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Linear instabilities in multi-ion plasmas

C P Anil Kumar[1]*, C Venugopal[1], B George[2] and G Renuka[2]

[1]School of Pure & Applied Physics, Mahatma Gandhi University, Priyadarshini Hills, Kottayam-686 560, Kerala, India

[2]Department of Physics, University of Kerala, Kariyavattom Campus Thiruvananthapuram-695 581, Kerala, India

E-mail : cpanil@cusat.ac in

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Abstract : In this paper, the stability of ion-cyclotron waves in multi-ion plasma consisting of electrons, positively and negatively charged oxygen ions and hydrogen ions are examined by simulations. In order to identify the modes in a mixed plasma and to gain physical insight into the nature of the instability, cold plasma theory is used, while the warm plasma theory accounts for finite temperature effects. The expression for the frequencies of lower hybrid and ion-ion hybrid modes are derived and solved by numerical methods. From the simulation studies it is concluded that lower hybrid mode is a weakly unstable mode while the ion-ion hybrid mode is almost freely propagating

Keywords : Ion cyclotron wave, Buchsbaum mode

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1. Introduction

Ion cyclotron waves were first studied by Kindel and Kennel [1] who showed that for current carried by a streaming electron distribution in the presence of stationary Maxwellian ions, ion-cyclotron waves have the lowest threshold among the current driven instabilities in an isothermal plasma ($T_e = T_i$). Later, Abdalla et al [2] showed that the addition of heavy ions in the plasma dispersion modified the lower - hybrid mode and also allowed an ion-ion hybrid mode *i.e* the Buchsbaum mode to exist [3, 4]. The ion-ion mode has a frequency between the proton and heavy ion gyro-frequency and is driven unstable at oblique wave angles, by an electron beam.

The plasma wave probe carried by the spacecraft *International Cometary Explorer (ICE)* detected a number of ion-acoustic and ion-cyclotron waves in the environment of comet Giacobini-Zinner. During ICEs nearest approach, the plasma wave instrument detected broad-band electrostatic noise and a changing pattern of weak electron plasma oscillations for the outer layers of the cold plasma tail. Near the tail axis, the plasma wave instruments also detected whistler mode ($f < 100$ Hz) and lower hybrid mode ($f < 10$ Hz) [5].

The comet literature contains many theoretical suggestions involving the generation of various plasma instabilities [6-15]. These studies either modelled cometary environments or were easily adaptable to them. The analysis presented here, illustrates the impressive degree to which ion-cyclotron instability and newborn ion species interact over a region, that extends beyond the comet boundaries (*i.e*, tail). The expressions for the frequencies of the 'lower hybrid' and 'ion-ion hybrid' modes are derived using cold plasma theory and considered the stability of these modes. It is observed that the lower hybrid mode is weakly unstable, while the ion-ion hybrid mode is an almost freely propagating mode. The model approximates the plasma environment of the tail of comet Giacobini - Zinner.

2. Method of analysis

In order to identify the modes in a mixed plasma and to gain physical insight into the nature of the instabilities, cold plasma theory can be used. The result from this analysis was then used to study the stability of these modes, using warm plasma theory.

*Corresponding Author
Address for Commincaion :
Deparment of Electronics, Cochin University of Science & Technology, Cochin, Trikkakara-682 022, Kerala, India

(a) Cold plasma theory :

We consider mixed plasma that includes electrons, negatively and positively charged oxygen ions (O^- and O^+) and hydrogen ions (H) which drift with a velocity U with respect to the O^+ ions. For simplicity, in the derivation of the frequencies of the mode, let $U = 0$. The cold plasma electrostatic dispersion relation for wave propagating perpendicular to the magnetic field is well known and is given by an extension of the result [16] as

$$1+\frac{\omega_{Pe}^2}{\Omega_e^2-\omega^2}+\frac{\omega_{PH}^2}{\Omega_H^2-\omega^2}+\frac{\omega_{PO^+}^2}{\Omega_{O^+}^2-\omega^2}+\frac{\omega_{PO^-}^2}{\Omega_{O^-}^2-\omega^2}=0. \quad (1)$$

In eq (1), the species indices are given by 'e' for electrons, H for hydrogen and O^+ and O^- respectively for positively and negatively charged oxygen ions. The gyro-frequencies for species α is denoted by Ω_α with $\Omega_\alpha = q_\alpha B / M_\alpha C$ (q_α, is the charge of the particle; B, the ambient magnetic field; M_α, the mass of the particle and C, the velocity of light). The plasma frequency is defined as $\omega_{P\alpha}$, with $\omega_{P\alpha}^2 = 4\pi n_\alpha q_\alpha^2 / m_\alpha$. The frequency is indicated by ω. For low frequencies such that $\omega^2 << \Omega_e^2$ and assuming $\omega_{Pe}^2 << \Omega_e^2$, eq. (1) reduces to

$$1+\frac{\omega_{PH}^2}{\Omega_H^2-\omega^2}+\frac{\omega_{PO^+}^2}{\Omega_{O^+}^2-\omega^2}+\frac{\omega_{PO^-}^2}{\Omega_{O^-}^2-\omega^2}=0. \quad (2)$$

The two positive solutions to (2) are given by

$$\omega_{LH}=\left[\omega_{PH}^2+\omega_{PO^+}^2+\omega_{PO^-}^2\right]^{1/2} \quad (3)$$

and

$$\omega_{IH}=4\Omega_O^+\,\frac{n_H\Omega_o^+ +\left(n_o^+ + n_o^-\right)\Omega_H}{n_H\Omega_H+\left(n_o^- + n_o^+\right)\Omega_{O^+}} \quad (4)$$

The densities of H^+, O^+ and O^- are given respectively by n_H, n_o^+ and n_o^-.

Eq. (3) is the expression for the lower hybrid frequency in the presence of the heavy ions O^+ and O^-, while eq. (4) represents the ion-ion hybrid (*i.e.* Buchsbaum) model. The mode is present only when another ion component is added to the plasma, as is evident from eq. (4).

(b) Warm plasma theory :

The cold plasma theory is very valuable for the mode identification. However, it cannot account for the finite temperature effect. Therefore, we consider the fully magnetized kinetic dispersion relation for electrostatic waves. The plasma, as already mentioned, is made up of hydrogen (H), oxygen ions (O^+ and O^-) and electrons. Thus, generalizing the well-known dispersion relation of [17] for electrostatic waves, we have

$$1+\sum_\alpha\frac{1}{K^2\lambda_n^2}\times\left[1+\frac{\omega}{\sqrt{2}K_\parallel V_{T\alpha}}e^{-Y_\alpha^2}\sum_{n=-\infty}^{\infty}Z\left(\frac{\omega+n\Omega_\alpha}{\sqrt{2}K_\parallel V_{T\alpha}}\right)I_n(Y_\alpha)^2\right]+\frac{1}{K^2\lambda_n^2}\left(1+\frac{\omega-K_\parallel U}{\sqrt{2}K_\parallel V_{TH}}e^{-Y_H^2}\sum_{n=-\infty}^{\infty}Z\left(\frac{\omega+n\Omega_H-K_\parallel U}{\sqrt{2}K_\parallel V_{TH}}\right)I_n(Y_H)^2\right)=0 \quad (5)$$

In (5), the species summation α is over electrons, O^+ and O^- ; the hydrogen contribution has been written out separately K is the wave vector which has parallel components $K_\parallel$ and perpendicular $K_\perp$ to the magnetic field. Z is the plasma dispersion function, which arise from the $dV_\parallel$ integration and is defined as [18]

$$Z(\xi)=\frac{1}{\sqrt{\pi}}\int\frac{\exp(-x^2)}{x-\xi}dx, \quad (6)$$

while $I_n(Y_H^2)$ is the modified Bessel function which arises from the $dV_\perp$ integration with an argument

$$Y_\alpha^2=\frac{K_\perp^2T_\alpha}{\Omega_\alpha^2M_\alpha}\quad \alpha=\mathrm{H, O^+,\ O^-}\ \text{or e}. \quad (7)$$

The temperature of T_α are related to the thermal velocity $V_{T\alpha}$ by the relation

$$V_{T\alpha}^2=\frac{T_\alpha}{M_\alpha}\quad \alpha=\mathrm{H, O^+, O^-}\ \text{or e}. \quad (8)$$

All the four constituents of plasma have been modelled by isotropic Maxwellian distributions (the hydrogen ions alone being modelled as an isotropic, drifting component). Assume that ω is complex and can be written as

$$\omega=\omega_r+i\gamma.$$

Expanding the plasma dispersion function as a Taylor series :

$$Z\left[\frac{\omega+n\Omega_\alpha}{\sqrt{2}K_\parallel V_{T\alpha}}\right]=Z\left[\frac{\omega_r+n\Omega_\alpha}{\sqrt{2}K_\parallel V_{T\alpha}}\right]+\frac{i\gamma}{\sqrt{2}K_\parallel V_{T\alpha}}Z'\left[\frac{\omega_r+n\Omega_\alpha}{\sqrt{2}K_\parallel V_{T\alpha}}\right]$$

$$=Z\left[\frac{\omega_r+n\Omega_\alpha}{\sqrt{2}K_\parallel V_{T\alpha}}\right]-\frac{2i\gamma}{\sqrt{2}K_\parallel V_{T\alpha}}\left[1+\frac{\omega_r+n\Omega_\alpha}{\sqrt{2}K_\parallel V_{T\alpha}}Z\left(\frac{\omega_r+n\Omega_\alpha}{\sqrt{2}K_\parallel V_{T\alpha}}\right)\right] \quad (9)$$

Substituting (9) in (5), we can write down the dispersion relation as

$$1+\sum_{\alpha}\frac{1}{k^2\lambda_\alpha^2}\left\{1+\sum_{n=-\infty}^{\infty} e^{Y_\alpha^2} I_n\left(Y_\alpha^2\right)\right.$$

$$\frac{\omega_r}{\sqrt{2}K_\parallel V_{T\alpha}} Z(\Gamma)+\frac{i\gamma}{\sqrt{2}K_\parallel V_{T\alpha}} Z(\Gamma)-2\frac{\omega_r}{\sqrt{2}K_\parallel V_{T\alpha}}$$

$$\frac{\omega_r(\omega_r+n\Omega_\alpha)}{\left(\sqrt{2}K_\parallel V_{T\alpha}\right)^2} Z(\Gamma) \quad \frac{}{k^2\lambda_L^2}\left\{1+\sum_{n=-\infty}^{\infty} e^{Y_H^2} I_n\left(Y_H^2\right)\right.$$

$$\left[\frac{\omega_r-UK_\parallel}{\sqrt{2}K_\parallel V_{TH}} Z(\Gamma)+\frac{i\gamma}{\sqrt{2}K_\parallel V_{TH}} Z(\Gamma')-2\frac{(\omega_r-UK_\parallel)}{}\right.$$

$$2\frac{(\omega_r-K_\parallel U)\;(\omega_r-K_\parallel U+n\Omega_H)}{\left(\sqrt{2}K_\parallel V_{TH}\right)^2} Z(\Gamma')\Big\}=0, \quad (10)$$

where $\Gamma=\dfrac{\omega_r+n\Omega_\alpha}{\sqrt{2}K_\parallel V_{T\alpha}}$ and $\Gamma'=\dfrac{(\omega_r-K_\parallel U+n\Omega_H)}{\sqrt{2}K_\parallel V_{TH}}$.

In eq (10), again the summation α includes e, o^+ and o^-.

One simplifying assumption has been made with regard to the electron contribution, that the species are over magnetized; that is $\Omega_e >> K_\perp V_{Te}$. Thus, $Y_e^2 << 1$ and among the infinite number of terms, only one survives, namely $I_n\left(Y_e^2\right)=1$; the other terms are not contributive as, $I_n\left(Y_e^2\right)=0$ for $n\neq 0$. In the asymptotic expansion for electrons and hydrogen ions, consider only the case where both o^+ and o^- ions are cold, thus necessitating the asymptotic expansion for Z for these ions. The asymptotic expansions for the plasma dispersion function is [18]

$$Z(\xi)=i\sqrt{\pi}\exp\left(-\xi^2\right)-\frac{1}{\xi}-\frac{1}{2}\frac{1}{\xi^3}-\frac{3}{4}\frac{1}{\xi^5}. \quad (11)$$

Eq. (11) is now substituted in (10) and the real and imaginary contributions of each species to the dispersion relation are separated. Writing each contribution in full and carrying out the involved algebra, the expression for the growth/damping rate which is normalized against the o^+ ion gyro-frequency, can be obtained as

$$\frac{\gamma}{\Omega_{0^+}}=\frac{-\sqrt{\pi}}{\Gamma_e+\Gamma_H+\Gamma_{0^+}+\Gamma_{0^-}}\left(\gamma_e+\gamma_H+\gamma_{0^+}+\gamma_{0^-}\right). \quad (12)$$

The contributions are :

$$\gamma_e=\theta Y_o^+\xi_e e_e^{-\xi^2},\ \xi_e=\frac{1}{\sqrt{2}}\frac{\omega_r}{K_\parallel V_{Te}};$$

$$\gamma_H=\frac{n_H T_e}{n_e T_H}\frac{M_H T_{0^+}}{M_{0^+} T_H}\sum^{\infty} e^{-Y_H^2} I_n\left(Y_H^2\right)e_H^{\xi_H^2},$$

$$\xi_H=\frac{1}{\sqrt{2}}\frac{\omega_R-K_\parallel U+n\Omega_H}{KV_{TH}};$$

$$\gamma_{0^+}=\frac{n_{0^+}T_e}{n_e T_{0^+}}\sum_{n=-\infty} e^{-Y_{0^+}^2} I_n\left(Y_{o^+}^2\right)e^{-\xi_{0^+}^2}\frac{\omega_r}{\Omega_+};$$

$$\xi_{o^+}=\frac{1}{\sqrt{2}}\frac{\omega_R+n\Omega_{0^+}}{K_\parallel V_{T_{o^+}}};$$

$$\gamma_0=\frac{n_0 T_e}{n_e T_0}\sum_{n=-\infty}^{\infty} e^{-Y_0^2} I_n\left(Y_o^2\right)e^{-\xi_0^2}\left(\frac{\omega_r}{\Omega_0}\right),$$

$$\xi_{0^-}=\frac{1}{\sqrt{2}}\frac{\omega_R+n\Omega_0}{K_\parallel V_T} \quad (13)$$

In eq. (13), $\theta=\dfrac{K_\parallel}{K_\perp}$ is the ratio of the wave vector parallel and perpendicular to the magnetic field, n_α is the species density while the remaining parameters are defined earlier.

In the denominator of (12), since $I_n(x)=I_{-n}(x)$, the summation over n form $-\infty$ to $+\infty$ can be reduced to a summation from 0 to ∞. Thus, carrying out the summation and writing down the contribution, we get

$$\Gamma_e=-\left(\frac{m_e}{m_{0^+}}\frac{T_e}{T_{0^+}}\left(\frac{1}{\xi_e^3}+\frac{3}{\xi_e^5}\right)\right),$$

$$\Gamma_H=2\frac{n_H}{n_e}\frac{T_e}{T_H}\frac{m_H}{m_{0^+}}\frac{T_o^+}{T_H}$$

$$\times\sum_{n=0}^{\infty}\left\{e^{-Y_H^2} I_n\left(Y_H^2\right)\right|\theta Y_H\frac{Z_*}{Z_*^2-n^2}+\left(\frac{\theta Y_H}{2}\right)^3\frac{Z_*^3+3Z_*n}{\left(Z_*^2-n^2\right)}$$

$$-\frac{3}{4}(\theta Y_H)^5\frac{Z_*^5+10Z_*^3n^2+5Z_*n^4}{\left(Z_*^2-n^2\right)^5}-\theta Y_H\frac{Z_*\left(Z_*^2+n^2\right)}{\left(Z_*^2-n^2\right)^2}$$

$$-\frac{3}{2}(\theta Y_H)^3\frac{Z\left(Z_*^4+6Z_*^2n^2+n^4\right)}{\left(Z_*^2-n^2\right)^4}$$

with

$$Z_*=\frac{\omega_r}{\Omega_0^+}-\frac{K_\parallel U}{\Omega_0^+},$$

$$\Gamma_{0^+} = 2\frac{n_{H_{0^+}}}{n_e}\frac{T_e}{T_{0^+}}$$

$$\times\sum_{n=0}^{\infty} e^{-Y_{0^+}^2} I_n\left(Y_{0^+}^2\right)\left[\theta Y_{0^+}\frac{Z}{Z^2-n^2}+\left(\frac{\theta Y_{0^+}}{2}\right)^3\frac{Z^3+3Zn^2}{\left(Z^2-n^2\right)^3}\right.$$

$$-\frac{3}{4}\left(\theta Y_{0^+}\right)^5\frac{Z^5+10Z^3n^2+5Zn^4}{\left(Z^2-n^2\right)^5}-\theta Y_{0^+}\frac{Z\left(Z^2+n^2\right)}{\left(Z^2-n^2\right)^2}$$

$$\left.-\frac{3}{2}\left(\theta Y_{0^+}\right)^3\frac{Z(Z^4+6Z^2n^2+n^4)}{\left(Z^2-n^2\right)^4}\right]$$

and $$\Gamma_0 = 2\frac{n_0}{n_e}\frac{T_e}{T_0}\sum_{n=0}^{\infty}\left\{e^{-Y_0^2} I_n\left(Y_0^2\right)\right.$$

$$\theta Y_0\frac{Z}{Z^2-n^2}+\left(\frac{\theta Y_0}{2}\right)^3\frac{Z^3+3Zn^2}{\left(Z^2-n^2\right)^3}$$

$$-\frac{3}{4}\left(\theta Y_0\right)^5\frac{Z^5+10Z^3n^2+5Zn^4}{\left(Z^2-n^2\right)^5}-\theta Y_0\frac{Z\left(Z^2+n^2\right)}{\left(Z^2-n^2\right)^2}$$

$$\left.-\frac{3}{2}\left(\theta Y_0\right)^3\frac{Z\left(Z^4+6Z^2n^2+n^4\right)}{\left(Z^2-n^2\right)^4}\right\} \qquad (14)$$

with $Z=\frac{\omega_r}{\Omega_{0^+}}$ and $y_\alpha^2=\frac{K_\perp^2 T_\alpha}{\Omega_\alpha^2 M_\alpha}$, where α = H, O^+ and O^-.

3. Results

The computation of the expression for the growth / damping rate with the density of hydrogen ions n_H = 3.0 at a temperature equivalent of T_H = 3.0 keV and a drift velocity of $U = 10V_{TO^+}$ has been considered. The oxygen ions n_{o^+} and n_{o^-} were assigned a value of 1.0 each and a temperature equivalent of 0.03 eV. In addition, the electrons were assigned a temperature equivalent of T_e = 1.0 keV; the number densities being calculated from the charge neutrality condition, $n_e + n_{o^-} = n_o + n_H$. The magnetic field used was the observed value of 60 × 10^{-5} Gauss[13].

The routines for the calculation of the modified Bessel functions that occur in the expressions (13) and (14) were obtained from Ref. [19]. The values of $e^{-x}I_n(x)$ were then checked against their values in Ref. [20], the agreement was found to be good. The programme was then completed to calculate the other terms in (13) and (14), and computations made. Though eqs. (13) and (14) contain an infinite summation over the modified Bessel functions, the summation was restricted to only twelve terms, from zero to eleven, since the value $e^{-x}I_n(x)$ of beyond this was found to be extremely small

We first consider the stability of the lower hybrid wave, corresponding to (3). Figure 1 is a plot of the growth rate *versus* frequency as a function of $\theta(=K_{||}/K_\perp$ = 0.6, 0.7 and 0.8) for n_H = 3.0, T_H = 1.0 keV, U = 10.0, $V_{T_{0^+}}$, $n_{o^+} = n_{o^-}$ = 1.0, $T_{o^+} = T_{o^-}$ = 0.03eV and T_e = 1.0 KeV. We find that the instability of the wave shifts towards higher frequencies with increasing θ, or for increasingly parallel propagation. Also the growth rate, in general, increases with θ.

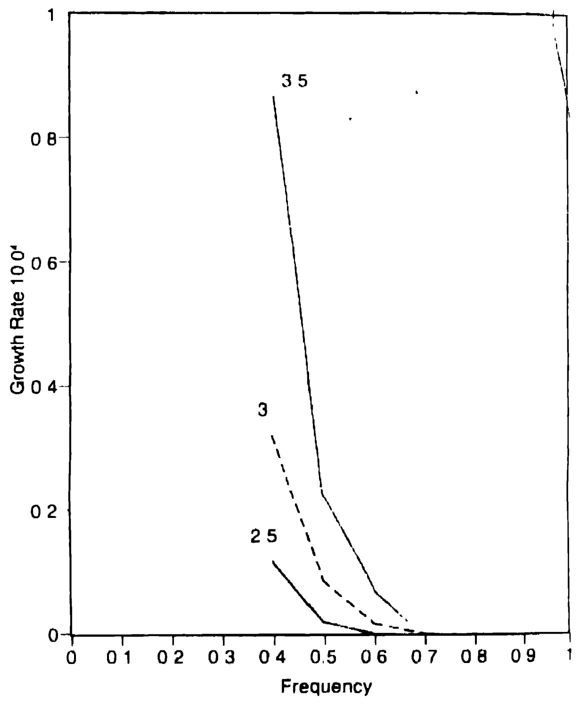

Figure 1. Plot of growth rate *versus* frequency for n_H = 3 0 (T_H = 1 0 KeV, U = 10 $V_{T_{0^+}}$), $n_0^+ = n_0^-$ = 1 0 ($T_{0^+} - T_{0^-}$ = 0.03 eV) and T_e = 1 0 KeV, as a function of θ (= $K_{||}/K_\perp$) = 0.6, 0 7 and 0 8

The stability of the wave was studied as a function of the hydrogen density n_H. Figure 2 depicts the variation of the growth rate *versus* frequency as a function of n_H (= 2.5,3.0 and 3.5) with θ = 0.6; the other parameters being the same as in above case. We find that the lower frequency has a marked effect on the stability of the wave, the growth rate increasing with hydrogen densities. As the frequencies increase, the lighter ion does not have any effect on the growth rate of the wave.

Figure 3 is a plot of the growth rate *versus* frequency as a function of the ratio of the temperatures T_{o^+} to T_{o^-} (= 2.0, 1 0 and 0.5), for $n_{0^+} = n_{0^-}$ =1 and θ = 0.6; the parameters of hydrogen and electrons are the same as in Figure 1. The instability of the wave starts at lower frequencies when the positively ionized oxygen is hotter ($T_{0^+} / T_{0^-} = 2.0$). Instability sets in at the same frequency when $T_{0^+} / T_{0^-} = 1.0$ and 0.5. However, the inspection of three curves shows that the growth rate is, in general, larger when $T_{0^+} \neq T_{0^-}$.

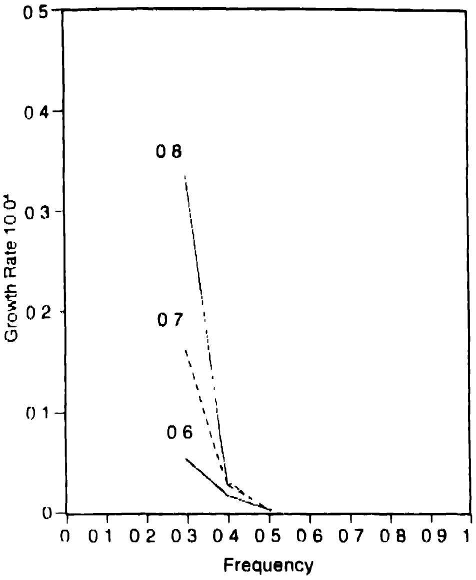

Figure 2. Plot of growth rate *versus* frequency as a function of $n_{//}$ (= 2.5, 3.0 and 3.5) with $T_{//}$ = 1.0 KeV, V = 10 $V_{T_{0+}}$ and θ = 0.6, the other parameters being the same as in Figure 1

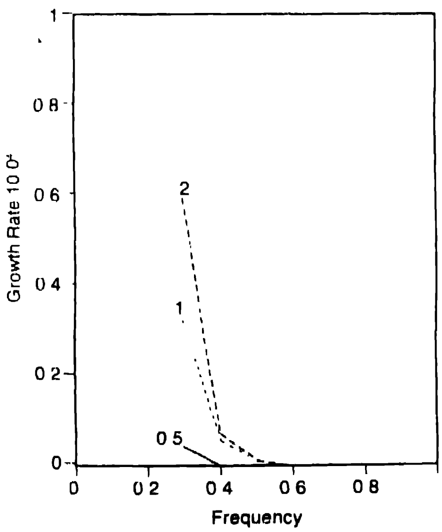

Figure 3. Plot of growth rate *versus* frequency as a function of the temperature ratio T_{o^+} / T_{o^-} (= 2.0, 1.0, 0.5) with $n_{o^+} = n_{o^-}$ = 1.0, $n_{//}$ = 3.0, ($T_{//}$ = 1.0 KeV, U = 10 $V_{T_{0+}}$) Tc = 1.0 KeV and θ = 0.6.

Figure 4 depicts the variation in the growth rate *versus* frequency as a function of Y_{0^+} [defined in (7)] for $\theta = 0.6$, $n_{//} = 3.0$ ($T_{//} \equiv$ 1.Kev, U = 10.0V T_{0^+}), $n_{0^+} = n_{0^-} = 1.0$, ($T_{0^+} = T_{0^-} \equiv$ 0.03eV) and $T_e \equiv$ 1.0KeV, Y_{O^+} is equal to $K_\perp \gamma_{LO^+}$ where γ_{LO^+} is the larmour radius of the O⁺ ions. For low Y_{0^+}, the instability occurs at lower frequencies, with a correspondingly low growth rate.

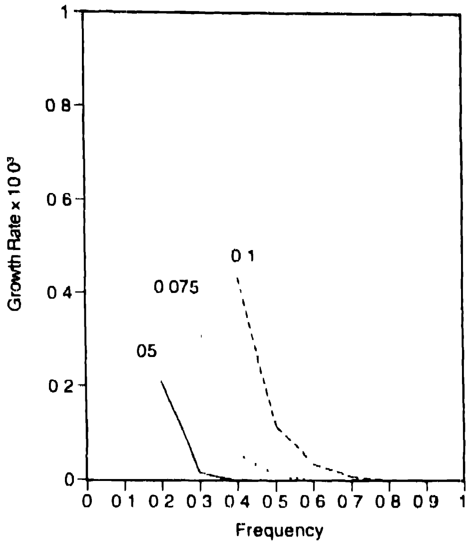

Figure 4. Plot of growth rate *versus* frequency as a function of Y_{0}^{+} (= .05, .075, 0.1) for θ = 0.6, $n_{//}$ = 3.0, ($T_{//}$ = 1.0 Kev, U = 10.0 V_{T0^+}), $n_{0^+} = n_{0^-}$ = 1.0 ($T_{0^+} \neq T_{0^-}$ = .03eV) and $T_e \equiv$ 1.0 KeV.

Finally, in Figure 5, the growth rate *versus* frequency as a function of n_{0^-}, the negatively ionized oxygen density is shown. The other parameters for the figures are $n_{0^+} = 1.0$ ($T_{0^+} = T_{0^-} \equiv$ 0.03eV), $n_{//} = 3.0$, ($T_H \equiv$ 1.0KeV and U = 10.0V T_{0^+}) and $T_e \equiv$ 1.0 KeV. It is noted that when $n_{0^-} = 0$, the growth rate is low with instability setting in at the higher end of the frequency spectrum. However, when $n_{0^-} \neq 0$ (n_{0^-} = 0.75 and 1.5), the instability sets in at very low frequencies. For the aforesaid parameters, the Buchsbaum mode given by (4) is almost a freely propagating mode.

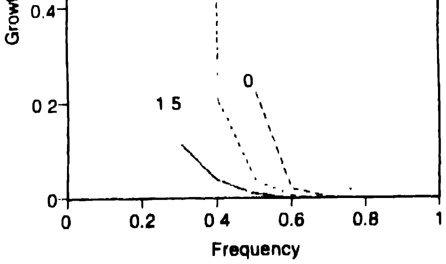

Figure 5. Plot of growth rate *versus* frequency as a function of n_{o^-} (= 0.0, 0.75 and 1.5); the other parameters being n_{o^+} = 1.0 ($T_{o^+} = T_{o^-}$ = 0.03 eV), $n_{//}$ = 3.0 and $T_{//}$ = 1KeV and V = 10.0 $V_{T_{o^+}}$, T_e = 1.0 KeV and θ = 0.6

4. Conclusions

We studied the stability of electrostatic waves in a multi-ion plasma made up of electrons, positively and negatively charged oxygen ions and hydrogen ions which drift with a velocity U with respect to the O^+ ions. The two wave modes were considered: the lower hybrid mode and the ion-ion hybrid (*i.e.* Buchsbaum) mode whose frequencies were derived using the cold plasma theory. The stability of the lower hybrid wave increases with parallel propagation and hydrogen densities. It is larger when $T_{0^+} \neq T_{0^-}$ than for the case with $T_{0^+} = T_0$ and it increases with increasing Y_{0^+} (when Y_{0^+} is the perpendicular wave vector normalised with O^+ ions Larmour radius). The mode is driven unstable at lower frequencies in the presence of n_0 and no variation is found with respect to the drift velocity U of the hydrogen ions. It may be due to the fact that no resonance conditions were satisfied for the set of parameters considered. The other mode, the Buchsbaum mode is a freely propagating mode.

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