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Multifractality and multifractal specific heat in fragmentation process in ^{24}Mg-AgBr interaction at 4.5 AGeV

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Abstract : We have investigated the multifractality of target fragments of ^{24}Mg-AgBr interaction at low energy (4.5AGeV) using a new method as proposed by Takagi. The analysis involves the step of measuring the generalised dimension D_q which in turn, deduce the multifractal behaviour of target fragments. Ultimately we determine multifractal specific heat. A comparison with other data is also done

Keywords : Heavy ion interaction, fragmentation, multifractality, specific heat

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1. Introduction

The analysis carried on multiparticle production process revealed the hint of possible existence of fractal properties in production process. Nuclear targets offer a unique property to learn about the space time development of the formation of secondary hadrons within very small distances and short times from the impact. Fractals are self-similar objects of a non-integral dimension. More complicated self-similar objects exist, consisting of differently weighted fractals with different non-integer dimensions. They are called multifractals and are characterized by generalized dimensions D_q which depend on the rank q of the moment of the probability distribution over such objects. Fractal geometry enables one to characterize a system mathematically, which is intrinsically irregular at all scales. The unique property of the fractal structure is that if a small portion of it is magnified, same complexity alike the entire system is observed. This property signifies the irregularity in the system. Fractals are of two types –uniform fractals and multifractals. A fractal can be imagined as a collection of fragments which may form a hierarchy or a tree, where there are $N_n = \mu^n$ fragments with sizes $l_1, l_2, ...l_N$ in the n-th generation. A multifractal is constructed by iteration of two or more 1st generation length scales. The fundamental characteristic of multifractality is that the scaling properties may be different for different regions of the system. The idea is therefore to construct a formalism that is able to describe systems with local properties of self-similarity. The earlier method to analyze fractality was G-moment method. But this method suffers from the problem that the experimental data sets do not show the linear behaviour in a log-log plot of moment against bin size as expected from the mathematical formulations. Due to these shortcomings, it looses its importance and new method was introduced to carry the ball to goal. Takagi proposed a method [1] in this context in which he overcomes all the defects associated with the earlier method. This method was applied to electron-positron annihilations [2,3] and UA5 data on proton-antiproton collisions [4] to extract fractal properties. Recently, Bershadskii [5] has reported a new method for calculating multifractal specific heat. Following that method, we also found out multifractal specific heat in our case of ^{24}Mg-AgBr interaction at 4.5 AgeV. Our study on this data reports about the fractal nature of particles which causes the black tracks and hence calculation of specific heat using Takagi's moment.

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2. Experimental details

The fractality in multiparticle production process is precisely studied using the interaction data initiated by ^{24}Mg-AgBr at 4.5 AGeV. The data set used in this present analysis, were obtained by irradiating stacks of NIKFIBR2 nuclear emulsion plates by an ^{24}Mg beam with incident energy 4.5 AGeV at JINR Dubna. The scanning of the plates was carried out with the help of a high resolution Leitz metalloplan microscope provided with an online computer system. The details of scanning and measurement are given in our previous work [6].

To carry out this analysis, we have considered only the slow target fragments known as black particles. In 'evaporation model', the particles corresponding to shower and grey tracks are emitted from the nucleus very soon after the instant of impact, leaving the hot residual nucleus in an excited state. Emission of particles from this state takes place relatively slowly. In order to escape from this residual nucleus, a particle must await a favourable statistical fluctuation, as a result of random collisions between the nucleons within the nucleus, which takes the particle close to the nuclear boundary, travelling in an outer direction. After evaporation of this particle, a second particle is brought to the favourable condition for evaporation and so on, until the excitation energy of the residual nucleus is so small that the transition to the ground state is likely to be affected by the emission of rays. According to the emulsion terminology, black tracks are identified by ionisation lying between $1.4I_0 < I \leq 10I_0$ where I_0 is the minimum ionisation of a singly charged particle. These Black tracks have the range <3mm.

The azimuthal angle (ϕ) of each black track with respect to the beam axis is calculated by noting the space coordinates of the interaction center (x_0, y_0, z_0), one point on the incident beam track and one point on the respective secondary track and applying simple coordinate geometry. The average multiplicity of produced black particles of the sample is 9.6 ± 0.47.

3. Method of study

Here, we consider the multiparticle production process at some incident energy. In such process, the particle distribution is considered in a phase space x. A single event contains n particles. The multiplicity n changes from event to event according to the distribution $P_n(x)$. The selected phase space interval of length x has been divided to M bins of equal size, the width of each bin being $\delta x = x/M$. Then the multiplicity distribution for a single bin is denoted as $P_n(\delta x)$ for n = 0,1,2,3......where we assume that the inclusive particle distribution dn/dx is constant and $P_n(\delta x)$ is independent of the location of the bin. n hadrons, contained in a single event, is distributed in the interval $x_{min} < x < x_{max}$. The multiplicity n changes from event to event according to the distribution $P_n(x)$, where $x = x_{max} - x_{min}$. If the number of independent event is Ω, then the particle produced in those events are distributed in ΩM bins of size δx. Let N be the total number of target associated slow particles produced in these Ω events and n_{aj} the multiplicity of black particles in the j-th bin of the a-th event.

The theory of multifractals [7] has been motivated to consider the normalized density P_{aj} defined by

$$P_{aj} = n_{aj}/N. \tag{1}$$

And to consider the quantity

$$T_q(\delta x) = \ln \sum_{a=1}^{\Omega}\sum_{j=1}^{M} P_{aj}^q \quad \text{for } q > 0. \tag{2}$$

Evaluating the double sum of P_{aj}^q, Takagi showed that for sufficiently large Ω,

$$\ln\langle n^q\rangle = A_q + \{(q-1)\,D_q + 1\}\ln\langle n\rangle \tag{3}$$

$$= A_q + (B_q + 1)\ln < n >,$$

a simple linear relation between $\ln\langle n^q\rangle$ and $\ln < n >$. The generalized dimension D_q can be obtained from the slope values using the relation

$$D_q = B_q/(q-1).$$

The case with $q = 1$ is obtained by taking an appropriate limit [8]. D_1 can be obtained from the relation

$$\langle n \ln n\rangle / < n > = C_1 + D_1 \ln < n >. \tag{4}$$

So far, the methodology is developed for non-overlapping x bins, but it can also be applicable for overlapping bins [9].

Multifractality can also be defined with so called fractal dimension $f(\alpha)$ [10]. B_q is related with $f(\alpha)$ by Legendre transformation as

$$B_q = q\alpha - f(\alpha). \tag{5}$$

From eq. (5), it is clear that D_q is evaluated by the particular value of $\alpha(q)$ determined by the extreme condition

$$df/d\alpha = q. \tag{6}$$

These relationships with the thermodynamic interpretation reveals that q can be related with an inverse temperature by the relation $q = T^{-1}$. Here, the spectrum $f(\alpha)$ and α play the role of the entropy and energy (per unit volume) correspondingly [10-12].

Bershadskii [5] has pointed out that in many important thermodynamical cases, the specific heats of solids and gases are constant, independent of temperature over a certain temperature interval [13]. The entropy $f(q)$ in this case, can be approximated by the following relation

$$f(q) = a - c\ln q, \tag{7}$$

where a is constant and c is constant specific heat (CSH). Using the constant a, one can get the generalised dimension D_q from CSH approximation as

$$D_q = (a-c) + c\ln q/(q-1). \tag{8}$$

If one plots D_q *versus* $\ln q/(q-1)$, the slope of the graph will give the value of specific heat. In this paper, we investigate the multifractal nature of ^{24}Mg-AgBr interaction at 4.5AGeV and from the knowledge of D_q, we calculate the specific heat using eq. (8). On the other hand, corresponding to (7), the information dimension is chosen as $D_1 \cong a$. It is well known that information dimension D_1 [14] is the exponent of the scaling of the number of bins containing the dominant contribution to the mass and could be very sensitive to the dynamics of particle production.

4. Analysis and discussions

To analyze our data, it is considered that a single event contains n black particles distributed in a particular interval in azimuthal angle phase space. This interval is divided into overlapping bins whose size is varied in steps of 15. The central value is 180.

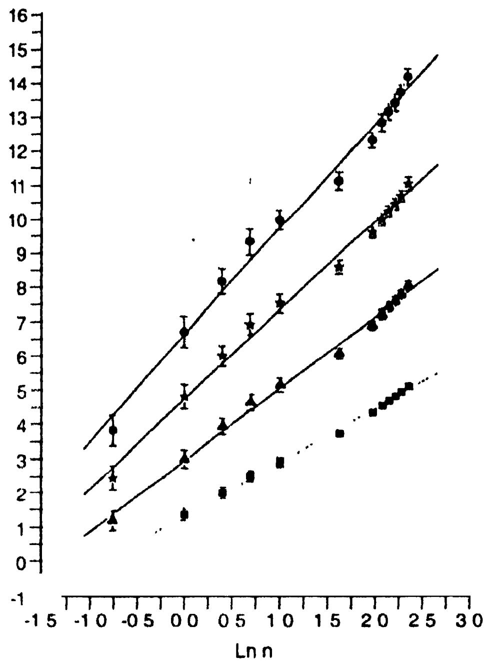

Figure 1. Variation of ln $<n^q>$ with ln $<n>$.

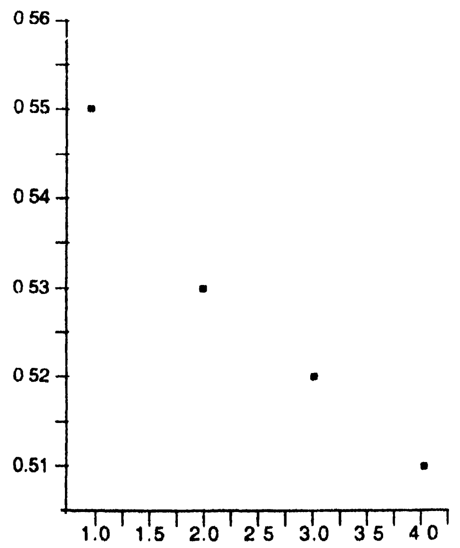

Figure 2. Variation of D_q with q.

In Figure 1, ln $< n^q >$ is plotted against ln $<n>$ for $q = 1,2,3,4$. A linear behaviour is obtained in all of the above log-log plot. The generalized dimensions D_2, D_3, D_4, D_5 are obtained from the slopes of such linear plots by using eq. (3). D_1 is obtained from eq. (4). In Figure 2, D_q is plotted against q. This plot shows that D_q decreases gradually with increase of q, which suggests multifractality in case of target evaporated particles. We also plot D_q against $\ln(q)/(q-1)$ which is a linear plot. From the slope of its linearity, we get the specific heat. The generalised dimensions are given in Table 1. In Table 2, specific heat is given. Moreover, for comparison, we also included the data of ^{32}S-AgBr interaction at 200AGeV(15), ^{16}O-AgBr interaction at 60AGeV[15] and ^{28}Si-AgBr interaction at 14.5AGeV(15) in Table-2. Thus, this analysis shows that even at very low energy (4.5AGeV), the target fragments show multifractal behaviour.

Table 1. Generalished dimensions values of D_q for different q values of ^{24}Mg – AgBr interaction

DATA	D_1	D_2	D_3	D_4	Ref
^{24}Mg-AgBr at 4 5AGeV	55	53	52	51	This work

Table-2. Specific heat for ^{24} Mg – AgBr interaction and comparison with different data sets at different energies

DATA	Specific heat
^{24}Mg-AgBr at 4 5 AGeV	0 20
^{28}Si-AgBr at 14 5 AGeV	0 26
^{16}O-AgBr at 60 AGeV	0 13
^{32}S-AgBr at 200 AGeV	0 21

It is also interesting to observe that multifractal specific heat values are different in interaction at different energies. No

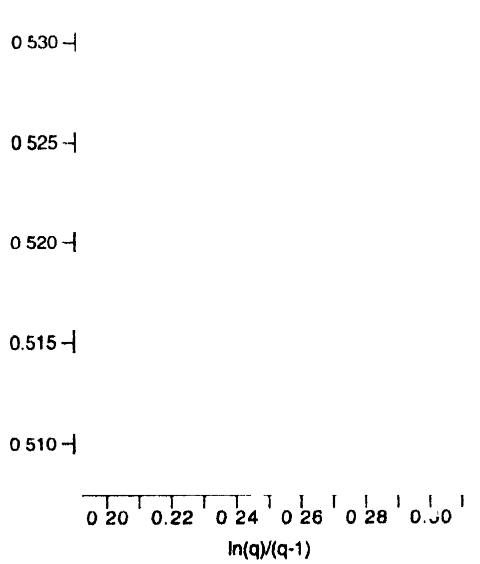

Figure 3. Variation of D_q with ln $(q)/(q-1)$.

systematic energy or projectile dependence is exhibited by the data.

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