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A Discrete and Improved Bat Algorithm for solving a Medical Goods Distribution Problem with Pharmacological Waste Collection

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Abstract

The work presented in this paper is focused on the resolution of a real-world drugs distribution problem with pharmacological waste collection. With the aim of properly meeting all the real-world restrictions that comprise this complex problem, we have modeled it as a multi-attribute or rich vehicle routing problem (RVRP). The problem has been modeled as a Clustered Vehicle Routing Problem with Pickups and Deliveries, Asymmetric Variable Costs, Forbidden Roads and Cost Constraints. To the best of authors knowledge, this is the first time that such a RVRP problem is tackled in the literature. For this reason, a benchmark composed of 24 datasets, from 60 to 1000 customers, has also been designed. For the developing of this benchmark, we have used real geographical positions located in Bizkaia, Spain. Furthermore, for the proper dealing of the proposed RVRP, we have developed a Discrete and Improved Bat Algorithm (DaIBA). The main feature of this adaptation is the use of the well-known Hamming Distance to calculate the differences between the bats. An effective improvement has been also contemplated for the proposed DaIBA, which consists on the existence of two different neighborhood structures, which are explored depending on the bat's distance regarding the best individual of the swarm. For the experimentation, we have compared the performance of our presented DaIBA with three additional approaches: an evolutionary algorithm, an evolutionary simulated annealing and a firefly algorithm. Additionally, with the intention of obtaining rigorous conclusions, two different statistical tests have been conducted: the Friedman's non-parametric test and the Holm's post-hoc test. Furthermore, an additional experimentation has been performed in terms of convergence. Finally, the obtained outcomes conclude that the proposed DaIBA is a promising technique for addressing the designed problem.

Keywords: Bat Algorithm, Medical Distribution, Rich Vehicle Routing Problem, Combinatorial Optimization, Traveling Salesman Problem

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1. Introduction

Transportation and logistics are important issues for the society these days, both for citizens and the business sector. We are perfectly aware that public transportation is used by almost all the population, and that it directly affects the people quality of life. In addition, business logistics can also be considered as transportation problem, which requires optimization techniques to solve. Therefore, this paper will focus on the logistic problems concerning medical device distribution and pharmacological waste collection.

In the business world, the fast advance of technology has made the logistic increasingly important in this area. Additionally, anyone in the whole world can be well connected. This situation has led transport networks to be very demanding, something that was less important in the past. Nowadays, a competitive logistic network can make the difference between some companies, and can crucially contribute to their success.

This work is focused on the proper modeling and treatment of a real-world logistic problem. Specifically, the real-world situation tackled in this paper is related to the distribution of medical goods. In this case, we center our attention in a SME[1] medical distribution enterprise, with regional influence. This company has an established logistic philosophy, which needs to be followed when they perform the daily distribution planning. All the characteristics that integrate this philosophy are explained in the following section. Finally, despite the object of this study is a company physically placed on Bizkaia (Spain), the main objective of this study is to propose a model which can be applied to every similar company.

Hence, the main objective of this work is to tackle efficiently this Drugs Distribution System with Pharmacological Waste Collection (DDSPWC). For reaching this goal properly, we have modeled the DDSPWC as a Rich Vehicle Routing Problem (RVRP). Currently, this type of complex problems is catching the attention of the scientific community, as can be read in several works, such as (Caceres-Cruz et al. (2015)) or (Doerner & Schmid (2010)). As we can be found in these surveys, RVRPs are special cases of the conventional Vehicle Routing Problem (VRP) (Golden et al. (2008)). These special cases are characterized for having multiple variables and constraints, and a complex formulation.

The principal reasons for the importance and popularity of these problems are twofold: the social interest they generate, and their inherent scientific interest. Firstly, RVRPs are usually designed for dealing with a specific real-world situation related to transport or logistics. This is the reason why their efficient resolution entails a profit, either business or social one. Secondly, most of RVRPs have a great computational complexity, and their resolution represents a major challenge for the scientific community.

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[1]SME: Small and medium-sized enterprise

Specifically, we present in this paper a Clustered Vehicle Routing Problem with Pickups and Deliveries, Asymmetric Variable Costs, Forbidden Roads and Cost Constraints (C-VRP-P*C) to tackle the proposed DDSPWC. As has been mentioned, RVRPs have caught the attention of the current community. In this sense, (Sicilia et al. (2016)) and (de Armas & Melián-Batista (2015)) are two examples of recently published RVRPs. The first of these works is related to the capillary transport of goods problem. The research project presented in that work was carried out for an important Spanish distribution company, and its main goal is to manage their resources in urban areas by reducing costs caused by inefficiency and ineffectiveness. The RVRP considered in that study comprises some constraints such as pick up and deliveries, backhauls, site-dependence, time-windows, capacities and openness. Authors proposed two different methods for its resolution: a variable neighborhood search (VSN) and a tabu search (TS). The second of the mentioned works presents also a VNS for the resolution of a dynamic RVRP. In that case, several real constraints have been considered, such as heterogeneous fleet of vehicles, multiple and soft time windows and customers priorities. Furthermore, it is worth mentioning that the software developed in that work has been incorporated into the fleet management system of a company in Spain. An additional example of recently developed RVRP is the one proposed in (Amorim & Almada-Lobo (2014)). In this paper, the authors present an RVRP to deal with the perishable food management. The RVRP designed in this case is a heterogeneous fleet site-dependent VRP with multiple time windows.

Furthermore, in 2016, Mancini presented in (Mancini (2016)) an interesting RVRP with multiple periods, multiple depots and heterogeneous fleet is presented. For tackling this challenging problem, the author developed an adaptive VNS based approach. The experimentation performed in that paper addresses 9 different datasets composed by 50 and 75 customers, and highlight the quality of the presented method. Besides that, in (Belmecheri et al. (2013)), Belchemeri el al. developed a particle swarm optimization (PSO) algorithm for solving a real-world based RVRP with heterogeneous fleet, time windows and mixed backhauls. In that paper, the results obtained by the presented approach are compared with a basic local search, and an ant colony optimization (ACO). For the experimentation, an ad-hoc modification of the well-known Solomon VRPTW Benchmark is used, with instances composed of 100 nodes. Finally, another interesting example was presented in (Penna et al. (2016)), in which an electric fleet size and mix VRP was designed, with recharging stations and time windows. For solving this novel problem, a hybrid iterative local search was implemented.

There are several appropriate approaches to deal with such complex optimization problems. Anyway, the most successful techniques to address the resolution of RVRP are heuristics and metaheuristics. In this paper, our attention focuses on the second of these categories: metaheuristics. In line with this, we propose a nature-inspired metaheuristic for the resolution of the designed C-VRP-P*C.

Lots of metaheuristics have been presented in the literature along the years (Fister Jr et al. (2013)). The implementation of new and classical methods, and their proper application still forms a hot topic in the scientific community (Precup et al. (2014); Mahdavi et al. (2015); Wari & Zhu (2016); Precup et al. (2015)). In fact, many novel approaches have been presented in the last decade, such as the Firefly Algorithm, proposed by Yang (Yang (2010a)), Charged System Search, presented by Kaveh and Talatahari in 2010 (Kaveh & Talatahari (2010)), or the Spider Monkey Optimization,

proposed by Bansal et al. in 2014 (Bansal et al. (2014)). Another kind of methods that have demonstrated a good performance applied to RVRPs are the memetic algorithms (Moscato et al. (1989)). Some examples of this good performance are (Bortfeldt et al. (2015)), in which a RVRP with clustered backhauls and 3D loading constraints is tackled, or (Zhang et al. (2013)), where a multiperiod VRP with profit is addressed. Additional works can be found in (Gutierrez et al. (2016); Rogdakis et al. (2017)).

This way, we have highlighted some methods which have been already used in the literature for solving RVRP problems: VNS, TS, PSO, ACO, local search methods, and memetic algorithms. Additional approaches can be found in the current literature to properly addressing this kind of problems, such as the genetic algorithm (Shi et al. (2017)), or the simulated annealing (Wang et al. (2017)). The Large Neighborhood Search has also been recently used in the literature for solving a RVRP (Talarico et al. (2017)). As can be logical, each of these methods have their advantages and disadvantages. In this specific paper, and with the aim of properly addressing the designed RVRP, we propose a nature-inspired metaheuristic based on the Bat Algorithm (BA). The BA was firstly presented by Yang in 2010 (Yang (2010b)), and it is based on the echolocation behavior of microbats, which can find their prey and discriminate different kinds of insects even in complete darkness. As has been highlighted in several studies, such as (Yang & He (2013)) or (Parpinelli & Lopes (2011)), the BA has been applied to different optimization fields and problems up to now. Furthermore, the fact that many research works focused on BA are being currently published proves that this approach is still interesting for the researchers, in different areas such as the continuous optimization (Chakri et al. (2017)), or the thermal engineering (Tharakeshwar et al. (2017)). Furthermore, the algorithm itself is also the focus of recent research, such as the works presented in (Perez et al. (2017a)) and (Perez et al. (2017b)), in which the parameter adaptation of the algorithm is studied.

Focusing on routing problems, several recently published papers have shown that the BA is a promising technique also in this field. For example, in (Taha et al. (2015)), which was published in 2015, an adapted variant of this algorithms for solving the well-known Capacitated VRP. The Adapted BA developed in that study allows a large diversity of the population and a balance between global and local search. Furthermore, in 2017, the same authors presented in (Taha et al. (2017)) an adaptation of the same technique for solving the well-known VRP with Time-Windows. Another interesting work is the one work proposed in (Zhou et al. (2016)) by Zhou et al., in which the Capacitated VRP is faced. In that paper, a hybrid BA with path relinking is described. This approach is constructed based on the framework of the continuous BA, in which the greedy randomized adaptive search procedure and path relinking are effectively integrated. Additionally, with the aim of improving the performance of the technique, the random subsequences and single-point local search are operated with certain probability.

Besides that, the BA has also been applied to the famous Traveling Salesman Problem several times in recent years. In (Osaba et al. (2016b)), Osaba et al. presented an improved adaptation of the BA for addressing both symmetric and asymmetric TSP. The results show that the improved version of BA could obtain promising results, in comparison with some reference techniques, such as an evolutionary simulated annealing, a genetic algorithm, a distributed genetic algorithm or an imperialist competitive algorithm. An additional example of this specific application is the one presented by Saji and Riffi in 2106 (Saji & Riffi (2016)). In that work, the performance of their discrete

version of the BA is compared with three different meta-heuristics: a discrete particle swarm optimization (PSO) (Chen & Chien (2011)), a genetic simulated annealing ant colony system with PSO techniques and a discrete cuckoo search (Ouaarab et al. (2014)).

Nevertheless, despite this interest, the BA has never been applied before to any kind of RVRP. This lack of works is one of the motivations behind using the BA for our study. There are additional reasons for the choosing of this technique, such as the growing scientific interest shown by the community in recent years, or the proper balance between exploration and exploitation shown by the technique for solving complex problems. Anyway, and most importantly, the good performance demonstrated since its first proposal, along with its fast execution, its reduced number of parameters, and its easy implementation are the crucial reasons which have motivated the using of BA.

With all this, the main contributions and novelties of the work presented on this paper are twofold. On the one hand, we have used an RVRP for dealing with the proposed DDSPWC. As will be explained later, similar problems have been previously presented in the scientific community, but never using an RVRP as complete as the one proposed in this study. In this sense, the main originality is not only the application of the BA to the medical distribution problem. In fact, the designed problem itself presents also a novelty, being the first time that an RVRP with these features is proposed in the literature.

On the other hand, in order to address the proposed problem, we have developed a discrete and improved version of the classic BA, named DaIBA. As far as we know, this is the first time that a BA is applied to such a complex RVRP. Additionally, the proposed technique is an adaptation of a recently proposed discrete (BA) (Osaba et al. (2016b)), which has only been applied for both Symmetric and Asymmetric Traveling Salesman Problem. With the intention of proving that the DaIBA is a promising method for solving the raised C-VRP-P*C, we have compared its results with the ones obtained by an evolutionary algorithm (EA), an evolutionary simulated annealing (ESA) (Yip & Pao (1995)), and a Firefly Algorithm (FA) (Yang (2009)).

The structure of this paper is as follows. The following Section 2 is devoted to the problem formulation. In this section, first, we describe the real-world problem that motivated this study. After that, we present the proposed RVRP. In Section 3, the designed DaIBA is deeply described. Furthermore, the experimentation performed is detailed in Section 4, along with the proposed benchmark. Finally, we end this paper with the conclusions of the study, and our planned future work (Section 5).

2. Problem formulation

This section is divided into two different parts. The first one, Section 2.1, is dedicated to the conceptual definition and description of the problem. The main intention is to contextualize the study and highlight its real-world application. After that, the designed C-VRP-P*C is deeply detailed in Section 2.2, in which an overall description of the problem is depicted, as well as its mathematical formulation.

2.1. Drugs distribution and pharmacological waste collection

As has been discussed in the introduction, the real-world problem addressed in this paper is related to the distribution of drugs to hospitals, neighborhood health centers and drugstores. Specifically, the problem arises in a regional pharmaceutical distributor.

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This distribution company serves the demand of hospitals, drugstores and health centers located in several cities and towns. The distribution company offers two services: delivery of prescription drugs and collection of pharmacological waste and expired or deteriorated medicines. The second service is aimed at collecting spoiled medicines and pharmacological wastes. These residues, like bio-sanitary waste, cannot be deposited in the usual trash containers since they must be processed in a special way.

The objective of the work presented in this paper is the design of an algorithm that plans the distribution and collection routes that minimize the operating costs of the distribution company. In addition to costs, the company's logistics planning is based on a series of principles. The first principle is to treat each city as a separate unit. In this sense, when a vehicle arrives in a city, it must take care of all the requests (distribution or collection) that the sanitary centers or drugstores of that location have. Therefore, a vehicle cannot enter a city and town if it does not have a sufficient capacity to attend all the requests of that location. The second principle is related to the schedule in which requests are handled. Requests are only served between 6:00 am and 3:00 pm. In addition, within this temporary window there is a range called "peak hours" (in this paper that range is set between 8:00 am and 10:00 am). The costs of traveling from one place to another are higher in the "peak hours". This range tries to simulate the temporary moments in which the traffic is denser in the cities and towns. Additionally, all vehicles must respect the rules of circulation. Therefore, the graph that configures the road map between the different locations will not be composed entirely of bi-directional links. Forbidden links will be defined, as if they were real roads. Finally, in order not to elaborate extremely long journeys for a single worker, all the routes have a maximum duration which cannot be exceeded.

Throughout the past few decades drug distribution problems have been modeled as classic VRP or as a variant to incorporate some constraints. For this reason, it is difficult to find works that focus specifically on vehicle routing for the drug distribution sector. More recently, with the rise of home health care systems, papers that address route planning for drug delivery can be found. In (Liu et al. (2013)), Liu et al. proposed a metaheuristic based on a Genetic Algorithm and a Tabu Search for home health care logistics. They model the problem as a VRPTW with delivery and pickup. The problem addressed has two types of delivery (from depot to patient and from hospital to patient) and two types of pickup (from patient to depot and from patient to medical lab). Authors test their new algorithm with instances derived from existing VRPTW benchmarks. Other work, also in the context of home health care, can be found in (Liu et al. (2014)). In this case, the problem used as reference is a Periodic Vehicle Routing Problem with Time Windows (PVRPTW). The problem involves 3 types of patient demands: transportation of drugs/medical devices between the depot and patients' homes, delivery of special drugs from the hospital to patients, and delivery of blood samples from patients to the medical lab. To solve the problem a metaheuristic based on the classical Tabu Search is defined.

The scope of application presented in the present work is novel. In addition, the problem of routing modeled also presents original aspects with respect to other problems of routing applied to problems of distribution in the sanitary field. In addition, as we will see in the rest of the paper, the RVRP proposed in this work has a great number of constraints, making easier its application to the real world.

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2.2. Clustered Vehicle Routing Problem with Pickups and Deliveries, Asymmetric Variable Costs, Forbidden Roads and Cost Constraints

As has been pointed in the introduction, the real-world situation faced in this paper has been modeled as a RVRP. Now, in this section, we describe in depth the presented RVRP problem. First, in Section 2.2.1, we detail the basic characteristics of the problem. Then, in Section 2.2.2, the mathematical formulation is represented.

2.2.1. Overall description of the proposed problem

The proposed RVRP has been modeled taking into account each and every condition mentioned in Section 2.1. Furthermore, it should be borne in mind that we have considered some additional restrictions in order to develop a model closer to real-world conditions. Hence, the proposed RVRP has the following general features.

1. *Clustered*: This feature means that the clients placed in the environment are grouped in several clusters. In this sense, every cluster corresponds to a city. Additionally, if any vehicle enters a city, it must meet the demand of every customer placed here. In other words, a vehicle is not allowed to enter a cluster if it cannot meet the demand of all the clients belonging to this city. This same feature has been used studied in several papers of the literature (Defryn & Sörensen (2017); Expósito-Izquierdo et al. (2016)).

2. *Pickup and Delivery*: This feature has been use in several studies previously (Männel & Bortfeldt (2016); Avci & Topaloglu (2016)). This characteristic contemplates two different kind of nodes: the *delivery nodes* and the *pickup nodes*. On the one hand, *delivery nodes* are those points where medical supplies are delivered. On the other hand, in *pickup nodes* the used medical stuff is collected, with the aim of taking them back to the warehouse. It is important to mention that this feature has a *simultaneous* nature. In this way, a drugstore or sanitary center can ask for both delivery and collection of material. For this reason, *delivery-pickup nodes* can also be found. Finally, it is assumed that all clients request the delivery of material. Therefore, drugstores or health centers demanding only the pickup of material are not present in our scenario.

3. *Asymmetric Variable Travel Times*: In real logistic problems, the travel between two different points does not always take the same time, or the same cost. In almost all the cases, this cost is under the influence of some external variables. With the aim of creating a more realistic model, we have represented this situation in the problem that we have proposed in this study. To this end, we have fixed a working-day between 6:00 am and 3:00 pm. Within this schedule, we have set two time-periods: "peak hours" and "off-peak hours". The first period is from 8:00 am to 10:00 am. All travels carried out at this time window will imply higher costs. On the other hand, the same trips will take less time if they are conducted in the "off-peak" period. Additionally, all the traveling costs are asymmetric, meaning that the effort of traveling from one node i to another node j implies different costs comparing with the reverse trip. This specific feature is appreciated in real-world applications, and it has been previously used on this kind of problem (Leggieri & Haouari (2016); Pham et al. (2014)).

4. *Forbidden Roads*: In real-world situations, it is quite common to find roads in which the traffic is allowed only in one direction. Furthermore, we can also find
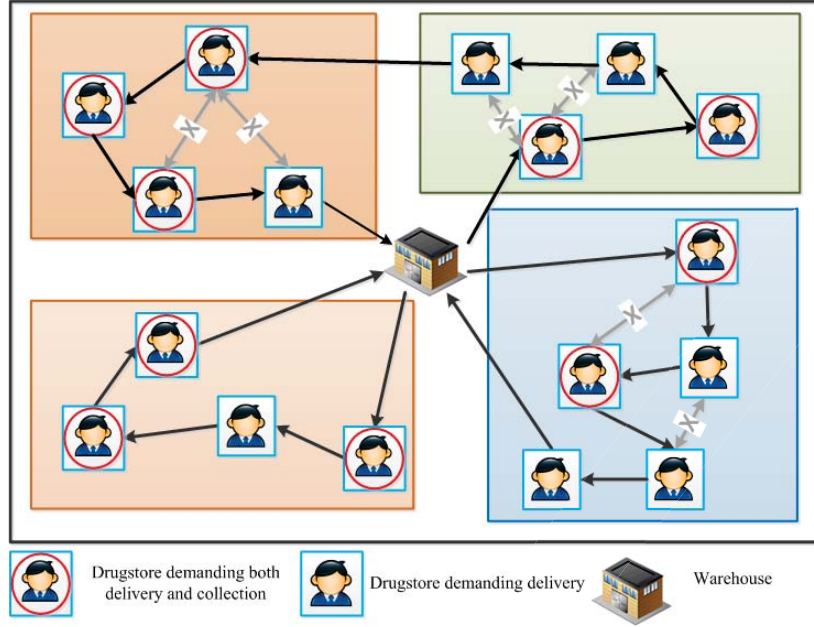
7

Figure 1: Possible C-VRP-P*C dataset composed of 15 nodes, and one feasible solution. Gray arcs represent forbidden paths.

    pedestrian streets, where vehicles are prohibited to go through. With the intention of recreating this kind of paths, the proposed C-VRP-P*C has certain arcs (i, j) which are not allowed to be used in the final solution.

5. *Cost-Constrained*: The last characteristic is related with the maximum cost that a route can afford. This constraint, as can be easily deduced, guarantees that the total cost of the arcs in a single route does not exceed a maximum route cost. This feature ensures the avoidance of long routes, giving priority to a more distributed planning between the fleet of vehicles. This same characteristic has been referenced many times in the literature (Li et al. (1992)).

    With all these features, the proposed C-VRP-P*C is an RVRP, whose main objective is to find a group of routes, taking into account the two different types of clients, trying to minimize the total traveling costs, not going through forbidden roads, and respecting the restrictions imposed by the capacity of the vehicles (C), the maximum allowed cost per route and the clusters. We show in Figure 1 a possible 15-noded dataset of the presented problem. We also show a feasible solution to this dataset in the same figure.

2.2.2. Mathematical formulation of the presented problem

    The proposed C-VRP-P*C can be represented as a complete graph $G = (V, A)$, where $V = \{v_0, v_1, \ldots, v_n\}$ is the set of vertex which depicts the drugstores and sanitary centers that comprise the system. Furthermore, $A = \{(v_i, v_j) : v_i, v_j \in V, i \neq j\}$ is the set of arcs which represents the interconnections between drugstores, hospitals and health centers. Each arc of the system has associated a c_{ij} cost. As we have mentioned above, the

presented problem has asymmetric costs, for this reason, the cost of traveling from i to j is always different from the cost of traveling from j to i. In a formal way, $c_{ij} \neq c_{ji}$. Furthermore, in order to contemplate forbidden arcs, we have fixed as infinite the cost of these paths. This way, we can ensure that these arcs will not appear in the final solution. Additionally, each route cannot exceed a maximum cost of D.

Additionally, the vertex v_0 represents the depot, and the rest are the visiting drugstores or sanitary centers. Besides this, V is divided into $cl+1$ mutually exclusive non-empty subsets, $CL = \{V_0, V_1, ..., V_{cl}\}$, each one for each cluster. These subsets are subject to these two conditions:

$$V_x \cap V_y = \emptyset, \quad x, y \in 0, 1, ..., cl, x \neq y \tag{1}$$

$$V = V_0 \cup V_1 \cup ... \cup V_{cl} \tag{2}$$

It should be borne in mind that V_0 contains only v_0. The remaining n hospitals, health centers and drugstores are distributed into cl different clusters. Furthermore, client i has two types of demands: one of them associated with the delivery of supplies, $d_i > 0$, and the other with the pick-ups $p_i \geq 0$.

Before showing the mathematical formulation of the proposed C-VRP-P*C, it should be highlighted that y_{ij} represents the demand picked-up in clients routed up to node i (including node i), and transported in the arc (i, j). Besides this, the total number of routes has been represented as k. Additionally, the parameter z_{ij} depicts the demand to be delivered to customers scheduled after node i and transported in arc (i, j) (Montané & Galvao (2006)). Furthermore, the binary variable x_{ij}^r is 1 if the vehicle r uses the arc (i, j), and 0 otherwise. Finally, w_s^r is a binary variable, which takes the value of 1 if the mobile unit r enters the cluster s, and 0 in other case. With all this information, the presented C-VRP-P*C can be mathematically formulated in the following way, where the main problem is now to minimize:

$$\sum_{i=0}^{n} \sum_{j=0}^{n} \sum_{r=1}^{k} c_{ij} x_{ij}^r \tag{3}$$

subject to:

$$\sum_{j=0}^{n} \sum_{r=1}^{k} x_{ij}^r = 1, \quad i = k, \ldots, n; j \neq i, \tag{4}$$

$$\sum_{i=0}^{n} \sum_{r=1}^{k} x_{ij}^r = 1, \quad j = 0, \ldots, n; i \neq j, \tag{5}$$

$$\sum_{i=0}^{n} \sum_{r=1}^{k} x_{i0}^r = k, \tag{6}$$

$$\sum_{j=0}^{n} \sum_{r=1}^{k} x_{0j}^r = k, \tag{7}$$

$$\sum_{j=0}^{n} x_{ij}^r - \sum_{l=0}^{n} x_{li}^r = 0, \ i = 0, \ldots, n; r = 1 \ldots k, \tag{8}$$

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$$\sum_{i=0}^{n} x_{ij}^r - \sum_{l=0}^{n} x_{jl}^r = 0, \ \ j = 0, \ldots, n; r = 1 \ldots k, \tag{9}$$

$$\sum_{i=0}^{n} \sum_{r=1}^{k} d_{ij} x_{ij}^r < \infty, \quad j = 0, \ldots, n; i \neq j, \tag{10}$$

$$\sum_{j=0}^{n} \sum_{r=1}^{k} d_{ij} x_{ij}^r < \infty, \quad i = 0, \ldots, n; j \neq i, \tag{11}$$

$$\sum_{r=1}^{k} w_s^k = 1 \quad s = 1, \ldots, c. \tag{12}$$

$$\sum_{i=0}^{n} z_{ji} - \sum_{i=0}^{n} z_{ij} = d_j, \quad j = 0, \ldots, n, \tag{13}$$

$$\sum_{i=0}^{n} y_{ji} - \sum_{i=0}^{n} y_{ij} = p_j, \quad j = 0, \ldots, n, \tag{14}$$

$$y_{ij} + z_{ij} \leq Q \sum_{r=1}^{k} x_{ij}^r, \quad i, j = 0, \ldots, n, \tag{15}$$

$$\sum_{j=0}^{n} \sum_{r=1}^{k} c_{ij} < D, \quad i = 0, \ldots, n; j \neq i, \tag{16}$$

where

$$y_{ij} \geq 0, \quad i, j = 0, \ldots, n, \tag{17}$$

$$z_{ij} \geq 0, \quad i, j = 0, \ldots, n. \tag{18}$$

$$w_s^r \in \{0, 1\}, \quad r = 1, \ldots, k; s = 1, \ldots, c, \tag{19}$$

$$x_{ij}^r \in \{0, 1\}, \quad i, j = 0, \ldots, n; i \neq j; r = 1 \ldots k, \tag{20}$$

The first formula depicts the objective function, which must be minimized, and which is the sum of all the costs associated with the routes that compose the solution. Conditions (4) and (5) guarantee that all the drugstores and sanitary centers are visited exactly once. Additionally, equations (6) and (7) assure that the total amount of vehicles leaving the depot, and the number of vehicles that return to it is the same. Furthermore, the proper flow of each route is ensured by restrictions (8) and (9), avoiding the generation of subloops.

On the other hand, formulas (10) and (11) guarantee that every trip between two different nodes has not an infinite cost. In this way, we ensure that forbidden paths will not form part of the final solution. Moreover, function (12) assures that only one vehicle enters every cluster. This constraint, along with the above described (4) and (5) ensures that all the customers belonging the same cluster are visited by the same mobile unit.

In addition, constraints (13) and (14) guarantee that the flows for the delivery and the pick-ups are properly conducted. These clauses ensure that both demands are correctly satisfied for every drugstore and sanitary center. Besides that, formula (15) assures that

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the total capacity of any vehicle is always respected. This same restriction also represents that both delivery and collection demands will only be transported using arcs included in the solution (Montané & Galvao (2006)). Furthermore, constraint (16) guarantees that the total cost of each route does not exceed the fixed maximum. Finally, the formulas (17), (18), (19) and (20) represent the domains of the variables y_{ij}, z_{ij}, w_s^r and x_{ij}^r.

It is interesting to highlight that all the constraints inherent to the problem make the generating of feasible solutions a very complex task. This complexity makes impossible to directly apply most of the operators used for solving the common VRP. For this reason, the developing of appropriate functions has been one of the main difficulties for its solving. On the other hand, it is also noteworthy that all the constraints reduce the size of the search space which comprises all the feasible solutions, but increments the probability of falling into local optima. In order to avoid this fact, a simple but effective improvement has been implemented in the proposed DaIBA, which help to enhance its exploration ability. This mechanism, which endows each bat with a certain intelligence for performing its movements, is explained in the following Section 3.2.

3. Bat algorithm

As we have mentioned in the introduction of this work, a Discrete and Improved Bat Algorithm (DaIBA) is presented in this paper to face the designed C-VRP-P*C. In the present section, we introduce first the classic version of the BA (Section 3.1). After that, we describe in detail in Section 3.2 the proposed DaIBA.

3.1. Classic Bat Algorithm

As it has been mentioned in previous sections, the BA is a nature-inspired metaheuristic based on the echolocation system of bats. In the nature, bats emit ultrasonic pulses to the surrounding environment with navigation and hunting purposes. After the emission of these pulses, bats listen to the echoes, and based on them they can locate themselves and also identify and locate preys and obstacles. Besides that, each bat is able to find the most "nutritious" areas performing an individual search, or moving towards a "nutritious" location previously found by any other component of the swarm.

It is important to mention that some rules have to be previously established with the aim of making an appropriate adaptation (Yang (2010b)):

1. All bats use echolocation to detect the distance, and they have one "magic ability" that permit them to distinguish between an obstacle and a prey.
2. All bats fly randomly with a velocity v_i at position x_i with a fixed frequency f_{min}, varying wavelength λ and loudness A_i to search for a prey. In this idealized rule, it is assumed that every bat can adjust in an automatic way the frequency (or wavelength) of the emitted pulses, and the rate of these pulses emission $r \in [0, 1]$. This automatic adjustment depends on the proximity of the targeted prey.
3. In the real world, the bats emissions loudness can vary in many different ways. Nevertheless, we assume that this loudness can vary from a large positive A_0 to a minimum constant value A_{min}.

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