Indian J. Phys. **77B** (4), 399-403 (2003)

AN INTERNATIONAL JOURNAL
IJP B

Theoretical and experimental investigations on the frequency selective property of an array of slotted dipoles

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Received 10 February 2003, accepted 8 May 2003

Abstract : A frequency selective surface (FSS) comprising of a two dimensional array of slotted dipoles resonant at a single frequency is proposed. A computationally efficient method for analyzing this FSS is presented. The formulation is carried out in the spectral domain where the convolution form of the integral equation for the induced current reduces to an algebraic one and the Spectral-Galerkin technique is used to solve the resulting equation. Entire- domain basis function that satisfies the edge condition is introduced to expand the unknown induced current on the complimentary structure *i.e.* an array of metal printed dipoles. Using Babinet's principle for complementary screen, the transmitted electric field for the structure of slotted dipoles have been calculated. This theoretical result has been compared with the experimental data also. The theoretical and experimental data indicate that this structure can be used as a band pass filter.

Keywords : Frequency selective surface, slotted dipoles, spectral-Galerkin technique, band pass filter

PACS Nos. : 52.25.Os, 84.30.Vn, 84.40.Ba

1. Introduction

Frequency selective surfaces are often employed in the reflector antenna system of a communication satellite [1-3] or a deep space exploration vehicle [4-7] for multi-frequency operations. Typically, an FSS consists of one or more grids supported by one layer or multiple layers of dielectrics. The grids are thin conducting periodic elements etched on thin substrates. As these grids can be designed to resonate at specified frequencies, the FSS will reflect waves at these frequencies and pass waves at other frequencies. Experimental investigations on the frequency selective property of an array of printed dipoles [8] have been carried out recently. In this paper, our goal is to design an FSS, which will act like a band pass filter and may be used in a multi-band reflector system. In the multi-band reflector antenna application [4], typically an FSS is employed as the subreflector and the different frequency feeds are optimized independently and placed at the real and virtual foci of the mainreflector. Hence, only a single main reflector is required for multifrequency operation. For example, the FSS on the high gain antenna (HGA) of the voyager spacecraft was designed to diplex S and X bands [4]. In that application the S band feed is placed at the prime focus of the main reflector and the X band feed is placed at the cassegrain focal point. The main advantage is that only the main reflector is required for this dual band operation. Thus, tremendous reductions in mass, volume and cost of the antenna system are achieved with the FSS subreflector.

2. Design of the FSS

An array of 7×5 aperture dipoles were fabricated within the copper screen on one side of a dielectric slab and the copper coating on the other side of the slab was completely removed (Figure 1. (a)). Dimensions of the dielectric slab were 140 mm × 140 mm × 3.16 mm. Its dielectric constant is 2.4. The FSS was designed in such a way that it may resonate at the frequency of 10 GHz without considering the dielectric loading effect. At this frequency, the corresponding free space wavelength is 30 mm. For resonance to occur, the length of each dipole aperture should be equal to half of the free space wavelength [9]. Hence, the

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length of each dipole aperture was made equal to 15mm. Its width was made 1.5mm. The spacing between two rectangular dipole apertures was so chosen that the rule governing a conventional array antenna be maintained. Here, the spacing between two adjacent rectangular apertures was chosen to be 15mm (half wavelength) in a row and 7.5mm (quarter wavelength) in a column (Figure 1. (a)).

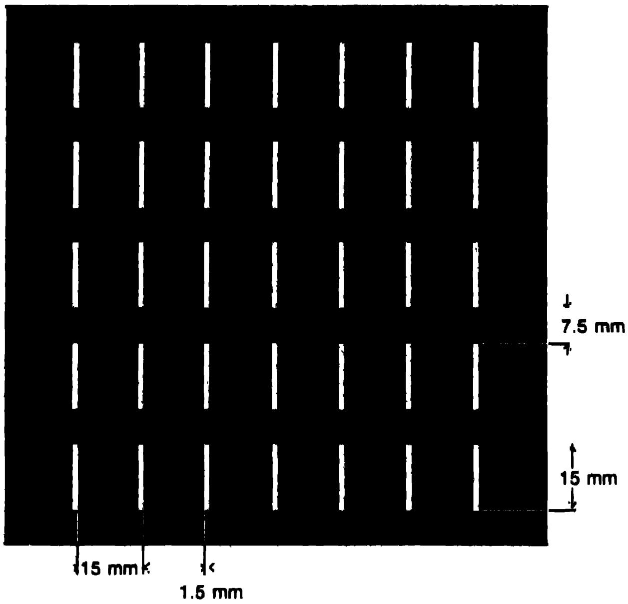

Figure 1. (a). Frequency selective surface under investigation (All Dimensions are in mm).

3. Formulation for theoretical investigation

In this Section, the integral equation is developed for the scattering from a frequency selective surface of the type shown in Figure 1 (a). Let J be the current density induced on the FSS due to a given incident field and k_0 is the free space wave number. The scattered electric field in the transformed domain $E^s(\alpha,\beta)$ *at* $z = 0$ can be derived from [10] :

$$\begin{bmatrix} \tilde{E}_x^s(\alpha,\beta) \\ \tilde{E}_y^s(\alpha,\beta) \end{bmatrix} = \frac{1}{j\bar{\omega}\varepsilon_0} \begin{bmatrix} k_0^2 - \alpha^2 & -\alpha\beta \\ -\alpha\beta & k_0^2 - \beta^2 \end{bmatrix} \left|\tilde{G}(\alpha,\beta)\right| \begin{matrix} \tilde{J}_x(\alpha,\beta) \\ \tilde{J}_y(\alpha,\beta) \end{matrix} \tag{1}$$

where

$$\tilde{G}(\alpha,\beta) = \frac{-j}{2\sqrt{k_0^2 - \alpha^2 - \beta^2}} I, \text{ for } k_0^2 > \alpha^2 + \beta^2$$

$$\frac{1}{2\sqrt{\alpha^2 + \beta^2 - k_0^2}} \cdot I, \text{ for } k_0^2 < \alpha^2 + \beta^2.$$

On the conducting patch, the tangential electric field vanishes and we get

$$E^s_{Tan} + E^{inc}_{Tan} = 0 \quad i.e. \quad E^s_{Tan} = -E^{inc}_{Tan}.$$

Since the structure is periodic, $\tilde{J}(\alpha,\beta)$ has a discrete spectrum in the transformed domain *i.e.* $\tilde{J}(\alpha,\beta)$ is non-zero for discrete values of α,β *viz.* α_{mm}, β_{mm} which correspond to the Floquet's modes. So while taking the inverse Fourier transform of (1), the integral form can be converted into a summation form.

Putting $E^s = -E^{inc}$ and taking inverse Fourier transform of (1), we can get

$$\begin{bmatrix} \tilde{E}_x^s(x,y) \\ \tilde{E}_y^s(x,y) \end{bmatrix} \quad \begin{matrix} E_x^{inc} \\ E_y^{inc} \end{matrix}$$

$$= \frac{1}{j\varpi\varepsilon_0} \times \frac{1}{(2\pi)^2 ab} \sum_{m=-\alpha}^{+\alpha} \sum_{n=-\alpha}^{+\alpha} \begin{bmatrix} k_0^2 - \alpha_m^2 & -\alpha_m\beta_n \\ -\alpha_m\beta_n & k_0^2 - \beta_n^2 \end{bmatrix}$$

$$\times \tilde{G}(\alpha_m,\beta_n) \begin{matrix} \tilde{J}_x(\alpha_m,\beta_n) \\ \tilde{J}_y(\alpha_m,\beta_n) \end{matrix} e^{j\alpha_m x} e^{j\beta_n y},$$

where

a is the periodicity in x direction,

b is the periodicity in y direction,

$$2m\pi \; + k^{inc}$$

$$\beta_n = \frac{2n\pi}{b} + k_y^{inc},$$

$$k_x^{inc} = k_0 \sin\theta \cos\phi,$$

$$k_y^{inc} = k_0 \sin\theta \sin\phi,$$

[where θ is an angle of incident wave with the Z-axis and ϕ is the angle of the projection of the incident wave on the XY-plane with the X-axis. (Figure 1 (b))].

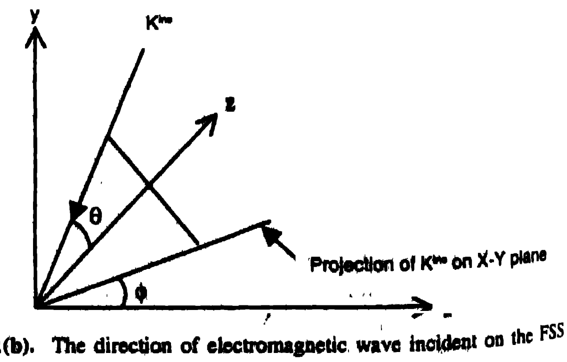

Figure 1(b). The direction of electromagnetic wave incident on the FSS

Next, our goal is to solve the eq. (2) using Galerkin's procedure and to calculate $\tilde{J}$ in the transformed domain. For this, we first express the unknown J in terms of an appropriate basis function.

Basis function selection and solution :

Let us write the matrix eq. (2) in the symbolic diadic form :

$= g$, where g is the known matrix $-\begin{bmatrix} E_x^{inc} \\ E_y^{inc} \end{bmatrix}$

and u is the matrix to be determined $\begin{bmatrix} \tilde{J}_x(\alpha_m,\beta_n) \\ \tilde{J}_y(\alpha_m,\beta_n) \end{bmatrix}$

and L is the total operator involved in matrix eq. (2).

Now we can write,

$$\left(\hat{x}\hat{x}L_{xx} + \hat{x}\hat{y}L_{xy} + \hat{y}\hat{x}L_{yx} + \hat{y}\hat{y}L_{yy}\right)\bullet\left(\hat{x}u_x + \hat{y}u_y\right)$$
$$= \hat{x}g_x + \hat{y}g_y \qquad (3)$$

is the basis function which is to be chosen. In generalised orm :

$$u = \sum_{i=1}^{N}\left(\hat{x}C_{x_i} f_{x_i} + \hat{y}C_{y_i} f_{y_i}\right). \qquad (4)$$

From eq. (3), we can write

$$L_{xx}u_x + L_{xy}u_y = g_x$$

$$L_{xx}\sum_{j=1}^{N} C_{x_j} f_{x_j} + L_{xy}\sum_{j=1}^{N} C_{y_j} f_{y_j} = g_x \qquad (5)$$

milarly,

$$L_{yx}\sum_{j=1}^{N} C_{x_j} f_{x_j} + L_{yy}\sum_{j=1}^{N} C_{y_j} f_{y_j} = g \qquad (6)$$

Now taking the inner product of f_{x_i} and eq. (5), we get

$$\left\langle f_{x_i}, L_{xx}\sum_{j=1}^{N} f_{x_j} C_{x_j}\right\rangle + \left\langle f_{x_i}, L_{xy}\sum_{j=1}^{N} f_{y_j} C_{y_j}\right\rangle = \left\langle f_{x_i}, g_x\right\rangle. \qquad (7)$$

Similarly, taking the inner product of f_{y_i} and eq. (6), we get

$$\left\langle f_{y_i}, L_{yx}\sum_{j=1}^{N} f_{x_j} C_{x_j}\right\rangle + \left\langle f_{y_i}, L_{yy}\sum_{j=1}^{N} f_{y_j} C_{y_j}\right\rangle = \left\langle f_{y_j}, g_y\right\rangle. \qquad (8)$$

From eq. (7), we get

$$\left\langle f_{x_i}, L_{xx} f_{x_j}\right\rangle C_{x_j} + \sum_{j=1}^{N}\left\langle f_{x_i}, L_{xy} f_{y_j}\right\rangle C_{y_j} = \left\langle f_{x_i}, g_x\right\rangle. \qquad (9)$$

From eq. (8), we get

$$\sum_{j=1}^{N}\left\langle f_{y_i}, L_{yx} f_{x_j}\right\rangle C_{x_j} + \sum_{j=1}^{N}\left\langle f_{y_i}, L_{yy} f_{y_j}\right\rangle C_{y_j} = \left\langle f_{y_i}, g_y\right\rangle. \qquad (10)$$

In short form, eqs. (9) and (10) are written in the following manner :

$$\sum_{j=1}^{N} a_{ij}^{xx} C_{x_j} + \sum_{j=1}^{N} b_{ij}^{xy} C_{y_j} = \lambda_i^x \qquad (11)$$

$$\sum_{j=1}^{N} a_{ij}^{yx} C_{x_j} + \sum_{j=1}^{N} b_{ij}^{yy} C_{y_j} = \lambda_i^y, \qquad (12)$$

where

$$a_{ij}^{xx} = \left\langle f_{x_i}, L_{xx} f_{x_j}\right\rangle;\ b_{ij}^{xy} = \left\langle f_{x_i}, L_{xy} f_{y_j}\right\rangle,$$

$$a_{ij}^{yx} = \left\langle f_{y_i}, L_{yx} f_{x_j}\right\rangle;\ b_{ij}^{yy} = \left\langle f_{y_i}, L_{yy} f_{y_j}\right\rangle,$$

$$\lambda_i^x = \left\langle f_{x_i}, g_x\right\rangle \text{ and } \lambda_i^y = \left\langle f_{y_i}, g_y\right\rangle.$$

As a whole, we can write in matrix form,

$$\begin{bmatrix} a_{11}^{xx}\ldots a_{1N}^{xx} & b_{11}^{xy}\ldots b_{1N}^{xy} \\ a_{21}^{xx}\ldots a_{2N}^{xx} & b_{21}^{xy}\ldots b_{2N}^{xy} \\ \ldots & \ldots \\ a_{N1}^{xx}\ldots a_{NN}^{xx} & b_{N1}^{xy}\ldots b_{NN}^{xy} \\ a_{11}^{yx}\ldots a_{1N}^{yx} & b_{11}^{yy}\ldots b_{1N}^{yy} \\ a_{21}^{yx}\ldots a_{2N}^{yx} & b_{21}^{yy}\ldots b_{2N}^{yy} \\ \ldots & \ldots \\ a_{N1}^{yx}\ldots a_{NN}^{yx} & b_{N1}^{yy}\ldots b_{NN}^{yy} \end{bmatrix} \begin{bmatrix} C_{x_N} \\ C_{y_1} \\ C_{y_2} \\ \\ C_{y_N} \end{bmatrix} = \begin{bmatrix} \lambda_1^x \\ \lambda_2^x \\ \cdot \\ \lambda_N^x \\ \lambda_1^y \\ \lambda_2^y \\ \cdot \\ \lambda_N^y \end{bmatrix} \qquad (13)$$

We shall have to calculate

$$C_{x_1}, C_{x_2}, C_{x_3}, \ldots C_{x_N} \text{ and } C_{y_1}, C_{y_2}, \ldots C_{y_N}$$

Then we can find out the function u or J from

$$u = J = \sum_{i=1}^{N}\left(\hat{x}C_{x_i} f_{x_i} + \hat{y}C_{y_i} f_{y_i}\right). \qquad (14)$$

For easy and faster calculation, Mittra *et al* [11] suggested the basis functions for different FSS structures. For the FSS structure consisting of printed dipoles, Basis function suggested by Mittra *et al* [11] is :

$$f_{x_i} = 0 \text{ i.e. } J_x = 0 \text{ and}$$

$$J_{y_i} = f_{y_i} = \sin\frac{i\pi}{L}\left(y+\frac{L}{2}\right)\times P_y(0,L)\,;\; i=1,2,3....$$

L = Length of the dipole.

$$P_y(0, L) = 1 \text{ for } |y| \leq \frac{L}{2}$$

= 0 otherwise.

Considering this basis function, we have calculated $\tilde{J}_x(\alpha,\beta)$ and $\tilde{J}_y(\alpha,\beta)$ assuming $E_y^{inc} = e^{-jk_0 y}$ and $E_x^{inc} = 0$. Next, the values of $\tilde{J}_x(\alpha,\beta)$ and $\tilde{J}_y(\alpha,\beta)$ have been put in eq. (2) to get the scattered electric field $E_x^s(x,y)$ and $E_y^s(x,y)$. Now from this scattered electric field, the transmission coefficients of the structure of mode *m, n* due to incident mode *k*, l may be readily calculated [10,12]. From the transmission coefficients transmitted electric field have been calculated for the array of printed dipoles. Then following Babinet's principle for complementary screen [12], the transmission coefficients for the array of slotted dipoles have been calculated and normalized transmitted electric fields in dB *vs.* frequency have been plotted as shown in Figure 2.

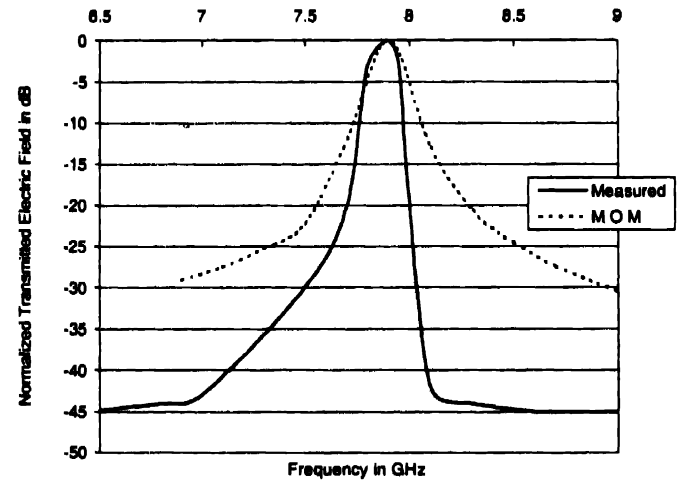

Figure 2. Normalized transmitted electric field *vs.* frequency.

4. Dielectric loading effect

Dielectrics are often used for structural support. A number of papers describing dielectric loading effects have been published. The resonant frequencies decrease as the dielectric thickness increases [9]. If f_r be the resonant frequency of the slotted dipoles etched on a thin copper sheet supported by thin dielectric substrate of dielectric constant ε_d then its effective resonant frequency f_{er} is $\dfrac{f_r}{\sqrt{\dfrac{\varepsilon_a+\varepsilon_d}{2}}}$ where ε_a = dielectric constant of air. This formula is valid for normal incidence of the electromagnetic wave. Here, $\varepsilon_a = 1$, $\varepsilon_d = 2.4$ and f_r = 10 GHz so, f_{er} = 7.7 GHz.

5. Measurement

Transmission and reflection tests for the FSS (Figure 1. (a)) were performed at J and X bands using standard microwave test bench. The FSS was placed between the transmitting and receiving horn antennas (Figure 3). The distance between these two horns is 25 times of the wavelength corresponding to the centre frequency (10 GHz). The transmitting horn antenna was connected to a Marconi microwave source. The receiving horn antenna was connected to a VSWR meter. Measurements have been made in the frequency range of 6 GHz to 9 GHz with an interval of 0.1 GHz. From these measured data, the normalised transmitted electric fields *versus* frequency were plotted in the same Figure 2 for comparison.

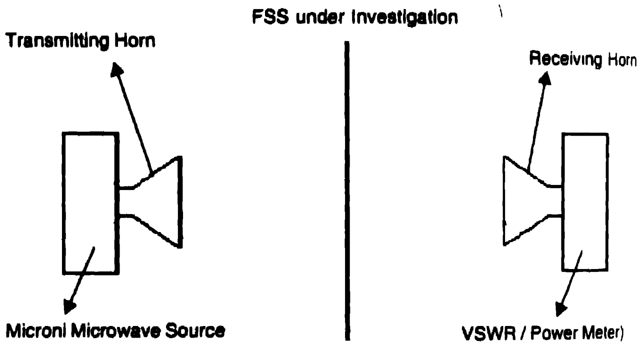

Figure 3. Experimental setup.

6. Conclusion

The experimental results show that the maximum transmission through the FSS occurs at the frequency range of 7.7GHz to 7.95GHz and almost total reflection from the FSS occurs at the frequencies above 8.1 GHz and below 7GHz. Theoretical investigation shows that maximum transmission occurs in the frequency range of 7.68GHz to 8.1 GHz. However, the plot of the computed result is not so steep as the plot of the measured result. This may be due to the fact that in eq. (13), the matrix size has been taken 20 X 20 instead of infinity.

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