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Simulation of *I-V* characteristics of Josephson junctions array magnetic field effect

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Abstract : The I-V characteristic of the Intrinsic stacked Josephson junctions (ISJJ) as a function of magnetic field are simulated. ISJJ has been modelled as one dimensional series array of Josephson junction. Each Josephson junction of the series is considered as resistively shunted junction model in which the effect of capacitance, inductance and a current source producing white noise are included. The critical currents of the ISJJ as function of magnetic field shows Fraunhoffer like pattern. The simulated results have been compared with the Tl2212 mesas experimental results. It has been found that the simulated results are in good agreement with the experimental one.

Keywords : Josephson junction array, intrinsic stack, superconductivity.

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1. Introduction

The Josephson junction (JJ) is a natural high frequency source tunable up to terahertz. However, the line width of the single JJ is large and maximum power is small. The line width can be narrowed and power can be increased by using phased locked arrays of many nearly identical JJ. In planar arrays if we increase the number of junctions, the lateral dimension soon exceed the wavelength, which restrict the tunability of device. This restriction can be avoided if vertical packing of the junction is done.

The single crystal and c-axis oriented thin films of high T_c superconductors (HTSC) forms natural stacks of Josephson junctions with CuO_2 layers acting as superconducting electrodes and the Bi_2O_3 or Tl_2O_3 or Y_2O_3 or (Ba,Ca)O layers acting as insulating barrier (Intrinsic Josephson effect) as shown in Figure 1.

However, the vertically stacked Josephson junction differs qualitatively from planar structure. The superconducting electrode thickness is much smaller than the London penetration depth, thus screening current in magnetic field or transport current flowing along the superconducting layer spread over λ_L couples many stacked junctions. The transport current flowing perpendicular to layer also coupled the adjacent junction

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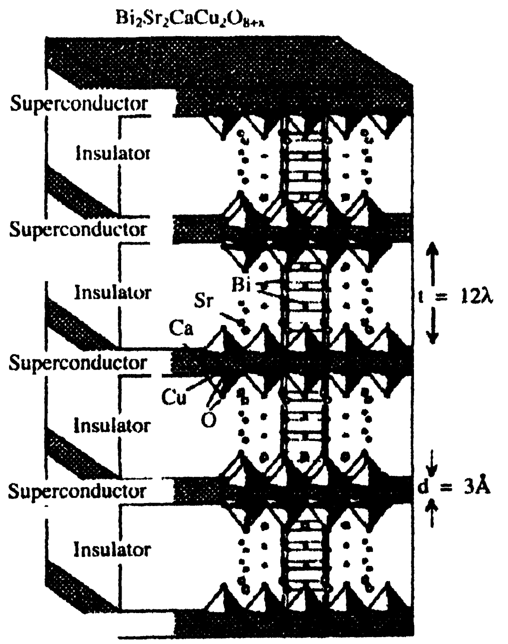

Figure 1. Crystal structure of the HT_C superconductor BSCCO (courtesy of R Kleiner).

via weakening the order parameter within the coherence length ξ, if ξ is greater than or comparable to electrode thickness. The third mechanism of coupling is quasi-particle diffusion (which takes part in transport current in the resistive state) over diffusion length, which is order of several microns.

To the first approximation we neglected the coupling effect in the vertical stacked JJs in the present *I-V* simulation. We simply treated each junction independently connected in series *i.e.* the same total current passes through each junction. The voltage developed across each junction in the stack was added up.

So far in *I-V* simulation of individual JJ the contribution of inductance were neglected. RSJ [1,2] and RCSJ [3] modelled were used in *I-V* simulation. Ignoring the contribution of inductance may be justified in SIS type of junction where capacitance is large in compression to inductance. Where in case of SNS proximity junctions, the value of capacitance is low and comparable to the inductance. In the present *I-V* simulation, contribution of both inductance and capacitance were taken into account, which is more general.

Large application potential in new generation devices such as sub-millimetre sources; detectors or mixer [4–6] and interesting physical properties of intrinsic stacked JJ encourage us to study the *I-V* characteristics of it.

2. Model and formulations

Intrinsic stacked Josephson junctions inside the single crystals or c-axis oriented thin film of high temperature super conductor (as shown in Figure 1) were modelled as a one-dimensional array of Josephson junctions connected in series as shown in Figure 2(b) and each Josephson junction is modeled as shown in Figure 2(a). Coupling between JJs has been neglected. *I-V* curve of each junction is simulated on the basis of Resistively Capacitively Inductively Shunted Junction (RCLSJ) model. RCLSJ model is the modified form of RCSJ model. In the RCSJ model inductance is connected in series with resistance as shown in Figure 2. The resistance R_n in RCSJ is replaced by the total impedance $\sqrt{R_n^2+\omega^2L^2}$

(a)

(b)

Figure 2. Schematic of Josephson junction and array in the RCLSJ model. The symbol ⊠ represents an ideal Josephson junction.

The response of the junction on the basis of RCLSJ model is given by

$$\frac{hC}{2e}\frac{d^2\phi}{dt^2}+\frac{h}{2eZ}\frac{d\phi}{dt}+I_C\sin\phi = I_{dc} + I_{ac}\sin\omega t + L(t), \quad (1)$$

where $Z=\sqrt{R_n^2+\omega^2L^2}$.

C, L, R_n and I_c denote the capacitance, inductance, normal resistance and critical current density of junction respectively. ω represents the frequency of microwave irradiation. On the right hand side, the driving current is given by dc part I_{dc} and microwave part I_{ac}. A random variable $L(t)$ representing white Gaussian process is added to equation for which $<L(t)L(t')> = 2\Gamma\delta(t-t')$, where noise parameter $\Gamma = 2ek_BT_c/\hbar I_c$ is the thermal energy normalized to Josephson coupling energy [7]. In our calculation white noise is approximated by series of rectangular pulses with Gaussian distributed amplitude and width is smaller than the every time constant of JJ.

Effect of magnetic field on *I-V* curve was simulated by introducing the variation of critical current (I_c) of JJ which follows the Fraunhoffer pattern. Thus, I_c in above equation was replaced by $I_cF(H)$. Where $F(H)$ is given by $F(H) = |\sin(k)/k|$ where $k = \pi H/H_0$ and $H_0 = \Phi_0/\mu_0 d.w$; H is external applied field and Φ_0 is flux quanta. $d = 2\lambda_L + t$, w = lateral dimension of junction, t = thickness of barrier.

Introducing a dimensionless time variable $\tau = \omega_c t$,

$$\omega_c = 2eI_cR_n/\hbar,\ \Omega = \omega/\omega_c,\ \beta_c = \omega_c.R_n.C,\ \beta_L = \frac{\omega L}{R},$$

$I_0 = I_{dc}/I_c$, $I_1 = I_{ac}/I_c$

The eq. (1) becomes

$$\beta\frac{d^2\phi}{d\tau^2}+\frac{d\phi}{d\tau}+F(H)\sin\phi = I_0 + I_1\sin\Omega\tau + L(\tau) \quad (2)$$

where $\beta = \beta_c(1+\beta_L^2)$.

β_c is the Stewart-McCumber parameter used in RCSJ model. The above differential equation has been solved using fourth order Runga Kutta method.

3. Results and discussion

In our present simulation, series array of two junctions were only taken. Taking more junctions does not change the physics but increases the calculation time. Figure 3 shows the simulated I-V curves of series array of two junctions in different magnetic fields mentioned against each curve in normalised form H/H_0, where H0 represent the field at which the 1st quanta of flux enter into the junction. The arrow shows the scan direction of the current. The values of current in forward at which the 1st and 2nd jumps appear represent the critical current (I_c) of the first and second junctions of array respectively. The steps in the backward scan of the current represent the retraping current (I_r). I-V curves in Figure 3 clearly show the variation of I_c as a function of magnetic field. Hysteresis effect is also pronounced in the simulation

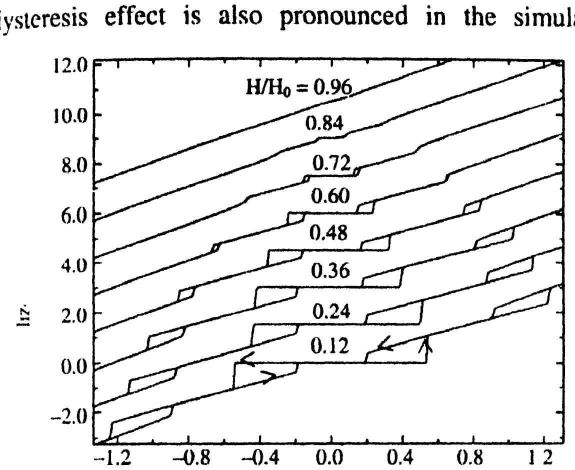

Figure 3. I V characteristics curves of an array of two Josephson junctions with the variation of Applied Magnetic Field. Parameters are $\beta_c = 20$, $\beta_1 = 0.001$, $I_1 = 0.001$, $\Omega = 1.0$, noise level = 0.01 The curves are 1 0 vertically shifted for clarity.

result. The junction parameter β_c =20 is taken in the simulation. The hysteresis is also decreasing with increasing H.

The Figure 4 shows the experimental I-V characteristics of Tl_2–Ba_2–Ca_1–Cu_2–O_8 mesas at different magnetic field [8]. The shape of the experimental I-V characteristic of the Tl2212 mesas as in Figure 4 clearly matches with the simulated results as shown in Figure 3 from the simulated results. It is very much clear that the I-V of mesas of Tl2212 is like the intrinsic stacked Josephson junction. The hysterisis effect is also pronounced in the experimental result and also it decreases with the field. Thus, it is clear that the β_c of JJs in the mesas of Tl2212 is of higher value *i.e.* (β >>1). The effect of noise is pronounced near H_0 where it starts rounding the steps in I-V curve as it is clear from simulated results in Figure 3. The same features are also present in the experimental results in Figure 4. Near H_0 the critical current of the junction becomes small and it is comparable to the value of the noise current and thus the effect of noise is pronounced.

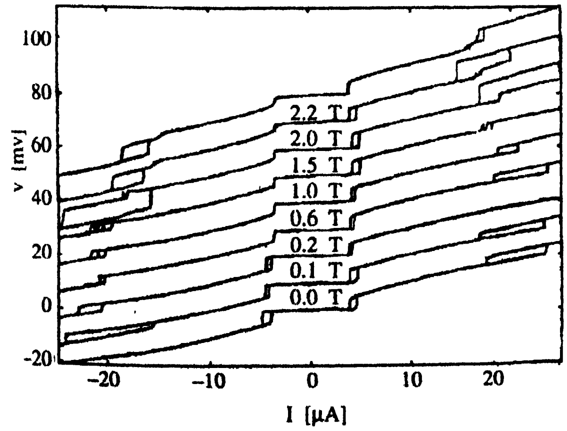

Figure 4. Experimental I-V curve as a function of magnetic field [8].

The variation of I_c, I_cR_n and I_r of a junction in array as a function of H from simulated I-V curve is shown in Figure 5. It is clear from the figure that I_c, I_cR_n and I_r of the junction follow like Fraunhofer pattern. The experimental results of the variation of critical current of one junction of intrinsic stacked JJ as function of field in the TBCCO mesas is shown in Figure 6. The experimental results also show Fraunhofer like variation with slight

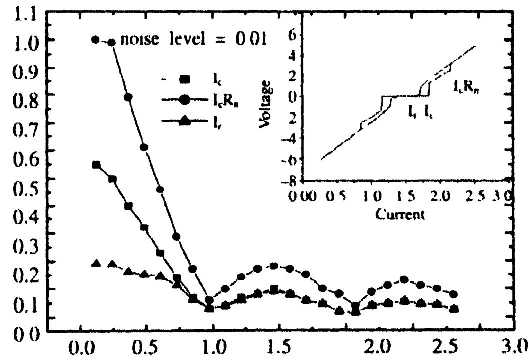

Figure 5. Effect of applied magnetic field on critical current (I_c), critical voltage (I_cR_n), and retrapping current (I_r) at noise level = 0.01. Inset shows the definition of I_c, I_cR_n and I_r.

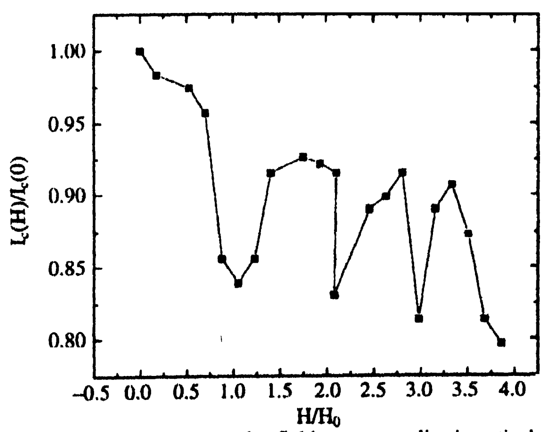

Figure 6. Normalized magnetic field *vs* normalized critical current graph drawn from experimental data [8].

deviation. The critical current does not become zero at H_0, $2H_0$ or $3H_0$ *etc*. This may be due to the small misalignment of magnetic field parallel to the ab- plane of mesas, which will leads to the formation of Abrikosov vortices penetrating the electrodes. These vortices will cause local variations of the critical current density in c-direction [9,10] and therefore strongly influence the Fraunhofer pattern. The simulated curve is in good agreement with the experimental results [8] of intrinsic stacked Josephson junction in mesas of TBCCO.

We conclude that the simulation of I-V characteristic of Intrinsic stacked Josephson junction as a function of magnetic field on the basis of series array of JJs and each junction modelled as RCLSJ, clearly reproduce the experimental I-V characteristic of Tl2212 mesas. But it differs in minute details. This discrepancy occurs due to neglecting the effect of vortices on I_c. Work is in progress to take the Abrikosov vortices effect.

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