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Energy and intensity distributions in double-photon Compton scattering*

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Abstract : The scattering process of a single incident photon by an electron into a final state consisting of two simultaneously emitted photons is called double-photon Compton scattering. The theory and elementary features of this higher order process are described. The reported experimental measurements so far on this higher order process, demonstrate conclusively its existence. Some elementary features of the theory of this process have also been confirmed in some of the measurements.

Keywords : Double-photon compton scattering, energy and intensity distribution, experimental data.

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1. Introduction

The phenomenon of Compton scattering is well known in atomic and nuclear physics. In the collision between a photon and an electron, it is impossible to state with certainty that the final state of the photon-electron system consists of one photon only. The scattering process of a single incident photon by an electron into a final state consisting of two photons is called double-photon Compton scattering and is believed to be well described by the standard quantum electrodynamics. Heitler and Nordheim [1] postulated the existence of this phenomenon and calculated order of magnitude of cross section in restrictive conditions, unfavourable for experimental verification. Eliezer [2] set up an expression for collision differential cross section of this process in the limiting case of one hard and one soft photon only. Mandl and Skyrme [3] using *S*-matrix formalism of quantum electrodynamics have provided an exact theory of this process. Their expression for the collision differential cross section can be regarded as double-photon Compton analog of the well-known Klein-Nishina relation for single-photon Compton scattering.

The phenomenon of double-photon Compton scattering is important because it provides a test of quantum electrodynamics implicitly, although QED has been tested to much higher accuracy. This higher order process provides a mechanism of photon multiplication [4] along with the bremsstrahlung in astrophysics and there is appreciable contribution of this effect to total scattering coefficients at extremely higher energies where this phenomenon is more likely to occur. Moreover, this phenomenon requires investigation in detail because it is a major background process to the study of another non-linear QED process namely photon splitting in the fields of heavy atoms, the first experimental confirmation of which has recently been reported by Akhmadaliev *et al* [5]. The data analysis of their experiment, in the energy region of 120–450 MeV, results in about 400 photon-splitting events for 1.6×10^9 photons incident on the BGO target. The significant background process contributing to the registered events being from double-photon Compton scattering of the incident photons by the atomic electrons.

2. Theory

Mandl and Skyrme [3], using *S*-matrix formalism of QED, have provided the currently acceptable theory of this process. The energy k_2 (in m_0c^2 units) of one of the two final photons, in terms of k_0 (incident photon energy), k_1 (energy of the other photon taken as independent final photon energy) and scattering angles is given by

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*Invited Talk

$$k_2 = \frac{k_0 - k_1[1 + k_0(1-\cos\theta_1)]}{1 + k_0(1-\cos\theta_2) - k_1(1-\cos\theta_{12})} \tag{1}$$

where the angles θ_1 and (θ_2, ϕ_2) define the directions of emission of two final photons having energies k_1 and k_2 respectively. θ_{12} being the angle between momentum vectors of the two emitted photons. The energy k_2 (k_0; k_1, θ_1, θ_2, ϕ_2) is a function of five variables in contrast to a function of two variables $k'(k_0;\ \theta)$ in case of single-photon Compton scattering.

The following expressions provide kinetic energy (T) and direction (θ_3, ϕ_3) of the recoil electron in this process.

$$T = \frac{k_0k_1(1-\cos\theta_1) + k_0(k_0 - k_1)(1-\cos\theta_2) + k_1(k_1 - k_0)(1-\cos\theta_{12})}{1 + k_0(1-\cos\theta_2) - k_1(1-\cos\theta_{12})} \tag{2}$$

$$\theta_3 = \cot^{-1} \frac{k_1\cos\theta_1 + k_2\cos\theta_2 - k_0}{(k_1^2\sin^2\theta_1 + k_2^2\sin^2\theta_2 + 2k_1k_2 \sin\theta_1\sin\theta_2\cos\phi_2)^{1/2}} \tag{3}$$

$$\phi_3 = \cot^{-1}\left[\frac{k_1\sin\theta_1 + k_2\sin\theta_2\cos\phi_2}{k_2\sin\theta_2\sin\phi_2}\right] \tag{4}$$

These expressions like k_2 (k_0; k_1, θ_1, θ_2, ϕ_2) are also functions of five independent variables.

The collision differential cross section for double-photon Compton scattering for the case in which a photon with energy in the interval k_1 and $k_1 + dk_1$ being ejected into an element of solid angle $d\Omega_1$ in the direction θ_1, the other emitted photon (whose energy being given by eq. (1)) being ejected into an element of solid angle $d\Omega_2$ in the direction (θ_2, ϕ_2), is given by the following equation :

$$\left.\frac{d^3\sigma_D}{d\Omega_1 d\Omega_2 dk_1}\right|_{\text{collision}} = \frac{\alpha r_0^2}{16\pi^2}\frac{k_1k_2}{k_0}\frac{X}{T_c} \tag{5}$$

where α is the fine structure constant and r_o is the classical electron radius. The parameters X and T_c are complicated functions of the photon energies and scattering angles, and are given elsewhere [3].

These collision cross sections refer to the number of collisions of any particular type and thus describe the number of pairs of photons scattered in a particular geometry as a fraction of the number of incident photons. In sharp contrast, double-photon Compton scattering cross sections [6] refer to the amount of energy scattered in a particular geometry, and thus describe the energy content of the photons which are scattered in a particular geometry as a fraction of incident intensity. The scattering differential scattering cross section representing scattering or mere deflection of gamma photons and absorption differential cross section representing true absorption of energy from the gamma beam are given elsewhere [6]. The complicated form of X/T_i (with $i = c$, s or a for collision, scattering and absorption cross sections respectively) is responsible for the difficulty of surveying the features of the cross section formulae [3 and 6].

3. Experimental data available on double-photon Compton scattering

The principle of measurements, reported on this process so far, is based upon detection of two simultaneously emitted gamma quanta using two gamma ray spectrometers working in coincidence. The double-photon Compton cross section involves two solid angles and one independent final photon energy. Thus, all the experimental observations on this process performed till now, are based on coincidence measurements [6–18]. In these experiments, the directions of both final photons are kept fixed and their coincidences are counted. The greatest difficulty in such experiments, lies in the low value of intensity to be measured; in fact, the cross section for this process is already low in itself, and in such experiments, one selects the pairs of photons emitted into two small solid angles around the fixed direction.

Prior to 1952, there had been no experimental data available on double-photon Compton scattering. Cavanagh [7] is first to confirm this phenomenon experimentally. He used a 200 mCi ^{60}Co gamma ray source and Ag, Be, Al, Cu scatterers of thickness varying from 40–400 mg-cm^{-2}. The double-photon Compton cross section, integrated over the energy range of 80–530 keV, is found to be 3 × 10^{-3} of the single-photon Compton cross section. A value for coincidence count rate per recorded quanta of 0.4 × 10^{-4} results and is to be compared with the experimental value of 1.0 × 10^{-4}. His basic assumption of isotropic emission of one of the two final photons is incorrect. In the experimental set-up used, both the detectors accept gamma quanta with scattering angles in the range of 45°–145° (resulting in poor geometry) and elimination of coincidences resulting from detector to detector scattering is not total.

A number of other workers have also performed their experiments to confirm the existence of this process and the details have been provided by McGie *et al* [8]. They also studied the collision differential cross section for double-photon Compton scattering as a function of independent final photon energy for 0.662 MeV incident photons from 6 Ci ^{137}Cs radioactive source, with two NaI(Tl) scintillation detectors placed at 90° to each other and also to the incident beam. The resolving time of coincidence set-up being 110 nsec and scatterer being of beryllium of 9.884, 18.794 and 49.162 mg-cm^{-2} thickness. The measured differential cross-sections are in agreement with theory. The total cross section for this process for the energy range of E_1, $E_2 \geq 31.1$ keV comes out to be $(4.00^{+0.20}_{-0.12}) \times 10^{-30}$ cm^2 Sr^{-2} which agrees with the theoretical value of 4.14×10^{-30} cm^2 Sr^{-2}. McGie and Brady [9] again measured the intensity distribution of the two photons emitted in this process at $\theta_1 = \theta_2 = \pi/2$ and $\phi_2 = 120°$. The estimated error in their measurements being ±10%.

Sekhon *et al* [10] reported measurements on double-photon Compton process with a beam of 661.65 keV photons from 110 mCi ^{137}Cs radioactive source at different scattering angles $\theta_1 = 30°$ to 150° and $\theta_2 = \phi_2 = \pi/2$. Two NaI(Tl) scintillation detectors of dimensions 51 mm × 51 mm and 45 mm × 25 mm, working in coincidence are used to detect the two scattered photons. A target of aluminium of thickness 40 mg-cm^{-2} and area 25 cm^2 (square in size) is used. The cross section values measured for E_1, $E_2 \geq 100$ keV show agreement with the theory of Mandl and Skyrme [3] at small scattering angles but at large scattering angles, these are somewhat higher than the theory predicts.

Sandhu *et al* [11–17] carried out measurements for energy, intensity and angular distributions of double-photon Compton process. An intense and collimated beam of gamma photons of 0.662 MeV energy, obtained from 8 Ci ^{137}Cs radioactive source, is made to impinge on an aluminium foil. The two simultaneously emitted photons are detected by two NaI(Tl) detectors of dimensions 51 mm × 51 mm and 45 mm × 25 mm placed at desired angular positions, subtending the solid angles at the scattering centre 0.13 and 0.27% respectively. The angular spread due to detector apertures are 8.3° and 11.9° respectively and are quite small in comparison to the 34° spread in an earlier measurement of McGie *et al* [8]. A timing electronics employing Canberra ARC timing amplifiers and of 30 nsec resolving time is used to record the coincidence spectra. The observed coincidence spectra are corrected for chance and coincidences unrelated to the target. The various sources contributing to false events are taken into account. The energy spread in the observed coincidence spectra due to angular aperture of the spectrometers is quite small compared to intrinsic resolution of the spectrometers while the spread due to finite energy window of the second photon is comparable to intrinsic resolution of NaI(Tl) scintillation spectrometer. The interdependence of energy between the two final photons in measurements [11, 13 and 15] are in agreement with theory [3] while the collision integral cross sections in measurements [11,12] are somewhat higher than theory as no method was applied to eliminate the Compton-bremsstrahlung (CB) coincidences from the observed coincidences due to double-photon Compton events. Later on, the aluminium foils of thickness 13.0, 27.5, 40.0 and 53.5 mg-cm^{-2} are used as scatterer for evaluation of coincidences resulting from Compton-bremsstrahlung events. A modified form of the experimental approach suggested by Cavanagh [7] is used to eliminate CB-events from the observed target-in coincidences. The collision differential cross sections in measurements [13–15], and scattering and absorption differential cross sections in measurements [6,14,16 and 17] are in agreement with theory within experimental estimated error of nearly 20%. The important features of theory like energy spectra of the emitted photons being continuous, the probability of occurrence of this process is higher when one of the emitted photon is soft and the emission of one of the photon in the forward direction is most likely to occur than that in the backward direction when second photon is emitted at 90° to the incident beam, are confirmed by the observed experimental results of double-photon Compton scattering.

Dewan *et al* [18] reported measurements on the energy distribution of gamma photons scattered in this process. An intense beam of gamma rays from 8 Ci ^{137}Cs radioactive source is made to fall on an aluminium target of thickness 17.48 mg-cm^{-2}. The two simultaneously emitted gamma quanta are detected using slow-fast coincidence set-up of 25 nsec resolving time in which the HPGe detector (56.4 mm × 29.5 mm) and NaI(Tl) scintillation detector (51 mm × 51 mm) are placed at 50° and 90° to the incident beam respectively. In one set of measurements, the two detectors are placed at 90° to each other ($\phi_2 = 90°$), while in the second, at 180° apart in the same plane ($\phi_2 = 180°$). Angular spreads due to two detector apertures are 11.1° and 10.4° respectively. The full energy peaks in the coincidence spectra corresponding to weighted energy values of one soft and hard photon are not symmetrical about their respective peak positions. This is because of the fact that double-photon Compton process being more probable with the emission of one hard and one soft photon rather than two photons of approximately equal energy. The experimental data on energy distribution do not suffer from inherent energy resolution of the gamma detector and confirm continuous nature of energy spectra for the emitted photons. A more faithful reproduction of the shape of distribution under the full energy peak favours the use of HPGe detector and contrary to this, the intensity measurements discourage the use because of its low intrinsic peak efficiency.

4. Conclusions

The expressions providing interdependence of energy between two final photons, energy and direction of the recoil electron, as well as for collision, scattering and absorption differential cross sections are quite complicated and their physical contents are difficult to display. So these expressions are computed for several experimentally realisable cases to get in depth knowledge about the features of this higher order process. The computational work and the experiments carried on this process, have revealed the following features of this process.

(i) The probability of occurrence of this process is quite small as compared to that of single-photon Compton scattering. Measurements [7–17] support this fact.

(ii) This process may or may not be coplanar. Some experimental measurements [12 and 18] confirm the coplanar nature, while the measurements [7–11 and 13–18] confirm non-coplanar nature.

(iii) The energy spectra in double-photon Compton scattering are continuous in contrast to the line spectra in case of single-photon Compton scattering. Measurements [8,11,13,15 and 18] have confirmed this elementary feature.

(iv) The energy available to two final photons in this process, is more than that available to the scattered photon in case of single-photon Compton scattering, except for the case when one of the final photon carries negligible amount of energy. Measurement [18] has confirmed this elementary feature.

(v) The kinetic energy of the recoil electron increases with the increase in the scattering angle of the hard photon and with the increase in incident photon energy. The plane of emission of the recoil electron lies close to the plane of emission of hard photon.

(vi) The electron recoils always in the forward hemisphere and more likely in the forward direction at higher incident energies and also when hard photon is scattered in the backward direction.

(vii) The probability of both the photons to be emitted in the direction of incident photon is zero. Both photons moving in direction of incident photon, implie that the incident photon splits into two photons with the electron remaining at rest, which according to QED is not allowed.

(viii) The process exhibits characteristic logarithmic divergence of the infrared catastrophe when one considers $E_1 \rightarrow 0$ or $E_2 \rightarrow 0$.

(ix) The probability of occurrence of one hard and one soft photon is more pronounced than that of two photons of approximately equal energies. This elementary feature has been confirmed in measurements [8, 13 and 14].

(x) The energy scattered into a particular geometry is more, hence higher scattering cross section, when the incident photon interacts with the free electron through double-photon Compton process in comparison to the case when the same interaction occurs through single-photon Compton process. This feature has been confirmed in measurements [6, 16 and 17].

(xi) The hard and soft photons are likely to be emitte into a narrow cone in the forward direction and this cone becomes much narrower with the increase in incident photon energy. The angular distribution of soft photon is not isotropic as assumed in measurement [7].

(xii) Double-photon Compton scattering has grea preference for small scattering angles, which becomes much more pronounced at higher inciden energies.

(xiii) At small scattering angles ($\theta_1 = \theta_2$), coplanar emission is less probable but at intermediate scattering angles, the coplanar emission is more probable. For large scattering angles, the probability of occurrence of this process is nearly independen of the azimuthal angle ϕ_2.

(xiv) The maxima of collision, scattering and absorption differential cross sections of this higher order process, shift towards small scattering angles θ_1, as the incident photon energy increases.

(xv) The probability of occurrence of this process increases, first sharply and then slowly with the increase in incident photon energy, while it decreases for single-photon Compton scattering.

The reported experimental measurements so far on this higher order process demonstrate conclusively its existence. Some elementary features of the theory of this process have also been confirmed in some of the measurements, but are confined to 0.662 MeV incident photons in a number of geometrical cases. It becomes desirable to obtain more experimental data on energy, intensity and angular distributions of the two emitted photons especially at small scattering angles and higher incident photon energies (such as 1.12 MeV and 6.14 MeV) to test the validity of currently acceptable theory.

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