School of Engineering

FSI of viscosity measuring mechanical resonators: theoretical and experimental analysis

rheonics

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Introduction and Goal

Measuring viscosity online is crucial to maintaining the quality of many chemical and biological processes. Within this work, the response of the fluid-structure interaction between sensor and viscous damping, which is induced by the surrounding fluid, is used to determine the viscosity of the fluid. Common mechanical resonators used for viscosity monitoring are probe style and create obstruction inside the piping system thus disturbing the flow.

The goal of this study is to elaborate the working principle of a tube-style sensor. Such a device will not create an obstruction nor a disturbance to the flow.

Tube or rod-style sensors, which underlie the same working principle, have experimentally been tested [1-3] for different applications. In case of the tube-style sensor, the flow is inside the tube/sensor which is part of the piping system and does not interfere with the flow. The first torsional mode is excited using an electromagnetic system. In order to get a deeper insight into the functionality of such a device, a mathematical model has been developed. It describes the mechanical vibration coupled with the fluid interaction.

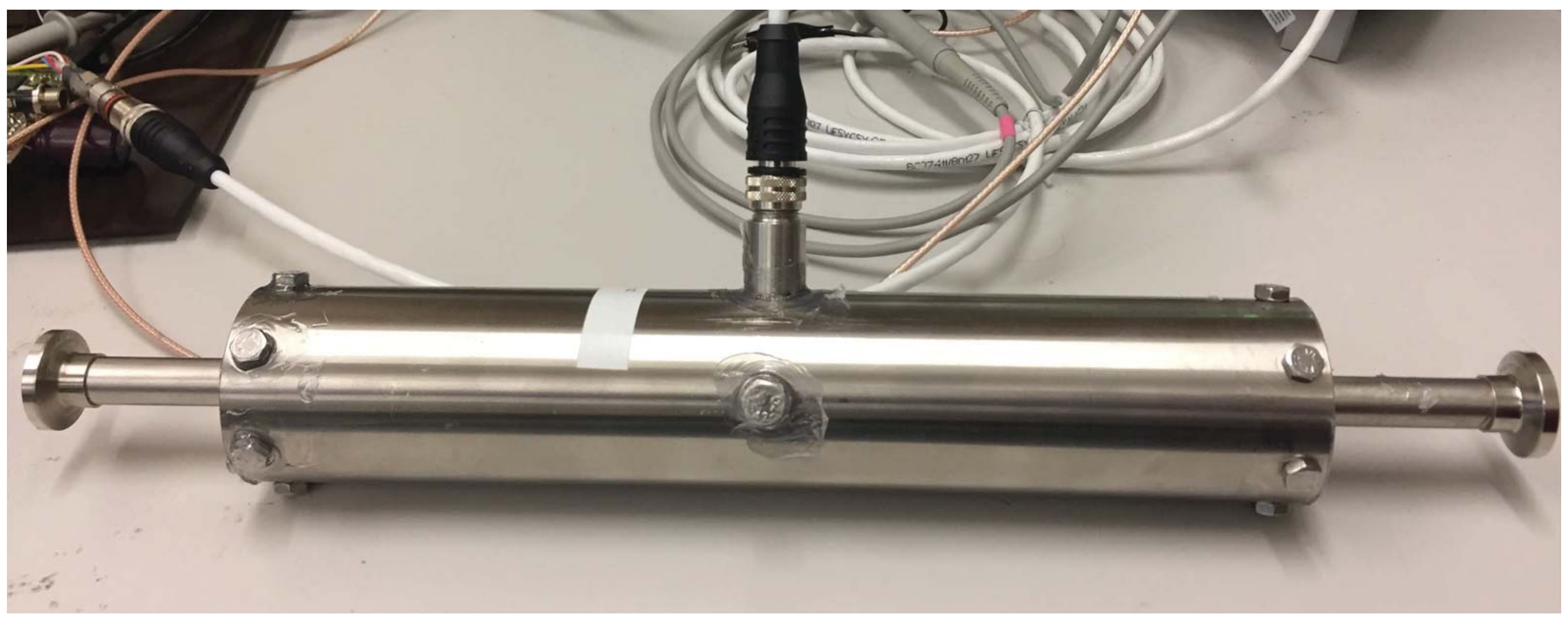

Fig 1: Experimental set-up of a tube-style sensor for measuring viscosity

Dynamic Viscosimetry

Any immersed mechanical resonator is subjected to shear stresses by the surrounding fluid. These shear stresses scale with $\sqrt{\rho\omega\eta}$ and induce viscous damping as well as mass loading. If the tube-style sensor is excited torsionally near the natural frequency of the mode of interest, the method of Eigenmode decomposition can be used and the resonator can be described by a 2nd order ODE for the temporal dependence.

This ODE describes the angular deflection with respect to a moment of inertia J, spring stiffness c, internal damping k and two source terms for excitation and viscous damping .

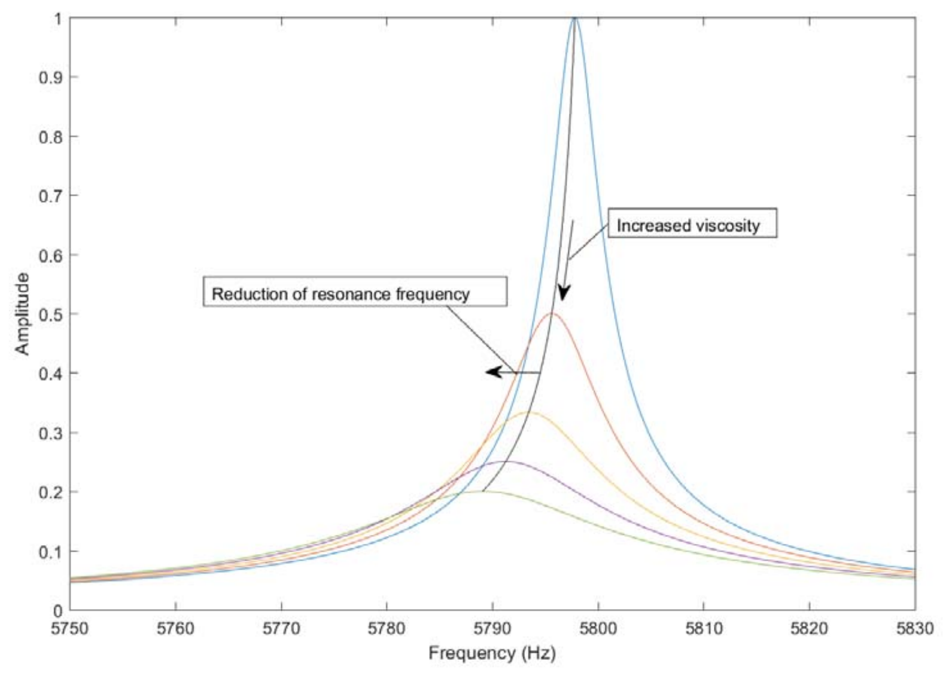

Fig 2: Amplitude dependency on viscous damping

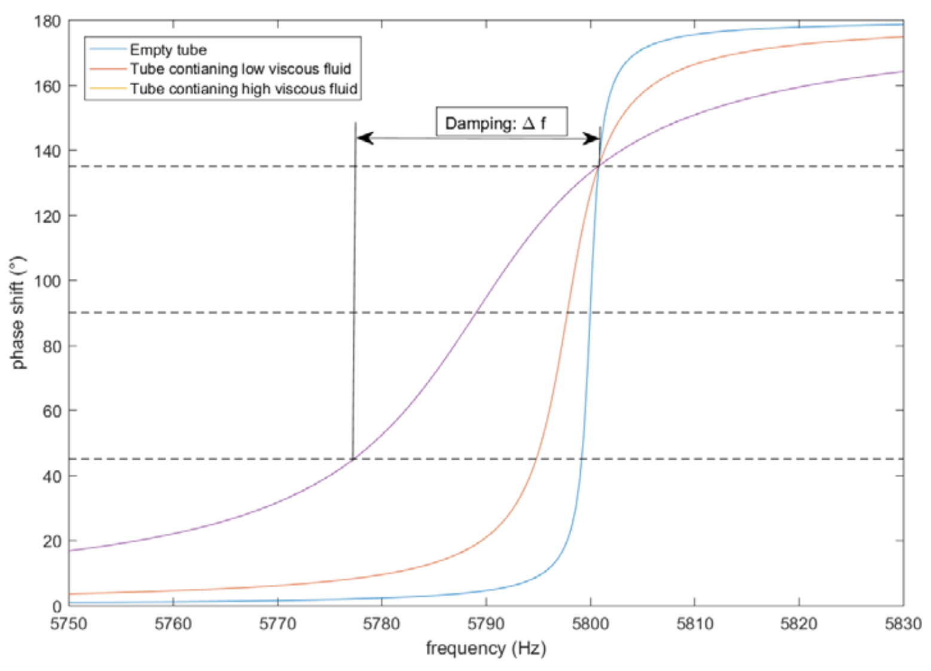

Fig 3: Phase shift due to viscous damping

Fig 2 & 3 show the response of the system to viscous damping.

In Fig 2 the amplitude is reduced with increasing viscosity at constant excitation-force. In Fig 3 the slope of the phase curve becomes smaller as viscosity increases. For the analysis, the slope of the phase curve has been considered, because it does not depend on the amplitude of the angular deflection nor the excitation force.

It is approximated by:

$$\Delta f = \omega(\Phi = 135°) - \omega(\Phi = 45°)$$

[1]Dual, J., 1989, “Experimental Methods in Wave Propagation in Solids and Dynamic Viscometry,” (8659).

[2]Dual, J., and O’Reilly, O. M., 1993, “Resonant Torsional Vibrations: An Application to Dynamic Viscometry,” Arch. Appl. Mech., **63**(7), pp. 437–451.

[3] Häusler, K., Reinhart, W. H., Schaller, P., Dual, J., Goodbread, J., and Sayir, M., 1996, “A Newly Designed Oscillating Viscometer for Blood Viscosity Measurements,” Biorheology, **33**(4–5), pp. 397–404.

Velocity field in the vicinity of the tube’s wall

The 1st torsional mode of the oscillating structure creates a tangential harmonically oscillating velocity on the inner wall of the tube. The tangential velocity amplitude (Fig 4) decreases exponentially with increasing distance from the wall, where $\delta = \sqrt{\rho\omega/\eta}$ is the thickness of the boundary layer. If δ is very small in comparison to the radius of the tube, i.e. 1000-times smaller, it can be considered as flat and the velocity field can be determined analytically. Fig 4 & 5 show the comparison of the analytical solution to the CFD simulation in OpenFOAM for the tube geometry. The velocity amplitude closed to the wall is shown in Fig 4. Fig 5 shows the normalized velocity with respect to time at three different locations. Because δ is very small, simulation and analytical solution are almost identical.

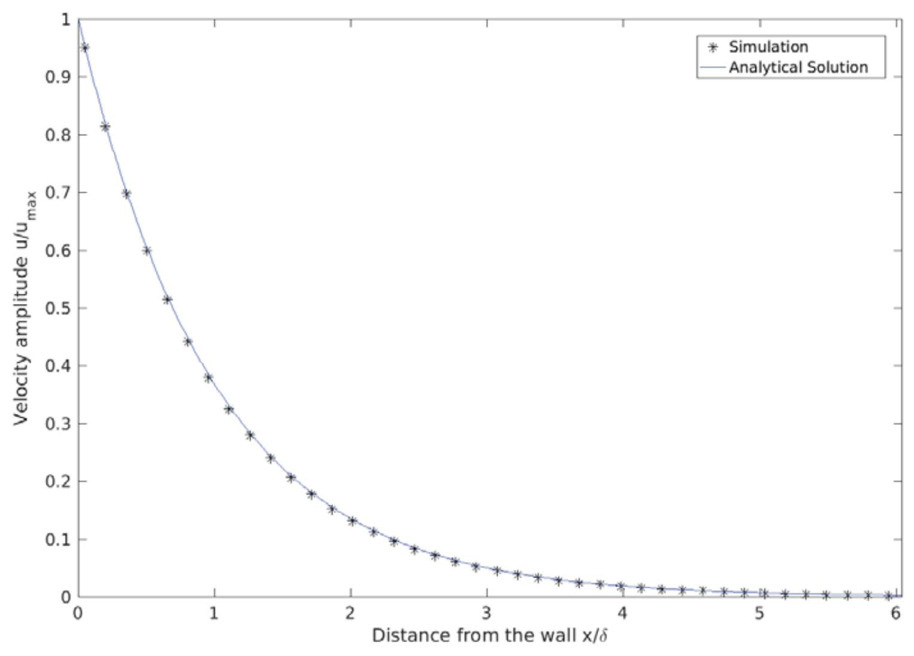

Fig 4: Normalized amplitude of tang. velocity

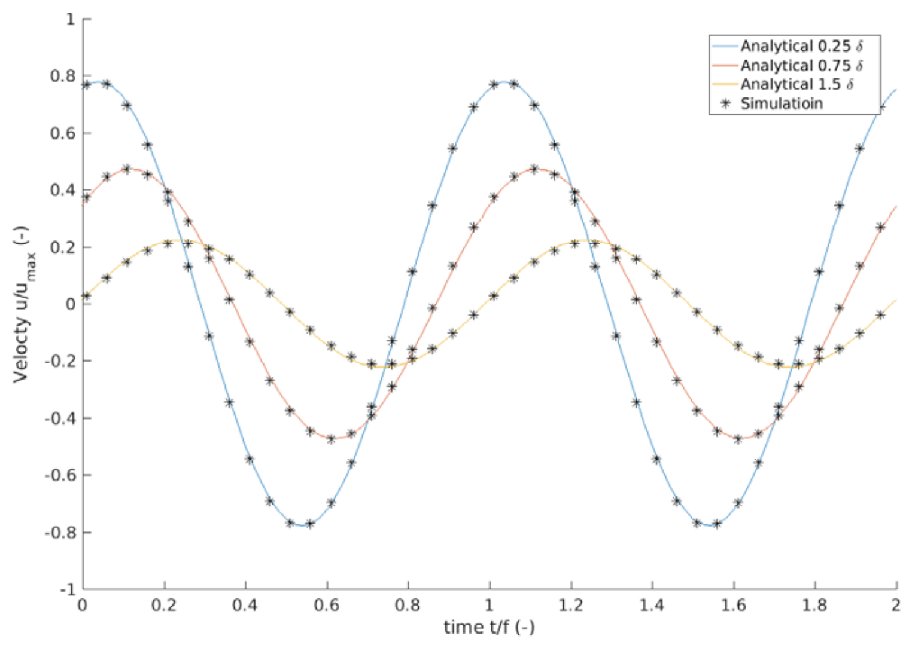

Fig 5: Transient velocity field near the wall.

Experimental validation

Experiments have been conducted for different viscosities using the device shown in Fig 1. The damping parameter Δf has been determined for the analytical flat plate solution imposed at the wall of the tube, the CFD simulation in OpenFOAM and experimental data. A correction coefficient $J_{kor} = 0.93$ has been used to fit the data to the computed results and to account for manufacturing tolerances, see Fig 6. The analytical solution deviates at higher viscosity because the curvature of the tube is not considered. However, both solutions match the experiments quite well. The damping parameter Δf can be described for low viscosities by the analytical solution:

$$\Delta f = \frac{1}{\sqrt{2}} \cdot \frac{l r^3 \sqrt{\rho\omega\eta}}{J_{kor} \cdot J_0} + \frac{k}{J_{kor} \cdot J_0 2\pi}$$

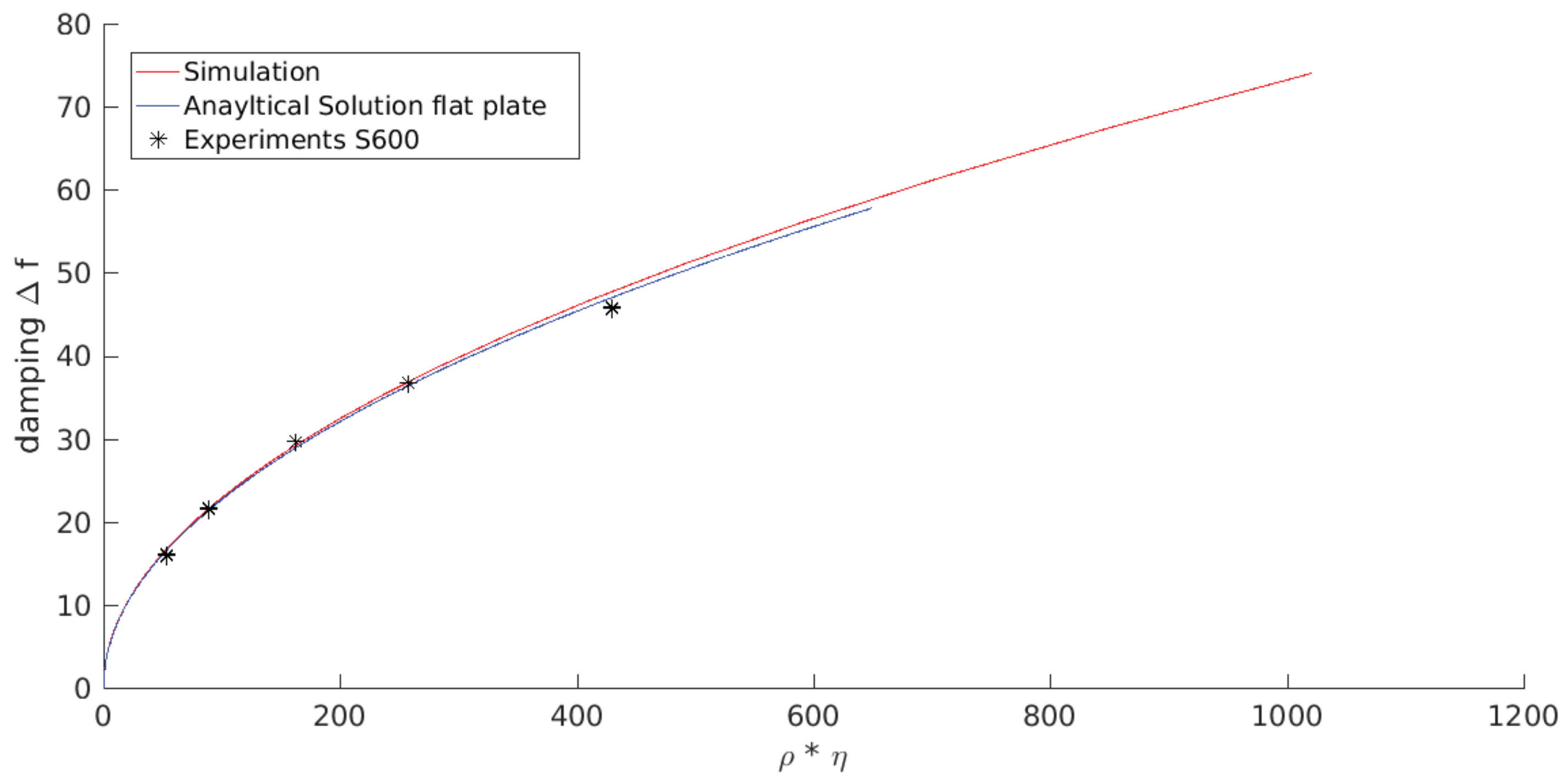

Fig 6: Damping of the tube-style sensor depending on the fluids viscosity-density product $\rho\eta$. Comparison between analytical solution (red), CFD simulation (blue) and Experiments (fluid S600, tube radius 5.5mm, length 180mm)

Conclusions

A tube-style sensor can be used to measure the viscosity-density product of the contained fluid. Two models, one based on the analytical solution on a horizontally oscillating plate, the other on a CFD-simulation have been used to describe the viscous damping of the system.

For the validation, a correction factor for the moment of inertia of 0.93 has been introduced in order to account for manufacturing tolerances. The relative deviation in the viscous damping is 5.7% for the uncorrected and 2% for the corrected moment of inertia, see Fig 6. Since the overall agreement is in acceptable range, the models can also be used as a design tool. Furthermore, this study has shown that a tube-style sensor can be used for measuring viscosity. Interferences with flow passing through the pipe as well as Non-Newtonian effects will be subject of further studies.