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# A Speed Planner Approach Based On Bézier Curves Using Vehicle Dynamic Constrains and Passengers Comfort

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**Abstract**—This paper presents a speed profile generation approach for longitudinal control of automated vehicles, based on quintic Bézier curves. The described method aims to increase comfort level of passengers based on the ISO2631-1 specification, while taking into account vehicle dynamics and traffic rules to keep high safety levels. The algorithm has been tested in an in-house tool for high accuracy vehicle dynamics simulations, called Dynacar. The considered scenario is a closed circuit inside Tecnalia facilities. The resulting profile has better properties (for example, rate of change) than a raw input based on traffic speed limits. When used as reference for the speed controller, it improves both comfort and safety.

**Index Terms**—Intelligent Transportation Systems, Automated Driving, Motion Planning, Longitudinal Control, Speed Profile

## I. Introduction

The interest in automated vehicles is considerably growing in research institutes and companies every year since mid-nineties, when a lot of demonstrations and projects in this topic have grown [1]–[5]. Drawbacks related to safety implementation in this technology have been partially solved in the last decade obtaining commercial solutions which are fully available [6], such as the Adaptive Cruise Control (ACC).

Nowadays, most automated driving projects are strongly supported by private and/or public funding, involving a large list of OEM and technologies' providers. Current automated driving run is pushing the limits of the technology and, safety and comfort criterion are playing an important role on acceptance of intelligent vehicles within the society.

The present work is based on the control architecture shown in [7], which consists of six blocks. Those consider all the domains in automated driving discipline: acquisition, perception, communications, decision, control and actuation. The decision block - in which is the main contribution of this work - has the goal of dealing with lateral (steering) and longitudinal (speed) motion planning. Longitudinal control is tightly related with keeping an optimal speed in terms of comfort and safety. Anticipation is a key capability for a robust longitudinal control algorithm that can be, nonetheless, difficult to embed in low level control due to its nature (fast controllers, typically PID). Some authors as [8]–[10] have made approaches on these

areas as Model Predictive Control, spline curves and jerk considerations to do speed planning.

The current work is focused on designing and implementing a longitudinal speed planner generating an optimal profile in terms of comfort and safety. The output of the planner feeds an automated vehicle fuzzy control system, that will be in charge of keeping the vehicle as close as possible to the speed reference.

The contribution of the work is simplifying the task of determining the optimal speed over time, simultaneously considering a customizable set of variables such as comfort and vehicle maximum acceleration and deceleration. Its output can be applied to any control strategy, effectively improving its response in the aforementioned terms.

This paper is organized as follows. Section II describes the speed profile design based on quintic Bézier curves, showing the corresponding mathematical expressions which support the implementation. In Section III, the scenario used for testing is detailed, with special attention in the acceleration constraints based on the ISO2631-1. Obtained results are published and explained in Section IV, comparing the speed planner in both constant and variable speed conditions. Lastly, Section V presents some conclusions and future work on terms of speed planning and its impact in automated driving.

## II. Speed profile approach

Automated speed regulators are usually influenced by sudden reference changes. These variations can result in undesired effects such as slow control responses, overshoots or instabilities.

Traditional control techniques use tunable parameters, which improve their robustness under expected inputs, e.g. overshoots can be reduced at the cost of underdamped behaviours producing slow responses in the controller. In most cases, the best achievable trade-off between both effects can still be considered a suboptimal solution. This problem can be improved working with the reference signal, so that the control system would be tuned for a smaller range of situations.

Hence, the primary goal of this work is to present a speed planning approach based on parametric curves for speed set point generation. The proposed planner

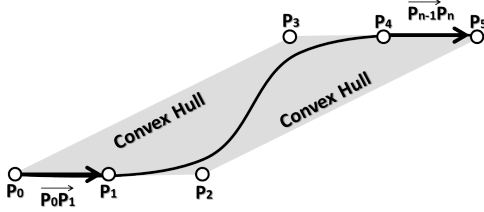


Fig. 1. Relevant characteristics of a Bézier curve.

considers the constrain of being under the speed limits at all time, as well as the dynamic restrictions of the vehicle, allowing the vehicle to keep the speed under safe conditions (acceleration criterion).

The speed planner approach is based on quintic Bézier curves (figure 1). The selection of this curves family is based on its convenient properties for designing continuous and safe paths under low computational cost. The most relevant characteristics [11] are described below:

- Departure and arrival vectors are defined by  $\overrightarrow{P_0P_1}$  and  $\overrightarrow{P_{n-1}P_n}$  respectively, which allow to connect curves at starting and ending points.
- The generated curve is confined within the convex hull formed by its control points, depicted as a grey area in figure 1.
- The generated curve is  $C^n$  continuous, with “ $n$ ” the order of the Bézier polynomial (number of control points minus 1).

These characteristics are contained in the general formulation of Bézier curves, based on the Bernstein Polynomials, where  $t_B$  is the control parameter of the curve:

$$B_{xy}(t_B) = \sum_{i=0}^n \binom{n}{i} P_i t_B^i (1-t_B)^{n-i}, t_B \in [0, 1] \quad (1)$$

This equation is general for any order Bézier curve in  $\mathbb{R}^2$  space, which curves will be limited to fifth order (6 control points). As an additional constrain, control points  $P_0, P_1, P_2$  will be collinear and aligned with the desired entry direction; Similarly, points  $P_3, P_4, P_5$ , will lie in the same line heading to the leaving direction (figure 2), ensuring starting and ending direction parallel to the abscissa axis.

Control points are equidistant along longitudinal abscissa axis. Distance between two consecutive points is defined as  $d_x(P_{n_x}, P_{n_x+1_x}) = D$ . Combining these designing characteristics the control points are defined as follows:  $P_0 = [0, V_0]$ ,  $P_1 = [D, V_0]$ ,  $P_2 = [2D, V_0]$ ,  $P_3 = [3D, V_0 + W]$ ,  $P_4 = [4D, V_0 + W]$  and  $P_5 = [5D, V_0 + W]$ . Where  $V_0$  is the initial speed and  $W$  is the speed span between final speed and  $V_0$ . The resulting  $x$  component of Bézier (equation 1) will be:

$$B_x = 5Dt_B \quad (2)$$

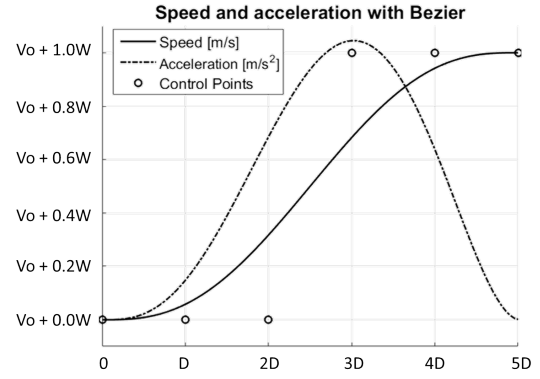


Fig. 2. Speed and acceleration using Bézier.

This equation shows that the  $x$ -distance between consecutive points is constant when the control parameter  $t$  of Bézier curve is sampled at regular intervals.

For speed planning generation, maximum acceleration/deceleration are defined as fixed designing parameters. Hence, it is mandatory to calculate those parameters for the generated curve. The Bézier speed profile is a function on the position domain where each point  $B_y$  correspond to a point  $B_x$  that has a constant separation from the precedent point. Equation 3 shows the application of the variable change  $v = ds/dt$  in the acceleration definition. The goal is to shift the calculation of the acceleration from time to position domain.

$$a_l(t) = \frac{dv_l(t)}{dt} \Rightarrow a_l(s) = v_l(s) \frac{dv_l(s)}{ds} \quad (3)$$

Where  $a_l$  represents the longitudinal acceleration,  $v_l$  longitudinal speed,  $t$  time and  $s$  position. Equation 4 shows the acceleration with the correspondence of the Bézier components, where  $B_x$  is the component of Bézier on the  $x$ -axis or position axis and  $B_y$  is the  $y$ -axis component or speed. Expression  $5Dt_B$  is converted into  $s$  for an easier interpretation (equation 5).

$$a_l(t_B) = B_y \frac{dB_y}{dB_x} = \frac{B_y}{5D} \frac{dB_y}{dt_B}, t_B \in [0, 1] \quad (4)$$

$$a_l(s) = B_y \frac{dB_y}{ds}, s \in [0, 5D] \quad (5)$$

The derivative of equation 5 determines the positions of maximum acceleration and deceleration.

$$\frac{da_l}{ds} = \left( \frac{dB_y}{ds} \right)^2 + B_y \frac{d^2B_y}{ds^2} \quad (6)$$

And finally, doing an approximation to a second order polynomial, equation 6 square roots are:

$$s_{a_{max}} = \begin{cases} \frac{2.02 - 2\frac{V_0}{W} + \sqrt{(2.02 - 2\frac{V_0}{W})^2 - 12.09(0.11 - \frac{V_0}{W})}}{6.04}, & \text{if } W > 0 \\ \frac{2.17 - 2\frac{V_0}{W} - \sqrt{(2.17 - 2\frac{V_0}{W})^2 - 12.69(0.13 - \frac{V_0}{W})}}{6.04}, & \text{if } W < 0 \end{cases}$$

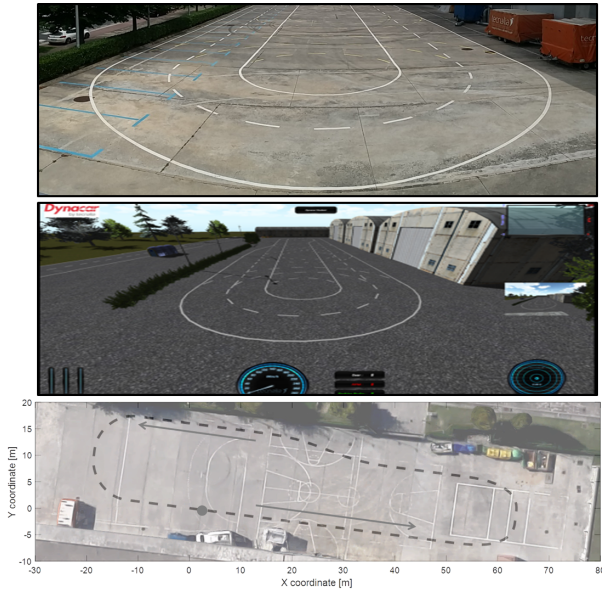


Fig. 3. Real scenario (top), Dynacar scenario (middle) and test trajectory (bottom).

All these steps are calculated for one segment. The process is iteratively repeated for all the segments in the path, considering speed and acceleration constrains, that is, being under both, road and comfort speed limit the whole time. In figure 2 is depicted (dotted line) the acceleration profile associated with the speed planning. The point of maximum acceleration or deceleration will not lay in the center even if the curve is symmetric.

### III. Test scenario

Simulation trials were performed in a 180 meters length circuit (80 meters long, 10 meters wide). It is composed by two roundabouts and a lane change segment, as shown in figure 3. The scenario has been modelled in the simulator based on the real test place (middle part of figure 3).

The selected simulation tool, Dynacar, models the vehicle with a multibody description for high accuracy. High precision simulation testing reduces the amount of trials in real vehicles, allowing a safe implementation of complex and hazardous manoeuvres.

The vehicle used is a Renault Twizy, due to its reduced dimensions, which suits the circuit size, while retaining most of the characteristics of a standard car.

The experimental part involves two different tests cases, i.e. first one sets the speed limit to a constant value of  $20[km/h]$ ; where as the second scenario considers variable speed limits, with a maximum value of  $30[km/h]$ .

It is relevant to notice, that in both cases, the speed limit will be related with a comfort parameter given by the equation based on the ISO2631-1 [12]:

$$a_\omega = \sqrt{(1.4a_x)^2 + (1.4a_y)^2 + a_z^2} \quad (7)$$

Where  $a_\omega$  is the total acceleration perceived by a passenger (comfort level),  $a_x$  is the longitudinal acceleration,  $a_y$  is the lateral acceleration and  $a_z$  is the vehicle's vertical acceleration.

Additionally, using a kinematic approximation of the bicycle model for the lateral acceleration and approximating the vertical and longitudinal acceleration to zero (small speed changes on  $x$  and  $z$  axis), the equation is reduced in:

$$a_\omega = V_l^2 K \quad (8)$$

Where  $V_l$  is the longitudinal speed and  $K$  is the curvature of an specific path segment. The equation implies a reduction of the speed limit when the comfort parameter is overpassed in curved segments. The speed limit will remain at the maximum value in case of straight lines (curvature equal to zero).

## IV. Simulation results

Tests have been performed with both constant and variable speed limit conditions. Additionally, the comfort criterion was defined according to the equation 8 with  $3.0[m/s^2]$  for the total acceleration  $a_\omega$ . The model includes the mechanical limits of the Twizy platform,  $1.15[m/s^2]$  for full throttle acceleration and  $3.5[m/s^2]$  deceleration achievable by the braking system.

### A. Test with constant speed limit

The first test was performed in the scenario shown in figure 3, with the speed limit set to  $20[km/h]$ , slowing down on curved segments so as to accomplish the comfort criterion with success.

On top of figure 4, the maximum speed based on the comfort criterion is shown in dotted line. That supposes a speed reduction in between 60 to 80 and from 145 to 170 meters given by the curvature change. Additionally, both maximum acceleration and deceleration used for solving the problem (dynamic constrains) are shown at the bottom of the figure.

The dashed line represents the generated speed profile (top part of figure 4). In this case the initial speed is greater than zero (around  $1[m/s]$ ) to overcome the starting vehicle inertia, thus, achieving the first point of movement faster. This is a necessary condition since the speed profile is generated as a function of the position and not the time. The plot at the bottom shows the acceleration associated with the speed planning. Its absolute value remains within the dynamic constraints during the whole trajectory.

Furthermore, the central figure displays the response of the simulated vehicle and its performance while tracking the reference speed (solid line). The maximum relative error is 12.5%, and it occurs in the last roundabout.

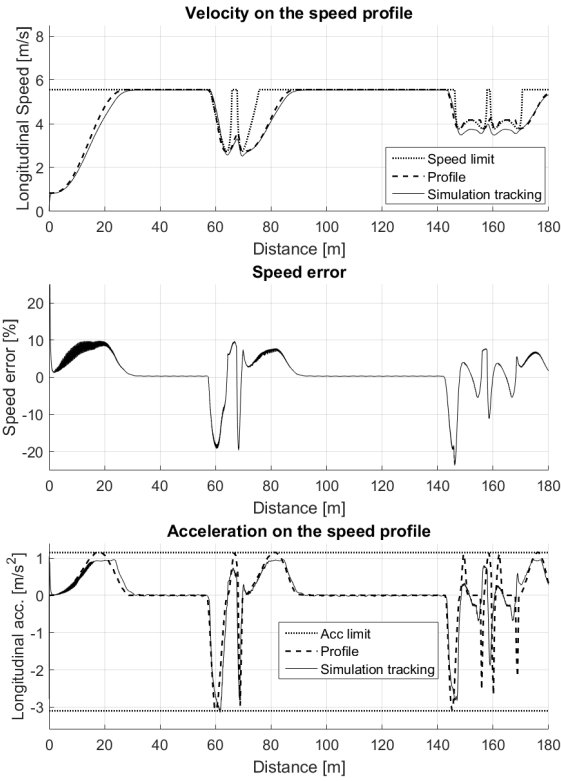


Fig. 4. Experiment with constant speed limit of 20 [km/h].

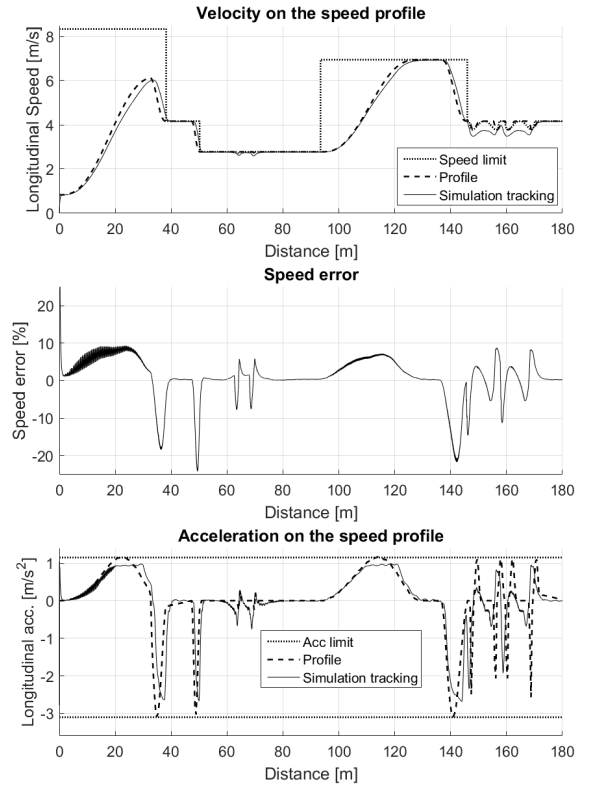


Fig. 5. Experiment with changes in set points.

## B. Test with variable speed limit

The second test consists in varying the speed limit at different times, including the consideration of comfort as in the first case (in figure 5 top the space between 145 and 170 meters). The maximum speed in the first segment is 30[km/h] (between 0 and 40 meters) and in the lane change side is 25[km/h] (between 95 and 145 meters).

Dotted line at the top of figure 5 represents the maximum allowed speed, considering the acceleration and deceleration constrains. Initial speed is different of zero to overcome the inertia of the vehicle at starting. As in the previous case, errors are kept under 12.5% for the speed tracking. The acceleration/deceleration obtained is within the dynamic constraints.

In this case, it is not possible to reach the maximum speed in the first 30 meters of the trajectory (bigger speed limit that in the first case). The algorithm anticipates the need of braking at maximum deceleration to reach the lower speed limit (this happens at 32 meters based on the figure 5).

## V. Conclusions and future works

The work presents a speed profile generator based on quintic Bézier curves, simultaneously considering three constraints: passenger comfort, vehicle dynamics and road speed limit.

Comfort is reached using the criterion of maximum acceleration on passengers feelings, which uses a kinematic

approximation for the lateral dynamics. The second criterion (vehicle's dynamics constrain) was achieved using maximum acceleration and deceleration of the vehicle. Lastly, the Bézier speed profile has the priority in reducing the speed in presence of future limits (avoiding overpass the maximum speed and keeping the numerical stability) keeping safety during automated driving.

Initial reference speed was set slightly greater than 0[m/s] to overcome the inertia of the vehicle when it is stopped. This effect is specifically related with the low level controllers (throttle and brake controllers) used during the tests.

Future works involve adding a prediction mechanism in cascade for improving the tracking of the speed. This is important when the system is subject to big actuation delays, as in the real vehicle conditions (around 0.5[s] delay for throttle and break).

Finally, the algorithm will be integrated in the real vehicle in order to compare the obtained results with the ones obtained using the simulation platform with the multibody model.

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