Pion Production in Neutrino-Nucleus Interactions

DISSERTATION

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Dedicated to

My Parents

POST GRADUATE DEPARTMENT OF PHYSICS

UNIVERSITY OF KASHMIR, SRINAGAR

Dated....................

CERTIFICATE

Certified that the Dissertation entitled "**Pion production in neutrino-nucleus interactions**" embodies the original work of **Ms. Qudsia Gani** carried out under my supervision. The work is worthy of consideration for the award of M. Phil. Degree in Physics.

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PREFACE

The discovery of neutrino oscillations is one of the most exciting recent developments in particle physics. Current and future neutrino experiments are aiming to make precise measurements of the oscillation parameters. Improving our understanding of neutrino-nucleus cross-sections is crucial to these precision studies of neutrino oscillations. Interactions in the neutrino energy region around 1 GeV are particularly important because this is the region of the expected oscillation signal in many experiments, but the cross-sections in this region are not very well known. This energy region is complicated due to overlapping contributions from quasi-elastic scattering, resonant single pion production, and deep inelastic scattering. This dissertation describes a measurement of the cross-section for resonant single charged pion production in quasi-elastic charged-current muon neutrino interactions with oxygen as target.

The results of this measurement are consistent with previous experiments and predictions based on a widely-accepted models.

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1 Introduction

1.1 A brief history of neutrino physics

The first manifestations of the weak interactions were observed at the very end of the 19th century. In 1896, Henri Becquerell discovered radiation coming from uranium salts [1], then in 1898 Pierre and Marie Curie isolated radioactive radium [2]. In 1899 - 1902, three types of radiation were established, which differ by their charge: alpha (positive), beta (negative) [3] and gamma (neutral) [4-7]. Moreover, in 1900 Becquerell showed that beta particles have a charge-to-mass ratio close to that of electrons [8], so they were identified as the latter. Since then, the beta decay process has been studied intensively. The first evidence for the existense of the neutrino was obtained in 1920-1927, when Charles Drummond Ellis along with colleagues established clearly that the electron spectrum in beta decays is continuous [9-14]. It was understood that a certain amount of nuclear energy is released in the decay, and thus in the two-body decay the outgoing electron energy should have been discrete. In order to save the energy conservation law Wolfgang Pauli proposed in 1930 the existence of a neutral particle that is emitted along with the electron in the beta decay process [15].

In February 1932, James Chadwick discovered the neutron [16], which was a prime candidate for the particle emitted in the beta decay. However, in 1933 Francis Perrin showed that the neutrino mass has to be significantly lower than the electron mass [17]. Since neutrons are heavy particles, they can not correspond to the particle proposed by Pauli. Later that year, Enrico Fermi proposed the name for the new particle, neutrino; the Italian word for the *"little neutral one"*. He published the first model of the beta decay in which the neutrino is produced [18].

In 1934, Hans Bethe and Rudolf Peierls calculated the neutrino interaction cross-section to be less than $10^{-44}cm^2$, stating that it was therefore impossible to directly observe these interactions [19]. The series of the experiments by Frederick Reines and Clyde Cowan were the first step to directly observe neutrinos through the inverse beta decay:

$$\nu^{-} + p \longrightarrow n + e^{+}$$

These workers used a new detection technology, a liquid scintillator counter, to detect the products of the decay [20]. The signal for this reaction would be the scintillation light from the primary positron, a delayed pair of gammas from the positron annihilation, and a 2.2 MeV gamma from the neutron capture on hydrogen. In their first experiment, the detector was placed near a plutonium-producing reactor at the Hanford Engineering Works near Richland, Washington. The experiment found an excess of events over the background which was consistent with the prediction of neutrino interactions [21]. However, the experiment had a signal-to-background ratio of only ~ 0.2.

In 1956, they performed a second experiment to confirm the existence of the neutrino. The detector was placed at the Savannah River Plant, South Carolina. It was separated into three regions to remove the appearance of signal events in all three tanks, which would signify cosmic ray muons. A neutrino signal was observed and was in $\sim 5\%$ agreement with the neutrino cross-section prediction, even though the latter had $\sim 25\%$ uncertainty [22]. The experiment had a signal-to-background ratio of 3/1. A second Savannah River experiment was held later during 1956-1959, with improved electronics; it also confirmed the neutrino signal [23].

In 1962, muon neutrinos were discovered by Leon Lederman, Mel Schwartz, Jack Steinberger and colleagues at Brookhaven National Laboratory (BNL) and it was confirmed that they were different from electron neutrinos [24].

In 1968, Ray Davis and colleagues collected the first radiochemical solar neutrino events by using neutrino capture on chlorine in a detector in the Homestake Mine in North Dakota. The result lead to the observation of a deficit in the neutrinos produced by the Sun [25], as predicted by John Bahcall [26]. The problem became known as the solar neutrino problem.

In 1988, a deficit of atmospheric muon neutrinos was observed by the Kamiokande experiment [27] in Japan and the IMB experiment (Irvine, Michigan, Brookhaven) [28] in the Morton salt mine in Mentor, Ohio. This was the first clue to the neutrino oscillations.

During 1993-1998, the LEP (Large Electron Positron) accelerator experiments in Switzerland studied the width of the Z^0 boson. It was determined that there are only 2.984 ± 0.008 active and light (relative to the Z^0 boson mass) neutrino species that may couple to the Z^0 [29-32].

In 1998, after analyzing more than 500 days of data, the Super-Kamiokande experiment reported finding atmospheric neutrino oscillations and, thus, indirect evidence for a non-zero value of theneutrino mass [33, 34].

In 2001, the DONUT (Direct Observation of NU Tau) experiment at the Fermi National Accelerator Laboratory (FNAL) observed ν_τ charge current interactions [35], the third neutrino flavor.

In 2002, the SNO (Sudbury Neutrino Observatory) experiment near Sudbury, Ontario, Canada reported observation of neutral-current and charged-current scatterings from solar neutrinos, which provided a convincing evidence that neutrino oscillations are the solution of the solar neutrino problem [36].

In 2003, the KamLAND (Kamioka Liquid Scintillator Antineutrino Detector) experiment in Kamioka, Japan observed reactor antineutrino oscillations consistent with the solar neutrino problem and allowed a precision measurement of solar neutrino oscillation parameters [37].

In 2003, the K2K (KEK to Kamioka) long baseline neutrino oscillation experiment published the first measurement of atmospheric oscillation parameters using an accelerator-based neutrino beam created at KEK, a 12 GeV Proton Synchrotron facility in Japan [38]. Later, in 2006, the MINOS (Main Injector Neutrino Oscillation Search) experiment reported results on the atmospheric oscillation parameters measurement using an accelerator-based neutrino beam at FNAL.

Presently efforts, theoretical as well as experimental, are made around the globe for understanding the nature of this particle in detail. India is making a good deal of lead in these efforts in terms of establishing an underground observatory "India based Neutrino Observatory" (INO) [39]. Certain interesting physical phenomena that neutrinos (and antineutrinos) may undergo if indeed they mix requires that neutrino and antineutrino interactions be separately identified. Charged-current interactions of neutrinos and antineutrinos with atomic nuclei produce leptons (such as negatively charged electrons and muons) and antileptons (such as positrons and mu-plus) respectively. The interactions of the neutrinos and antineutrinos in the detector is thus identified by the track of this charged particle. These will be detected by means of an iron calorimeter (ICAL) which will be constructed in horizontal layers. INO is proposed to be located under the Bodi West Hills, about 110 km west of Madurai city in South India.

1.2 Neutrinos in the Standard Model

The Standard Model is not complete with regard to neutrinos. For decades there has been a problem that the number of measured neutrinos coming from the sun is inconsistent with the prediction from the standard solar model [40-44]. The resolution to this problem, neutrino oscillations, has led to many interesting consequences. The first is that neutrinos have mass, implying that these must have a right-handed component. The second is that the mass states differ from the neutrino flavor states which allows for the flavor states to oscillate [45,46] as a neutrino propagates. The third is that the violation of charge conjugation and parity, may occur in the leptons. The fourth is that neutrinos may be their own anti-particles.

1.3 An introduction to Monte Carlo event generators

The expression "Monte Carlo method" is actually very general. Monte Carlo (MC) methods are stochastic techniques [47] based on the use of random numbers and probability statistics to investigate problems. We can find MC methods used in everything from economics to chemistry to nuclear physics to regulating the flow of traffic. Of course, the way these are applied varies widely from field to field. The use of MC methods to model physical problems allows us to examine more complex systems than we otherwise can do. Solving equations which describe the interactions between two atoms is fairly simple but solving the same equations for hundreds or thousands of atoms is not so easy. With MC methods, a large system can be sampled in a number of random configurations and that data can be used to describe the system as a whole. Advancing our understanding of fundamental neutrino properties will require building a more complete picture of neutrino interactions. This will pose a series of important theoretical and experimental challenges. Neutrino generators [48-54] are an interface between theory and experiments. As such, these play a variety of important roles in neutrino experiments from conception to the final physics publication. These are used to evaluate the feasibility and physics reach of proposed experiments, optimize the detector design, analyse the collected data samples and evaluate systematic errors. This multitude of roles makes neutrino generators impressively polymorphic tools.

In the neutrino experiments, event generators are used to provide information how the signal and the background events are observed in the detectors. Therefore, each generator is expected to simulate all the possible interactions and the simulations of each interaction have to cover entire kinematical region using appropriate models. Of course, it is not possible to simulate all the neutrino interactions perfectly and thus, there are always simplifications and assumptions in the actual implementation of the simulation programs

There are several neutrino event generators available in the market. In the early days, each experiment developed their own event generators. NEUT[55], NUANCE[56,57] and NEUGEN[58] are in this category. NEUT was initially developed or the Kamiokande experiment and continuously updated for the Super-Kamiokande, the K2K, the SciBooNE[59] and the T2K experiments. NUANCE was developed for the IMB experiment and used in the other experiments. For example, MiniBooNE [60] has been using NUANCE as the official generator and improved it using the high statistics data of this experiment. NEUGEN was developed for the SOUDAN experiment[61] and has been updated to be used in the MINOS experiment.

Then, there are attempts to develop general purpose generators. FLUKA

[62,63] and GENIE [64] are in this category. These event generators are not the simple interaction simulation programs but have additional functionalities like the geometry handling and so on. FLUKA is the general purpose simulation program which simulates interactions of wide variety of particles and it also handles neutrino interactions. GENIE is a generator which is intended to be used in various neutrino experiments. GENIE is designed to be a new universal generator and actually used in several experiments like ArgoNeut[65], MicroBooNE[66], MINOS, MINERνA[67], and T2K. The GENIE collaboration is continuously working to include the latest interaction models.

Recently, there are another kind of event generators, which were developed by theorist groups. GIBUU[68] and NuWro[69] are in this category. GIBUU is aiming to provide an unified transport framework in the MeV to GeV energy regimes for elementary reactions on nuclei, e.g. electron - nucleus, photon - nucleus, hadron - nucleus, heavy ion and neutrino - nucleus collisions. This program library simulates particle transportation in nucleus with numerous nuclear effects with up to date models. NuWro is another event generators developed by Wroclaw group. The main motivation of the authors of NuWro was to have tools to investigate the impact of nuclear effects on directly observable quantities with all the final state interactions included. Now, NuWro simulates all the essential interactions and it is possible to be used in the experiments.

In the present study, we have used the NuWro event generator and as such some of its characteristics are discussed below.

The NuWro event generator:

NuWro is the Monte Carlo generator of neutrino interactions constructed by a group of physicists C. Juszczak et al from the Wrocaw University [69,70] during last decade and is light weight but full featured. It handles all interactions types important in neutrino-nucleus interactions as well as DIS hadronization and intra nuclear cascade. NuWro serves as a tool to assess the relevance of various theoretical models [17] being investigated currently.

This MC generator as already defined, is organized around the event structure which contains three vectors of particles: incoming, temporary and outgoing. It also contains a structure with all the parameters used and a set of boolean flags tagging the event as quasi-elastic (QEL), resonance excited scattering (RES), deep inelastic (DIS), charged-current (CC), neutral-current (NC) etc. The input parameters are read at start-up from a text file and the events are stored in the ROOT tree file to simplify further analysis.

The basic algorithms of NuWro follow better known codes (NEUT, NUANCE, NEUGEN/GENIE). In order to facilitate comparisons, NuWro allows running simulations choosing easily the values of parameters, sets of form-factors, models of nucleus etc. The distinguished features of NuWro are: fine hadronization model, description of resonance region without Rein-Sehgal approach [71] and effective implementation of spectral function in order to describe correctly the distribution of nucleon momenta and binding energies in the impulse approximation scheme.

1.4 Motivation for the present work

neutrino-nucleus interactions are currently a topic of great interest as these offer unique opportunities to explore some fundamental questions in physics leading to explain the various thought provoking phenomena in nature and also in determining the structure of matter. Neutrinos are the cleanest probe of nuclear matter as they are light and electrically neutral particles and hence do not interact through the strong nuclear force. When encountering nuclear matter, these penetrate deeply into a nucleon before occasioning a weak interaction after which these either escape unchanged, retaining their flavour or change into their associated charged lepton partners (μ, e, τ). Weak interactions can proceed via charged-current (CC) or neutral-current (NC) channels. In charged-current interactions, a $W^{\pm}$ boson is emitted as the neutrino converts into its charged lepton partner. neutral-current interactions are facilitated by the exchange of a Z^0 boson that leaves the neutrino flavor unchanged. As the weak force maximally violates parity, the handedness of the neutrino is fixed and they are all left handed. The Feynman diagrams for neutrino interactions with matter are shown in Figure 1 below.

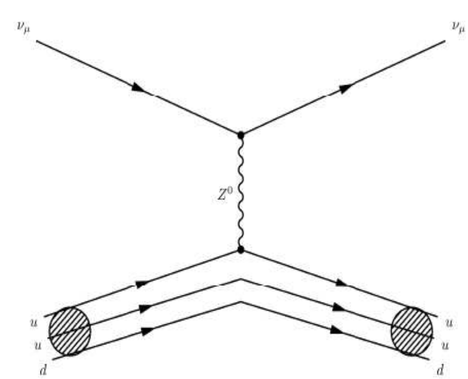

Figure 1: Feynman diagrams for neutrino-nucleus interactions

The feature of weak interaction interests us because it means that neutrinos can be used to probe environments that other radiations such as light or radio waves cannot penetrate. The properties that make neutrinos such good probes of nucleons also make them extremely difficult to work with. Using neutrinos as a probe was first proposed early in the twentieth century [72].

Another important use of the neutrinos is in the observation of supernovae [73], the explosions that end the lives of highly massive stars. The core collapse phase of a supernova is an extremely dense and energetic event. It is so dense that no known particles are able to escape the advancing core front except for neutrinos. Consequently, supernovae are known to release approximately 99 percent of their radiant energy in a short burst of neutrinos. These neutrinos are a very useful probe for core collapse studies.

The rest mass of the neutrino is an important test of cosmological and astrophysical theories . The neutrino's significance in probing cosmological phenomena is as great as any other method, and is thus a major focus of study in astrophysical communities [74].

The study of neutrinos is equally important in particle physics. Neutrinos typically have the lowest mass, and hence are examples of the lowest energy particles theorized in extensions of the standard model of particle physics. However, neutrinos are still the least understood of the fundamental particles. For half a century physicists thought that neutrinos, like photons, had no mass. But recent observations [45] have overturned this view and confirmed that the Standard Model of particle physics is incomplete. To extend the Standard Model so that it incorporates massive neutrinos in a natural way will require far-reaching changes. For example, some theorists argue that extra spatial dimensions are needed to explain neutrino mass, while others argue that the hitherto sacred distinction between matter and antimatter will have to be abandoned. Neutrino masses remain one of the greatest puzzles in elementary particle physics.

In recent years, a number of positive neutrino oscillation signals [46] made irrefutable claims of non zero neutrino masses and increased the interest in this issue.There are quite a number of experiments [40-44], addressing intriguing questions in current neutrino physics. For instance, in hadronic and nuclear physics neutrino scattering experiments can shed light on electroweak form factors [75], the strange quark content of nucleon etc.

1.5 Plan of the dissertation

In this survey we have focussed on neutrino induced pion production on nucleons up to energies of 2 GeV which occurs as the neutrinos are inelastically scattered off the nucleus producing a nucleon excited state (Δ, N^*) dominated by the excitation and subsequent decay of the $\Delta(1232)$ resonance [76] but, depending on the channel, non resonant pion production is not negligible; at higher energies, heavier resonances become increasingly important [77]. For nucleons bound in a nucleus, the cross-sections are modified due to Fermi motion, Pauli blocking, mean field potentials and collisional broadening of the particles [26]. we shall also improve our knowledge of the energy fluxes, backgrounds and detector responses in order to minimise systematic errors. An understanding of the nuclear effects is also essential for the interpretation of the data.

On nuclei, pions can be produced either coherently, leaving the nucleus intact or incoherently. While the former one has attracted considerable attention, the literature on incoherent processes is limited. A full description of the pion production requires a realistic treatment of the final state interactions FSI [78] as well.

Keeping in view the fact that there is a spurt in the studies on pion production in nucleus interactions, it is worthwhile to look into some of the aspects of these interactions which we have tried to do in the present work. We have calculated cross-sections for coherent pion production in nuclei induced by neutrinos of the muon types due to their larger availability. The analogies and differences between this process and the related ones of coherent pion production has been compared and discussed. Neutrino-induced pion production on nuclear targets is the major inelastic channel in all present-day neutrino-oscillation experiments. It has to be understood quantitatively in order to reconstruct neutrino energy in different experiments. In our method, we have included quasi-elastic scattering, resonance excited scattering and deep inelastic scattering all in a unitary, common theoretical framework and code for a range of energies to select a proper event for our study.

2 Neutrino-nucleus interactions

2.1 Introduction

The interaction of neutrinos with nuclei at intermediate energies plays an important role in the precise determination of neutrino properties such as their masses and mixing parameters. It can also provide relevant information on the axial hadronic currents. The statistical significance of the experiments is rapidly improving. However, the data analysis needs to consider a large number of nuclear effects that distort the signals and produce new sources of background that are absent in the elementary neutrino nucleon processes. In this context, it is clearly of interest the elaboration of a theoretically well founded and unified framework in which the electroweak interactions with nuclei could be systematically studied. Furthermore, the recent measurements of the cross-sections for several channels [79-82] provide a serious benchmark to the theoretical models.

A suitable theoretical model should include, at least, three kinds of contributions: (i) quasi-elastic (QE) for low energy transfers, (ii) pion production and two-body processes from the QE region to that around the $\Delta(1232)$ resonance peak, and (iii) double pion production and higher nucleon resonance degrees of freedom induced processes at even higher energies. The QE processes have been abundantly studied. Simple approaches using a global Fermi gas for the nucleons and the impulse approximation are good enough to describe qualitatively electron scattering but more sophisticated treatments of the nuclear effects are necessary to get a detailed agreement with data. There are different kinds of models like those based on the use of proper nucleon spectral functions [83-85], others in which nucleons are treated in a relativistic mean field

[86,87] and models based on a local Fermi gas including many body effects such as spectral functions [88] and RPA [89-91]. The predicted cross-sections for QE scattering are very similar for most models. On the other hand, the theoretical results are clearly below the recently published MiniBooNE data [80]. The discrepancy is large enough to provoke much debate and theoretical attention. In another line of research, the role of meson exchange currents [92] and superscaling [93] have been also estimated recently. Finally, another idea has been explored [94,95], which include two nucleon mechanisms (and others related to Δ excitation) and reproduce MiniBooNE QE data. These latter results suggest that much of the experimental cross-section can be attributed to processes that are not properly QE, stressing again the need of a unified framework dealing with all relevant mechanisms, namely π production and multinucleon excitation.

The matter of π production induced by neutrinos is also of much interest [96-98]. The elementary reaction on the nucleon, at low and intermediate energies, includes both background and resonant mechanisms. The background terms can be obtained from the chiral lagrangians. The resonant terms contain some free parameters that have been adjusted to ANL and/or BNL old bubble chamber data. In nuclei, several effects are expected to be important for the π production reaction. First, the elementary process is modified by Fermi motion, by Pauli blocking and more importantly by the changes of the spectral function of the Δ resonance in the medium. In addition, the final pion can be absorbed or scattered by one or more nucleons.

2.2 An overview of neutrino-nucleus interactions

Neutrino interactions with nuclei have received a considerable attention in recent years stimulated by the needs of neutrino oscillation [46] experiments. A variety of theoretical calculations have been performed for the different reaction channels. At the same time new high quality data are becoming available from MiniBooNE [99], MINOS [100], NOMAD [101], SciBooNE [102] and more is expected from MINERνA [103], an experiment fully dedicated to cross-section measurements. Nuclei are most often used as neutrino detectors, providing relatively large cross-sections that offer a broad variety of information. The properties of neutrinos can only be inferred by detecting the secondary particles they create when interacting with matter. Neutrino-nucleus interactions are broadly classified as elastic scattering, quasi-elastic scattering, resonance excited scattering, inelastic scattering and deep inelastic scattering. Each process can occur through two schemes viz; neutral-current interaction and charged-current interaction. For the case of charged-current processes, one of the lepton has electric charge and thus besides the weak interactions, it would also interact with nucleus via the static coulomb interaction which should be incorporated. The interaction of neutrinos with nuclei at intermediate energies plays an important role in the precise determination of neutrino properties such as their masses and mixing parameters. The statistical significance of the experiments is rapidly improving. However, the data analysis needs to consider a large number of nuclear effects that distort the signals and produce new sources of background that are absent in the elementary neutrino-nucleus processes. Revisiting this type of neutrino scattering physics seems interesting as new data is challenging our thinking and turning up a few surprises. To look for energy distribution of all the three flavours of neutrinos as they are scattered off the nucleus in order to get an insight into their mass values, we make use of the energy-time uncertainty relation.

$$\Delta t(E) = 0.515(m/E)^2 D$$

Where $\Delta t(E)$ is the uncertainty in time for a neutrino of mass m in eV having energy in GeV to arrive at a distance D from the source. This will further enlighten us on oscillation parameter. The pursuit of ν oscillations has unfortunately forced us to do physics at GeV energies where our experimental knowledge of ν interactions is limited. A suitable theoretical model should include, at least, three kinds of contributions: (i) quasi-elastic (QE) for low energy transfers, (ii) pion production from the QE region to that around the $\Delta(1232)$ resonance peak, (iii) double pion production and higher nuclear resonance degrees of freedom induced processes at even higher energies. we use the following plots as our guide as we make a survey.

There are extra contributions coming from multi nucleon correlations in the nucleus [77]. Moreover other nuclear effects such as mean field potential, pauli blocking and fermi motion etc. are also important and has necessitated a dedicated campaign of new measurements [104]. Therefore new experiments are making improved cross-section measurements covering a broad energy range. The study of neutrino oscillations has necessitated a new generation of neutrino experiments that are exploring neutrino-nucleus scattering processes. The charged-current quasi-elastic scattering is a particularly important channel that has been extensively investigated both in the bubble chamber era and by current experiments. Recent results [99-102] have led to theoretical re-examination of this process.

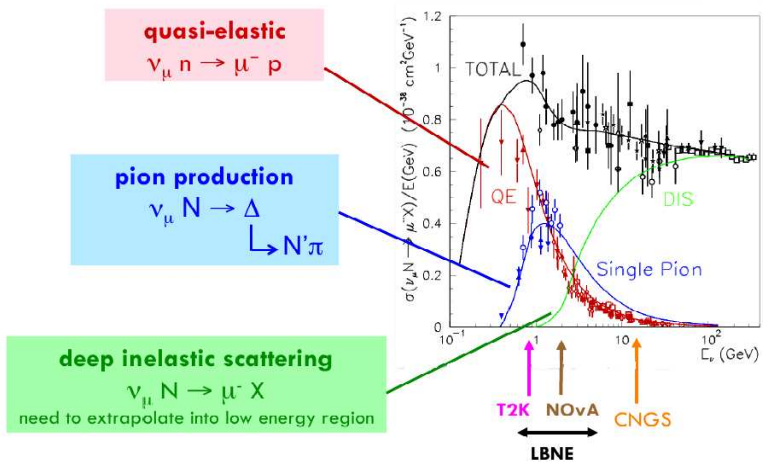

Figure 2: Neutrino-nucleus cross-sections

Understanding QE charged-current (CC) neutrino-nucleus interactions in the few GeV region is very important for many current and future neutrino experiments. The study of neutrino-nucleus reactions in this region is complicated and requires many intermediate steps, such as a description of the nuclear model, understanding the neutrino-nucleon cross-sections, modelling of hadronization, as well as the modelling of intra-nuclear hadron transport [105] and other secondary interactions. These can all play a significant role in how we understand the nature of neutrinos as well as providing useful information about nuclear phenomena. The modelling of neutrino-nucleus interactions is complex and requires linking together many different pieces of theory. The total cross-section for neutrino-nucleon scattering has contribution from all processes.

2.3 Pion production in neutrino-nucleus interactions

When neutrino interactions take place in the nucleus, the particles that are produced, can interact with the nuclear medium, thus modifying the observed characteristics of the interaction. In this direction, the matter of π production induced by neutrinos is of much interest. The pion production processes from nucleons and nuclei at intermediate neutrino energies are important tools to study the hadronic structure [106] and play an important role in analysis of the present neutrino oscillation experiments, where they constitute a major source of uncertainty in the identification of electron and muon events. The elementary reaction on the nucleon, at low and intermediate energies, mainly includes resonant mechanisms which can be shown as under:

$$\nu_l + N \longrightarrow L + N + \pi^+$$

$$\nu_l + N \longrightarrow N + \pi^0 + \pi^+$$

We will therefore include π production in a well established framework that has been tested. At neutrino energies below a few GeV, the most common neutrino interactions are those that minimally affect the interaction target. For charged-current interactions with baryon targets, the baryon must, at a minimum, undergo a change in its electric charge to accommodate the exchange of the charged $W^{\pm}$ boson; these are called charged-current quasi-elastic interactions. If, instead of simply altering the charge of the target baryon, the W$^{\pm}$ transfers enough momentum to promote the target into a low-mass resonance state, the decay of the resonance will typically produce a nucleon and a pion.

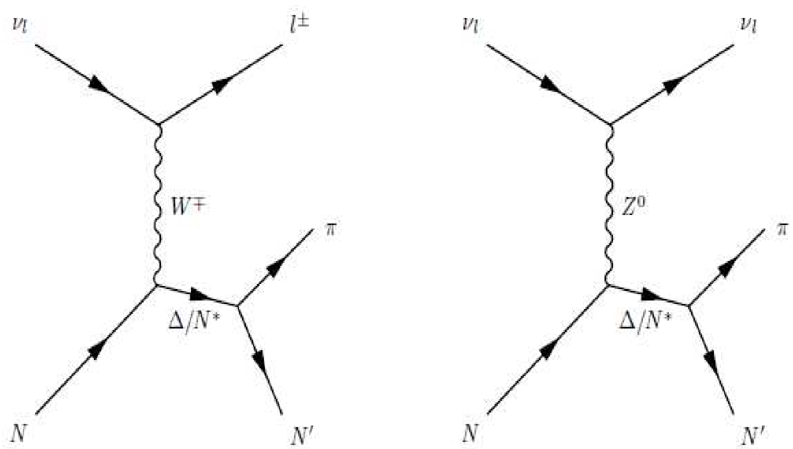

Figure 3: pion production in charged-current and neutral-current resonant interactions

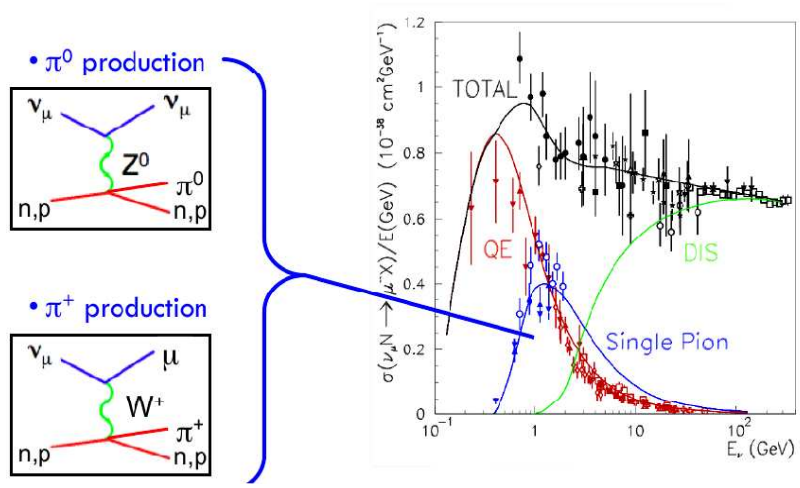

Figure 4: Single pion production in charged-current and neutral-current resonant interactions respectively

Such processes are referred to as charged-current pion production. In these processes, the pion always decays to a muon and a muon neutrino [34] (well, almost always, 99.99 percent of the time).

$$\pi^+ \longrightarrow \mu^+ + \nu_\mu$$

Since pion decays into two particles, the conservation of momentum and energy gives definite energies to the final products. The decay proceeds by the weak interaction and can be visualized in terms of Feynman diagrams as shown below.

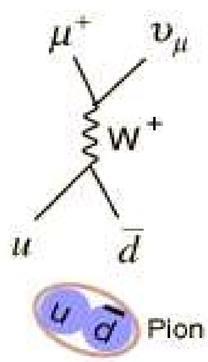

Figure 5: Feynman digram for muon decay

The muon neutrino interacts with a nucleus to make a muon, not an electron. This is also called conservation of lepton number. But it seems the neutrino is composed of a combination of two or three different mass states. The way a neutrino propagates from one place to another depends on the mass states. But quantum mechanics tells us that if two or more neutrinos are composed of the same mass states but in different combinations then the neutrino can oscillate from one flavor to another while it travels through space. Which flavor will it be when it interacts in the detector? The answer must be calculated quantum mechanically, and therefore it involves a probability.

3 Results and Discussion

3.1 Introduction

In this Chapter, we present the results on the various classes of neutrino-nucleus interactions at different energies with oxygen as target. For the present analysis, NuWro event generator has been used to simulate the event samples for these interactions.

3.2 Neutrino-nucleus interaction cross-sections

In the present study, cross-sections for 10,000 events of muon neutrino interactions with oxygen for neutrino energies around 1 GeV for all the three processes enabled by NuWro i.e., resonance excited scattering (RES), deep inelastic scattering (DIS) and quasi-elastic (QE) scattering each through both schemes of interaction, charged-current (CC) and neutral-current (NC) have been obtained and these results are exhibited in Table 1.

The understanding of neutrino-nucleus interaction around 1 to 2 GeV energy is of great importance in analysing neutrino oscillation experiments [46]. In this energy region, the main reaction mechanisms are quasi-elastic scattering, resonance excited scattering and single pion production through the Δ excitation [105].

The results are much in confirmation with the previously established results [105,106]. It is clear from Table 1 that the cross-sections around 1 GeV of neutrino energy have overlapping contributions from both schemes of interactions viz; charged-current and neutral-current for RES and DIS processes. These processes are, thus, not preferred for investigation in the present study. However, QE interactions have marginally different values of cross-sections for

Energy (GeV)	**QE**		**RES**		**DIS**	
	CC	**NC**	**CC**	**NC**	**CC**	**NC**
0.2	0.89770	0.52870	0	0.000019	0	0
0.4	3.29464	1.58127	0.008824	0.072860	0	0
0.6	4.97448	2.20443	0.253052	0.427116	0	0
0.8	5.84243	2.55954	0.669427	0.848842	0	0.0000305
1.0	6.26241	2.75051	1.152660	1.263050	0.001382	0.0183309
1.2	6.44748	2.85169	1.679710	1.559430	0.057111	0.1186240
1.4	6.51031	2.90985	2.160680	1.837420	0.263569	0.3227250
1.6	6.50404	2.94959	2.561360	1.982990	0.632025	0.6083780
2.0	6.33238	2.67122	3.110070	2.177180	1.76413	1.3251000
5.0	5.83771	2.61397	3.855310	2.450550	14.7171	7.9024400

Table 1: cross-section for various processes (x 10^{-39} cm^2)

the two schemes of interactions. Hence this process is considered for further investigations in the present study. At 1 GeV of neutrino energy, we are also in a transition region where QE and RES processes dominate but where there is also a significant DIS component being switched on as we increase the energy. The plots obtained for these process through both modes ; CC and NC are shown in Figures 6-8.

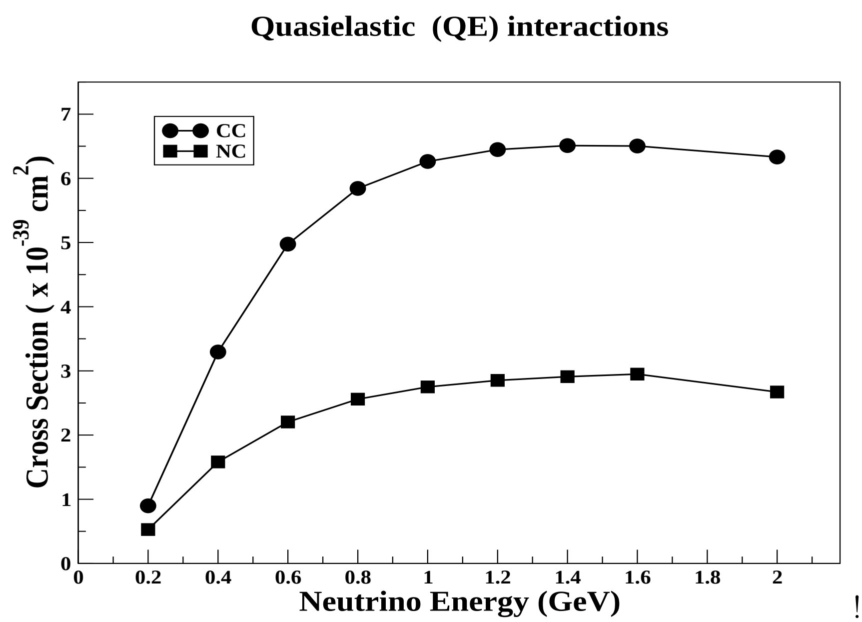

Figure 6:

One well established fact as evident from the tabulated values is that, of the six possible interaction channels, QE CC has the largest cross-section by far. Quasi-elastic cross-section is well known in which we can consistently describe some experimental data. The very fact that the predicted cross-sections for QE scattering are very similar for most models [49] is the reason that QE processes have been abundantly studied. However, more sophisticated treatments of the nuclear effects are necessary to get a detailed agreement with data. For example the theoretical results are clearly below the recently published MiniBooNE data [99]. The discrepancy is large enough to provoke much debate and theoretical attention. This suggests that much of the experimental cross-section can be attributed to processes that are not properly QE, stressing again the need of a unified framework dealing with all relevant mechanisms, namely π production and multi-nucleon excitation. This again underlines the need for studying pion production.

The inelastic scattering of neutrinos produces a nucleon excited state (Δ,N*). Such baryonic resonances quickly decay most often to a nucleon and a single pion in the final state.

The cross-sections for the production of pions in the neutrino interaction with oxygen target using NuWro are shown in Figure 9. The figure gives the results similar of a fit to the elementary pion production data, extrapolated to higher energies, and compared with the NEUT and GENIE [78] as shown in Figure 10.

The region of neutrino energies around 1 GeV is particularly troublesome. It is in this region that many of the above cross-sections are similar in magnitude.

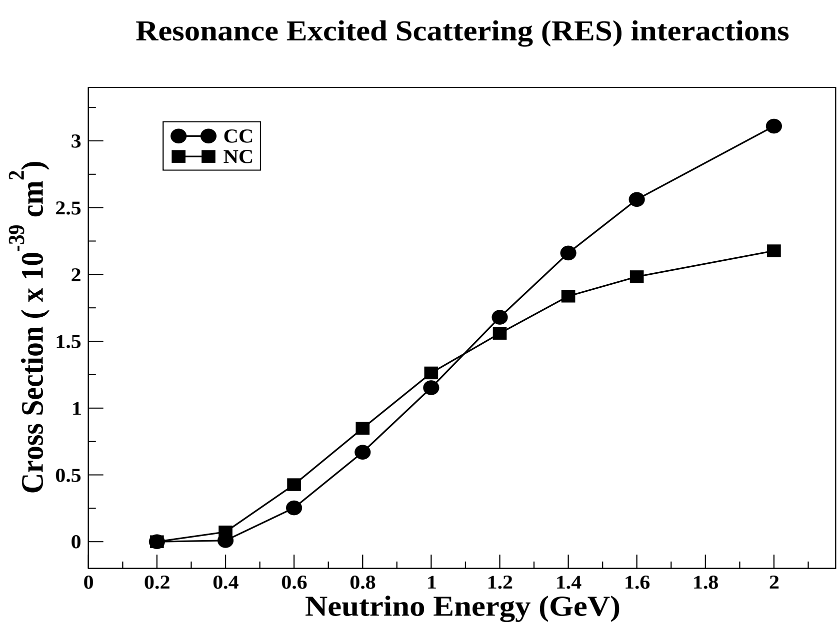
Resonance Excited Scattering (RES) interactions
CC
NC
Cross Section (x 10^-39 cm^2)
Neutrino Energy (GeV)
0
0.5
1
1.5
2
2.5
3
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

Figure 7:

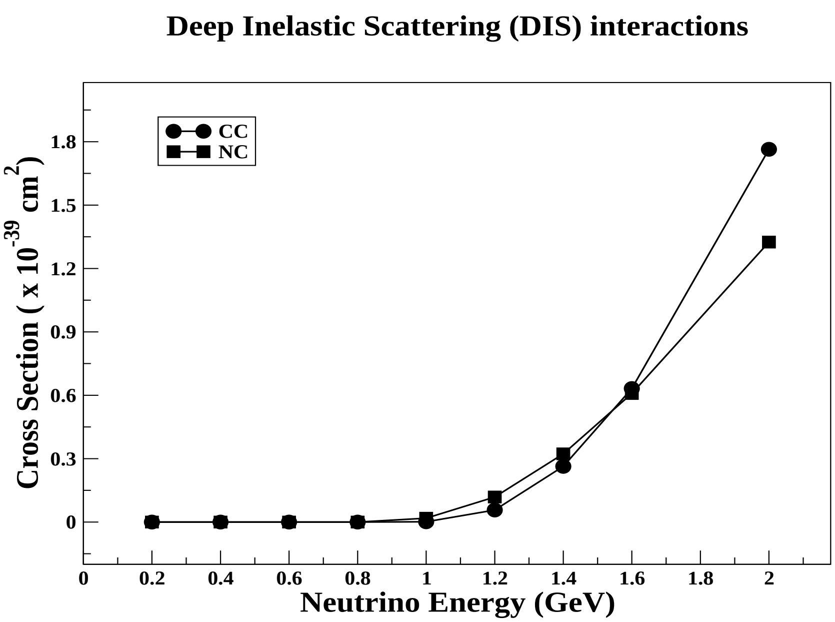
Deep Inelastic Scattering (DIS) interactions
CC
NC
Cross Section (x 10^-39 cm^2)
Neutrino Energy (GeV)
0
0.3
0.6
0.9
1.2
1.5
1.8
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

Figure 8:

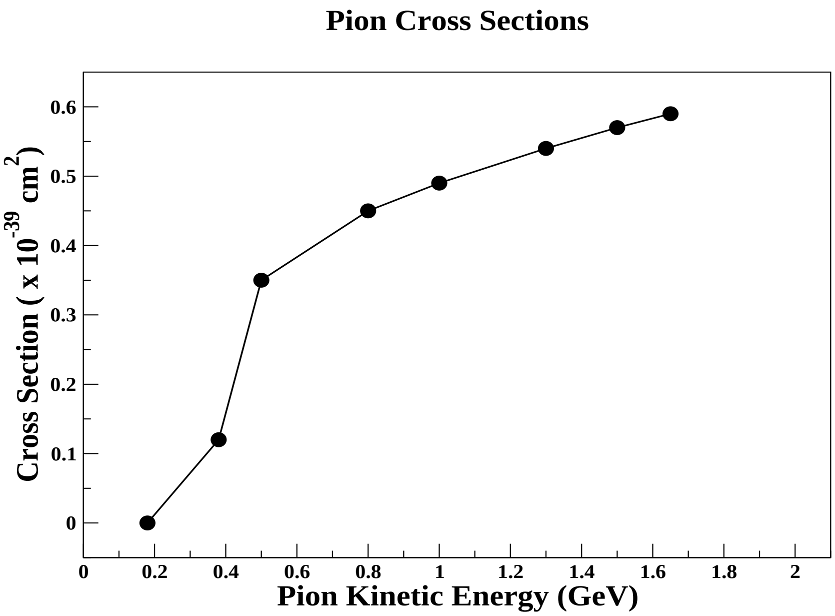

Figure 9:

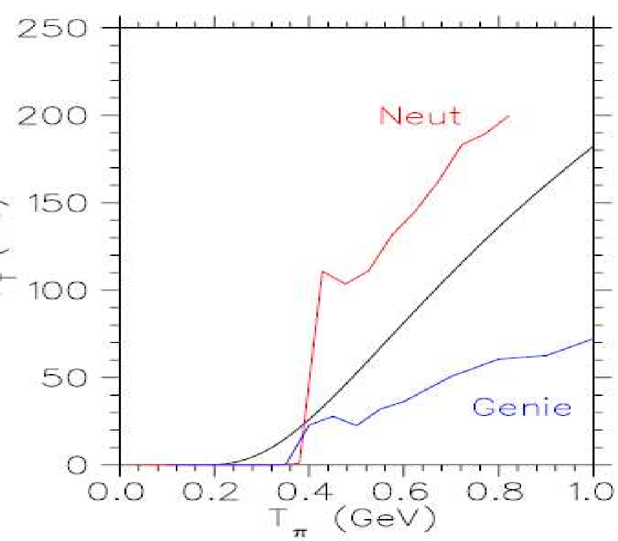

Figure 10: Pion production cross-sections as predicted by NEUT and GENIE

We have plotted the cross-sections as percentage for all the three processes viz; quasi-elastic (QE), single pion production (SPP) and deep inelastic scattering (DIS) each through CC mode in Figure 11.

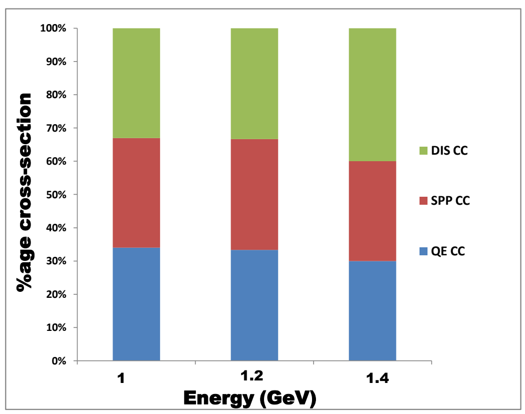

Figure 11: Contribution to the pion production by various processes; percentage cross-section of QE (blue), SPP (red) and DIS (green)

Here resonance single pion production contributes nearly around 30 percent of the total cross-section which is more or less similar to the contributions from QEL and DIS processes. Such results have earlier been obtained [107] as well. This is a problem experimentally as RES events can have indistinguishable signatures to DIS events in a detector, making it hard to measure each process exclusively. This is also amply supported by the recent results of the three experiments running at Fermilab [108] which show a more or less equal contribution of the three processes to cross-section as the neutrino energy varies between 1 to 2 GeV as shown in Figure 12.

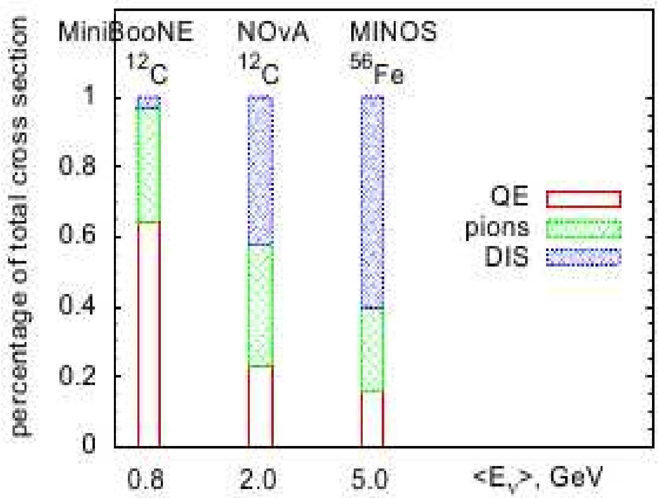

Figure 12: Fermilab results

3.3 Final state interactions

Most of the Monte Carlo neutrino interactions simulations are based on the impulse approximation scheme: the degrees of freedom are quasi-free nucleons and a primary interaction occurs on one of them [109]. This is followed by final state interactions (FSI), i.e., the hadrons propagate through nucleus before they can be detected. As we are mainly restricting ourselves to SPP and QE processes , We have plotted, in Figure 13, these processes without FSI using NuWro event generator for muon neutrino beam energy around 1 GeV energy to see how it looks different from the universal graph [110] shown in Figure 14, where FSI is taken into consideration.

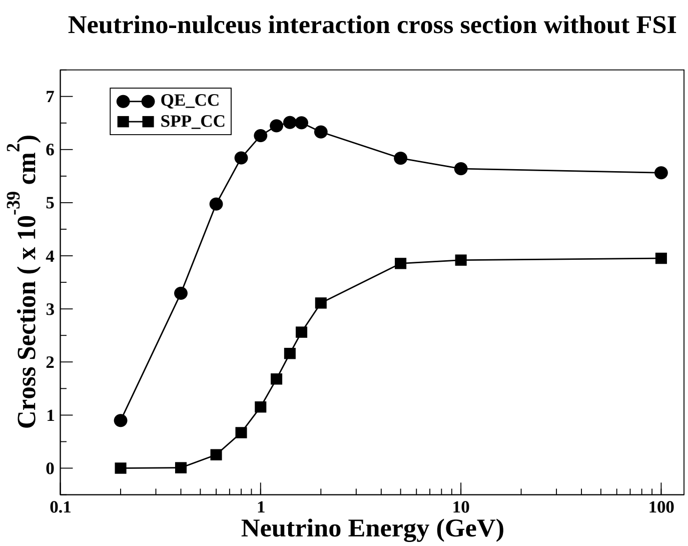

Figure 13:

The clear inference is that cross-section saturates at larger energies as FSI is left off and as FSI is kept on, the interactions of produced pions lead to pions being absorbed in the nucleus or re-scattered, thus reducing the cross-section. Overall, the impact of nuclear effects impact on observables is considerably significant.

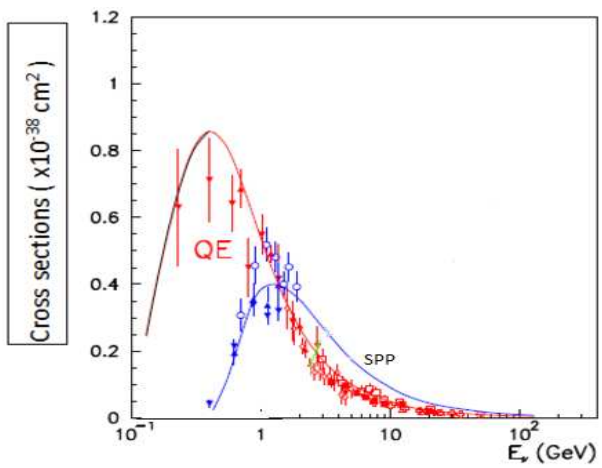

Figure 14: FSI On

Monte Carlo generators use rather simple semi-classical models of FSI: particles propagate through nucleus along straight lines between possible re-interactions. The only quantum mechanical effects are Pauli blocking and formation time/zone: hadron needs some time to be formed and become able to re-interact [78,104]. The NuWro FSI code has recently been updated by implementing the Oset model [111] of effective pion-nucleon cross-sections. The neutrino interaction point is selected inside nucleus according to the nuclear matter density. All secondary hadrons propagate through nucleus and can interact with nucleons inside. At each point of their path it is decided if there was an interaction or not. This is done based on an effective cross-section model, using NuWro.

The FSI effects are known to be large. They mix QE (quasi-elastic) and SPP (single pion production) primary vertex events and give rise to QE-

CC only	with FSI	without FSI
0π	337431	292705
π^0	35033	35532
π^+	121242	169365
π^-	3737	0
$2\pi^0$	390	630
$2\pi^+$	214	100
$2\pi^-$	8	0
$\pi^0\pi^+$	801	774
$\pi^0\pi^-$	180	0
$\pi^+\pi^-$	817	892
$\geq 3\pi$	147	2

Table 2: Pion statistics

like (no pion in the final state) and SPP-like (a single pion in the final state) events with the nomenclature referring to the particles which leave nucleus. Table 2 contains the pion statistics for two separate samples independently, where one had FSI turned off and in the other they were left on. It is clear that the final state can be very different than it would be with no FSI. One of the worst cases comes when a pion production principal interaction appears to be quasi-elastic if the pion is absorbed in the final state interactions. Depending on cuts, this effect is thought to account for about 10-20 percent [112] of quasi-elastic events. Because FSI effects involve complicated nuclear physics effects, there are uncertainties associated with any approach.

3.4 Pion absorption

Pion initiated reactions with no pions in the final state are called pion absorption processes. If the initial hadron is a pion and has enough energy, a second pion can be produced in the nucleus. We call those events as pion production. The following table is based on the simulations carried out by various workers [109] where the probabilities of fate of single pions produced at the primary vertex in 1 GeV muon neutrino interactions on oxygen target through both charged-current and neutral-current modes have been evaluated.

$\pi^0 \longrightarrow \pi^0$	50%
$\pi^+ \longrightarrow \pi^+$	59%
$\pi^0 \longrightarrow$ no pions	29%
$\pi^+ \longrightarrow$ no pions	30%
$\pi^0 \longrightarrow \pi^+$	9%
$\pi^0 \longrightarrow \pi^-$	8%
$\pi^+ \longrightarrow \pi^0$	8%

Table 3: Rate of events with single pion or no pion in final state if there was single pion in initial state.

In general QE processes give rise to topologies with no pions in the initial and final states whereas DIS and RES are more likely to result in events with pions in the primary and final state. Generally all the generators have a larger number of zero pion topologies in the final state than were in the primary state. This indicates that pions are more likely to be absorbed than created. In order to verify it further, the following plot has been prepared using NuWro which

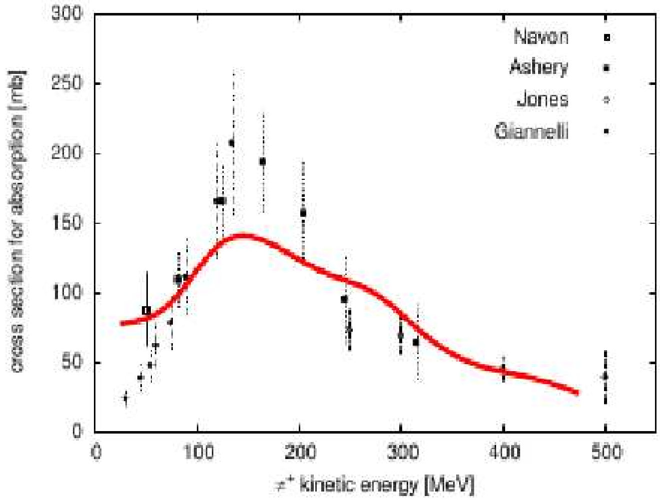

Figure 15: Pion absorption cross-section on oxygen using NuWro

shows the absorption cross-section of pions on oxygen target. The data points are taken from: Ashery [113, 114], Navon [115], Jones [116] and Giannelli [117]. The solid line shows NuWro predictions.

Pions are particularly susceptible to the effects of the nuclear medium, since they interact via the strong nuclear force. Charged pions can either be absorbed or converted into neutral pions via:

$$n + \pi^{+} \longrightarrow p + \pi^{0}$$

The nuclear medium can also influence whether a pion is even created. All these factors have a direct bearing on the cross-section.

4 Summary and concluding remarks

The study of neutrino interactions is an exciting field because of the many ways in which neutrinos can participate in physics beyond the Standard Model. There are a good number of unanswered questions relating to neutrinos, viz., are there any right-handed neutrinos? what could be the implications of neutrinos having mass? are these Dirac or Majorana particles? do neutrinos decay? and lastly the reason behind neutrino oscillations. All of these phenomena are active areas of research presently.

One of the most straightforward approaches to have an understanding of the various characteristics of neutrinos is to investigate the interactions of neutrinos with nuclei. Keeping in view the importance of such studies, an attempt has been made in the present study to measure the cross-section for single pion production via resonance in charged-current neutrino interactions with oxygen. The result serves to test the predictions of the Rein Sehgal model and verify old experimental measurements. While the results do not improve on precision, these serve as a useful cross check in a region with few measurements.

Cross-sections of neutrinos on nucleons and nuclei are tiny and many processes contribute simultaneously. This fact makes the analysis of data and theoretical predictions more challenging. But it is critically important to know the cross-sections because we need these in order to estimate how many events we should expect and the kind of signals, i.e., final states, we will observe. The cross-sections are reasonably well known at low and at very large energies. However, in the intermediate energy range of around 1 GeV, they are known only crudely. Yet it is this energy range that is crucially important in neutrino oscillation studies.

While most event generators are similar in their treatment of the initial neutrino-nucleus interactions, these differ substantially in their treatment of the final-state interactions in the target nucleus. Based on calculations with GENIE and NEUT, we can make a rough statement that about 40 percent of the pions with energies close to the Δ resonance have FSI for oxygen. For low energy and high energy pions, this fraction falls to about 20 percent. For protons in oxygen, the fraction is about half, slightly lower at low energies and slightly higher at higher energies. The question is should we focus on pion-nucleus cross-sections? How useful is the pion transparency data? But there is also quite a lot of implicit FSI data coming from the neutrino experiments. We have made a priliminary study in terms of the cross-sections of the various processes using the NuWro event generator.

REFERENCES

[1] A. H. Becquerel, Compt. Rend. Math.**122**, 420 (1896).

[2] P. Curie and M. Curie, Comptes rendus **127**, 175 (1898).

[3] E. Rutherford, Philosophical Magazine Ser. **5**, 109 (1899).

[4] P. Villard, Comptes rendus **129**, 244 (1899).

[5] P. Villard, Comptes rendus **130**, 1010 (1900).

[6] P. Villard, Comptes rendus **130**, 1178 (1900).

[7] P. Villard, Seances de la Societe francaise de physique, 45 (1900) .

[8] H. Becquerel, Comptes rendus **130**, 809 (1900).

[9] C. Ellis, Proc. Roy. Soc. A **99**, 261 (1921).

[10] Ellis and Skinner, Proc. Roy. Soc. A **105**, 185 (1924).

[11] L. Meitner, Ergebnisse der Exakten Naturwissenschaften **3**, 160 (1924).

[12] K. Emeleus, Proc. Camb. Phil. Soc. **22**, 400 (1924).

[13] R. Gurney, Proc. Roy. Soc. A **109**, 540 (1925).

[14] Ellis and Wooster, Proc. Roy. Soc. A **117**, 313 (1927).

[15] W. Pauli, Phys. Today **31N9**, 27 (1978).

[16] J. Chadwick, Nature **129**, 312 (1932).

[17] F. Perrin, Comptes Rendus **197**, 1625 (1933).

[18] E. Fermi, Z. Phys. **88**,161 (1934).

[19] H. Bethe and R. Peierls, Nature **133**, 532 (1934).

[20] C. L. Cowan, F. Reines, F. B. Harrison, E. C. Anderson, and F. N. Hayes, Phys. Rev. **90**, 493 (1953).

[21] F. Reines and C. L. Cowan, Phys. Rev. **92**, 830 (1953).

[22] C. L. Cowan, F. Reines, F. B. Harrison, H. W. Kruse, and A. D. McGuire, Science **124**, 103 (1956).

[23] F. Reines and C. L. Cowan, Phys. Rev. **113**, 273 (1959).

[24] G. Danby et al., Phys. Rev. Lett. **9**, 36 (1962).

[25] J. Davis, Raymond, D. S. Harmer, and K. C. Hoffman, Phys. Rev. Lett. **20**, 1205 (1968).

[26] J. N. Bahcall, Phys. Rev. Lett. **12**, 300 (1964).

[27] K. S. Hirata et al., Phys. Lett. **B205**, 416 (1988).

[28] W. Gajewski, in Proceedings of High energy physics, Munich, 1988, p.1305.

[29] L. Arnaudon et al., Phys. Lett. **B307**, 187 (1993).

[30] R. Akers et al., Z. Phys. C **65**, 47 (1995).

[31] P. Abreu et al., Z. Phys. C **74**, 577 (1997).

[32] M. Acciarri et al., Phys. Lett. **B431**, 199 (1998).

[33] Y. Fukuda et al., Phys. Lett. **B433**, 9 (1998).

[34] Y. Fukuda et al., Phys. Lett. **B436**, 33 (1998).

[35] K. Kodama et al., Phys. Lett. **B504**, 218 (2001).

[36] Q. R. Ahmad et al., Phys. Rev. Lett. **89**, 011301 (2002).

[37] K. Eguchi et al., Phys. Rev. Lett. **90**, 021802 (2003).

[38] M. H. Ahn et al., Phys. Rev. Lett. **90**, 041801 (2003).

[39] http://www.imsc.res.in/ ino/

[40] T. Bruce and Cleveland et al., Astrophysical Journal **496**, 496 (1998).

[41] J. N. Abdurashitov et al., J. Exp. Theor. Phys. **95**, 18 (2002).

[42] W. Hampel et al., Phys. Lett. **B447**, 127 (1999).

[43] Y. Fukuda et al., Phys. Rev. Lettz. **82**, 2430 (1999).

[44] B. Aharmim et al., Phys. Rev. C **72**, 055502 (2005).

[45] R. N. Mohapatra and Palash Pal, Massive neutrinos in Physics and Astrophysics (World Scientific, Singapore, 2004).

[46] G. Rajasekaran, Pramana **55**, 19 (2000).

[47] H. L Anderson, Los Alamos Science **14**, 96 (1986).

[48] C. Juszczak, J. A. Nowak, and J. T. Sobczyk, Nucl. Phys. B Proc. Suppl. **159**, 211 (2006).

[49] J. A. Nowak, Phys. Scr. **T127**, 70 (2006).

[50] J. Dobson and C. Andreopoulos, Acta Phys. Pol. B **40**, 2613 (2009).

[51] H. Gallagher, Acta Phys. Pol. B **40**, 2607 (2009).

[52] M. Nakahata et al; J. Phys. Soc. Jps. **55**, 3786 (1986).

[53] A. Ferrari, P. R. Sala, A. Fasso, J. Ranft, CERN-2005-010, INFN/TC-05/11(2005).

[54] D. Casper, The Nuance neutrino physics simulation and the future.

[55] Y. Hayato, Acta Phys. Polon. B **40**, 2477 (2009).

[56] D. Casper, Nucl. Phys. Proc. Suppl. **112**, 161 (2002).

[57] P. Przewlocki, Acta Phys. Polon. B **40**, 2513 (2009).

[58] H. Gallagher, Nucl. Phys. Proc. Suppl. **112**, 188 (2002).

[59] Y. Kurimoto et al., Phys. Rev. D **81**, 111102 (2010).

[60] A. A. Aguilar-Arevalo et al., Nucl. Instrum. Meth. A **599**, 28 (2009).

[61] T. Kafka, J. Phys. Conf. Ser. **39**, 310 (2006).

[62] G. Battistoni et al., AIP Conference Proceeding **896**, 31 (2007).

[63] A. Fass, A. Ferrari, J. Ranft, and P. R. Sala ‚FLUKA a multi-particle transport code', CERN-2005-10 (2005), INFN TC-05-11, SLAC-R-773.

[64] C. Andreopoulos et al., Nucl. Instrum. Meth. A **614**, 87 (2010).

[65] C. Anderson et al., Phys. Rev. Lett. **108**, 161802 (2012).

[66] C. M. Ignarra, arXiv:1110.1604 [physics.ins-det].

[67] D. W. Schmitz, AIP Conf. Proc. **1405**, 243 (2011).

[68] O. Buss et al., Phys. Reports. **512**, 1 (2012).

[69] T. Golan, C. Juszczak, and J. T. Sobczyk, Phys. Rev. C **86**, 015505 (2012).

[70] C. Juszczak, J. A. Nowak, and J. T. Sobczyk, Nucl. Phys. B Proc. Suppl **159**, 211 (2006).

[71] D. Rein and L.M. Sehgal, Nucl. Phys. B **223**, 29 (1983).

[72] F. Reines and C. L. Cowan Jr., Nature **178**, 446 (1956).

[73] A. K. Mann, Shadow of a star: The neutrino story of Supernova 1987A (W. H. Freeman and Company, 1997), p. 122.

[74] A. Heger et al., astro-ph/0307546.

[75] S. Schael, et al., Phys. Rev. **427**, 257 (2006).

[76] E. Hernandez, J.Nieves, and M. Valverde, Phys. Rev. D **76**, 033005 (2007).

[77] O. Lalakulich, E. A. Paschos, and G. Piranishvili, Phys. Rev. D **74**, 014009 (2006).

[78] "Review of Final State Interactions in Neutrino Interactions," S.A. Dytman, Prepared for 6th International Workshop on Neutrino-Nucleus Interactions in the Few-GeV Region (NUINT 2009), Sitges, Spain, 18-22 May 2009. AIP Conf. Proc. **1189**, 51 (2009).

[79] A. A. Aguilar-Arevalo et al., Phys. Rev. D **81**, 092005 (2010).

[80] A. A. Aguilar-Arevalo et al., Phys. Rev. D **82**, 092005 (2010).

[81] A. A. Aguilar-Arevalo et al., arXiv:1011.3572 [hep-ex].

[82] Y. Nakajima et al., Phys. Rev. D **83**, 012005 (2011).

[83] O. Benhar, N. Farina, H. Nakamura, M. Sakuda, and R. Seki, Phys. Rev. D **72**, 053005 (2005).

[84] A. M. Ankowski and J. T. Sobczyk, Phys. Rev. C **77**, 044311 (2008).

[85] A. M. Ankowski, O. Benhar, and N. Farina, Phys. Rev. D **82**, 013002 (2010).

[86] C. Maieron, M. C. Martinez, J. A. Caballero, and J. M. Udias, Phys. Rev. C **68**, 048501 (2003).

[87] M. C. Martinez, P. Lava, N. Jachowicz, J. Ryckebusch, K. Vantournhout, and J. M. Udias, Phys. Rev. C **73**, 024607 (2006).

[88] M. C. Martinez, P. Lava, N. Jachowicz, J. Ryckebusch, K. Vantournhout and J. M. Udias, Phys. Rev. C **73**, 024607 (2006).

[89] T. Leitner, O. Buss, L. Alvarez-Ruso, and U. Mosel, Phys. Rev. C **79**, 034601 (2009).

[90] J. Nieves, M. Valverde , and M. J. Vicente Vacas, Phys. Rev. C **73**, 025504 (2006).

[91] S. K. Singh, E. Oset, Nucl. Phys.**A542**, 587 (1992).

[92] J. E. Amaro, M. B. Barbaro, J. A. Caballero, T. W. Donnelly, and C. F. Williamson, Phys. Lett. **B696**, 151 (2011).

[93] J. E. Amaro, M. B. Barbaro, J. A. Caballero, and T. W. Donnelly, arXiv:1012.4265.

[94] M. Martini, M. Ericson, G. Chanfray, and J. Marteau, Phys. Rev. C **80**, 065501 (2009).

[95] M. Martini, M. Ericson, G. Chanfray, and J. Marteau, Phys. Rev. C **81** , 045502 (2010).

[96] L. Alvarez-Ruso, S. K. Singh, and M. J. Vicente-Vacas, Phys. Rev. C **59**, 3386 (1999).

[97] T. Sato, D. Uno, and T. S. H. Lee, Phys. Rev. C **67**, 065201 (2003).

[98] K. M. Graczyk, D. Kielczewska, P. Przewlocki, and J. T. Sobczyk, Phys. Rev. D **80**, 093001 (2009).

[99] A. A. Aguilar-Arevalo et al., Phys. Rev. D **81**, 013005 (2010).

[100] D. G. Michael et al., Nucl. Instrum. Meth. A **515** 800 (2003).

[101] P. Astier, et al., Phys. Lett. **B570**, 19 (2003).

[102] K. Hiraide et al., Phys. Rev. D **78**, 112004 (2008).

[103] D. D. Stancil et al., Mod. Phys. Lett. A **27**, 1250077 (2012).

[104] S. K. Singh, M. J. Vicente-Vacas, and E. Oset, Phys. Lett. B **416**, 23 (1998).

[105] K. S. Kuzmin, V. V. Lyubushkin, and V. A. Naumov, Phys. Atom. Nucl. **69**, 1857 (2006).

[106] Riazuddin, Physical Review **114**, 1184 (1959).

[107] C. Andreopoulos, Acta Phys. Pol. B **40**, 2461 (2009).

[108] U. Moseland and O. Lalakulich; arXiv:1211.1977v1nucl-th 8 Nov (2012).

[109] M. Antonello et al., Acta Phys. Pol. B **40**, 2519 (2009).

[110] G.P. Zeller: Particle Data Group Review (2012).

[111] L. L. Salcedo, E. Oset, M. J. Vicente-Vacas, and C. Garcia-Recio, Nucl. Phys. **A484**, 557 (1988).

[112] T. Leitner, Phys. Rev. C **81**, 064614 (2010).

[113] D. Ashery et al, Phys. Rev. C **23** 2173 (1981)

[114] D. Ashery et al, Phys. Rev. C **30** 946 (1984)

[115] I. Navon et al, Phys. Rev. C **28** 2548 (1983)

[116] M. K. Jones et al, Phys. Rev. C **48** 2800 (1993)

[117] R. A. Giannelli et al, Phys. Rev. C **61** 054615 (2000)