Thermodynamics and Correlation Functions in Clustering of Galaxies in an Expanding Universe

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Certificate

This is to certify that the dissertation entitled "***Thermodynamics and Correlation Functions in Clustering of Galaxies in an Expanding Universe*** " submitted by ***Naveel Ahmad Wani***, in partial fulfilment for the award of the degree of ***Master of Philosophy*** in ***Physics***, is the original research work carried out by him under our supervision and guidance. It is further certified that the dissertation has not been submitted for the award of M. Phil. or any other degree to this University or any other University. The scholar has attended the department for statutory period as required under rules.

Dr. Naseer Iqbal	**Professor Sheikh Javid Ahmad**
(*Supervisor*)	(*Head of the department*)

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Chapter 1

Introduction

The major mysteries of the universe is that on the one side, there is the inhomogeneous distribution of matter throughout space, while on the other side there is the homogeneous distribution of matter throughout space [1]. This apparent contradiction (puzzle) typifies our uncertainty about the origin of structures in the universe as both statements are true. In order to measure the inhomogeneity of matter, we need only look around us and indeed the closer we look the greater the density contrasts usually seem to be. Considering the enormous development in terms of observational techniques, we are now in a position to answer some of the questions regarding the origin and evolution of the universe. The study of the structure of our universe is one of the most active and exciting research fields in cosmology. In the recent decades, new and sensational facts in cosmology have been unveiled and our understanding of the large scale structure of the universe is improving rapidly. There are various cosmological models and the purpose of these models is to explain the origin of the structure (processes) to form the present universe and observations have added enormous knowledge for understanding the large-scale structures in the present universe. The inhomogeneity of the universe has been a major aspect of cosmology over the last 25 years. We have learned a great deal, especially from red-shift surveys, and although things turn out to be fairly complicated in the sense that the universe is not simply a pile of clusters distributed at random, nevertheless possesses some systematics upon which we can build models.

The evolution of a self-gravitating many-body system involves the long-range nature of attractive gravity and is fundamentally connected with statistical mechanics and thermodynamics. Historically, the important consequence from the thermodynamical arguments had arisen in the 1960s, known as the gravo-thermal catastrophe, i.e., thermodynamic instability due to the negative specific heat [2][3]. Originally, the gravo-thermal catastrophe had been investigated in a very idealized situation, i.e., a stellar system confined in a spherical cavity [4][5]. In order to describe the thermodynamic information about the system, it is useful to emphasize one or another parameter of the thermodynamic system in different situations. For this purpose, it is necessary to summarize the inter-relations between different descriptions (like T, P, μ as a function of S, V, N) which however contains all the basic information about the system. These equations of state does not contain much information about the system, so it is necessary to consider each equation of state as a partial differential equation given as Maxwell's thermodynamic equations. Although, thermodynamics mainly applies to equilibrium systems, but self-gravitating systems continually evolve towards more singular states, so they are never in equilibrium. The main reason for this inadequacy is the long range nature of the gravitational force and the fact that it does not saturate. In order to avoid this inadequacy, attempts have been made and the assumption is that the universe is in a state of quasi-equilibrium, so the thermodynamics can be used to understand the phenomena of galaxy clustering. There have been a clear indication that this assumption (quasi-equilibrium) was quite good as thermodynamics worked surprisingly well for describing the instabilities and slow evolution of gravitating systems. These results also agreed reasonably well with computer simulations.

An interesting technique called n-particle correlation function between galaxies is one of the approach for understanding the galaxy clustering. The technique of correlation function (measure of deviation from randomness) was first introduced by [6] and later was popularised by many workers like [7][8]. The evaluation of n-particle correlation functions can be studied by using Bogollubov-Born-Green-Kirkwood-Yvon (BBGKY) hierarchy, but it is too complicated to handle it for higher order correlations. However, the lower order correlation function called two-point correlation function was introduced by [7] and is presently the most widely used as a statistical indicator. The shape of the two-point correlation function for different clusters of galaxies have been measured and the simple power law led many workers to describe the theoretical results and the N-body numerical simulations. The two-point correlation function (ξ_2) governed by the amplitude and scale length has become very popular as same contains information about clustering on all higher scales [16][17][18].

1.1 Plan of Dissertation

In the second chapter, we describe an overview of the formation of galaxy clusters in the early phase of universe. Cosmological parameters and the properties of the dark matter help to determine the growth of density perturbations and eventually the formation of massive dark halos on large scale and on smaller scales. The particular importance of studying such processes will help to know about the mass distribution. Also, the effects of non-linearity and smoothing processes in the evolution of clustering of galaxies in an expanding universe can be seen. We also see how correlation functions, thermodynamics and N-body simulation models are used to understand the clustering phenomena of galaxies in an expanding universe.

In the third chapter, we start with the detailed studies of correlation function (ξ) and see how correlation function has been used so far to understand the clustering phenomena of galaxies in an expanding universe. In one of the sections in this chapter, we understand how the thermodynamics can be used for gravitating systems (assuming to be point mass) and how two-point correlation function determines the thermodynamic properties of an infinite system. From the thermodynamic point of view, Vander Waal's equation of state is used with the fact that in gravitating systems their is no repulsive interaction between gravitating particles (galaxies). From this equation, we describe the correlation parameter (b), which in turn depends on correlation function, hence will be helpful to determine and study the clustering phenomena of galaxies. A differential equation is developed relating two-point correlation function (ξ_2) with the average number density $(\bar{n})$, temperature (T) and the inter-particle distance(r) and the unique solution chosen have important consequences for galaxy clustering. Also, we see how the form of three point correlation function has been proposed and is in good agreement with the data of Zwicky and Shane-Virtanen catalogues. Finally, the evolution of two-point correlation function on the basis of simulations are extensively being used to examine the distribution functions of gravitating systems.

In the fourth chapter, we extend the results of two-point correlation function to extended mass structures. It should be noted that galaxies with point mass consideration is only an approximation. In fact, galaxies have real extended structures, where dark matter especially is having an important contribution. A differential equation is developed here which again relates the two-point correlation function (ξ) with average number density $(\bar{n})$, temperature (T) and the inter-particle distance (r) for extended mass structures. The extended nature of galaxies is studied by using the softening parameter (ϵ) which avoids the divergence of $(r^2 + \epsilon^2)^{-1/2}$ in the limit of $r \to 0$. The solution of this differential equation developed in this chapter on the basis of variable separation method by introducing the constants Z and Z_N is one of the important research findings of my M. Phil. work. We use boundary conditions to fit the required values of Z_N in the solution for understanding the galaxy clustering phenomena.

In the fifth chapter, we plot correlation parameter (b and b_e) verses cell size (R) for a well defined cluster by taking equation (4.45) into consideration from chapter 4th. The graphs are obtained for some specific cluster (e.g; A2048) having a well defined mass and number of galaxies.

Finally we summarize the dissertation in the form of discussion and explore various research findings.

Chapter 2

Formation of Galaxy Clusters

As the universe continued to expand and cool (3000 K), electrons no longer have enough energy to overcome the attractive force of atomic nuclei, and become bound to atoms. The stage was set for the structures to form. The large-scale structures of the cosmos we observe today were formed as a consequence of the growth of the primordial fluctuations i.e; small changes in the density of the universe in a confined region. As the universe cooled, clumps of dark matter began to condense, and within them gas began to condense due to primordial fluctuations of gravitationally attracted gas and dark matter in the denser areas, and thus the structures that would later become galaxies were formed, which constituted the formation of first galaxies of the universe. At this point, the universe was almost exclusively composed of hydrogen, helium, and dark matter. Soon after the first protogalaxies formed, the hydrogen and helium gas within them began to condense and make the first stars. Thus the first galaxies were then formed. The discovery of a galaxy more than 13 billion years old, which existed only 480 million years after the Big Bang, was reported in January 2011. The first galaxies may have formed much earlier than thought, as new study suggests just after 200 million years after the universe's birth. Using several different telescopes, astronomers have discovered a distant galaxy whose stars appear to have formed 200 million years after the Big Bang i.e, about 300 million years earlier than the oldest previously known galaxies. The universe itself is estimated to be 13.7 billion years old. This observation challenges the theories of how soon galaxies formed and evolved in the early phase of the universe. It could even help to solve the mystery of how the hydrogen fog that filled the early universe was cleared. The universe was very violent in its early epochs, and galaxies grew quickly, evolving by accretion of smaller mass galaxies. The result of this process is left imprinted on the distribution of galaxies in the nearby universe (2dF Galaxy Redshift Survey). Galaxies are not isolated objects in space, rather galaxies are distributed in a great cosmic web of filaments throughout the universe. Galaxies come in a variety of shapes, from round, featureless elliptical galaxies to the pancake-flat spiral galaxies consisting of stars, interstellar gas, dust, etc.

It has been assumed that gravity acted on minute density variations in matter, gases, and the mysterious ”dark matter” of the universe after the Big Bang in order to form this early stage of universe. The study of clustering of galaxies in an expanding universe is considered to be one of important challenge in the modern cosmology. in this chapter, we will focus on various aspects which are important to understand the structure formation in an expanding universe.

2.1 Mass Distribution

The measurement of galaxy clustering has long been a primary tool in constraining structure formation models and cosmology. However, the power of galaxy surveys to discriminate between models is partially compromised by the fact that they provide an indirect measure of the mass distribution. The presently observed clustering of galaxies suggests that their motions have been dominated by mutual gravitational dynamics. At present, the structure formation of galaxies is heuristically divided into two parts. On large scales, cosmological parameters and the properties of the dark matter determine the growth of density perturbations and the eventual formation of massive dark halos. On smaller scales, hydrodynamic and other processes shape how luminous galaxies form within dark matter halos and how they evolve as haloes accrete and merge. The natural consequence of this picture is that distribution of galaxies is related, but differs from mass distribution. This difference in distribution arises in parts because halos are more strongly clustered than the dark matter as a whole, and more massive halos are more strongly clustered than less massive ones. The cold dark matter successfully describes the formation of structures in the universe as properties of galaxies such as their stellar mass, colour and morphology are closely related to the inferred mass of their host haloes. It has been found that observed clustering of galaxies and effectiveness of this clustering in constraining galaxy formation models and cosmological parameters is very sensitive to the host halo mass in which galaxies live.

2.2 Clustering Phenomena of Galaxies

The description of gravitational clustering in an expanding universe must account the actual growth and evolution of correlated structure in a gravitating system, and also the effect of the smoothing process with which structure is viewed. When the clustering has evolved significantly away from initial conditions, it is said to be non-linear, and the operations of non-linear evolution and smoothing need not commute [9][10]. It should be noted that complete description of the growth of clustering must be able to describe the non-linear evolution and then the effects of smoothing the non-linear evolved structure. When the gravitational evolution is not highly evolved, it is possible to provide such a complete description of the evolved system, after it has been smoothed on some given large scale. These linear and quasi-linear analyses attempt to solve the equations of motion directly, in the limit of small changes from the initial conditions [7][11][12]. When the changes from the initial conditions are substantial, the evolution of clustering in the non-linear regime is more difficult to describe. So the non-linear clustering phenomenon is determined by physical processes involving a lengthy and complex sequence of events. Various approaches for understanding the clustering phenomena are;

One of the way to understanding the galaxy clustering in the universe deals with the evaluation of n-particle correlation functions between galaxies. This can be done by solving system of Liouvilles equations or BBGKY-hierarchy equations and have been discussed by many workers like [13][14][15][16]. But BBGKY-hierarchy equations are too complicated to handle for higher order correlation functions. However, the lowest order i.e; two-point correlation function can also be pursued for discussing the phenomenon of galaxy clustering which contains information on all the higher n-particle correlations in full BBGKY-hierarchy [8][17][18][19]. An alternative simple and more effective statistical approach to two-point correlation functions for non-linear galaxy clustering has been developed by [20] with the help of gravitational thermodynamic results. This approach discussed by [8], assumes that clustering evolves through a sequence of quasi-equilibrium states. This assumption allows them to use statistical thermodynamics to describe the growth of clustering, particularly in the highly non-linear regime. It should be noted that quasi-equilibrium approach appears to provide a good description of the growth of clustering from an initially poission distribution and this approach is not so accurate when the initial conditions are significantly different from poission [21][22][23][24]. When mutual gravitational interactions of individual galaxies dominate, clustering can be described by quasi-equilibrium thermodynamics [8][17][18]) and statistical mechanics [19][20]. This is the another way of using thermodynamics and statistical mechanics. These theories are found to be in good agreement with observations given by [30] and as long as the evolution is in quasi-equilibrium, we may be able to use thermodynamics. The main aspect in which thermodynamics helps us to move forward is that expansion of the system of galaxies is to a good approximation adiabatic [18]. It should be noted that gravitational clustering may be adiabatic, but it is not necessary that it will be either isoentropic or reversible [8][18]. The applicability of thermodynamics to the cosmological many-body problem suggests that statistical mechanics should also apply. This close relation occurs because statistical mechanics is the microscopic (and therefore perhaps more fundamental) description of system (galaxy) positions and motions, whose ensemble averages provide the macroscopic thermodynamic description of the system. The statistical mechanical theory of N-body galaxy clusters has been developed by [31], where the relevant partition function has been solved.

The second approach first discussed by [25] assumes that on average gravitating systems will collapse spherically. This assumption allowed to compute the distribution of non-linear, virialized clump masses, given some initially gaussian density field, as a function of time. It should be noted that their original derivation of the mass multiplicity function was controversial as it has been redefined and improved by a number of authors [23][24][26][27][28]. The Press-Schechter distribution of clump masses appears to be in good agreement with that measured in N-body simulations of clustering from arbitrary gaussian initial conditions given by [29]. Although Press-Schechter approach provides information about the distribution of virialized clump masses, but it does not provide information about the internal structure. Of these clumps, nor does it describe how these clumps are distributed relative to each other in space. The clumps may be correlated with each other, or distributed uniformly at random. Thus, one cannot compute the n-point correlation functions of the clustered distribution, nor one can construct the non-linear counts in cells distribution function. In this respect, the Press-Schechter distribution of mass clumps provides a good, but by no means perfect fit to the virialized clump size distribution measured in N-body simulations.

2.3 Effects of Clustering

The clustering of galaxies encodes important information about the values of the cosmological parameters as it can be related to the spatial distribution of the underlying dark matter, and also about the physical processes behind galaxy formation. In the cold dark matter (CDM) hierarchical structure formation theory, the evolution of galaxies takes place inside dark matter haloes [32][33]. The formation and evolution of CDM is governed by gravity and can be modelled accurately using N-body simulations. We do not yet have the same level of knowledge of the fate of the baryons, which depends on the physics of gas accretion, star formation and feedback processes. Recent improvements and techniques in astronomical instrumentation have led to a wealth of new information becoming available on galaxy clustering, both locally and at earlier epochs. In particular, the unprecedented size and number of galaxies in the 2dF Galaxy Redshift Survey [34] and the SDSS [35] make it possible to quantify how the clustering signal depends on intrinsic galaxy properties, such as luminosity, stellar mass or colour. The variation of clustering strength with an intrinsic galaxy property encodes important information about how galaxies populate haloes. As we know galaxies cluster on very large scales under the influence of their mutual gravitation and the characterization of this clustering is a problem of current interest. Different techniques such as percolation [36][37], minimal spanning trees [38][39]), fractals [6], correlation functions [13][40]), and distribution functions [17] have been introduced to understand the large-scale structure in the universe. However, the description of correlations and distribution functions have been related most directly to physical theories of gravitational clustering and the consequences of these theories also agree well with observations.

Thus far, theories of the cosmological many-body galaxy distribution function have been developed mainly from a thermodynamic point of view. This starts from the first two laws of thermodynamics and, for quasi-equilibrium evolution, derives gravitational many-body equations of state in the context of the expanding universe. Application of the thermodynamic fluctuation theory to these equations of state by considering the galaxies as point gravitating masses gives their distribution function. Comparisons of gravitational thermodynamics to the cosmological many-body problem have been discussed on the basis of N-body computer simulation results [41][42][43]. Comparisons with the observed galaxy clustering [44][45][46] along with other theoretical arguments [14][15] support it further.

The general conditions under which statistical mechanics may describe the cosmological many-body problem are closely related to those for the applicability of thermodynamics, described in detail by [8][17][18]. When the ensemble averaged thermodynamic quantities change more slowly than local dynamical crossing or clustering timescales, then the form of the statistical distribution functions remains essentially the same, and only their macroscopic variables evolve. In this quasi-equilibrium evolution, equilibrium statistical mechanics provides a good approximation to the distribution of particles and velocities at any given time with the values of the macroscopic variables at that time. In equilibrium, all permissible microstates of the systems in the ensemble have an equal a priori probability. This is the fundamental postulate of statistical mechanics which implies that the approximate probability of finding a specified macro-state in the system is proportional to the number of permissible micro-states having the macro-state's properties.

Cosmological many-body systems generally satisfy the time-scale criterion of quasi-equilibrium statistical mechanics since macroscopic global variables such as average temperature, density, and the ratio of gravitational correlation energy to thermal energy change on time-scales at least as long as the Hubble time, whereas local dynamical time-scales in regions of clustering are shorter. The criterion of equal a priori probabilities for any micro-state or configuration is less well understood, and its rigorous derivation remains an important unsolved problem even in classical statistical mechanics. It is closely related to statistical homogeneity and the absence of extensive very non-linear structures over scales comparable with the system. A more detailed analysis of the ranges of initial conditions that form the basis of attraction can be considered as an important problem for future.

To investigate the problem of non-linear gravitational galaxy clustering from the point of view of statistical mechanics, the statistical mechanics of N-body systems is based on the N-body Hamiltonian described by [31] as;

$$H = \sum_{i=0}^{N} \frac{P_i^2}{2m} + \phi(r_1, r_2, ...r_N) \tag{2.1}$$

Where p_i is the momentum of the ith particle and $\phi(r_1, r_2, ...r_N)$ is the function of the relative position vectors. If the system occupies a volume V and R the size of each spherical cell, then the partition function $Z_N(V, T)$ of a system of particles interacting gravitationally by making use of above equation is given by;

$$Z_N(V,T) = \frac{V^{3N}}{\Lambda^{3N}N!}\int \exp\left[-\left(\sum_{i=0}^{N}\frac{P_i^{\,2}}{2m} + \phi(r_1, r_2, ...r_N)\right)T^{-1}\right]d^{3N}pd^{3N}r \quad (2.2)$$

The evaluation of above integral is generally very complicated and lengthy process. However, the partition function for the cosmological many-body problem have been evaluated analytically, which has proved a big success in understanding the clustering phenomena of galaxies in an expanding universe. The details of its evaluation has been worked out by [31] in which the partition function is described as;

$$Z_N(V,T) = \frac{1}{N!}\left(\frac{2\pi mT}{\Lambda^2}\right)^{\frac{3N}{2}} V^N(1+\beta\bar{n}T^{-3})^{N-1} \quad (2.3)$$

where β is given as;

$$\beta = \frac{3}{2}(Gm^2)^3 \quad (2.4)$$

This serves as a basic result for rigorously evaluating all the thermodynamic properties of the system, starting with the free energy. It is particularly interesting that the correlation parameter 'b', which is the ratio of the gravitational correlation energy to twice the kinetic energy of peculiar motion, emerges directly in the partition function and in the equations of state. So their is no need to make any assumptions in the derivation of the functional form of $b(\bar{n}T^{-3})$ as was done earlier by [11][12]. Once the many-body partition function is known, there is no difficulty in evaluating the grand canonical partition function, which represents the exchange of both particles and energy. From the grand canonical partition function, the distribution function of galaxies follows directly. The proper thermodynamic dependence of the correlation parameter 'b' emerges directly in the equations of state and one can calculate then distribution function ($f(N)$) simply. One should note that it has been found that all these results agree exactly with earlier ones derived using thermodynamic arguments. The following are the equations of state, which can be directly calculated from the free energy described by $A = -TlnZ_N$

$$U = \frac{3}{2}NT(1-2b) \tag{2.5}$$

$$P = \frac{NT}{V}(1-b) \tag{2.6}$$

In addition to the above two equations, other important thermodynamical quantities like entropy (S), chemical potential (μ) which are helpful in describing the macroscopic state of a system can also be evaluated and the final expressions are given as;

$$\frac{S}{N} = ln(\bar{n}^{-1}T^{3/2}) - ln(1-b) - 3b + S_0 \tag{2.7}$$

where $S_0 = \frac{5}{2} + \frac{3}{2}ln(\frac{2\pi m}{\Lambda^2})$ is an arbitrary constant.

$$\frac{\mu}{T} = ln(\bar{n}T^{-3/2}) + ln(1-b) - b - \frac{3}{2}ln\left(\frac{2\pi m}{\Lambda^2}\right) \tag{2.8}$$

Also the distribution function which represents the overall clustering of galaxies is characterised by the full set of $f(N)$. A simple objective description of the distribution function is to count their number in cells of a given size, which are distributed uniformly over the sky. The galaxy distribution function calculated on the basis of the partition function Z_N is written as;

$$f(N) = \frac{\bar{N}(1-b)}{N!}\left[\bar{N}(1-b) + Nb\right]^{N-1}\exp(-\bar{N}(1-b) - Nb) \tag{2.9}$$

In addition to its generality and rigour, the main advantage of this approach is that it can easily be extended to non-point mass systems. Actually galaxies have extended structures and haloes, and the introduction of a softening parameter (ϵ) enables us to include effects of large haloes of dark matter around galaxies. The analytical solution of the configuration integral for the cosmological gravitational systems has been developed by [31] and this integral may be applied to systems containing either point or extended masses. Then, one can analytically calculate the partition function and for non-point mass structures the partition function developed by [31] is given as;

$$Z_N(V,T) = \frac{1}{N!}\left(\frac{2\pi mT}{\Lambda^2}\right)^{\frac{3N}{2}} V^N \left[1+\beta\bar{n}T^{-3}\alpha(\epsilon/R)\right]^{N-1} \tag{2.10}$$

The partition function obtained above is again helpful in evaluating all the thermodynamic quantities associated with the system. The constant α used in above equation depends upon the softening parameter ϵ and the cell size R, which contains large number of non-point mass galaxies is given as;

$$\alpha(\epsilon/R) = \sqrt{1+\epsilon^2/R^2} + \frac{\epsilon^2}{R^2} ln \frac{\epsilon/R}{1+\sqrt{1+\epsilon^2/R^2}} \tag{2.11}$$

On the basis of equation (2.10), one can also evaluate the equations of state, distribution function, and also other thermodynamical quantities like chemical potential, etc. The details of the evaluation of all the important equations can be seen from the work of [31]. The final equations are written as;

$$U_e = \frac{3}{2}NT - \frac{2\pi Gm^2N^2}{V}\int \xi(r,\bar{n},T)\left(1+\frac{\epsilon^2}{r^2}\right)^{\frac{-1}{2}} \frac{dV}{4\pi r} \tag{2.12}$$

$$P_e = \frac{NT}{V} - \frac{2\pi Gm^2N^2}{3V^2}\int \xi(r,\bar{n},T)\left(1+\frac{\epsilon^2}{r^2}\right)^{\frac{-3}{2}} \frac{dV}{4\pi r} \tag{2.13}$$

$$f(N,\epsilon) = \frac{\bar{N}(1-b_e)}{N!}\left[\bar{N}(1-b_e)+Nb_e\right]^{N-1} \exp(-\bar{N}(1-b_eJ)-Nb_eJ) \tag{2.14}$$

$$\frac{\mu}{T} = ln(nT^{-3/2}) + ln(1-b_e) - b_eJ - \frac{3}{2}ln\left(\frac{2\pi m}{\Lambda^2}\right) \tag{2.15}$$

Where J is given as;

$$J = \frac{5}{3} - \frac{2}{3\alpha\sqrt{1+\epsilon^2/R^2}} \tag{2.16}$$

2.4 Simulation Models

Over the past decade, models of galaxy clustering have evolved which allow us to interpret observational data and learn more about how galaxies are distributed between dark matter haloes. In the cold dark matter (CDM) hierarchical structure formation theory, galaxies grow inside dark matter haloes [47][48]. The formation of structure in the dark matter is governed by gravity and can be modelled accurately by using N-body simulations [49]. N-body simulations show that relaxation to the observed distributions of quasi-equilibrium statistical mechanics occurs for initial power-law perturbation spectra with power-law indices between about -1 and +1. Systems with much stronger global initial correlations or anti-correlations relax only after many expansion time-scales, or not at all. However, the fate of baryonic material is much more complicated as it involves a range of often complex and non-linear physical processes. The efficiency of galaxy formation is expected to depend on the mass of the host dark matter halo [49][50]. Modelling the dependence of galaxy clustering on intrinsic properties such as luminosity offers a route to establish how such properties depend upon the mass of the host halo and hence to improve our understanding of galaxy formation. This development has been led by semi-analytical models, which can populate large volumes with galaxies in a short time using physically motivated prescriptions [51][52][53][54][55]. Such studies also inspired empirical approaches, which involve fitting halo occupation distributions (HODs) [56][57][58] and conditional luminosity functions [59] describing the number of galaxies per halo and the luminosity of galaxies within a halo, respectively. Recent advances in astronomical instrumentation have also produced a wealth of information on galaxy clustering. The enormous volume and number of galaxies in the two-degree field Galaxy Redshift Survey (2dFGRS) [23] and the SDSS [24] have made possible accurate measurements of clustering for samples of galaxies defined by various intrinsic properties [60][61][62][63][64][65]. The variation of clustering strength with luminosity tells us how galaxies populate haloes and hence about the physics of galaxy formation. Any discrepancy between the observational measurements of clustering and theoretical predictions points out the need to improve the models, either by refining existing ingredients or adding new ones.

The dependence of galaxy clustering on luminosity has been measured accurately in the local universe [60][61][62][63][65][66]. Over the period spanned by these studies, galaxy formation models have evolved significantly, particularly in the treatment of bright galaxies [67]. The majority of current models invoke some form of heating of the hot gas atmosphere to prevent gas cooling in massive haloes, in order to reproduce the bright end of the galaxy luminosity function. This has implications for the correlation between galaxy luminosity and host dark matter halo mass, which in turn has an impact on the clustering of galaxies. [68] compared the semi-analytical galaxy formation models of [69] and [70], which are against the measurements of clustering from the SDSS. Qualitatively, the models displayed similar behaviour to the real data, but did not match the clustering measurements in detail as [65] and [68] have shown that as the luminosity varies the predictions of [70] model change the clustering amplitude by a similar amount to the observations. Thus, these models demonstrated that the clustering predictions could be improved, but not fully reconciled with the data.

Chapter 3

Correlation Functions

A correlation is the relation between random variables at two different points in space or time, usually as a function of the spatial or temporal distance between the points. If one considers the correlation between random variables representing the same quantity measured at two different points, then the correlation is referred to as an autocorrelation. However, correlation between different random variables are called as cross correlation because to emphasize that different variables are considered, they are made up of cross correlations.

3.1 Correlation Function

Correlation function in astronomy is a tool that describes the distribution of galaxies in the universe and the lowest order correlation function generally refers to the two-point correlation function. For a given distance, the two-point correlation function ξ_2 is a function of one variable (distance), which describes the probability that two galaxies are separated by this particular distance.

The spatial correlation function $\xi(r)$ is defined by the joint probability $dP(r)$ of finding two objects (galaxies) separated by a distance r and within volume elements dV_1 and dV_2, with $\bar{n}$ as the average space density of objects in the sample, such that

$$dP(r) = \bar{n}^2[1 + \xi(r)]dV_1 dV_2 \tag{3.1}$$

Positive correlation, i.e; $\xi > 0$, refers to the clustering phenomena, negative correlation i.e; $\xi < 0$, refers to anti-clustering and for the zero correlation, i.e; for a random distribution of points, there is no clustering.

The spatial distribution of rich clusters of galaxies and the clustering properties of clusters have been the subject of considerable interest over the past two decades, with a wide range of claims to the nature and properties of such clustering phenomena. Since rich clusters can be used rather efficiently in surveying the structure in large volumes of space, they have become an important tool in tracing the large-scale structure of the universe. The Abell (1958) catalogue of rich clusters has been analysed by many investigators [71][72][73][74] using different techniques in an attempt to determine the spatial distributions of rich clusters. The studies deal primarily with the surface distribution of clusters and, in some cases, used approximate estimates for cluster red-shifts. Moreover, [72][73] have used red-shift measurements of complete samples of clusters to determine directly the spatial distribution of rich clusters. The results indicate that rich clusters of galaxies cluster very strongly in space, forming clusters of clusters of galaxies or superclusters. The clustering strength of clusters was observed to be much higher than the clustering strength of galaxies. The clustering or correlation scale for rich clusters was found to be about five times larger than the correlation scale of galaxies. Similar investigations have shown that the results provide strong constraints on models for the formation and evolution of galaxies and structure.

3.2 Stellar Mass Correlation Function

The observed clustering of galaxies and the effectiveness of this clustering in constraining galaxy formation models and cosmological parameters is very sensitive to the host halo mass in which galaxies live. The two-point correlation function (2PCF) of galaxies has been accurately measured by large surveys, such as the two-degree Field Galaxy Redshift Survey [75], the Sloan Digital Sky Survey [76] at z = 0, and the DEEP 2 Galaxy Redshift Survey at z = 1 [77]. In all these cases, the 2PCF appears to be a power law over a wide range of scales, and it depends on red-shift, luminosity, colour and morphology of galaxies [78][79][80][81][82]. Recently, the traditional 2PCF has been extended [83] by measuring the stellar mass correlation function from the SDSS data, the so-called stellar mass correlation function (SMCF). This quantity provides additional constraint on galaxy formation models, as it depends on the relative mass of galaxies at given scales. In order to predict the 2PCF and the stellar mass correlation, we need to produce a large sample of galaxies in a large cosmological volume. To achieve this, galaxy catalogues have been build by using N-body simulations of different cosmologies.

The semi-analytical models (SAMs) for galaxy formation [84] are often used to study galaxy formation, and the early studies found that these models predicted clustering which marginally agreed well with the data [85]. The main advantage of this model allows one to easily investigate how galaxy properties vary as the underlying assumptions regarding the baryonic physics are changed. However, recent deep surveys (including more faint galaxies) have shown that currently SAMs predict too high clustering amplitude at small scales [85][86]. The excessive clustering in these studies was primarily due to the over abundance of faint galaxies in these models. The recent model given by [87] removed this over-abundance and was able to reproduce the local stellar mass function (SMF) down to very low mass end.

Another way to model galaxy clustering is the Abundance Matching Method (AMM) given by [88] with the assumption that there is a monotonic relation between a galaxy's stellar mass and its halo's mass and the stellar mass can be obtained by matching the halo abundance to the observed SMFs. The biggest advantage of this approach is that the observed SMF is perfectly reproduced and can well reproduce the properties of galaxy clustering seen in the SDSS at z=0 [89].

3.3 2-PCF for Point Mass Particles (Galaxies)

The formation of large-scale structures in the universe is one of the most important and interesting problems in cosmology. It is generally believed that these structures have developed by a process of gravitational instability from small initial fluctuations in the density of a largely homogeneous early universe. Hence, it is very important to clarify the physical mechanism of the evolution of density fluctuations. The inflationary scenario predicts a random gaussian field possessing the properties of statistical homogeneity and isotropy as the primordial fluctuations [90][91][92].

Classical thermodynamics is the theory of great scope and generality as it survived the relativity and quantum mechanical revolutions of physics nearly intact. The one of the reasons behind this was that among all theories of physics, thermodynamics has the least physical content. Its statements relate very general quantities which one can know through equation of state, etc. for specific application. Thus, it is natural to use thermodynamics for gravitating systems, but results of gravitational thermodynamics(GTD) are often surprising compared to thermodynamics of ordinary gases. These surprising results are caused by the long range, unsaturated (unshielded) nature of gravitational forces, and as a result understanding of GTD is less certain than ordinary thermodynamics.

However, the basic premise of classical thermodynamics is that systems in equilibrium can be characterised by a finite set of macroscopic parameters. These macroscopic parameters may be averages of microscopic parameters, which may not be known in detail and may vary in nature among different types of systems. It should be noted that equilibrium is always an idealisation for a system, which means that macroscopic disturbances occur on time-scales very long compared to microscopic relaxation time-scales. The macroscopic parameters which describe the system normally include the total internal energy(U), entropy(S), volume(V), and the number of particles(N) system contains. But, if it contains more than one type of particles, then each specie is characterised by its number(N_i), and volume(V_i), which each specie contains.

The gravitational clustering can be characterised by the distribution of voids, and in turn are related to the higher order correlations. One another method is the BBGKY-hierarchy approach to understand higher order correlation function. As the clustering evolves self consistently the methods like distribution of voids and BBGKY-hierarchy approach is of no help at all because these methods become very complicated. The two-point correlation function ξ_2 determines the thermodynamic properties of an infinite system and these thermodynamic properties are equations of state, which include gravitation and in turn determine the quantum fluctuations.

In order to describe the thermodynamic information of any system, it is useful to emphasize one or another parameter of the thermodynamic system in different situations. So it is necessary to summarize the inter-relations between different descriptions, which however contains all the basic information about the system. In this direction, models are attempts to find solvable mathematical descriptions which contain the essential physics of complicated problem. In the early descriptions of an imperfect gas, the Vander Waal's equation of state provide a simple model, which could be related to the phase transitions. As this model represents the large class of physically similar equation of state for uniform systems, so it is natural to use such a model for gravitating systems.

The Vander Waal's equation of state in the simple form is given as;

$$\left(P + a\frac{N^2}{V2}\right)\left(1 - b\frac{N}{V}\right) = \frac{NT}{V} \tag{3.2}$$

In the equation (3.2), value of 'b' refers to the short-range, hard-core part of the particle repulsive interaction potential giving rise to the excluded volume of the particles, and the value of 'a' refers to the long range part of the attractive potential producing a reduced pressure. But we know in case of gravitating systems, there is no repulsion, i.e; $b = 0$, so above equation reduces to

$$P = \left(\frac{NT}{V} - a\frac{N^2}{V^2}\right) \tag{3.3}$$

The above equation for pressure (P) can also be written as;

$$P = \frac{NT}{V}(1 - b) \tag{3.4}$$

Similarly, the equation for internal energy (U) can be written as;

$$U = K + W = \frac{3}{2}NT(1 - 2b) \tag{3.5}$$

Hence, the equations (3.4) and (3.5) represent the equation of state for gravitating systems. In the equation (3.5), $K = m < v^2 > /3$ is the temperature of the system, $< v^2 >$ represents the average of the square of the peculiar motions relative to the expansion, N=$\bar{n}$V is the average number of particles in volume V and b is a dimensionless variable known as correlation parameter and for gravitating systems, it is defined by [8] as;

$$b = -\frac{W}{2K} = \frac{2\pi G m^2 \bar{n}}{3T} \int_V \xi(\bar{n}, T, r) \frac{dV}{4\pi r} \tag{3.6}$$

In the gravitational thermodynamics, the measure of correlation (clustering) is studied on the basis of correlation parameter ’ b’. The correlation parameter ’b’ is defined as the ratio of correlation energy to twice the kinetic energy. In the correlation parameter ’b’, $\bar{n} = \frac{N}{V}$ is the average number density of the system of particles each of mass m, T is the temperature, V is the volume, G is the universal constant of gravitation, $\xi(\bar{n}, T, r)$ is the two-point correlation function and r is the inter-particle distance.

Hence, on using equation (3.6), equations (3.4) and (3.5) can also be written as;

$$P = \frac{NT}{V} - \frac{2\pi G m^2 N^2}{3V^2} \int_V \xi(\bar{n}, T, r) \frac{dV}{4\pi r} \tag{3.7}$$

$$U = \frac{3}{2} NT - \frac{2\pi G m^2 N^2}{V} \int_V \xi(\bar{n}, T, r) \frac{dV}{4\pi r} \tag{3.8}$$

From equations (3.7) and (3.8), we write

$$\left(\frac{\partial U}{\partial V}\right)_T = -\frac{2\pi G m^2 N^2}{V} \frac{\xi(\bar{n}, T, r)}{4\pi r} + \frac{2\pi G m^2 N^2}{V^2} \int_V \frac{\xi(\bar{n}, T, r)}{4\pi r} dV \tag{3.9}$$

$$\left(\frac{\partial P}{\partial V}\right)_V = \frac{N}{V} - \frac{2\pi G m^2 N^2}{3V^2} \int_V \frac{\partial \xi(\bar{n}, T, r)}{\partial T} \frac{dV}{4\pi r} \tag{3.10}$$

Now the differential equation for $\xi(\bar{n}, T, r)$ in terms of $\bar{n}$, T and r can be obtained from the Maxwell’s thermodynamic equation given as;

$$\left(\frac{\partial U}{\partial V}\right)_{T,N} = T \left(\frac{\partial P}{\partial T}\right)_{V,N} - P \tag{3.11}$$

On using equations (3.7), (3.9) and (3.10) in equation (3.11), we have

$$-3V\frac{\xi(\bar{n},T,r)}{4\pi r}+3\int_V\frac{\xi(\bar{n},T,r)}{4\pi r}dV=-T\int_V\frac{\partial\xi(\bar{n},T,r)}{\partial T}\frac{dV}{4\pi r}+\int_V\xi(\bar{n},T,r)\frac{dV}{4\pi r} \tag{3.12}$$

Differentiating above equation with respect to V,

$$\begin{aligned}\frac{-3V}{4\pi r}\frac{d\xi(\bar{n},T,r)}{dV}-\frac{3\xi(\bar{n},T,r)}{4\pi r}&\\ &-\frac{3V\xi(\bar{n},T,r)}{4\pi}\frac{\partial}{\partial V}\left(\frac{1}{r}\right)-3\frac{\xi(\bar{n},T,r)}{4\pi r}\\ &=-T\frac{\partial\xi(\bar{n},T,r)}{\partial T}\frac{1}{4\pi r}+\frac{\xi(\bar{n},T,r)}{4\pi r}\end{aligned} \tag{3.13}$$

As the two-point correlation function ξ depends on $\bar{n}$,T and r, so we can write

$$\xi=\xi(\bar{n},T,r) \tag{3.14}$$

$$\implies d\xi=\frac{\partial\xi}{\partial\bar{n}}d\bar{n}+\frac{\partial\xi}{\partial T}dT+\frac{\partial\xi}{\partial r}dr \tag{3.15}$$

$$(or)\frac{d\xi}{dV}=\frac{\partial\xi}{\partial\bar{n}}\frac{d\bar{n}}{dV}+\frac{\partial\xi}{\partial T}\frac{dT}{dV}+\frac{\partial\xi}{\partial r}\frac{dr}{dV} \tag{3.16}$$

Assuming

$$\frac{dT}{dV}=0 \tag{3.17}$$

Also

$$\frac{d\bar{n}}{dV}=\frac{-N}{V^2},and\frac{dr}{dV}=\frac{r}{3V} \tag{3.18}$$

Using equations (3.17) and (3.18) in equation (3.16), we get;

$$\frac{d\xi}{dV}=\frac{-N}{V^2}\frac{\partial\xi}{\partial\bar{n}}+\frac{r}{3V}\frac{\partial\xi}{\partial r} \tag{3.19}$$

Also we can write;

$$\frac{\partial}{\partial V}\left(\frac{1}{r}\right)=\frac{\partial}{\partial r}\left(\frac{1}{r}\right)\frac{\partial r}{\partial V}=\frac{-1}{3Vr} \tag{3.20}$$

Using equations (3.19) and (3.20) in equation (3.13), we have

$$\frac{-3V}{4\pi r}\left(\frac{-N}{V^2}\frac{\partial\xi}{\partial\bar{n}}+\frac{r}{3V}\frac{\partial\xi}{\partial r}\right)-\frac{3\xi(\bar{n},T,r)}{4\pi r}$$
$$\begin{aligned} &- \frac{3V\xi(\bar{n},T,r)}{4\pi}\frac{-1}{3Vr}-3\frac{\xi(\bar{n},T,r)}{4\pi r} \\ &= -T\frac{\partial\xi(\bar{n},T,r)}{\partial T}\frac{1}{4\pi r}+\frac{\xi(\bar{n},T,r)}{4\pi r} \end{aligned} \tag{3.21}$$

From the above equation, the final differential equation for ξ in terms of $\bar{n}$, T and r for point mass particles (galaxies) looks like[93];

$$3\bar{n}\frac{\partial\xi}{\partial\bar{n}}+T\frac{\partial\xi}{\partial T}-r\frac{\partial\xi}{\partial r}=0 \tag{3.22}$$

Equation (3.22) is a first order differential equation for two particle correlation function. It is characterised by average number density $\bar{n}$, temperature T and inter-particle distance r. The solution of the above differential equation on using set of physically valid boundary conditions is given as;

$$\xi_2(\bar{n},T,r)=\left(\frac{C_1\bar{n}T^{-3}}{1+C_1\bar{n}T^{-3}}\right)^2\frac{1}{C_1\bar{n}T^{-3}}\left(\frac{1}{C_2Tr}\right)^2 \tag{3.23}$$

Also the correlation parameter by using equation (3.23) in equation (3.6) for the point mass galaxies is given as;

$$b=\frac{\beta\bar{n}T^{-3}}{1+\beta\bar{n}T^{-3}} \tag{3.24}$$

Where $\beta=\frac{3}{2}(Gm^2)^3$ is a positive constant.

The unique solution chosen above clearly indicates the dependence of two-point correlation ξ_2 on $\bar{n}$, T and r and the self evaluation of b clarifies that b has a complete dependence on the combination $\bar{n}T^{-3}$ as was suggested earlier by [8].

3.4 Proposed form of 3-PCF

The statistical properties of the gaussian field are completely characterized by the two-point correlation function (2PCF) or the power spectrum, while higher order correlations vanish and the 2PCF has a certain value [94]. If the galaxy distribution have been entirely gaussian, then 2PCF would definitely provide a complete description of galaxy clustering. As the analyses of CMB suggest that the primordial mass fluctuation in our universe appear extremely gaussian, and non-gaussian signatures produced in the galaxy distribution are due to gravitational collapse. As such 2PCF provides only partial view of the full distribution and cannot sufficiently probe non-gaussian signals. In order to investigate non-gaussian structure as well as shape in these galaxy distributions, we require higher order clustering statistics. In the hierarchy of n-point correlation functions, the three-point correlation function is the lowest order statistic to provide information as this enables probes of triaxial nature of haloes and extended filaments within the cosmic web. Also, non-linear gravitational clustering gives rise to non-zero values for the higher order correlations, even if the primordial density fluctuations are set as gaussian. This is reasonable, since phases between fourier modes in the bi-spectrum are considered to be distributed non-randomly but correlated in some ways in the non-linear regime, where galaxies or galaxy groups are strongly clustered around each other [95][96][97][98][99][100][101]. Hence, the higher order statistics, such as the three-point correlation function (3PCF), become essential when the evolution of the density fields is governed by non-linear gravitational clustering. So, the three-point correlation function (3PCF) provides an important view into the clustering of galaxies that is not available to its lower order, i.e; two-point correlation function (2PCF). The non-gaussian information about structure and shape of galaxy or cluster is thus characterised by higher order statistics, such as the 3PCF. Therefore, it is quite important for the understanding of non-linear evolution to investigate the properties of the 3PCF. However, it is difficult to deal with the exact formula of the 3PCF both theoretically and numerically, since it is much more lengthy and complicated than the 2PCF. However, it has been proposed on the basis of simple assumption by [102][103], that the 3PCF can be expressed as;

$$\zeta_{abc} = Q\left(\xi_{ab}\xi_{bc} + \xi_{bc}\xi_{ca} + \xi_{ca}\xi_{ab}\right) \tag{3.25}$$

where ζ , ξ and Q are the 3PCF, 2PCF and a certain constant respectively.

The equation (3.25) is often called the hierarchical form given by [104]. The data from the Zwicky and Shane-Virtanen catalogues are in good agreement with this assumption when we choose $Q = 0.85$ for Zwicky and $Q = 1.24$ for Shane-Wirtanen [102][103]. However, there is a serious fault with the proposed equation (3.25) as it has no theoretical grounds, although some observational or numerical data can be well fitted. One explanation may be that equation (3.25) for $Q = 1$ coincides with the Kirkwood superposition approximation, which is familiar in liquid physics and turbulence theory, if $\xi_{ab}\xi_{bc}\xi_{ca} \ll 1$ [105][106]. However, this approximation is not appropriate for the non-linear regime, since the distribution of the gravitational sources is strongly correlated in such regimes. Hence, this resemblance is helpless to compensate for the theoretical defect. Indeed, there have been many theoretical investigations of the 3PCF [107][108]. Nevertheless, most of the analyses have been based on the hierarchical form assumption. The coefficient Q under the hierarchical form was estimated by assuming self-similarity [107]. The BBGKY hierarchy was analysed by assuming the hierarchical form, as well as self-similarity [108]. In the first place, however, there is no reason why the 3PCF should necessarily depend only on the second power of the 2PCF, even if we accept the self-similarity. Indeed, the work done by [109] suggest the possible existence of solutions that do not satisfy the hierarchical form on the basis of the BBGKY equations. To study the connection between the 2PCF and the 3PCF in the non-linear gravitational clustering regime, we can analyse these functions by using a scaling hypothesis. It is expected that the time evolution of the statistics also will fundamentally obey some self-similar rules, since gravity is scale free [105][107][108][110]). The self-similar investigation of the 2PCF or the power spectrum can be studied on the basis of N-body simulations [111][112]. The results have shown that the self-similar evolution of the power spectrum can be satisfied when the initial power spectrum is scale free and the power spectrum in the non linear regime obeys some scaling laws and the scaling hypothesis itself is very familiar.

Chapter 4

Correlation Function for Extended Mass Galaxy Clusters

The universe is dominated by matter in gravitational interaction into the spatial configuration consisting of galaxies, group of galaxies, super-clusters and even larger structures [113][114]. The standard way of understanding the structures in the universe (i.e; the departure from randomness and homogeneity) is described by means of correlation functions. From observation the galaxy-to-galaxy correlation function ξ_{gal} is known to scale as $\xi_{gal} = r^{-\gamma}$, where the exponent γ ranges from 1.6 to 1.8. The calculation of γ tacitly assumes the universe to have evolved to the stage, where the initial primordial matter has been formed into the observed galaxies and that these galaxies are coupled to the expansion of the universe. We take our clue by analogy with the well established theory of interacting gases [115], where the relative arrangements of the atoms and molecules (galaxies) are accurately described in terms of the principles and methods of thermodynamics. The approach of thermodynamics has already been discussed at length and breadth, by considering each galaxy to be a constituent particle of an infinite gas. We apply the same techniques to a system made up of many galaxies in gravitational interaction, subject to fluctuations emerging from the intrinsic properties of the gravitational interaction. A number of theories of the cosmological many body problem have been developed mainly from the thermodynamic point of view [15][16][93]. The theme of the approach is to describe different instabilities and evolution of gravitating systems. Of all the models used for understanding the clustering of galaxies, thermodynamic model provides the simplest model for the galaxy clustering in an expanding universe. We make use of the equations of state along with the correlation functions for the extended mass structures for the development of a semi-analytical model. This can even be done by solving a system of Liouville's equation or BBGKY-hierarchy equations and have been discussed by [11][15]. But BBGKY-hierarchy equations are too complicated to handle for higher order correlation functions. We extract the possible information about lowest order i.e; two-point correlation function (ξ_2) for galaxies with real extended mass structures clustering gravitationally in an expanding universe. It is important to note that galaxies with point mass consideration is only an approximation. In fact, galaxies have real extended structures, where dark matter especially is having an important contribution. The extended nature of galaxies is introduced by the softening gravitational potential proportional to $(r^2 + \epsilon^2)^{-1/2}$. The softening parameter (ϵ) represents the finite size of a galaxy and is sometimes a more realistic approximation for particular problem than the r^{-1} potential.

4.1 Equations of state for Extended Mass Structures

The gravitational clustering of galaxies in an expanding universe on the basis of two-point correlation function (ξ_2) for point mass particles (galaxies) have earlier been developed by [93] by using equations of state. Here, we study and gather the related information about ξ_2 for extended mass structures also by using the same approach i.e; making use of equation of state. The general pair of equations of state i.e, internal energy (U_e) and pressure (P_e) for extended mass structures are given by [93];

$$U_e = \frac{3}{2}NT - \frac{2\pi G m^2 N^2}{V} \int_V \xi(r, \bar{n}, T) \left(1 + \frac{\epsilon^2}{r^2}\right)^{\frac{-1}{2}} \frac{dV}{4\pi r} \tag{4.1}$$

$$P_e = \frac{NT}{V} - \frac{2\pi G m^2 N^2}{3V^2} \int_V \xi(r, \bar{n}, T) \left(1 + \frac{\epsilon^2}{r^2}\right)^{\frac{-3}{2}} \frac{dV}{4\pi r} \tag{4.2}$$

Equations (4.1) and (4.2) represent the equations of state, where the measuring correlation parameter b_e for extended mass structures is given by [93] as;

$$b_e = \frac{2\pi G m^2 N}{3VT} \int_V \xi(\bar{n}, T, r) \left(1 + \frac{\epsilon^2}{r^2}\right)^{\frac{-1}{2}} \frac{dV}{4\pi r} \tag{4.3}$$

Here $\bar{n} = \frac{N}{V}$ is the average number density of extended mass particles (galaxies) with ϵ as a softening parameter and is taken between 0.01 to 0.05 (in the units of total radius), T is the temperature, V the volume, G is the universal constant of gravitation, $\xi(\bar{n}, T, r)$ is the two-point correlation function for extended mass structures and r the inter-galactic distance. It should be noted that these expressions assume a large volume V for their validity.

4.2 Development of the differential equation of two-point Correlation Function for Extended Mass Structure

In this section, we develop the differential equation in terms of correlation function $\xi(\bar{n}, T, r)$, by making use of the Maxwell's thermodynamic equation relating the internal energy (U) and pressure (P) given as;

$$\left(\frac{\partial U}{\partial V}\right)_{T,N} = T \left(\frac{\partial P}{\partial T}\right)_{V,N} - P$$

But for extended mass structures, we use internal energy U_e and pressure P_e, so that above equation goes like;

$$\left(\frac{\partial U_e}{\partial V}\right)_{T,N} = T\left(\frac{\partial P_e}{\partial T}\right)_{V,N} - P_e \tag{4.4}$$

Now from equations (4.1) and (4.2), we have

$$\begin{aligned}\left(\frac{\partial U_e}{\partial V}\right)_{T,N} &= \frac{-2\pi Gm^2N^2}{V}\frac{\xi(\bar{n},T,r)}{4\pi r}\left(1+\frac{\epsilon^2}{r^2}\right)^{\frac{-1}{2}} \\ &+ \frac{2\pi Gm^2N^2}{V^2}\int_V \frac{\xi(\bar{n},T,r)}{4\pi r}\left(1+\frac{\epsilon^2}{r^2}\right)^{\frac{-1}{2}} dV \end{aligned}\tag{4.5}$$

and

$$\left(\frac{\partial P_e}{\partial T}\right)_{V,N} = \frac{N}{V} - \frac{2\pi Gm^2N^2}{3V^2}\int_V \left(1+\frac{\epsilon^2}{r^2}\right)^{\frac{-3}{2}} \frac{\partial \xi(\bar{n},T,r)}{\partial T}\frac{dV}{4\pi r} \tag{4.6}$$

Using equations (4.2), (4.5) and (4.6) in equation (4.4), we have

$$\begin{aligned}\frac{-2\pi Gm^2N^2}{V}\frac{\xi(\bar{n},T,r)}{4\pi r}\left(1+\frac{\epsilon^2}{r^2}\right)^{\frac{-1}{2}} + \frac{2\pi Gm^2N^2}{V^2}\int_V \frac{\xi(\bar{n},T,r)}{4\pi r}\left(1+\frac{\epsilon^2}{r^2}\right)^{\frac{-1}{2}} dV &= \frac{NT}{V} \\ -\frac{2\pi Gm^2N^2T}{3V^2}\int_V \left(1+\frac{\epsilon^2}{r^2}\right)^{\frac{-3}{2}} \frac{\partial \xi(\bar{n},T,r)}{\partial T}\frac{dV}{4\pi r} \\ -\frac{NT}{V} + \frac{2\pi Gm^2N^2}{3V^2}\int_V \xi(r,\bar{n},T)\left(1+\frac{\epsilon^2}{r^2}\right)^{\frac{-3}{2}}\frac{dV}{4\pi r}\end{aligned}\tag{4.7}$$

Equation (4.7) can also be written as;

$$\begin{aligned}\frac{-3V\xi(\bar{n},T,r)}{4\pi r}\left(1+\frac{\epsilon^2}{r^2}\right)^{\frac{-1}{2}} + 3\int_V \frac{\xi(\bar{n},T,r)}{4\pi r}\left(1+\frac{\epsilon^2}{r^2}\right)^{\frac{-1}{2}} dV &= \\ -T\int_V \left(1+\frac{\epsilon^2}{r^2}\right)^{\frac{-3}{2}} \frac{\partial \xi(\bar{n},T,r)}{\partial T}\frac{dV}{4\pi r} \\ \int_V \xi(r,\bar{n},T)\left(1+\frac{\epsilon^2}{r^2}\right)^{\frac{-3}{2}}\frac{dV}{4\pi r}\end{aligned}\tag{4.8}$$

Differentiating equation (4.8) with respect to V, and using equations (3.19) and (3.20), the two-point correlation differential equation for extended mass structure takes the form as [93];

$$3\bar{n}\frac{\partial \xi_2}{\partial \bar{n}} + T\left(\frac{r^2}{r^2+\epsilon^2}\right)\frac{\partial \xi_2}{\partial T} - r\frac{\partial \xi_2}{\partial r} = 0 \tag{4.9}$$

Equation (4.9) is a first order differential equation for two particle correlation function and is characterised by average number density $\bar{n}$, temperature T and inter-particle distance r

4.3 Solution of the differential equation of two-point Correlation Function for Extended Mass Structure

Equation (4.9) is a first order partial differential equation for two particle correlation function. The above equation is characterised by average number density $\bar{n}$, temperature T and inter-particle distance r. Therefore, two-point correlation function $\xi(\bar{n}$,T,r$)$ will depend on the values of $\bar{n}$ and T as well as on the spatial co-ordinate r in a statistically homogeneous distribution of galaxies clustering gravitationally in an expanding universe. Hence, the thermodynamic description of a two point correlation function for describing the galaxy clustering can be defined by the physical behaviour of equation (4.9). After looking for the possible solution of the equation (4.9), we are able to extend the work of [93] and study it in accordance with the prescribed boundary conditions. The detailed study of this solution will provide a new insight in the clustering problem phenomena of galaxies.

From the equation (4.9), it is clear that the two point correlation function (ξ_2) depends on three variables $\bar{n}$, T and r. As correlation function (ξ_2) is directly related to probability, so in order to look for the possible solution of the equation (4.9), we write two point correlation function (ξ_2) as product of three variable $\Theta(\bar{n})\Gamma(T)R(r)$.
Where
$\Theta(\bar{n})$ is function of $\bar{n}$ only,
$\Gamma(T)$ is function of T only and

$R(r)$ is function of r only.

Thus, we write;

$$\xi_2(\bar{n}, T, r) = \Theta(\bar{n})\Gamma(T)R(r) \tag{4.10}$$

From equation (4.10), it follows that

$$\frac{\partial \xi_2}{\partial \bar{n}} = \Gamma(T)R(r)\frac{d\Theta(\bar{n})}{d\bar{n}} \tag{4.11}$$

$$\frac{\partial \xi_2}{\partial T} = \Theta(\bar{n})R(r)\frac{d\Gamma(T)}{dT} \tag{4.12}$$

$$\frac{\partial \xi_2}{\partial r} = \Theta(\bar{n})\Gamma(T)\frac{dR(r)}{dr} \tag{4.13}$$

After using equations (4.11), (4.12) and (4.13) in equation (4.9), we have

$$\begin{aligned} 3\bar{n}\Gamma(T)R(r)\frac{d\Theta(\bar{n})}{d\bar{n}} &+ T\left(\frac{r^2}{\epsilon^2 + r^2}\right)\Theta(\bar{n})R(r)\frac{d\Gamma(T)}{dT} \\ &= r\Theta(\bar{n})\Gamma(T)\frac{dR(r)}{dr} \end{aligned} \tag{4.14}$$

Dividing equation (4.14) both sides by $\Theta(\bar{n})\Gamma(T)R(r)$, we get

$$\frac{3\bar{n}}{\Theta}\frac{d\Theta(\bar{n})}{d\bar{n}} + \frac{T}{\Gamma}\frac{r^2}{\epsilon^2 + r^2}\frac{d\Gamma(T)}{dT} = \frac{r}{R}\frac{dR(r)}{dr} \tag{4.15}$$

After rearranging equation (4.15), we have

$$\frac{3\bar{n}}{\Theta}\frac{d\Theta(\bar{n})}{d\bar{n}} = \frac{r}{R}\frac{dR(r)}{dr} - \frac{T}{\Gamma}\frac{r^2}{\epsilon^2 + r^2}\frac{d\Gamma(T)}{dT} \tag{4.16}$$

As both sides of equation (4.16) are functions of different variables, so equation (4.16) will be true only, if both sides are equal to the same constant say 'Z', and we write;

$$\frac{3\bar{n}}{\Theta}\frac{d\Theta(\bar{n})}{d\bar{n}} = Z \tag{4.17}$$

and

$$\frac{r}{R}\frac{dR(r)}{dr} - \frac{T}{\Gamma}\frac{r^2}{\epsilon^2 + r^2}\frac{d\Gamma(T)}{dT} = Z \tag{4.18}$$

In order to find the solution of equation (4.17), we proceed as;

$$\frac{d\Theta}{\Theta} = \frac{Z}{3}\frac{d\bar{n}}{\bar{n}}$$

Integrating on both sides of above equation, we have

$$\int \frac{d\Theta}{\Theta} = \frac{Z}{3}\int \frac{d\bar{n}}{\bar{n}} + lnC_1 \tag{4.19}$$

$$\Rightarrow ln\Theta = \frac{Z}{3}ln\bar{n} + lnC_1 \tag{4.20}$$

$$\Rightarrow \Theta = C_1(\bar{n})^{\frac{Z}{3}} \tag{4.21}$$

This represents the solution of one part of the differential equation. Similarly, in order to find the solution of equation (4.18), we proceed as;

$$\frac{\epsilon^2 + r^2}{r^2}\left(\frac{r}{R}\frac{dR}{dr} - Z\right) = \frac{T}{\Gamma}\frac{d\Gamma}{dT} \tag{4.22}$$

Equation (4.22) can be correct only, if both sides of it are equal to the same constant say 'Z_N', so

$$\frac{\epsilon^2 + r^2}{r^2}\left(\frac{r}{R}\frac{dR}{dr} - Z\right) = Z_N \tag{4.23}$$

and

$$\frac{T}{\Gamma}\frac{d\Gamma}{dT} = Z_N \tag{4.24}$$

In order to find the solution of equation (4.23), we proceed as;

$$\frac{r}{R}\frac{dR}{dr} - Z = Z_N\frac{r^2}{\epsilon^2 + r^2} \tag{4.25}$$

(or)

$$\frac{dR}{R} = \left(Z + Z_N\frac{r^2}{\epsilon^2 + r^2}\right)\frac{dr}{r} \tag{4.26}$$

On integrating both sides of above equation, we have

$$\int \frac{dR}{R} = \int \left(Z + Z_N \frac{r^2}{\epsilon^2 + r^2} \right) \frac{dr}{r} + lnC_2 \tag{4.27}$$

(or)

$$lnR = Zlnr + \frac{Z_N}{2} ln(\epsilon^2 + r^2) + lnC_2 \tag{4.28}$$

(or)

$$R(r) = C_2 r^Z (\epsilon^2 + r^2)^{\frac{Z_N}{2}} \tag{4.29}$$

Now in order to find the solution of the equation (4.24), let us integrate it as;

$$\int \frac{d\Gamma}{\Gamma} = \int Z_N \frac{dT}{T} + lnC_3 \tag{4.30}$$

(or)

$$ln\Gamma = Z_N lnT + lnC_3 \tag{4.31}$$

(or)

$$\Gamma(T) = C_3 T^{Z_N} \tag{4.32}$$

Using equations (4.21), (4.29) and (4.32) in equation (4.10), we get the two point correlation function (with unknown parameters Z and Z_N) as;

$$\xi_2(\bar{n}, T, r) = [C_1(\bar{n})^{\frac{Z}{3}}][(C_3 T^{Z_N}][C_2 r^Z (\epsilon^2 + r^2)^{\frac{Z_N}{2}}] \tag{4.33}$$

(or)

$$\xi_2(\bar{n}, T, r) = C_1 C_2 C_3 (\bar{n})^{\frac{Z}{3}} T^{Z_N} r^Z (\epsilon^2 + r^2)^{\frac{Z_N}{2}} \tag{4.34}$$

(or)

$$\xi_2(\bar{n}, T, r) = C(\bar{n})^{\frac{Z}{3}} T^{Z_N} r^Z (\epsilon^2 + r^2)^{\frac{Z_N}{2}} \tag{4.35}$$

where

$$C = C_1 C_2 C_3 \tag{4.36}$$

Hence, equation (4.35) is the required solution of the equation (4.9).

4.4 Functional form of two-point Correlation Function and Correlation Energy

The solution defined by equation (4.35) can have variety of forms, depending upon the parameters Z and Z_N, but we are interested in such a solution that is physically valid. The physically valid solution can be verified when a set of boundary conditions assigned for two-point correlation function are satisfied. The set of boundary conditions assigned for two-point correlation function are;

(1). The gravitational clustering of galaxies in a homogeneous universe requires correlation function (ξ_2) to have a positive value, which obviously depends upon the limiting values of $\bar{n}$, T and r.

(2). When $\bar{n}$, T and r are very small (approximately tending to zero), the two-point correlation function (ξ_2) will increase expect for the number density $\bar{n}$.

(3). When two-point correlation function increases, the clustering of galaxies becomes dominant because of virial equilibrium, which suggests that at low temperatures and high densities more and more clusters are formed. In other words, when $\bar{n}T^{-3}$ is very large (approximately tending to infinity), the two-particle correlation function (ξ_2) will increase and measuring correlation parameter (b_ϵ) defined by equation (4.3) will also increase, and vice versa.

So depending upon these boundary conditions of the correlation function, we will choose the values of Z and Z_N as per requirement. Let us substitute first equation (4.35) in equation (4.3), so that correlation parameter (b_e) for extended mass structure becomes as;

$$b_e = \frac{2\pi G m^2 N}{3VT} \int_V C(\bar{n})^{\frac{Z}{3}} T^{Z_N} r^Z \left(\epsilon^2 + r^2\right)^{\frac{Z_N}{2}} \left(1 + \frac{\epsilon^2}{r^2}\right)^{\frac{-1}{2}} \frac{dV}{4\pi r} \tag{4.37}$$

$$b_e = \frac{2\pi G m^2 N}{3VT} \int_0^R C \left(\frac{3N}{4\pi r^3}\right)^{\frac{Z}{3}} T^{Z_N} r^Z \left(\epsilon^2 + r^2\right)^{\frac{Z_N}{2}} \left(r^2 + \epsilon^2\right)^{\frac{-1}{2}} r \frac{4\pi r^2 dr}{4\pi r} \tag{4.38}$$

$$b_e = \frac{2\pi G m^2 N}{3VT} \int_0^R C \left(\frac{3N}{4\pi r^3}\right)^{\frac{Z}{3}} T^{Z_N} r^Z \left(\epsilon^2 + r^2\right)^{\frac{Z_N - 1}{2}} r^2 dr \tag{4.39}$$

$$b_e = \frac{2\pi G m^2 \bar{n}}{3T} C \left(\frac{3N}{4\pi}\right)^{\frac{Z}{3}} T^{Z_N} \int_0^R \left(\epsilon^2 + r^2\right)^{\frac{Z_N - 1}{2}} r^2 dr \tag{4.40}$$

Now we can see from equation (4.40), that for different values of Z_N, we have different values for the integral. But we will use only such values of Z_N, which has some physical significance for correlation parameter (b_e). So, the following cases can be taken into consideration;

Case 1. Let us take $Z_N = 1$, then the equation (4.40) gives us;

$$b_e = \frac{2\pi G m^2 \bar{n}}{3T} C \left(\frac{3N}{4\pi}\right)^{\frac{Z}{3}} T \int_0^R r^2 dr \tag{4.41}$$

$$\Rightarrow b_e = \frac{2\pi G m^2 \bar{n}}{3} C \left(\frac{3N}{4\pi}\right)^{\frac{Z}{3}} \frac{R^3}{3} \tag{4.42}$$

Equation (4.42) relates the variation of correlation parameter (b_e) for extended mass structures with the size of cluster. For a given cluster cell size with more dimensions of R, we can study the clustering rate without involving the thermodynamic quantities . However, there is need to show the temperature dependence of b_e also as we have assumed the system in quasi-equilibrium state. This is achieved by testing for other values of Z_N . It is interesting to note here that the validity of Z_N is based on $Z_N \leq 1$, because for $Z_N \geq 2$ the correlation parameter (b_e) shows direct power law dependence on T, meaning velocities are increasing and clustering is also increasing, which is absurd.

Case 2. let us take $Z_N = 0$, then the equation (4.40) gives us;

$$b_e = \frac{2\pi Gm^2\bar{n}}{3T}C\left(\frac{3N}{4\pi}\right)^{\frac{Z}{3}}\int_0^R \left(\epsilon^2 + r^2\right)^{\frac{-1}{2}} r^2 dr \tag{4.43}$$

$$b_e = \frac{2\pi Gm^2\bar{n}}{3T}C\left(\frac{3N}{4\pi}\right)^{\frac{Z}{3}}\int_0^R \frac{r^2}{(\epsilon^2 + r^2)^{\frac{1}{2}}} dr \tag{4.44}$$

$$b_e = \frac{2\pi Gm^2\bar{n}}{3T}C\left(\frac{3N}{4\pi}\right)^{\frac{Z}{3}}\frac{1}{2}\left(R\sqrt{\epsilon^2 + R^2} - \epsilon^2 log\left|\frac{R}{\epsilon} + \sqrt{1 + \frac{R^2}{\epsilon^2}}\right|\right) \tag{4.45}$$

Case 3. Let us take $Z_N = -1$, then the equation (4.40) gives us;

$$b_e = \frac{2\pi Gm^2\bar{n}}{3T}C\left(\frac{3N}{4\pi}\right)^{\frac{Z}{3}} T^{-1}\int_0^R \left(\epsilon^2 + r^2\right)^{\frac{-2}{2}} r^2 dr \tag{4.46}$$

$$b_e = \frac{2\pi Gm^2\bar{n}}{3T}C\left(\frac{3N}{4\pi}\right)^{\frac{Z}{3}} T^{-1}\int_0^R \frac{r^2}{(\epsilon^2 + r^2)} dr \tag{4.47}$$

$$b_e = \frac{2\pi Gm^2\bar{n}}{3}C\left(\frac{3N}{4\pi}\right)^{\frac{Z}{3}} T^{-2}\left(R - \epsilon tan^{-1}\left(\frac{R}{\epsilon}\right)\right) \tag{4.48}$$

Case 4. let us take $Z_N = -2$, then the equation (4.40) gives us;

$$b_e = \frac{2\pi Gm^2\bar{n}}{3T}C\left(\frac{3N}{4\pi}\right)^{\frac{Z}{3}} T^{-2}\int_0^R \left(\epsilon^2 + r^2\right)^{\frac{-3}{2}} r^2 dr \tag{4.49}$$

$$b_e = \frac{2\pi Gm^2\bar{n}}{3T}C\left(\frac{3N}{4\pi}\right)^{\frac{Z}{3}} T^{-2}\int_0^R \frac{r^2}{(\epsilon^2 + r^2)^{\frac{3}{2}}} dr \tag{4.50}$$

$$b_e = \frac{2\pi Gm^2\bar{n}}{3}C\left(\frac{3N}{4\pi}\right)^{\frac{Z}{3}} T^{-3}\left(\log\left|\frac{R}{\epsilon} + \sqrt{1 + \frac{R^2}{\epsilon^2}}\right| - \frac{R}{\sqrt{R^2 + \epsilon^2}}\right) \tag{4.51}$$

The equations (4.45), (4.48) and (4.51) are in good agreement with a set of boundary conditions. The correlation is maximum, if galaxies are treated as point mass objects ($\epsilon \rightarrow 0$) and goes on decreasing as softening parameter (ϵ) goes on increasing, thus clustering decreases. It has been found that the dispersion in the radial velocity in the coma cluster [116] could be up to $1000 Kms^{-1}$ and is sufficient to throw galaxies in the surrounding voids. The cause for decrease of correlation for galaxies with extended structures, may be due to dispersion in radial velocity. The precise nature of such dependence between b_e and the increase in radial velocity can be treated as one of the important problem to study in contact with the correlation function studies for galaxy clusters. Equation (4.51) is important in sense that it clarifies the earlier result as well as justifies the significance of b_e and b. In the earlier work of [8] and [93], it has been understood that b has a specific dependence on the combination $\bar{n}T^{-3}$. Here in our study, it is also true that b_e has also the specific dependence on $\bar{n}T^{-3}$, which means that the clustering takes place at moderate level. Z may have always positive values because of the reason that for large values of N, b_e increases.

Chapter 5

Role of Correlation Energy in Galaxy Clusters

The correlation energy plays an important role in the evolution of galaxy clustering phenomena in an expanding universe. A number of workers like [1], [8], [13], [19], [20], [31], [40], [41], [42], [43], [45] and [93] have over all discussed the evolution of correlation energy, which is defined as the ratio between the gravitational potential to twice the kinetic energy of the system. This correlation energy 'b' is measured in the scale of $0 - 1$ and is assumed that for un-clustered system of galaxies, i.e; a system having no interaction of particles (galaxies) $b = 0$ like particles of an ideal gas . As the galaxies cluster together under the phenomena of mutual interaction, the clustering rate goes on increasing, which means that as b starts increasing, the clustering scale is more and more. The aim of this chapter is to study the evolution of galaxy clustering rate with the size of the cluster. To treat the problem very simple, it is assumed that all galaxies can be considered to be co-moving in spatial homogeneous cell and the relevant study is done on the basis of N-body Hamiltonian, which defines the sum of the Kinetic energies and Potential energies of all the particles in a system (ensemble concept). This ensemble is considered to be made up of large number of cells of varying sizes. The size of the cell can be varied and the corresponding clustering rate b, which is in the scale of $0-1$ is plotted against the different cell sizes. The results are then extended to galaxies having extended structures. The extended nature of the galaxies and their correlation function details are discussed [117] in chapter 4.

5.1 Effect of b_e (Correlation Energy for Extended Mass Galaxies) on cell size R

In the first attempt, we study the effect of b_e (clustering scale for extended mass galaxies) with the cell size 'R' of a cluster. Recently, the analytical treatment for studying the two-particle correlation function for extended structures has been discussed by [117]. We take equation (4.45) of chapter 4th into consideration and test this equation for a well defined cluster like **A2048**. The cluster A2048 belongs to an Abell cluster have the following parameters as;

Number of galaxies (N)=59

Mass of each galaxy (m)=$5.7 \times 10^{42} Kg$.

The equation (4.45) is given as;

$$b_e = \frac{2\pi G m^2 \bar{n}}{6T} C \left(\frac{3N}{4\pi}\right)^{Z/3} \left(R\sqrt{\epsilon^2 + R^2} - \epsilon^2 log \left|\frac{R}{\epsilon} + \sqrt{1 + \frac{R^2}{\epsilon^2}}\right|\right) \tag{5.1}$$

$$b_e = \frac{2\pi G m^2 \bar{n}}{6T} C \left(\frac{3N}{4\pi}\right)^{Z/3} R^2 \left(\sqrt{1 + \left(\frac{\epsilon}{R}\right)^2} - \left(\frac{\epsilon}{R}\right)^2 log \left|\frac{R}{\epsilon} + \sqrt{1 + \frac{R^2}{\epsilon^2}}\right|\right) \tag{5.2}$$

The appropriate value of Z is taken to be 3 and using the value of softening parameter

$\epsilon = 0.05$ in the units of total radius R.

For simplicity the cell is considered to be spherical type so that, $\bar{n} = \frac{N}{V} = \frac{3N}{4\pi R^3}$

On making use of all these values in equation (5.2), the equation goes like;

$$b_e = \frac{2\pi G m^2}{6T}\frac{3N}{4\pi R^3}C\frac{3N}{4\pi}R^2\left(\sqrt{1+(0.0025)} - (0.0025)log|20+\sqrt{1+400}|\right) \quad (5.3)$$

$$b_e = \frac{21C}{352}\frac{Gm^2N^2}{TR}\left(\sqrt{1.0025} - (0.0025)log|20+\sqrt{401}|\right) \quad (5.4)$$

$$b_e = \frac{21C}{352}\frac{Gm^2N^2}{TR}\left(1.0012 - (0.0025)log|20+20.025|\right) \quad (5.5)$$

As we know that b_e is also measured in the scale of $0-1$, so as to make the value of b_e less than 1, the constant C defined in equation (5.5) is chosen of the order of 10^{-56}. The other parameter T can be taken as $T = 1, 10, 100$, but in the present case $T = 1$. It may be noted that in one of the earlier paper [8], the entropy studies with respect to b for galaxy clustering problem have already been discussed on these temperatures. With all these substitutions and $G = 6.67\times10^{-11}Nm^2Kg^{-2}$, the result leads to;

$$b_e = \frac{21\times10^{-56}}{352}\times\frac{6.67\times10^{-11}(5.7\times10^{42})^2(59)^2}{1\times R}\left(1.0012-(0.0025)\times log|40.025|\right) \quad (5.6)$$

$$b_e = 45004.52681\times\frac{10^{17}}{R}\left(1.0012-(0.0025)\times(1.602)\right) \quad (5.7)$$

$$b_e = 45004.52681\times\frac{10^{17}}{R}\left(1.0012-0.0040\right)) \quad (5.8)$$

$$b_e = 45004.52681\times\frac{10^{17}}{R}\left(0.9972\right)) \quad (5.9)$$

$$b_e = 44878.51414\times\frac{10^{17}}{R} \quad (5.10)$$

The various values of cell size(R) of this cluster are fixed at;

R=0.1Mpc, 0.2Mpc, 0.3Mpc and so on.

The computation values of b_e for these cell sizes are calculated from equation (5.10).

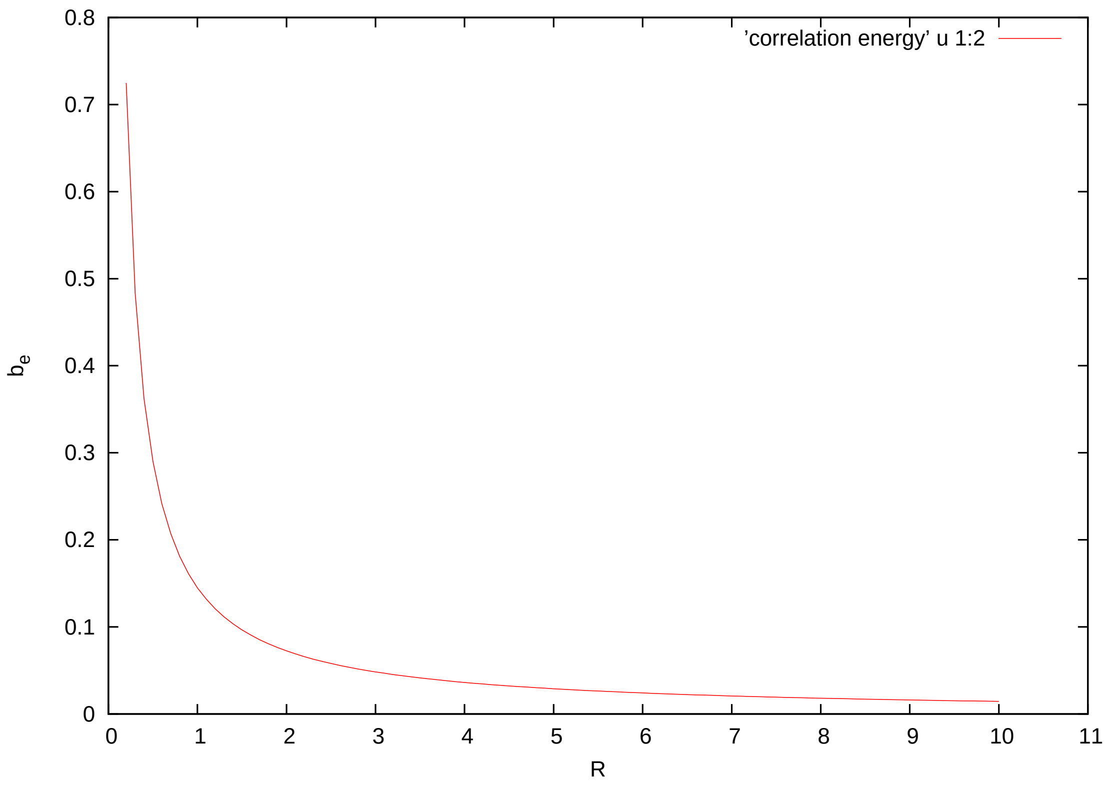

Figure 5.1: Variation between the correlation parameter b_e and the cell size R for the cluster A2048 on the basis of equation (5.10).

5.2 Effect of b (Correlation Energy for Point Mass Galaxies) on cell size R

As we have already studied the effect of correlation energy on cell size for extended mass structure in the above section. In the present section, we study the effect of correlation energy on cell size for point mass galaxies. We use directly equation (5.2) of above section with the limit $\epsilon \to 0$ and test this equations for a well defined cluster like **A2048**. The cluster A2048 belongs to an Abell cluster have the parameters defined in section (5.1).

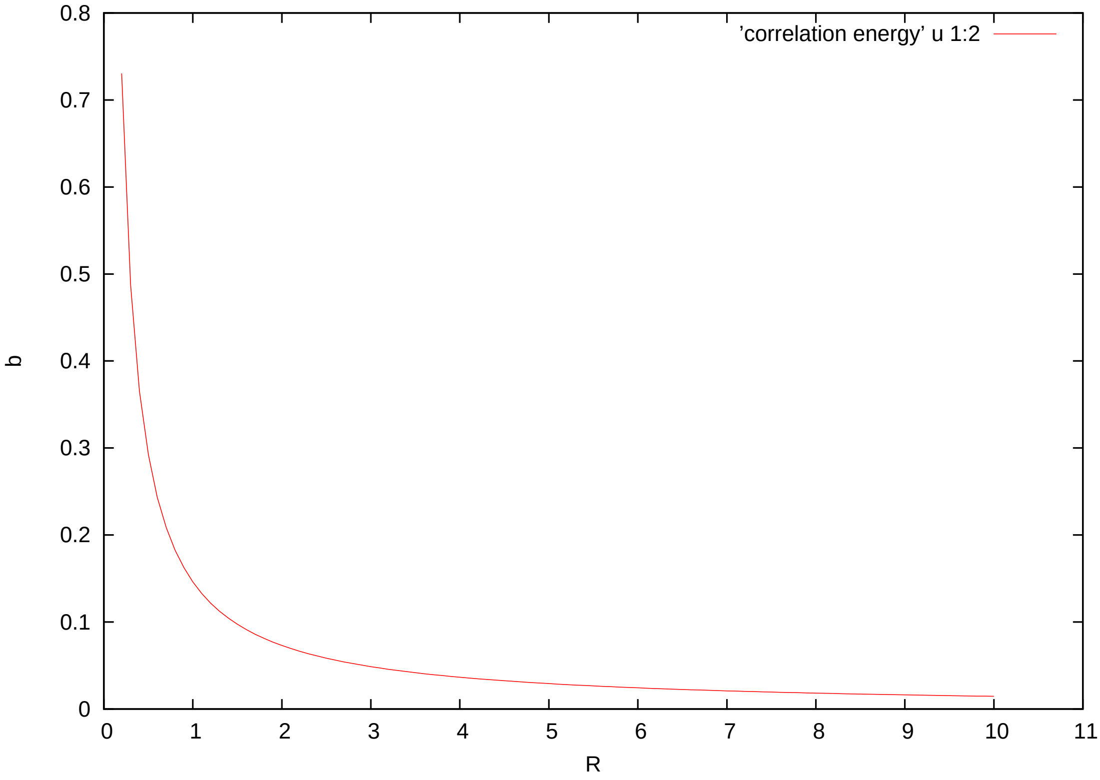

Figure 5.2: Variation between the correlation parameter b and the cell size R for the cluster A2048 on the basis of equation (5.15).

Now the equation (5.2) with $\epsilon \to 0$ gives;

$$b = \frac{2\pi G m^2 \bar{n}}{6T} C \left(\frac{3N}{4\pi}\right)^{Z/3} R^2 \tag{5.11}$$

On same substitutions as in section (5.1), equation (5.11) takes the form as;

$$b = \frac{2\pi G m^2}{6T} \frac{3N}{4\pi R^3} C \frac{3N}{4\pi} R^2 \tag{5.12}$$

$$b = \frac{21C}{352} \frac{G m^2 N^2}{TR} \tag{5.13}$$

As we know that b is measured in the scale of $0-1$, so as to make the value of b less than 1, the constant C defined in equation (5.13) is chosen of the order of 10^{-56}. The other parameter T can be taken as $T = 1, 10, 100$, but in the present case $T = 1$. With all these substitutions and $G = 6.67 \times 10^{-11} Nm^2 Kg^{-2}$, the result leads to;

$$b = \frac{21 \times 10^{-56}}{352} \times \frac{6.67 \times 10^{-11}(5.7 \times 10^{42})^2 (59)^2}{1 \times R} \tag{5.14}$$

$$b = 45004.52681 \times \frac{10^{17}}{R} \tag{5.15}$$

The various values of cell size(R) of this cluster are fixed at;

R=0.1Mpc, 0.2Mpc, 0.3Mpc and so on.

The computation values of b for these cell sizes are calculated from equation (5.15).

5.3 Effect of b_e on b

The correlation energy for extended mass galaxies is defined by [31] as;

$$b_e = \frac{\beta \bar{n} T^{-3} \alpha(\epsilon/R)}{1 + \beta \bar{n} T^{-3} \alpha(\epsilon/R)} \tag{5.16}$$

and is related to the point mass galaxy system [31] by;

$$b_e = \frac{b\alpha(\epsilon/R)}{1 + b(\alpha(\epsilon/R) - 1)} \tag{5.17}$$

We can see the effect of b_e on b by using different values of the softening parameter (ϵ), cell size (R) and the ratio ϵ/R as shown in tables (5.1) and (5.2). The effect shows that b_e has a strong dependence on ϵ and the value decreases, if ϵ is large and increases for smaller values of ϵ.

Table 5.1: Comparison between the b_e and b at different values of ϵ and $R = 0.0002$.

b	b_e	b_e	b_e
$\epsilon = 0$	$\epsilon = 0.01$	$\epsilon = 0.02$	$\epsilon = 0.03$
0.0	0.000	0.000	0.00
0.1	0.001	0.0007	0.0005
0.2	0.003	0.002	0.001
0.3	0.006	0.003	0.002
0.4	0.009	0.004	0.003
0.5	0.013	0.007	0.004
0.6	0.019	0.009	0.007
0.7	0.030	0.015	0.010
0.8	0.051	0.026	0.017
0.9	0.107	0.057	0.038
1.0	1.000	1.000	1.000

Table 5.2: Comparison between the b_e and b at different values of ϵ and $R = 0.0004$.

b	b_e	b_e	b_e
$\epsilon = 0$	$\epsilon = 0.01$	$\epsilon = 0.02$	$\epsilon = 0.03$
0.0	0.000	0.000	0.000
0.1	0.003	0.001	0.0009
0.2	0.007	0.003	0.002
0.3	0.011	0.006	0.004
0.4	0.017	0.009	0.006
0.5	0.026	0.013	0.009
0.6	0.038	0.019	0.013
0.7	0.058	0.030	0.020
0.8	0.096	0.051	0.030
0.9	0.193	0.107	0.074
1.0	1.000	1.000	1.000

Chapter 6

Discussion and Conclusion

The problems of the origin and the evolution of large-scale matter (galaxy clusters) are quiet different from other cosmological problems. The construction of theoretical models for the evolution of large scale structure in the universe have been the major growth area in the astrophysics of galaxy-clusters. The evolutionary history of the constituent particles (galaxies) need to be known and then the evolution of the system as a whole can be worked out on the basis of different assumptions and techniques. The galaxy clustering is considered to be one of the major study used in the study of large scale structures in the universe. In the universe, matter distribution has a hierarchical appearance as galaxies tend to group together to form clusters, and clusters are clamped into super-clusters. Some major techniques during the recent years have been developed in understanding the complicated process of gravitational clustering, like clusters of galaxies and super-clusters. The detailed analysis of the galaxy cluster distribution is given by the correlation functions. The galaxy correlation function is a measure of the degree of the clustering in either the spatial distribution $(\xi(r))$ or the angular distribution $(\omega(\theta))$ of galaxies. The most important one is the two-point correlation function, which is observed to decline with distance at a power law shape $(r_0/r)^\gamma$ having the same slope $\gamma = 1.6 - 1.8$ for both galaxy systems and cluster systems, although with different amplitudes. The sameness of the two indices suggests a simple underlying dynamics on all scales. Actually, the cosmological N-body computer simulations using gravitation as the only force, reproduces many of the features of clustering rather well. The hierarchical clustering and the power law correlation function suggest matter distribution be a grouping fractal or multi-fractal. Since a cluster is a cluster of galaxies, therefore an interesting technique called n-particle correlation function is one of the approaches for studying the galaxy clustering. However, it is too complicated to handle higher order correlation functions. The lower order correlation function is presently the most widely used as a statistical indicator. The use of two-point correlation function to express the statistical properties of galaxies has become very popular as the same contains information about clustering on all higher scales.

The approach of the applicability of thermodynamics has been discussed at length and breadth by a number of workers like [50][51][52][53], in which each galaxy is considered to be a constituent particle of an infinite gas. The physical validity of the application of thermodynamics in the clustering of galaxies and galaxy clusters has been discussed on the basis of N-body computer simulations results [54]. The gravitational galaxy clustering carried out by [50][53] ensures a more fundamental statistical mechanical description of the cosmological many-body problem. We have developed the differential equation for the clustering of galaxies for extended mass structures in an expanding universe (equation (4.9)). The characteristic solution of this differential equation (in accordance with prescribed set of boundary conditions) provides a new insight for understanding the clustering phenomena. Hence, the non-linear gravitational clustering for extended mass structures in an expanding universe can be studied with the help of two-point correlation function, which depends on average number density $\bar{n}$, temperature T, and inter-particle distance r. One of the appropriate solution of this differential equation is find out on the basis of variable separation method (equation (4.35)) with unknown parameters Z and Z_N. The values of these parameters Z and Z_N depends on the set of boundary conditions assigned for two-point correlation function. Most important boundary condition is that clustering becomes dominant at low temperatures and high densities as more and more clusters are formed. From equation (4.40), we can clearly understand that the correlation parameter b_e for extended mass structures depends on the limiting values of Z and Z_N. Hence, equation (4.40) serves as a basic equation for evaluating correlation parameter for extended mass structures b_e for the different galaxy clusters. It is clear that with the involvement of softening parameter (ϵ), the role of dark matter comes into play and one can study the effectiveness of clustering rate, where b_e is studied on the basis of the effective range of softening parameter (ϵ) in the limits of total radius (R) of a cluster of galaxies. We notice from the equations (4.45), (4.48) and (4.51) that b_e has a strong dependence on the softening parameter (ϵ). Thus, softening parameter (ϵ) introduces a correction term that lowers the correlation energy, with the result b_e decreases. Some mathematical tricks have off course been involved like the varying values of Z_N, as a result of which the description of ξ_2 for extended mass structures (galaxies) gets well defined by equations (4.42), (4.45), (4.48) and (4.51).

In the chapter 5th, we have plotted graphs between correlation energy (b and b_e) with the cell size (R) for a cluster. From these graphs, it is understood that for smaller values of the size of the cluster correlation energy is more, while as for larger values of the size of cluster correlation energy is less. We have also studied the effect of b_e on b and have found that with the increase in the softening parameter (ϵ), the effect of b_e shows a decreasing value from 0.1-1.0 for a given cell size. However, on increasing the cell size of a cluster, the effect of b_e shows an increasing trend.

The whole discussion of this dissertation is summarised in the form of following points;

1. To understand the formation of galaxy clusters, various models on the basis of mass distribution plays a vital role like the cold dark matter (CDM) sufficiently describes the formation of structures in the universe. Cosmological parameters add a considerable knowledge for studying the mass distribution.

2. The statistical mechanics and thermodynamics is an important tool in studying the phenomena of galaxy clusters in an expanding universe. However, the use of quasi-equilibrium thermodynamics provides sufficiently a good understanding of studying the behaviour of a system consisting of galaxies and various statistical properties are explored to understand the basic equations of state.

3. The two-particle correlation function (ξ_2) has proved to be a lowest possible tool in understanding the correlation between various particles (galaxies). We have reconfirmed the complete dependence of correlation parameter 'b' on the combination $\bar{n}T^{-3}$ on the basis of a partial differential equation relating ξ_2 with $\bar{n}$, T and r. Although, this study had earlier been described by [93], but the main aim of this work was to extend the same analytical model for a system of galaxies clustering gravitationally in which each galaxy is treated to be an extended mass instead of being treated a galaxy as mere a point particle. The extended nature of a galaxy is described by using the value of softening parameter (ϵ) from 0.01 to 0.05 in the units of total radius (R) of a cluster.

4. The extension of the earlier work [93] has been successfully made applicable to extended nature of galaxies in which a partial differential equation developed for extended structures (equation (4.9)) has been analysed in detail. An interesting kind of solutions are obtained by making use of certain constants like Z and Z_N, whose appropriate values has been chosen keeping the necessary boundary conditions into consideration. The basic objectives in exploring the results for b_e was to develop a new approach, whose analytical results can be tested for various systems. The extended results (equation (4.51)) have again shown that the measuring correlation parameter 'b_e' for extended mass galaxies has a complete dependence on the combination $\bar{n}T^{-3}$. This clearly indicates that the theory developed in this work as well as in previous studies [8][11][12][93] is applicable to a moderately dense system. The details of these results shown in our work (chapter 4th) has been recently published in **MNRAS Letters (May, 2012 issue)**[doi:10.1111/j.1745-3933.2012.01281.x].

5. The overall dependence of b (correlation parameter for point mass galaxies) and b_e (correlation parameter for extended mass galaxies) can be clearly understood by studying the effects of these two correlation parameters with the size of a cluster. e.g; In case of **A2048** cluster with known parameters for number of galaxies (N) and galaxy mass (m), we have found that for both b and b_e, small clusters have more correlation energy to build a cluster, while as bigger clusters have less correlation energy to build a cluster. It is important to understand here that the role of b and b_e is very much related with the size of cluster.

The overall impression of this M. Phil. work is that we have reasonably described an alternative kind of approach exclusively meant for extended mass structures to understand its clustering phenomena on the basis of two-point correlation function (ξ_2).

Bibliography

[1] Saslaw W. C. (1985) *Gravitational Physics of Stellar and Galactic Systems, Cambridge University Press*

[2] Binney J. and Tremaine S. (1987) *Galactic Dynamics, Princeton University Press*

[3] Splitzer L. (1987) *Dynamical evolution of Globular Clusters, Princeton University Press*

[4] Antonov V. A. (1962) *MNRAS* **7**, 135

[5] Lynden-bell D. and Wood R. (1968) *MNRAS* **138**, 495

[6] Totsuji H. and Kihara T. (1969) *Astro. Soc. Japan* **21**, 221

[7] Peebles P. J. E. (1980) *The large scale Structure of the Universe, Princeton Univ. Press*

[8] Saslaw W. C. and Hamilton A. J. S. (1984) *ApJ* **276**, 13

[9] Nityananda R. and Padmanabhan T. (1994) *MNRAS* **271**, 976

[10] Bernardeau F. and Kofman L. (1995) *ApJ* **443**, 479

[11] Fry J. N. (1984) *ApJ* **279**, 499

[12] Bernardeau F. (1992) *ApJ* **392**, 1

[13] Saslaw W. C. (1972) *ApJ* **177**, 19

[14] Inagaki S. (1976) *Astro. Soc. Japan* **28**, 463

[15] Fall S.M. and Saslaw W.C. (1976) *ApJ* **204**, 631

[16] Zhan Y. (1989) *ApJ* **340**, 23

[17] Zhan Y. and Dyer C. (1989) *ApJ* **343**, 107

[18] Hamilton A. J. S. (1993) *ApJ* **417**, 19

[19] Saslaw W. C. (2000) *The Distribution of the Galaxies, Cambridge University Press, New York*

[20] Saslaw W. C. and Fang F. (1996) *ApJ* **460**, 16

[21] Suto Y., Itoh M. and Inagaki S. (1990) *ApJ* **350**, 492

[22] Bouchet F. and Hernquist L. (1992) *ApJ* **400**, 25

[23] Sheth R. K. (1995) *MNRAS* **274**, 213

[24] Sheth R. K. (1995) *MNRAS* **276**, 796

[25] Press W. and Schechter P. (1974) *ApJ* **187**, 425

[26] Epstein R. I. (1983) *MNRAS* **205**, 207

[27] Bond J. R., Cole S., Efstathiou G. and Kaiser N. (1991) *ApJ* **379**, 440

[28] Jedamzik K. (1995) *ApJ* **448**, 1

[29] Lacey C. and Cole S. (1994) *MNRAS* **271**, 676

[30] Sivakoff G. R. and Saslaw W. C. (2005) *ApJ* **626**, 795

[31] Ahmad F., Saslaw W. C. and Iqbal N. (2002) *ApJ* **571**, 576

[32] White S. D. M. and Rees M. J. (1978) *MNRAS* **183**, 341

[33] York D. G., Adelman J., Anderson S. F., Annis J., Bahcall N. A., Bakken J. A. and Barkhouser R. (2000) *ApJ* **120**, 1579

[34] Colless M., Dalton G., Maddox S., Sutherland W., Norberg P., Cole S., Bland-Hawthorn J. and Bridges T. (2001), *MNRAS* **328**, 1039

[35] Zeldovich Ya. B., Einesto J. and Shandarin S. (1982) *MNRAS* **300**, 407

[36] Grimmet G. (1989) *Percolation*

[37] Pearson R. C. and Coles P. (1995) *MNRAS* **272**, 231

[38] Bhavasar S. P. and Splinter R. J. (1996) *MNRAS* **282**, 1461

[39] Mandelbrot B. B. (1982) *The Fractal Geometry of Nature*

[40] Saslaw W. C., Chitre S. M., Itoh M. and Inagaki S. (1990) *ApJ* **365**, 419

[41] Itoh M., Inagaki S. and Saslaw W. C. (1988) *ApJ* **331** 45

[42] Itoh M., Inagaki S. and Saslaw W. C. (1993) *ApJ* **403**, 476

[43] Sheth R. K., Mo H. J. and Saslaw W. C. (1994) *ApJ* **427**, 562

[44] Fang F. and Zou Z. (1994) *ApJ* **421**, 9

[45] Raychaudhury S. and Saslaw W. C. (1996) *ApJ* **461**, 514

[46] White S. D. M. and Frenk C. S. (1991) *ApJ* **379**, 52

[47] Cole S. (1991) *ApJ* **367**, 45

[48] Springel V., Frenk C. S. and White S. D. M. (2006) *Nature* **440**, 1137

[49] Eke V. R., Frenk C. S., Baugh C. M., Cole S., Norberg P., Peacock J. A., Baldry I. and Bland-Hawthorn J. (2004) *MNRAS* **355**, 769

[50] Baugh C. M. (2006) *RPPh* **69**, 310

[51] Kauffmann G., Nusser A. and Steinmetz M. (1997) *MNRAS* **286**, 795

[52] Kauffmann G., Colberg J. M., Diaferio A. and White S. D. M. (1999) *MNRAS* **303**, 188

[53] Benson A. J., Cole S., Frenk C. S., Baugh C. M. and Lacey C. G. (2000) *MNRAS* **311**, 739

[54] Benson A. J., Frenk C. S., Baugh C. M., Cole S. and Lacey C. G. (2001) *MNRAS* **327**, 1041

[55] Hatton S., Devriendt J. E. G., Ninin S., Bouchet F. R., Guiderdoni B. and Vibert D. (2003) *MNRAS* **343**, 75

[56] Peacock J. A. and Smith R. E. (2000) *MNRAS* **318**, 1144

[57] Seljak U. (2000) *MNRAS* **318**, 2039

[58] Cooray A. and Sheth R. K. (2002) *Physics Reports* **372**, 1

[59] Bosch F. C., Mo H. J. and Yang X. (2003) *MNRAS* **345**, 923

[60] Norberg P., Baugh C. M., Hawkins Ed., Maddox S., Peacock J. A., Cole S., Frenk C. S. and Bland-Hawthorn J. (2001) *MNRAS* **328**, 64

[61] Norberg P., Baugh C. M., Hawkins Ed., Maddox S., Madgwick D., Lahav O., Cole S. and Frenk C. S. (2002) *MNRAS* **332**, 827

[62] Zehavi I., Blanton M. R., Frieman J. A., Weinberg D. H., Mo H. J., Strauss M. A., Anderson S. F. and Annis J. (2002) *ApJ* **571**, 172

[63] Zehavi I., Zheng Z., Weinberg D. H., Frieman J. A., Berlind A. A., Blanton M. R., Scoccimarro R. and Sheth R. K. (2005) *ApJ* **630**, 1

[64] Madgwick D. S., Hawkins E., Lahav O., Maddox S., Norberg P., Peacock J. A., Baldry I. K. and Baugh C. M. (2003) *MNRAS* **344**, 847

[65] Li C., Kauffmann G., Jing Y. P., White S. D. M., Borner G. and Cheng F. Z. (2006) *MNRAS* **368**, 21

[66] Norberg P., Baugh C. M., Gaztanaga E. and Croton D. J. (2009) *MNRAS* **396**, 19

[67] Benson A. J., Bower R. G., Frenk C. S., Lacey C. G., Baugh C. M. and Cole S. (2003) *ApJ* **599**, 38

[68] Li G. L., Mao S., Jing Y. P., Lin W. P. and Oguri M. (2007) *MNRAS* **378**, 496

[69] Kang X., Jing Y. P., Mo H. J. and Borner G. (2005) *ApJ* **631**, 21

[70] Croton D. J., Springel V., White S. D. M., De Lucia G., Frenk C. S., Gao L., Jenkins A. and Kauffmann G. (2006) *MNRAS* **365**, 11

[71] Abell G. O. (1958) *ApJ* **50**, 241

[72] Bahcall N. A. and Soneria R. M. (1983) *ApJ* **270**, 20

[73] Klypin A. A. and Kopylov A. (1983) *Sov. Al* **9**, 41

[74] Bouchet F. and Lachieze-Rey M. (1986) *ApJ* **302**, L37

[75] Colless M., Dalton G. and Maddox S. (2001) *MNRAS* **326**, 255

[76] York D. G, Adelman J. and Anderson J. E. (2000) *ApJ* **120**, 1576

[77] Davis M., Faber S. M. and Newman J. A. (2003) *Proc. SPIE* **4834**, 161

[78] Norberg P. (2002) *MNRAS* **332**, 827

[79] Coil A. L., Davis M. and Madgwick D. S. (2004) *ApJ* **609**, 525

[80] Zehavi I., Blanton M. R. and Frieman J. A. (2002) *ApJ* **571**, 172

[81] Zehavi I., Zheng Z. and Weinberg D. H. (2005) *ApJ* **630**, 1

[82] Li C., Kauffmann G. and Jing Y. P. (2006) *MNRAS* **368**, 21

[83] Li C. and White S. D. M. (2009) *MNRAS* **398**, 2177

[84] White S. D. M. and Frenk C. S. (1991) *ApJ* **379**, 52

[85] Weinmann S. M., Van den Bosch F. C. and Yang X. (2006) *MNRAS* **372**, 1161

[86] Li C., Jing Y. P. and Kauffmann G. (2007) *MNRAS* **376**, 984

[87] Guo Q., White S. and Boylan Kolchin M. (2011) *MNRAS* **413**, 101

[88] Vale A. and Ostriker J. P. (2004) *MNRAS* **353**, 189

[89] Moster B. P., Somerville R. S. and Maulbetsch C. (2010) *ApJ* **710**, 903

[90] Guth A. H. (1981) *Phys. Rev. D* **23**, 347

[91] Guth A. H. and Pi S. Y. (1982) *Phys. Rev. Lett.* **49**, 1110

[92] Bardeen J. M., Bond J. R., Kaiser N. and Szalay A. S. (1986) *ApJ* **304**, 15

[93] Iqbal N., Ahmad F. and Khan M. S. (2006) *ApJ* **27**, 373

[94] Adler R. J. (1981) *The Geometry of Random Fields*

[95] Scherrer R. J., Melott A. L. and Shandarin S. F. (1991) *ApJ* **377**, 29

[96] Chiang L. (2001) *MNRAS* **325**, 405

[97] Chiang L., Naselsky P. and Coles P. (2004) *ApJ* **602**, 1

[98] Coles P. and Chiang L. (2000) *Nature* **406**, 376

[99] Watts P. and Coles P. (2003) *MNRAS* **338**, 806

[100] Watts P., Coles P. and Melott A. (2003) *ApJ* **589**, L61

[101] Matsubara T. (2003) *ApJ* **591**, L79

[102] Peebles P. J. E. and Groth E. J. (1975) *ApJ* **196**, 1

[103] Groth E. J. and Peebles P. J. E. (1977) *ApJ* **217**, 385

[104] Kauffmann G., Colberg J. M., Diaferio A. and White S. D. M. (1999) *MNRAS* **303**, 188

[105] Davis M. and Peebles P. J. E. (1977) *ApJ* **34**, 425

[106] Ichimaru S. (1973) *Basic Principles of Plasma Physics, Benjamin*

[107] Fry J. N. (1984) *ApJ* **279**, 499

[108] Ruamsuwan L. and Fry J. N. (1992) *ApJ* **396**, 416

[109] Yano T. and Gouda N. (1997) *ApJ* **487**, 473

[110] Yano T. and Gouda N. (1998) *ApJ* **118**, 267

[111] Colombi S., Bouchet F. R. and Hernquist L. (1996) *ApJ* **465**, 14

[112] Couchman H. M. and Peebles P. J. E. (1997) *ApJ* **970**, 8230

[113] Shane C. D. and Wirtanen C. A. (1967) *Publ. Lick Obs.* **22**, 675

[114] Maddox S. J. (1990) *MNRAS* **242**, 3

[115] Goodstein D. L. (1985) *States of matter, Dover New York*

[116] Barbara R. (2003) *Introduction to Cosmology, Addison Wesley*

[117] Iqbal N., Ahmad N., Hamid M. and Masood T. (2012) *doi:10.1111/j.1745-3933.2012.01281.x*