

## Dynamical fluctuation of proton emission in heavy ion interactions

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**Abstract** : This paper deals with a study of intermittent behaviour of medium energy knocked out protons in emission angle and azimuthal angle space for the data of  $^{28}\text{Si-AgBr}$  interaction at 14.5 AGeV. From intermittency exponent, the anomalous fractal dimension  $d_q$  and generalized dimension  $D_q$  are calculated. The variation of  $D_q$  with order  $q$  suggests monofractal and multifractal proton emission.

**Keywords** : Relativistic heavy ion interaction, medium energy protons.

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### 1. Introduction

In cosmic ray studies, it was reported that there were large concentration of particles (spikes) in small pseudo rapidity intervals for nuclear interactions at ultra high energies. To explain large fluctuations of multiplicity in restricted rapidity intervals observed in the spectacular JACEE cosmic ray event, Bialas and Peschanski [1,2] introduced the concept of intermittency, in analogy with turbulence theory in fluid dynamics [3]. The main tool of intermittency study, as suggested by the pioneers, is a statistical counting variable called the scaled factorial moment  $F_q$ . According to them, power law growth of the factorial moments with decreasing phase space interval size, signals the onset of intermittency in the context of high energy interactions. The observation of such behaviour suggests that the mechanism for particle production has a self-similar property. The self-similar nature of the dynamics directly implies a connection between intermittency and fractality. The speciality of this method is that it enables one to measure the non-statistical fluctuation filtering out the statistical fluctuation.

The existence of intermittency in critical phenomena were found by Wosiek [4]. Naturally, it was then conjectured that intermittent pattern of fluctuation is a manifestation of QGP phase transition. Now, a number of alternative suggestions like self-similar random cascading mechanism [5], formation of jets and minijets [6], B-E interference [7], conventional short range

correlation[8] *etc.* are available, but none of them is accepted universally.

Experimental data on intermittency do not rule out the possibility of something nontrivial. On the other-hand, theoretical interpretation has not been able to take a definite shape. In fact, the quest for the proper dynamics which can produce such chaotic structure, is still going on.

In high energy interactions, it is believed that relativistically produced particles are the most informative about the collision dynamics. But taking only the information about the produced particles, we can not know the dynamics of the nuclear interaction process completely. So, it is very essential to study the target-associated particles also. Since the medium energy protons are emitted immediately after the collision, they are expected to carry relevant information about the collision dynamics.

The intermittent behaviour of produced particles (pions) has been studied for different interactions[9-13]. This has also been studied for low energy target fragments[14]. However, such study for medium energy protons is extremely rare. So in this article, we have studied the intermittent behaviour for medium energy protons in  $^{28}\text{Si-AgBr}$  interactions at 14.5 AGeV.

### 2. Experimental details

The data were obtained from Illford G5 emulsion stacks exposed horizontally to  $^{28}\text{Si}$  beam of energy 14.5 AGeV from Alternating

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Gradient Synchrotron at Brookhaven National Laboratory (BNL AGS) [15]. A Leitz Metaloplan microscope with a 10X objective and 10X ocular lens provided with a semi-automatic scanning stage, was used to scan the plates. Each plate was scanned by two independent observers to increase the scanning efficiency. The final measurements were done using an oil-immersion 100X objective. The measuring system fitted with it, has  $1\ \mu\text{m}$  resolution along the  $X$  and  $Y$  axes and  $0.5\ \mu\text{m}$  resolution along the  $Z$  axis.

The events were scanned according to the following criteria:

- i) The incident beam track should not exceed more than  $3^\circ$  from the main beam direction in the pellicle. It was done to ensure that we have taken the real projectile beam.
- ii) Events showing interactions within  $20\ \mu\text{m}$  from the top or bottom surface of the pellicle were rejected. It was done to reduce the loss of tracks as well as to reduce the error in angle measurement.
- iii) The tracks of the incident particles, which induce interactions, were followed in the backward direction to ensure that it is a projectile beam starting from the beginning of the pellicle.

Generally, the relation between the energy loss due to ionization and velocity of the secondary particles according to the Bethe- Bloch formula, is given by

$$-(dE/dx) = 2\pi N_a r_e m_e c^2 \rho(Z/A) \left( z^2 / \beta^2 \right) \times \left[ \ln \left\{ \frac{2m_2 \gamma v^2 W_{\max}}{2 - 2\beta^2} \right\} \right],$$

where  $r_e$ : classical electron radius,  $m_e$ : electron mass,

$N_a$ : Avogadro's number,  $I$ : mean excitation potential,

$Z$ : atomic number of absorbing material,  $c$ : speed of light,

$\rho$ : density of the material,  $A$ : Atomic wt. of material.

$z$ : charge of the incident particle,  $\beta = v/c$  of incident particle,

$W_{\max}$ : maximum energy transfer in a single collision,  $\gamma = 1/(1 - \beta^2)^{1/2}$ .

According to the emulsion terminology, the particles emitted after interactions are classified as :

- a) **Black particles** : They are the target fragments with ionization greater or equal to  $10I_0$ ,  $I_0$  being the minimum ionization of a singly charged particle. Range of them are less than 3 mm and velocity less than  $0.3c$  and energy less than 30 MeV.

- b) **Grey particles** : They are mainly fast target recoil protons with energy upto 400 MeV. They have ionization  $1.4I_0 < I < 10I_0$ . Their ranges are greater than 3 mm and having velocities  $0.7c > V \geq 0.3c$ .
- c) **Shower particles**: The relativistic shower tracks having velocity  $> 0.7c$  with ionization less than or equal to  $1.4I_0$  are mainly produced by pions and are not generally confined within the emulsion pellicle.
- d) **The projectile fragments** are a different class of tracks with constant ionization, long range and small emission angle.

To ensure that the targets in the emulsion were silver or bromine nuclei, we had chosen only the events with at least eight heavy ionizing (black + grey) tracks. According to the above selection procedure, we had chosen 350 events of  $^{28}\text{Si}$ -AgBr interactions at 14.5 AGeV. The emission angle ( $\theta$ ) and azimuthal angle ( $\phi$ ) with respect to the beam direction were measured for each grey track by taking the coordinates of the interaction point ( $X_0, Y_0, Z_0$ ), coordinates ( $X_1, Y_1, Z_1$ ) at the end of the linear portion of each secondary track and coordinate ( $X_i, Y_i, Z_i$ ) of a point on the incident beam. It is worthwhile to mention that emulsion technique possesses very high spatial resolution which makes it very effective detector for studying correlation phenomena.

### 3. Scaled factorial moment method

We consider that  $\Delta X$  is a phase space interval in which the intermittency analysis is to be carried out.  $\Delta X$  has been subdivided into  $M$  bins, each having phase-space width  $\delta X = \Delta X / M$ . The normalized factorial moment  $F_q$  of order  $q$  is defined as [2]

$$F_q(\delta X) = M^{q-1} \frac{\sum n_m(n_m-1)\dots(n_m-q+1)}{\langle n_m \rangle^q}, \quad (1)$$

where,  $n_m$  is the multiplicity in the  $m^{\text{th}}$  bin.  $\langle \dots \rangle$  Indicates average over the whole sample of events. For given  $q$  and  $M$  values,  $F_q$ 's are calculated for all the events and then averaged over the whole sample of events to obtain  $\langle F_q \rangle$ . The unique feature of this moment is that it can detect and characterize the non-statistical density fluctuations in particle spectra, which are intimately connected with the dynamics of particle production.

If the non-statistical fluctuations are self-similar in nature, in the limit of small bin size, factorial moment follow a power law behaviour like

$$\langle F_q \rangle \propto M^{\alpha_q}$$

$$\ln \langle F_q \rangle = \alpha_q \ln M + e. \quad (2)$$

This property, in analogy with turbulent fluid dynamics, is called 'intermittency'.  $\alpha_q$  measures the strength of the intermittency and is called the intermittency exponent and  $e$  is a constant. The intermittency exponents  $\alpha_q$  is obtained by performing best fits according to eq (2).

The generalized dimension  $D_q$  related to the intermittency exponent as

$$\begin{aligned} D_q &= 1 - \alpha_q / (q - 1) \\ &= 1 - d_q, \end{aligned} \quad (3)$$

where  $d_q = \alpha_q / (q - 1)$ .

#### 4. Results and discussion

In this paper, analysis is done on multifragmentation data in the light of scaled factorial moment method discussed above and an enquiry has been made to see whether the intermittent behaviour can be obtained in the target fragmentation process. In the analysis of the intermittent behaviour, usually, the variable rapidity is used. But for emulsion studies, it is very difficult to measure the rapidity of each particle and the pseudorapidity ( $\eta = -\ln \tan \theta/2$ ) is used because rapidity reduces to pseudorapidity for the relativistic particles. Since for target-associated particles, pseudorapidity can not be defined due to the larger mass of particles, the emission angle  $\cos\theta$  and the azimuthal angle  $\phi$  are used as the basic variables for this data analysis.

For scaled factorial moment analysis (F-moment), we have used the experimental data of medium energy protons emitted in  $^{28}\text{Si-AgBr}$  interaction at 14.5 A GeV. We have divided  $\cos\theta$  and  $\phi$  space into  $M$  bins with  $M = 2, 3, 4, \dots, 20$  in each case. The normalized scaled factorial moments of order 2 and 3 are evaluated

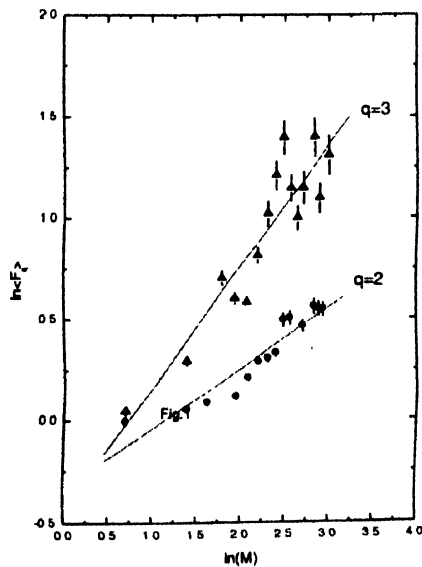


Figure 1. Plot of the dependence of logarithm of factorial moment of orders  $q = 2$  and  $3$  in emission angle space( $\cos\theta$ ).

using formulae (1) and (2). Figures 1 and 2 manifest the nature of variation of reduced normalized scaled factorial moments of order 2 and 3 with number of bins  $M$  for  $\cos\theta$  and  $\phi$  space respectively. Here, since  $\lambda^2$ /degrees of freedom is not less than one, the log-log plot does not show perfect linear behaviour. However, this data may suggest the intermittent behaviour of medium energy protons. The error bars shown in Figures 1 and 2 are nothing but standard statistical errors calculated from the standard deviation of the event-wise factorial moments. The intermittency exponents  $\alpha_q$  are evaluated by performing linear fits according to eq (2) for transformed variable  $\cos\theta$  and  $\phi$  variables. From the values of  $\alpha_q$ , we have calculated  $d_q$  and  $D_q$  using eq (3). In case of medium energy knocked out protons, the values of  $\alpha_q$ ,  $\alpha_q$  and  $D_q$  for order  $q = 2$  and  $q = 3$  for transformed variables  $\cos\theta$  and  $\phi$ , are tabulated in Tables 1 and 2 respectively. For comparison, the same values for low energy target fragments [14] are given in the same tables.

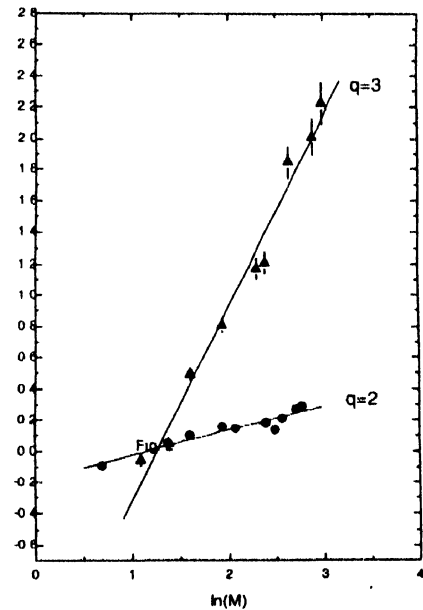


Figure 2. Plot of the dependence of logarithm of factorial moment of orders  $q = 2$  and  $3$  in azimuthal space( $\phi$ )

Figures 1 and 2 clearly suggest the intermittent behaviour of particles emission. Tables 1 and 2 show that for both slow target fragments and fast target protons, the intermittency exponents  $\alpha_q$  increase with the order of moment. The study of scaled factorial moment of the medium energy protons from  $^{28}\text{Si-AgBr}$  interactions at 14.5 A GeV reveals the intermittent pattern of fluctuation in both emission angle and azimuthal angle phase space, suggesting self-similarity of the emission process.

From Tables 1 and 2, it is also seen that for medium energy protons, the values of  $D_q$  decrease with order of  $q$  in  $\cos\theta$ -space but the values of  $D_q$  does not change with order of  $q$  in  $\phi$ -space. Again for low energy target fragments[ 14], the values

**Table 1.** Values of intermittency exponents for different orders in emission angle space ( $\cos\theta$ ).

Interaction	Types of particles	$q$	$D_q$		
$^{28}\text{Si-AgBr}(14.5 \text{ AG eV})$	Medium energy protons	2	$0.16 \pm 0.018$	$0.16 \pm 0.018$	$0.84 \pm 0.018$
		3	$1.23 \pm 0.067$	$0.66 \pm 0.033$	$0.34 \pm 0.033$
$^{28}\text{Si-AgBr}(14.5 \text{ AG eV})$	Low energy target fragments	2	$0.19 \pm 0.007$	$0.14 \pm 0.007$	$0.86 \pm 0.007$
		3	$1.41 \pm 0.040$	$0.57 \pm 0.020$	$0.43 \pm 0.020$

**Table-2.** Values of intermittency exponents for different orders in azimuthal space( $\phi$ ).

Interaction	Types of particles	$q$	$D_q$		
$^{28}\text{Si-AgBr}(14.5 \text{ AG eV})$	Medium energy protons	2	$0.30 \pm 0.031$	$0.30 \pm 0.031$	$0.70 \pm 0.031$
		3	$0.60 \pm 0.070$	$0.03 \pm 0.035$	$0.70 \pm 0.035$
$^{28}\text{Si-AgBr}(14.5 \text{ AG eV})$	Low energy target fragments	2	$0.05 \pm 0.006$	$0.05 \pm 0.006$	$0.95 \pm 0.006$
		3	$0.28 \pm 0.019$	$0.09 \pm 0.009$	$0.91 \pm 0.009$

of  $D_q$  decrease with the order of  $q$  in both  $\cos\theta$  and  $\phi$  spaces. From this observation, we may conclude that medium energy protons emitted in  $^{28}\text{Si-AgBr}$  interaction at 14.5 AGeV, show multifractal behaviour in  $\cos\theta$ -space and monofractal behaviour in  $\phi$ -space. But for the same interaction, low energy target fragments have multifractal nature in both  $\cos\theta$ - and  $\phi$ - spaces.

## 5. Conclusions

Although the data does not show perfect linear behaviour, this analysis may be an indication of intermittent type fluctuation of medium energy protons in  $^{28}\text{Si-AgBr}$  interactions at 14.5 A GeV.

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