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MEASURING EQUIVALENT CIRCUITS

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In this paper it will be shown that an equivalent which takes proper account of transformation ratios will give dependable results. A step-by-step procedure will be described for measuring such an equivalent without disturbing a network analyzer setup.

TYPES OF EQUIVALENTS

The type of equivalent circuit selected for a particular study will depend largely on the importance of representing loads, capacitors and other shunt paths connected to neutral throughout the network. For example, in a short-circuit study it may suffice to represent part of a system by a Thevenin's equivalent, ignoring completely the effects of loads. The assumption of neglecting loads might not be valid in the case of a power flow study and a more complex equivalent circuit may be needed. Among equivalents that take account of loads, a very useful one is derived by considering loads as impedances. The network then can be represented rigorously by the mesh equivalent of all of the impedances including loads and line charging capacitors, retaining the neutral bus as one of the terminals of the equivalent.[1] Generation busses also are retained. Another such circuit involves the obtaining of distribution factors for all loads and generation so that an equivalent load or generator can be assigned to each terminal of the equivalent.[2] As the first mentioned circuit, this one also uses a mesh equivalent of the actual network, but the impedances in this case are measured after all paths to neutral have been opened. These methods may be elaborated to make an equivalent which is useful for stability as well as load-flow studies.[3]

After a suitable type of equivalent circuit is selected, the transfer impedances may be measured when the system is set up on a network analyzer, or they may be calculated on a digital computer. Whichever method is used, transformer ratios within the system should be properly evaluated. This is nothing new, but it seems that the importance of transformer ratios is sometimes underestimated. A common practice at network analyzers is to reset all tapechangers to unity before transfer impedances are measured. Then the equivalent circuit is used without tapchangers. These prac-

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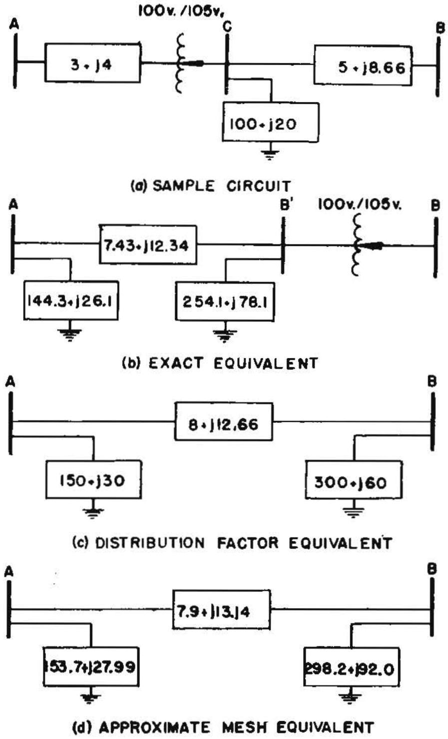

Fig. 1—Impedance diagrams of a circuit and its equivalents.

tices may lead to serious error, and probably some of the dissatisfaction with equivalent circuits is due to this error. The voltage ratios referred to here are off-nominal ratios which remain after the network impedances are expressed in terms of suitable nominal base voltages.

To illustrate the nature of the error and how it may be avoided, several types of equivalents have been calculated for a sample circuit. These are shown in Fig. 1. In this illustration both lines and loads are given as impedances expressed in percent R and X on any suitable base. The sample circuit, Fig. 1(a), is a realistic part of a power system connecting the major busses, A and B through a lower voltage network. The actual operating voltage at Bus C is used in converting the load watts and vars to R and X values. Bus C is to be eliminated in the equivalent. The existence of a transformer ratio other than unity within this circuit will illustrate the importance of accounting for such off-nominal ratios, both when measuring equivalent circuits and when using them. An equivalent will be obtained by transferring the load and the line impedance C-B to the 100 volt side of the transformer and replacing the resulting wye circuit by its equivalent mesh, in this case a delta. This results in the equivalent circuit shown in Fig. 1(b). It should be noted that the transformer with its proper ratio has been retained and is an integral part of this circuit. It is interesting to compare this circuit with circuits obtained by other methods.

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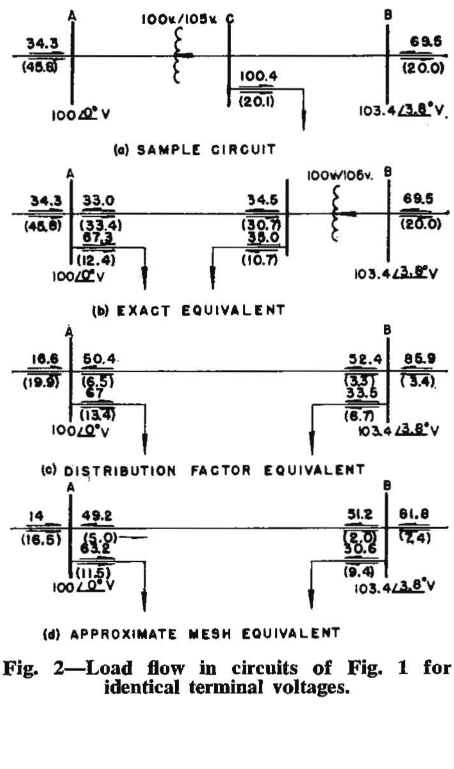


Fig. 2—Load flow in circuits of Fig. 1 for identical terminal voltages.

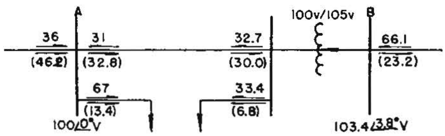


Fig. 3—Load flow in "Distribution Factor" equivalent with transformer added.

for all loads and generation within the network with the tapchangers set on unity. The impedances between terminals of the equivalent are measured with all shunt paths to neutral disconnected. For the sample circuit this method results in the equivalent shown in Fig. 1(c). It should be noted that the ratio of watts to vars in the equivalent loads at the terminals will be the same as at the original load since the same distribution factors are applied to both watts and vars. Reference to Fig. 1(b) will show that the ratio of watts to vars at any given bus depends not only on the watt-var ratio of the load, but the impedance angles on the lines feeding the load.

Next consider an approximate mesh equivalent, which includes the load, made from the circuit in Fig. 1(a) by setting the tapchanger on unity, and converting the wye to a mesh (delta) circuit. The result of this operation is shown in Fig. 1(d). Again it is noted that discrepancies exist compared with the equivalent in Fig. 1(b).

PERFORMANCE COMPARISONS

Admittedly, it is difficult to look at the various circuits shown in Fig. 1 and determine what effect the discrepancies noted might have when using these circuits. However, if the systems external to Busses A and B are considered stiff enough to maintain constant voltage and phase angle, each of the equivalent circuits may be tested by using it to replace the sample circuit. Figure 2 shows the assumed voltages and phase angles and the resulting flows.

When the exact equivalent replaces the sample circuit the result is show in Fig. 2(b). Note that the flows, both watts and vars, in System A and System B are the same as with the sample circuit Fig. 2(a).

Using the conventional distribution factor equivalent in place of the sample circuit results in flows shown in Fig. 2(c). Watts and vars are both greatly in error in System A and in System B.

Using the equivalent of Fig. 1(d) to replace the sample circuit results in the flow shown in Fig. 2(d). The errors are very much the same as in Fig. 2(c).

The errors in Fig. 2(c) and Fig. 2(d) can be shown to be mostly the result of not using a tapchanger in the equivalent, although there is also an error due to omitting the tapchanger when measuring the impedances. For example, adding the tapchanger to the distribution factor equivalent of Fig. 1(d), gives the results shown in Fig. 3, where the flows are much improved over those shown in Fig. 2(d).

MEASURING THE EQUIVALENT

The three impedances shown in Fig. 1(b) are the mesh equivalent of the entire network between A and B, including loads at the impedance they were adjusted to have for a particular operating voltage. The measurement of such an equivalent on a network analyzer will be made with loads, tapchangers, and capacitors (including pi-line capacitors), adjusted for the desired load condition and left connected. The steps in such a measurement are as follows:

Step 1 Record, from the load-flow study, the scalar voltages for the busses to be retained in the equivalent.

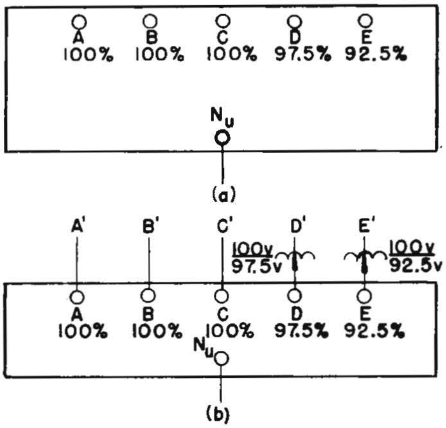


Fig. 4—(a) Actual circuit (b) Circuit additions for measurement.

Step 2 Disconnect the circuit from the rest of the system. Disconnect the generators within the circuit. The tie points thus opened, plus the generator busses (with possible exceptions discussed later), plus the neutral will be the terminals of the equivalent circuit. In Fig. 4 the terminals are A, B, C, D, E, and N, and the rectangle symbolizes a complex circuit which has been disconnected from the rest of the system. Loads at the terminals are disconnected and added in again after the equivalent loads are determined.

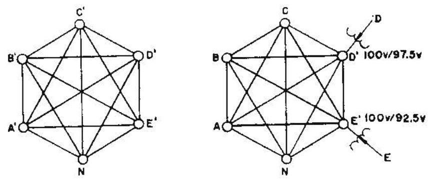
Step 3 Using one terminal as reference, determine the net turns ratio through the network analyzer tapchangers between this terminal and each of the other terminals. This may be done rather easily by tracing the principal low-impedance paths on a diagram, averaging the ratios found if there are alternate low-impedance paths with slightly different ratios. These ratios expressed in percent are shown in Fig. 4 at the terminals. In this case, either A, B, or C might have been picked as the reference and marked 100 percent. Theoretically, these turns ratios can be measured as open circuit voltages on the network analyzer with 100 percent volts applied at the reference terminal and with all shunt paths to neutral open. Besides requiring that every load and pi-line capacitor be switched off, and later switched back on, experience has shown that the results are not as good as with the inspection method recommended. This is because with the network energized at 100 percent volts the autotransformer magnetizing current and circulating currents affect the so-called open circuit voltages.

Step 4 Insert in series with the terminals autotransformers as required to make a new set of terminals so that between any of these new terminals the net ratio is unity. These are the primed terminals in Fig. 4, A′, B′, C′, D′, and E′.

Step 5 Measure the mesh equivalent impedances using neutral and the primed terminals. If the two possible measurements are made for each branch in the mesh, the two measurements should check each other. An exception may be that branches from the neutral terminal may not appear the same when driving voltage is on the neutral as when the neutral is grounded. This is usually because the load units on the analyzer are not linear down to the very low voltages they have when the neutral terminal is grounded. The shunt paths to the neutral are best measured, therefore, with driving voltage on the neutral terminal. The mesh equivalent as measured will have the branches shown in Fig. 5.

Step 6 The network determined in Step 5 is used with autotransformers connected at the appropriate terminals to restore the original turns ratios between terminals. Thus in Fig. 5, autotransformers are shown at D′ and E′. In effect, the autotransformers restore the original terminals, and Fig. 5 looking in at terminals A, B, C, D, E, and N is the equivalent of the original circuit looking in at the similarly named terminals. The autotransformers in Fig. 5 are a necessary integral part of the equivalent circuit.

Step 7 In the equivalent circuit, branches from the several terminals to neutral represent loads and losses, modified by line charging kva. In most studies, it is best to consider these branches as constant loads instead of constant impedances. These loads are set up by determining the watts and vars these impedances would take if the voltages recorded in Step 1 are applied to terminals A, B, C, D, and E in Fig. 5. The voltages at D′ and E′, needed to find the flow in branches D′-N and E′-N, are calculated, using the autotransformer ratios. This method neglects the slight effect on line charging kva of voltage variations which may show up in using the equivalent. Line charging is held constant along with



(a) EQUIVALENT CIRCUIT OF FIG. 4b (b) EQUIVALENT CIRCUIT OF FIG. 4a

Fig. 5—Mesh equivalent circuits.

loads, but the error is slight. In the steps just outlined, it was suggested that generator busses be retained if possible. This avoids the approximations in using distribution factors to assign generation to other busses. In cases where closely related generators may be combined without affecting the study, this is done just by omitting the busses in question from the terminals of the equivalent. If loads are netted with generation on a bus, a resultant net power input may be transferred to other generators to eliminate this bus from the equivalent, but a net reactive input would best be represented by a capacitor and not transferred to remote busses. If it is necessary to determine distribution factors they should be measured at the primed terminals of Fig. 4.

CONCLUSIONS

Mesh equivalent circuits determined as outlined above have given better results than those using distribution factors. In a recent study, the mesh circuit was substituted for the original network from which it was determined, with the results shown in *Table I.* The base case was the one for which load impedances were adjusted when the equivalent was measured. The outage case was with a heavily

TABLE I
FLOWS AND VOLTAGES AT TERMINALS
Original Network and Equivalent Substituted for It

Case and Terminal		Original			Equivalent		
		Watts	Vars	Volts	Watts	Vars	Volts
Base Case	A 100%	77.2	9.9	105	76.5	10.0	105
	B 100%	28.0	10.5	96	28.8	11.3	96.5
	C 100%	18.0	14.1	99.6	18.1	14.5	99.6
	D 97.5%	24.2	8.3	100.8	24.6	8.4	100.5
	E 92.5%	51.2	34.0	101.2	51.0	35.0	101.5
Outage Case	A 100%	98.6	14.7	105	95.0	15.0	105
	B 100%	30.0	12.0	93.5	29.5	13.2	93.5
	C 100%	−4.0	+11.0	92	−1.9	+11.7	92
	D 97.5%	22.8	7.5	101	23.3	9.1	100.5
	E 92.5%	56.0	41.0	101.2	53.0	38.0	101.5

loaded 138 kv line (not in the equivalent) out of service between terminals A and C. The nominal net turns ratios are given with the letter names of the terminals as in Fig. 5, and they happen to be the same as in Fig. 5.

The original network from which the *Table 1* equivalent was measured consisted of 51 lines, 25 loads, 10 tapchangers, and one generator, all connected up on 42 busses. In preparing for the measurements, four of the tapchangers were reset from the load-flow study settings to rationalize the turns ratios at the five terminals. This did not change terminal voltage or flows significantly.

In making impedance measurements, the characteristics of the particular network analyzer should be considered. For example, too small a kva base may result in having to use much less than 100 percent voltage for the measurements and some load units are not linear at low voltages. The equivalent loads depend on measuring small currents to a sufficient number of significant figures to get the desired accuracy.

The following conclusions are indicated by this investigation:

1. Off-nominal transformer ratios must be considered, as several other authors have suggested. In general, autotransformers must be used with the equivalent circuit.
2. Better results at less expenditure of time are obtained if loads are included in the impedance measurements rather than determining them by distribution factors. Line charging kva is accounted for and becomes part of the equivalent loads in the recommended method.
3. Retaining all important generator busses improves the accuracy of the equivalent in actual use in load-flow studies, and permits use in transient stability studies.

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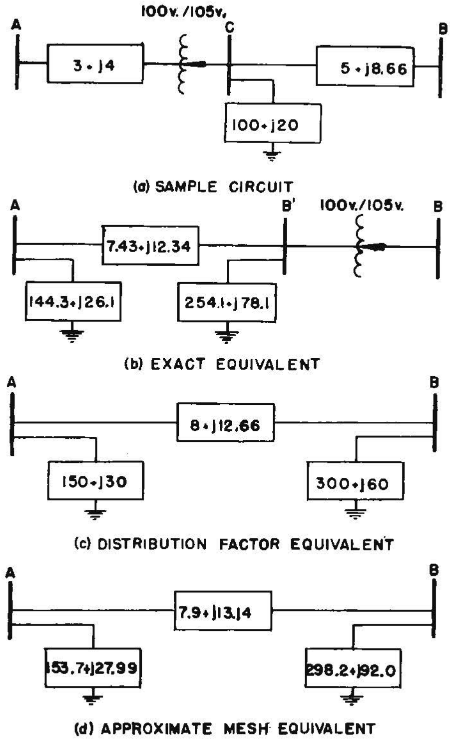

Fig. 1—Impedance diagrams of a circuit and its equivalents.

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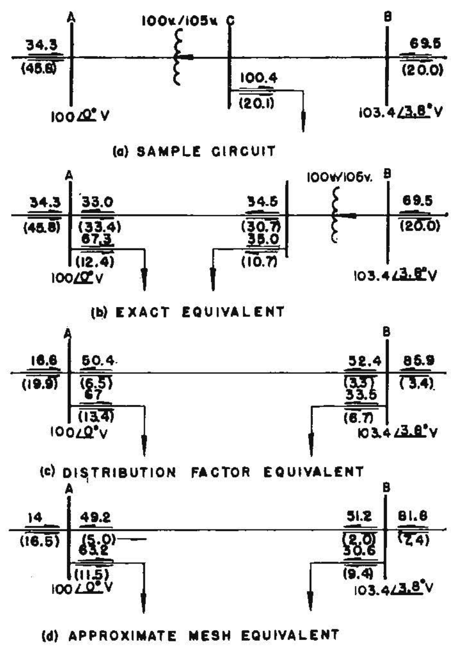
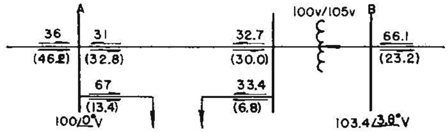


Fig. 2—Load flow in circuits of Fig. 1 for identical terminal voltages.



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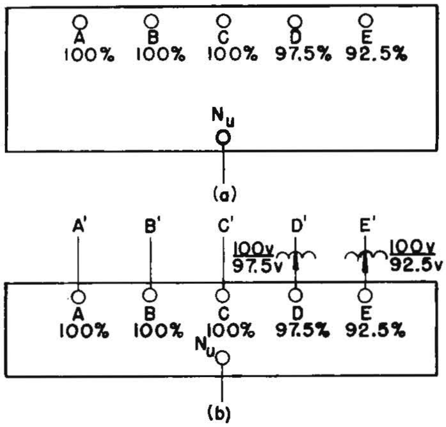

Fig. 4—(a) Actual circuit (b) Circuit additions for measurement.

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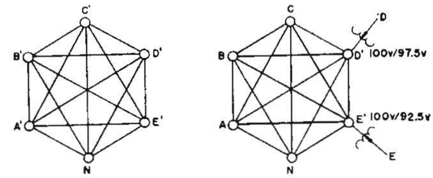

(a) EQUIVALENT CIRCUIT OF FIG. 4b (b) EQUIVALENT CIRCUIT OF FIG. 4a

Fig. 5—Mesh equivalent circuits.

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Outage Case	A 100%	98.6	14.7	105	95.0	15.0	105
	B 100%	30.0	12.0	93.5	29.5	13.2	93.5
	C 100%	−4.0	+11.0	92	−1.9	+11.7	92
	D 97.5%	22.8	7.5	101	23.3	9.1	100.5
	E 92.5%	56.0	41.0	101.2	53.0	38.0	101.5

loaded 138 kv line (not in the equivalent) out of service between terminals A and C. The nominal net turns ratios are given with the letter names of the terminals as in Fig. 5, and they happen to be the same as in Fig. 5.

The original network from which the *Table I* equivalent was measured consisted of 51 lines, 25 loads, 10 tapchangers, and one generator, all connected up on 42 busses. In preparing for the measurements, four of the tapchangers were reset from the load-flow study settings to rationalize the turns ratios at the five terminals. This did not change terminal voltage or flows significantly.

In making impedance measurements, the characteristics of the particular network analyzer should be considered. For example, too small a kva base may result in having to use much less than 100 percent voltage for the measurements and some load units are not linear at low voltages. The equivalent loads depend on measuring small currents to a sufficient number of significant figures to get the desired accuracy.

The following conclusions are indicated by this investigation:

1. Off-nominal transformer ratios must be considered, as several other authors have suggested. In general, autotransformers must be used with the equivalent circuit.
2. Better results at less expenditure of time are obtained if loads are included in the impedance measurements rather than determining them by distribution factors. Line charging kva is accounted for and becomes part of the equivalent loads in the recommended method.
3. Retaining all important generator busses improves the accuracy of the equivalent in actual use in load-flow studies, and permits use in transient stability studies.

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