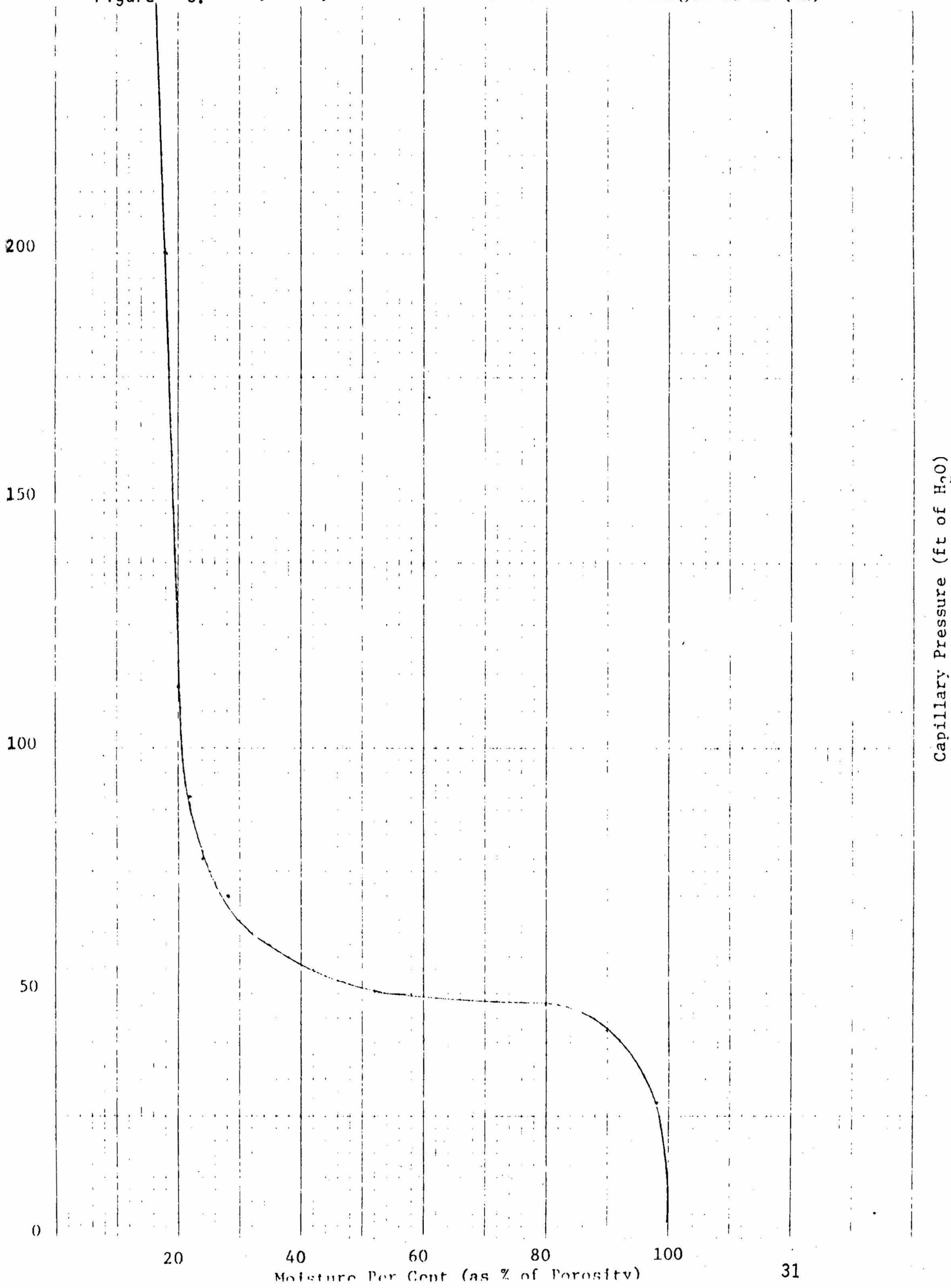
i) Typical values of absolute permeability for Sharkey Clay (clayey-silt) were obtained from Bedinger et al (41) and estimated at about 0.036 darcies. A reasonable range is 0.005-0.1 darcies.

ii) The capillary pressure-water saturation curve was input as a table of 100 values of water saturation at equally spaced intervals of capillary pressure over a range of 0-108 psi. The curve was obtained from Bedinger et al (41) for work done on soil from the Arkansas River Valley. The curve is shown in Figure 3.

iii) The water relative permeability-water saturation curve was obtained from the capillary pressure data using the formula from Fatt et al (49):

$$K = \frac{\int_0^S \frac{dS}{Pc^3}}{\int_0^{100} \frac{dS}{P_c^3}}$$

Figure 3. Capillary Pressure vs Saturation From Bedinger et al (41)



where:

K = relative permeability, dimensionless

S = water saturation, percent

P_c = capillary pressure at the given water saturation, psi.

Values of water relative permeability were obtained by integration of (36) using an IBM Continuous System Modeling Program (CSMP) technique. Air relative permeability-water saturation data was obtained from Green et al (1) who used the same technique. Unlike water relative permeability, air relative permeability is only a slight function of soil type and can be assumed to be the same in our case as Green't. Both air and water relative permeability-saturation data was input as 20 values evenly spaced from 0 to 100 percent saturation.

iv) Initial water saturation versus depth was estimated at 71=89% for wetlands.

v) Soil temperature versus time was estimated to lag 3-5^{o}F behind average monthly air temperature which was obtained as actual climatological data from the Stuttgart U.S. Weather Station. The nearest station reporting soil temperature for 1972 was Warren. They reported quarterly values which approximately check with the estimates.

Plant Parameters:

i) An estimate of water saturation in the soil is about 15% at the permanent wilting point as obtained from limiting values of capillary pressure data from Bedinger et al (41) and for small flux values of root uptake according to Hillel (42).

ii) An estimate of plant conductivity was obtained from the literature; e.g., Jensen et al (43) and Green et al (1). Values investigated range from 0.00025 - 0.1 kbm/ft^2 hr/psi.

iii) The height of vegetation above the datum plane was set at 20 ft, considerably less than the height of most trees since undergrowth is concentrated at lower heights.

iv) Critical leaf water potential was found to be about -225 psi for most plants according to Briggs (44), Cowan (27), and Hillel (42).

v) An extensive literature search was made to obtain root densities and very little actual data was found. The only data for hardwood forests came from Assmann (50), who reported a root density of 76.8 ft/ft^3. Root densities for greater depths were estimated since some trees have roots distributed approximately 40%-30%-20%-10% if the root depth is divided into quarters per Mois (45).

vi) Per cent ground coverage by vegetation was set at 95%. according to Sims (46) established the test plot. Also, root density for the surface (and lower) was varied from the calculated value 90 ft/ft^3 to 900 ft/ft^3.

Other Parameters:

i) Viscosities and densities of air and water as a function of pressure were obtained from Perry (47).

ii) Estimates of mass-transfer coefficients for bare soil and leaf surfaces were obtained from Treybal (48), Perry (47), and others. Values were investigated from 0.1-0.25(lbm $ft^{-2}hr^{-1}$) $(hr\ mile^{-1})^{0.8}$(psi^{-1}) for the leaf-air coefficient.

Atmosphere Parameters:

Climatological data was obtained from Sims who obtained actual data, averaged monthly, from the U.S. Weather Station at Stuttgart or as twenty-nine year average (1931-60) from the U.S. Dept. of Agriculture (51). Relative humidity and wind velocity were monthly averages over two years. Air temperature and precipitation were twenty-nine year averages.

Program Parameters:

These were similar to Green et al (1) with some added flexibility built into the time step.

The time step is set according to:

$$DT = DT + TIMEMU * DT$$

where,

DT = time step, hrs

Timemu = time step multiplier

After the total time simulation reaches a preset value (after convergence has been well established), the time step can be increased to give a greater total time simulated per computer run.

RESULTS AND DISCUSSION

The applicability of a mathematical model to simulate soil moisture profiles fifty or more years into the future is, at least, determined by its ability to:

1) Simulate unsaturated ground-water flow including effects of evapotranspiration,
2) Operate economically (low computer time),
3) Simulate termination of transpiration for the non-growing season,
4) Simulate water run-off when the soil moisture becomes saturated.

Green (1) has shown that the previously described computer model is capable of simulating unsaturated ground-water flow including effects of evapotranspiration over short periods of time (less than one year). They found that the model behaved well by simulating field experiments and then comparing calculated versus actual data. This was accomplished by what Green called a history-matching technique. The history-matching procedure consisted on a trial and error determination of a set of input data parameters that resulted in a good match between the computer and measured moisture profiles.

Alterations to the model and to the input data were made in order to achieve the four specifications listed above for application of the model to a wetlands environment. A discussion of the alterations and results follows.

Computer Efficiency

This model was developed so that water saturation profiles could be calculated starting at time A and progressing to time B using an increment time step procedure. Green (1) used time increments of ten hours. When using Green's soil data and a ten hour maximum time increment, the model consumed 47.2 minutes of computer time to generate one year of soil moisture data. At this rate of usage, it would require 39.3 hours of computer time to predict soil moisture fifty years into the future. The first step in this project was to lower the computer time required by increasing the size of the time increment.

The time step size can be increased by either increasing the program parameter DLTMAX (i.e., from 10 hr to 100 hr) or by changing the time base (i.e., from hours to days or to months). The changing of the time base requires changing of all data that contains the units of time. Table I is a list of the data that required changing and the values used to give time bases of hours, days, and months.

A search for the best time increment was conducted using Green's soil data. Time increments of ten hours, one-hundred hours, three days, and 0.5 months were tried. Over two years of moisture data was generated for each time increment. Table II presents the computer time used per year of data and a comparison of the amount of cumulative evapotranspiration predicted. A time increment of ten hours was considered the base for difference comparisons. Table III presents the moisture profiles predicted at the end of one year for time increments of ten hours, three days, and 0.5 months.

TABLE I

VARIABLE DATA USED FOR TIME BASE CHANGE STUDY

NAME	HOURS	DAYS	MONTHS
AIRK	0.0175	0.420	12.60
Plant conductivity	0.001	0.024	0.720
Air-leaf evap. coef.	0.125	3.00	90.00
Max Del T	10.0	3.00	0.50
K (X) Darcy	2.0	48.0	1440.0
DX**	720.0	30.0	1.0
TOLW*	0.0001	0.0024	0.0720
TOLG*	0.0001	0.0024	0.0720
PMCHW*	0.0001	0.0024	0.0720
PMCHG*	0.0001	0.0024	0.0720

* Convergence criteria

** Time increment for climate tables

TABLE II

COMPARISON OF MACHINE EFFICIENCIES AND ACCURACY
FOR FOUR DATA TIME BASES

TIME INCREMENT ΔTMAX	CUMULATIVE, lbm/yr EVAPOTRANSPIRATION	MACHINE TIME USE MUNUTES PER YEAR
10 hr.	249.1	47.20
100 hr.	259.1	23.288
3 days	247.7	12.4333
0.5 months	252.8	21.925

TABLE III

MOISTURE DISTRIBUTION AFTER ONE YEAR FOR THE DIFFERENT TIME BASES

	% SATURATION (AT 360) DAYS		
SOIL DEPTH, ft	ΔTMAX=10 hr	3.0 DAYS	0.5 MONTHS
0.0	0.09513	0.09495	0.05490
1.0	0.03855	0.03627	0.05738
2.0	0.02077	0.01957	0.01543
3.0	0.02154	0.02133	0.01696
4.0	0.05765	0.05760	0.05422
5.0	0.06686	0.06720	0.06530
6.0	0.06491	0.06473	0.06392
7.0	0.07880	0.07870	0.07410
8.0	0.08832	0.08851	0.08544
9.0	0.09273	0.09303	0.09013
10.0	0.09042	0.09065	0.08855
11.0	0.08793	0.08812	0.08618
12.0	0.08965	0.08986	0.08763
13.0	0.09158	0.09183	0.08938
14.0	0.09354	0.09358	0.09139
15.0	0.09527	0.09565	0.09347
16.0	0.09651	0.09694	0.09536
17.0	0.09703	0.09746	0.09680
18.0	0.09656	0.09687	0.09748
19.0	0.09450	0.09407	0.09695
20.0	0.08916	0.08721	0.09376
21.0	0.08308	0.08177	0.08689
22.0	0.08078	0.08054	0.08154
23.0	0.08372	0.08384	0.08356
24.0	0.10300	0.10300	0.10300
Average	0.07993	0.07974	0.07799

TABLE IV

BASE CASE -HYPOTHETICAL VALUES FOR THE KEY RESISTANCES

KEY RESISTANCES*	VALUE
Absolute Perm.	0.864
Plant Cond.	0.024
Air-Ground Ev. Coef.	0.42
Air-Leaf Coef.	3.0
Root Density	78.2

*Units are given in Subroutine Main, Appendix I-B.

As can be seen from the Tables II and III, the day increment data reproduces the hour increment data very well. Evapotranspiration differed from the hour data by 0.56%, and average percent moisture differed from the hour data by 0.24%. While reproducing the hour data calculations closely, day data cut the machine time from 47.2 min/year to 12.43 min/year. On the basis of this data, it was decided to check the day data out further by running a sensitivity test on the key physical parameters.

<u>Variation of Physical Parameters</u>

The effect of varying key parameters was investigated using wetlands data with the time base in hours and also in days. The purpose of this sensitivity test was to demonstrate that a model will react in the same manner using day data as when using hour data when a parameter is changed.

A parameter which was thought to be a key resistance was chosen in each portion of the soil-plant-atmosphere system. Table IV gives the original hypothetical values for the key resistances. The five main cases are:

1) Case I (Base Case) - Original hypothetical data
2) Case II - Base case with the plant conductivity increased six fold
3) Case III - Base case with the leaf-air mass transfer coefficient doubled
4) Case IV - Base case with the absolute soil permeability increased by six fold
5) Case V - Base case with the root density increased by six fold

TABLE V

CASE RESULTS

TIME DAYS	CUMULATIVE EVAPORATION lbm CASE I	II	III	IV	V	CUMULATIVE TRANSPIRATION lbm CASE I	II	III	IV	V
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
10.66	0.066	0.066	0.066	0.066	0.066	8.88	8.93	17.66	8.88	8.88
33.44	0.227	0.227	0.227	0.227	0.227	30.52	30.68	60.66	30.52	30.52
69.44	0.586	0.586	0.586	0.586	0.586	76.83	79.30	156.5	78.83	78.67
105.44	0.861	0.881	0.881	0.881	0.880	118.50	119.2	234.9	118.5	118.3

TABLE VI

SOIL DEPTH VS. SATURATION ($ft^3 H_2O/ft^3$ soil) AT 106 DAYS

DEPTH ft	INITIAL All Cases	CASE I	CASE II	CASE III	CASE IV	CASE V
0	0.3843	0.3232	0.3213	0.1241	0.3049	0.2259
1	0.3843	0.2375	0.2352	0.1010	0.2974	0.2178
2	0.3843	0.2728	0.2707	0.1007	0.3131	0.2738
3	0.3843	0.3041	0.3025	0.1009	0.3214	0.3139
4	0.3843	0.3243	0.3230	0.1024	0.3254	0.3382
5	0.3843	0.3344	0.3333	0.1031	0.3280	0.3510
6	0.3843	0.3426	0.3417	0.1096	0.3310	0.3578
7	0.3843	0.3466	0.3458	0.1144	0.3337	0.3624
8	0.3843	0.3615	0.3609	0.1434	0.3390	0.3661
9	0.3843	0.3689	0.3684	0.1775	0.3428	0.3693

Table V presents the cumulative evaporation and transpiration for each case. Table VI gives soil saturation versus depth for all cases, initially and after one-hundred-five days of simulation.

Cases I, II, and IV produced simulation results to within one percent agreement between the routine base and the day time base. Problems of convergence were encountered in Cases III and V using the hour time base, while there were no convergence problems using the day time base. These results indicated that by changing the data base from hours to days, an advantage of better computer efficiency and less convergence problems can be obtained while sacrificing very little in model reliability. The remainder of this project was performed using a data base of days.

Transpiration

While Green (1) was history matching model calculations with field data, difficulty was encountered with his October data. To obtain reasonable agreement in October, he found that it was necessary to "shut-off" transpiration prior to that data. If transpiration were continued in the model much beyond that date, the water loss to transpiration was excessive. Because of this difficulty, Green concluded that the model was not designed to handle this situation. Thus he used a single cut-off date in the fall and a single initiation date in the spring. The same difficulty was encountered when trying to run this program for an extended time.

It was decided to not use the on-off approach to seasonal changes in transpiration for two reasons. First, to use one initiation statement and one cut-off statement for each year would be too clumsy for a fifty year run. Second, transpiration really increases gradually at the beginning of the season and decreases gradually at the end. To overcome these problems, the following function generator was used.

```
QT    = (TTOT/360.0)*(6.254)-A
QTT   = (B)*(Sin(QT))+C
QTran = QTT*QTran
```

Where:

TTOT = Current simulation time (days)

QTran = Transpiration lbm/day

A = Constant to shift function left or right on time scale

B = Constant to control function magnitude

C = Constant to shift function up or down magnitude scale

For this project the constants A, B, and C were chosen such that transpiration starts at about the average date of the last frost of winter and ended at about the average data of the first frost in the fall. A graph of the function as used in this project is shown in Figure 4.

Drainage

Several long simulation runs were attempted using the above transpiration function. When using clay soil data, the upper soil layers quickly became super-saturated as transpiration went to zero. By the start of the next growth season the saturation of the top layer (1 foot) exceeded 130%. Thus, in effect, the forest was being flooded. To stop

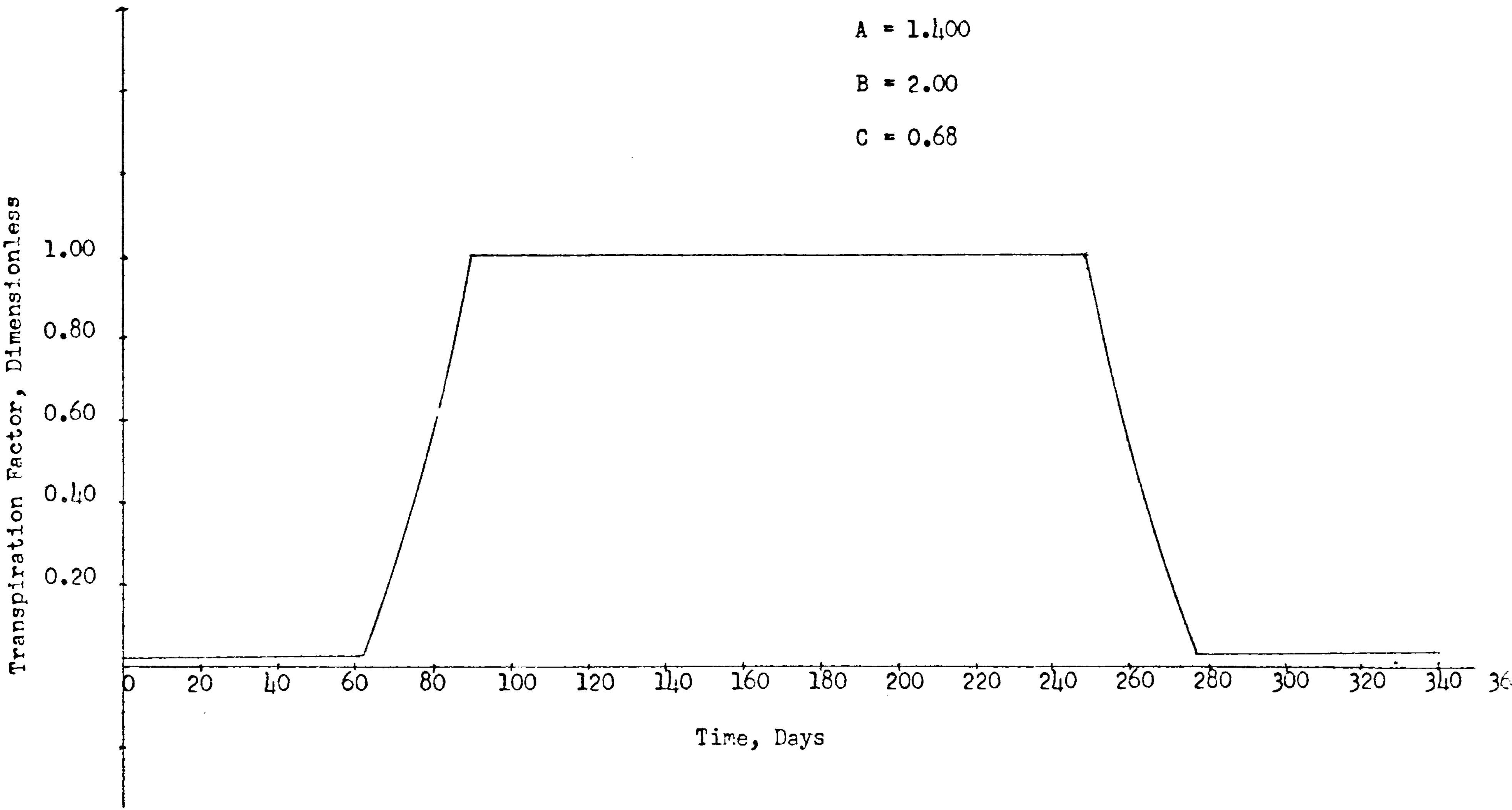


Figure 4. Transpiration Function

this excess moisture build-up in the top layer the following "drainage" function was added.

QWF = 10.0*(D-S(N))*QWF

Where:

QWF = Rainfall, in/ft^{2}day

S(N) = Percent moisture, top ft.

D = Constant

The "drainage" function will go into effect only if the percent moisture exceeds (D-0.100). Precipitation was set equal to zero if percent moisture exceeds "D". "D" was chosen by trial and error to allow the percent moisture of the top layer to just approach one hundred percent. Figure 5 is a graph of the percent rainfall allowed in the soil versus percent saturation of the top layer.

Final Form of the Numerical Model

The final form of the numerical model of unsaturated groundwater flow including the effects of evapotranspiration, seasonal changes in transpiration, and drainage of excess rain water is presented in Appendix I-B. Simulation of more than thirty years of moisture data has been accomplished with this model using data listed in Appendix I-C. The model generated reproducible results without convergence problems.

Computer time usage varied from less than two minutes per year to about six minutes per year of simulation time. The time required per year of data was very dependent on the percent moisture in the soil. If the soil moisture was within the fifteen to ninety-six percent range, convergence was very fast and simulation time was low. If the soil moisture varied from this range, simulation time greatly increased. While

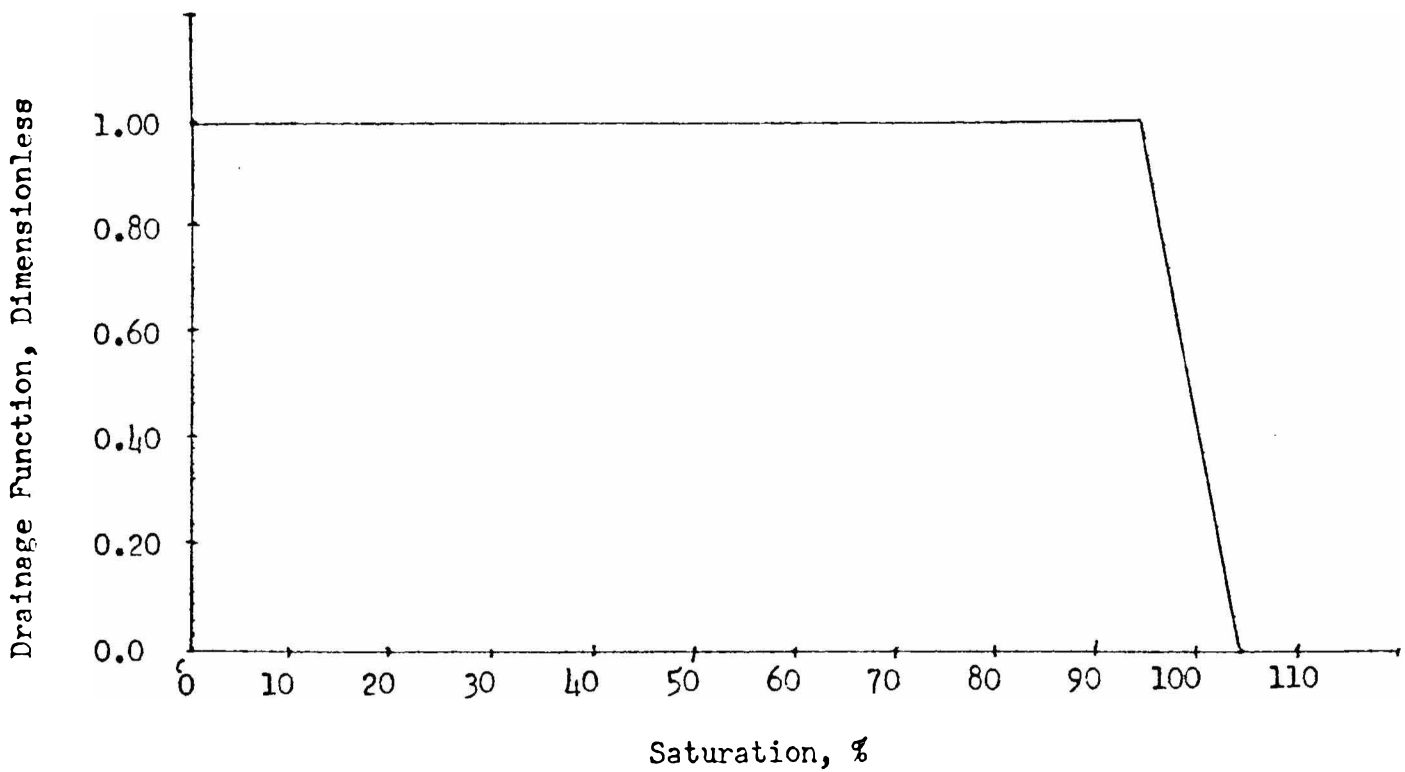


Figure 5. Drainage Function, "0"=1.04

running the model with the drainage function constant "D" equal to 1.00, the soil moisture stayed within fifteen to ninty-six percent range and simulation was 1.92 minutes per year of data. But, using a drainage constant of 1.04, soil moisture approached or equaled one hundred percent and the model required 5.88 minutes per year of data.

Two years of soil moisture data (average of the top give feet) are presented in Figure 6. The data was generated using drainage constant equal to 1.04. The predicted seasonal changed in the soil saturations were plausible and may, with some adjustments in soil and plant parameters, approach actual field values.

Figure 7 shows two years of soil moisture data (average of top five feet) using base data, and using base data with the air-leaf coefficient increased by one-third. Both sets of data were with a drainage constant of 1.00. This figure demonstrates the dramatic effect the air-leaf coefficient has on the moisture profile. The air-leaf coefficient will be a powerful tool to be used during any history matching procedure.

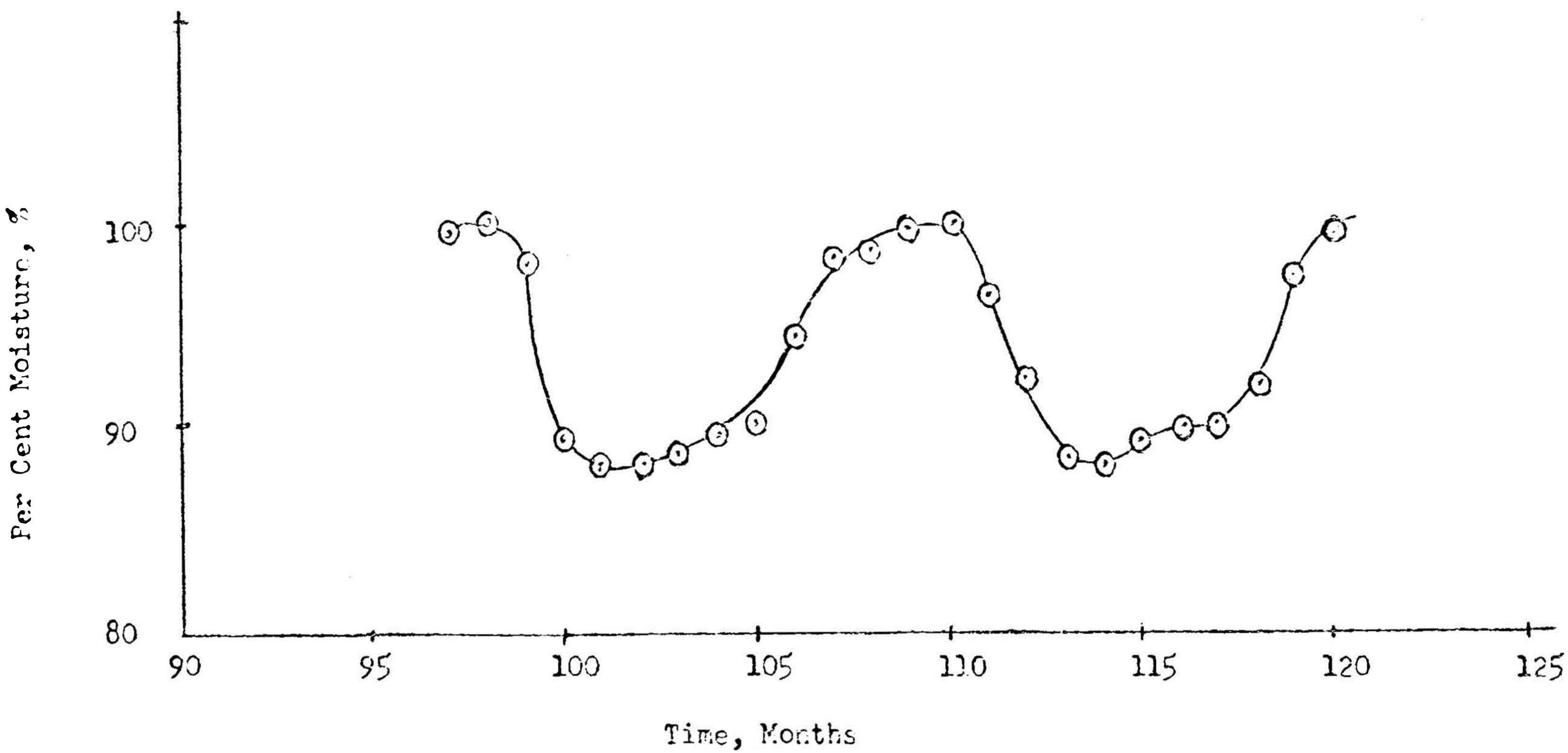


Figure 6. Average % Moisture for Top 5 ft, using Base Data

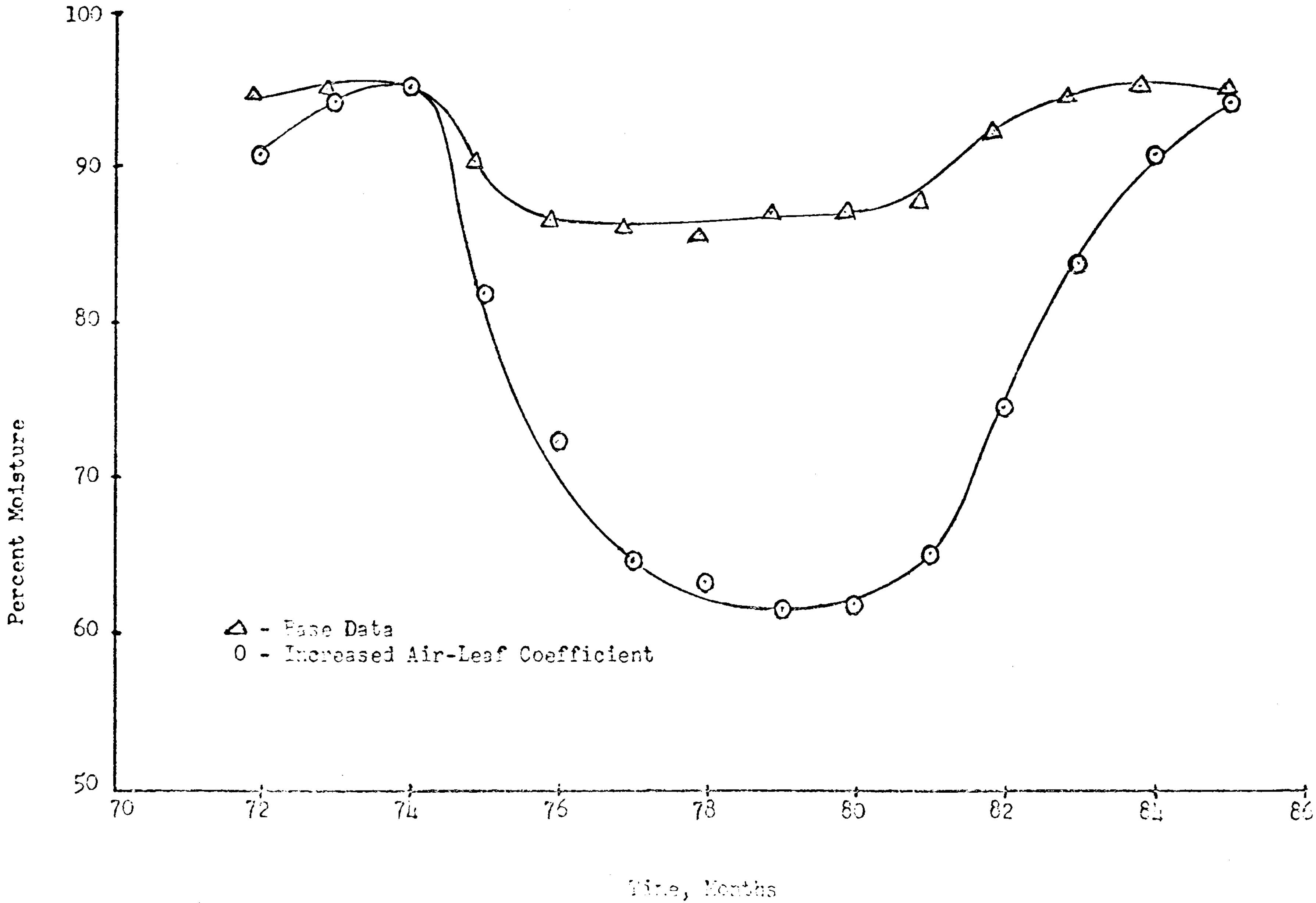


Figure 7. Moisture Profile, Showing Effect of Increasing Air-Leaf Coefficient by One-Third

CONCLUSIONS AND RECOMMENDATIONS

From the above discussion, it is possible to draw some conclusions and recommend proposals for future applications of the model to wetland forests.

Conclusions

1) Functions describing seasonal changes in transpiration and drainage of excess precipitation have been successfully adapted to a digital computer model to simulate the vertical movement of soil moisture through the zone of aeration.
2) Computer time usage is at minimum when day based data are used.
3) The model responds well to hypothetical wetland forest data for extended periods of time and for all seasons.
4) The leaf-air mass transfer coefficient seems to be the controlling resistance affecting transpiration.
5) Root densities appear to have little effect on total transpiration.

Recommendations

It is recommended for future applications of the model that:

1) As much actual data be obtained from the test plot as is feasible.
2) This field data be used in a history matching procedure similar to that done by Green et al (1). During the

history matching, the constants in the transpiration and drainage functions should be considered.

3) The simulation be extended into the future to predict how proposed changes in flood control and drainage will affect the water available for vegetative use.

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NOMENCLATURE

a	-	volumetric air content of the medium
A	-	root density, i.e., length of roots per volume of soil
A	-	constant in water saturation equation
B	-	constant in water saturation equation
B	-	constant which contains the geometrical factors in the flow equation
D	-	function of conductivity and derivative of the capillary pressure - water saturation curve
D_{atm}	-	molecular diffusivity of water vapor in air
E	-	total rate of transpiration
g	-	acceleration of gravity
h	-	thickness of a discrete layer in the root zone
k_i	-	capillary conductivity of the soil
K_e	-	effective soil conductivity
k	-	absolute permeability
k	-	conductivity
L	-	length of root per volume of soil
m	-	mass of water in the system
p	-	partial pressure of water vapor
P_o	-	water vapor pressure at $\underline{4} = 0$
P	-	total gas pressure
P_{ws}	-	water vapor pressure at surface temperature
P_{wa}	-	partial pressure of water vapor in the atmosphere
p_1	-	water vapor partial pressure at the porous media surface
P_a	-	partial pressure of water in the air

P_s^* - saturation water vapor pressure at soil temperature

p_1^* - saturated water vapor pressure at leaf temperature

P_{cl} - difference between air pressure and water pressure in the pores of the leaf

P_c - capillary pressure

Q_e - evaporation rate

q_{rap} - vapor flux density

$Q^1{}_w$ - rate of removal of water by evapotranspiration per unit volume of soil

r - resistance

r - radius of sand grains in porous media

R - universal gas constant

R_1 - rate of evaporation per unit area

S - volumetric water saturation

t - time

T - temperature

V - model water volume

V - wind velocity

W - rate of water uptake per unit cross-section of soil

X - space coordinate

Z - position

Z_i - average depth of the ith soil layer

Z - plant impedence

α - tortuosity factor allowing for extra path length (=0.62)

γ - "mass-flow factor" introduced to allow for the mass flow of _____ arising from the difference in boundary conditions governing the air and vapor components of the diffusion system

ρ - density of water vapor

τ_r - soil water potential at the root surface

Ψ - water potential based on pure water at atmospheric pressure and temperature as a reference

ϕ - porosity