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Effects of non-thermal electrons and negative ions on ion-acoustic solitary waves in a bounded plasma

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Abstract The effects of non-thermal electrons and negative ions on ion-acoustic solitary waves bounded in a finite geometry have been investigated theoretically using pseudo-potential technique It is found that finite geometry of the bounded plasma has significant contribution to the lity of the wave The structure of the solitary wave is then analysed as function of the parameter β which measures the deviation from the ialised state It is seen that when one considers non-thermal electrons and negative ions, both compressive and rarefactive solitons are found In ibsence of negative ions, solitary wave will not exist in bounded plasma for any value of β for non-thermal electrons. But, the existence of solitary is possible in unbounded plasma in presence of the non-thermal electrons for large values of β

Keywords Non-thermal electrons, compressive solitons, rarefactive solitons

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Introduction

Theoretical as well as experimental investigations on the excitation in plasma have been made by various authors in last years because these are important in many physical situations Washimi and Taniuti [1] were the first to study the ion-acoustic solitary waves in cold, collisionless, unmagnetised plasma through the derivation of Korteg-deVries (K-dV) equation using reductive perturbation method. The solitons being a compression of plasma density, have positive potential structure and this compressive soliton is described by the K-dV equation. Subsequently, experimental investigations on the ion-acoustic solitary wave were performed by Ikezi *et al* [2], Ikezi 3], Lonngren [4] and others from which it was observed that de and width are different from theoretical values. In last e decades, a lot of theoretical works on the propagation of n-acoustic waves have been done by various authors incorporating different parameters in the plasma *e.g.* non-isothermality [5, 6], inhomogeneity [7], temperature gradient [8], ion-temperature [9], two-temperature electrons [10, 11], ion beam [12, 13] *etc*. It is found that each of the above parameters has important role on the modification of the structure of the solitary waves. However, the solitary waves are found to have most interesting characteristics in a plasma in presence of negative ions together with positive ions. In a plasma consisting of negative ions, both the compressive as well as the rarefactive solitary wave may exist. The detailed study of ion-acoustic solitary wave in presence of negative ions was done by Das [14] and the remarkable conclusion was that negative ions may play an effective role in order to prevent the breaking of solitary wave into many solitons. Later, Das and Tagare [15], Watanabe [16] and other authors [17-19] theoretically studied the ion-acoustic solitary wave in negative-ion plasma and obtained important results. Experimental investigations on the existence and behaviour of ion-acoustic waves in multicomponent plasma

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with negative ions based on the theoretical observations were made by Nakamura *et al* [20], and other authors [21-23]. It was seen that the presence of negative ions in the plasma, give rise to interesting characteristics on the propagation of ion-acoustic waves. The presence of streaming ions have also significant contribution on the excitation of ion-acoustic solitary waves and double-layers in a negative-ion plasma [24-29].

Recently, Cairns *et al* [30] have shown that solitary waves exit in a plasma consisting of non-thermal electrons with an excess of energetic particles and ions. This investigation was motivated by the recent observation of solitary structures with density depletion made by *Freja* satellite [31]. It is to be mentioned here that many authors have studied the propagation of ion-acoustic wave in a bounded plasma [32-36], but the actual structure of the solitary wave in a bounded plasma having non-thermal electrons has not yet been done by any author.

This paper is organized as follows The basic equations for the plasma having cold positive ions, negative ions and non-thermal electrons in a bounded system are given in Section 2. The stability of ion-acoustic waves in bounded plasma has been analyzed in Section 3. In Section 4, the expression for the Sagdeev potential has been derived and graphically analyzed with the variation of the plasma parameters. The solution for the ion-acoustic solitary waves in bounded plasma is obtained in Section 5. The behaviour of the potential structure of the solitary waves and the width of the solitons have been discussed in this Section. Summary and concluding remarks of the paper are given in Section 6.

2. Formulation

We consider a collisionless, unmagnetised plasma consisting of non-relativistic ions, negative ions and non-thermal electrons confined in a cylindrical waveguide with its axis along the x-axis. The basic system of equations governing the plasma dynamics propagation can be written in the non-dimensional parameter as

For positive ions and negative ions :

$$\frac{\partial n_s}{\partial t}+\frac{\partial}{\partial x}(n_s u_s)=0, \quad (1)$$

$$\frac{\partial u_s}{\partial t}+u_s\frac{\partial u_s}{\partial x}=-\frac{\psi_s}{Q_s}\frac{\partial \phi}{\partial x}, \quad (2)$$

where the subscript 's' stands for $s = i$ for positive ions, $s = j$ for negative ions , $\psi_i = 1$, $\psi_j = -1$; $Q_i = 1$, $Q_j = Q$.

Poisson's equation :

$$\nabla_\perp^2\phi+\frac{\partial^2\phi}{\partial x^2}=n_e-\Sigma\psi_s n_s, \quad (3)$$

where

$$\nabla_\perp^2=\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}$$

is the transverse Laplacian, $Q = m_j / m_i$.

In the above equations, m_i, m_j denote the masses of positive ions and negative ions respectively. n_e, n_i, n_j and u_i, u_j are the densities and the velocities of the corresponding species In the equations, the velocities are normalised by $(K_BT_e/m_i)^{1/2}$, the densities by the equilibrium ion density n_0 and all length by the Debye length $(K_BT_e/4\pi n_0e^2)^{1/2}$, whereas the potential ϕ by K_BT_e/e, K_B being the Boltzmann constant.

For non-thermal electrons, we can choose the electron distribution function as [30]

$$f_e(v)=\frac{}{(1+3p)(2\pi v_e^2)^{1/2}}\left[1+p\left(\frac{v}{v_e^2}-2\phi\right)\right]$$

$$\exp\left[-\frac{1}{2}\left(\frac{v^2}{v_e^2}-2\phi\right)\right] \quad (4a)$$

where $v_e=(K_BT_e/m_e)^{1/2}$ and m_e is the electron mass The parameter 'p' determines the presence of fast particles in the model Therefore the electron density n_e normalised to the equilibrium ion number density n_0 is given by the following equations

$$n_e=\left(1-\beta\phi+\beta\phi^2\right)e^{\phi}, \quad (4b)$$

where $\beta = 4p/1+3p$, β measures the deviation from the thermalised state.

3. Dispersion relation

For linear analysis, we consider the variables in (1)–(4) ar perturbed as

(5

$$u_s=u_{s0}+u_{s1}, \quad (6$$

$$\phi=\phi_0+\phi_1, \quad (7$$

where n_{s0}, u_{s0} and ϕ_0 are the equilibrium values. n_{s1} u_{s1}, ϕ are the first order perturbed values. $\phi_0 = 0$ in the equilibrium state.

We assume the spatial and temporal dependence of th perturbed part to be of the form $f(r)\exp[i(kx-\omega t)]$, where 'k' is the wave number and 'ω' is the frequency of the wave. W now impose the condition that electric potential on the surfa of the cylinder is zero *i.e.* $\phi(r) = 0$ for $r = R$. Therefore, using (5)–(7) in (1)–(4), we get the dispersion relation as

$$p_{0n}^2 = (kR)^2 \left[\frac{n_{s0}}{Q_s(\omega - k u_{s0})^2} + \frac{(2-\beta)}{k^2} - 1 \right] \quad (8)$$

where p_{0n} are the roots of $J_0(x) = 0$. On simplification, relation (8) gets transformed into

$$a_4 k^4 + a_3 k^3 + a_2 k^2 + a_1 k + a_0 = 0, \quad (9)$$

where

$$a_4 = (2-\beta) u_0^2 Q R^2,$$

$$a_3 = -2u_0 (2-\beta) \omega Q R^2,$$

$$a_2 = \left\{ (2-\beta)\, Q\, R^2 \left(\omega^2 + u_0^2\right) - p_{0n}^2\, Q\, u_0^2 - \left(Q n_{i0} + n_{j0}\right) R^2 \right\},$$

$$a_1 = -\left\{ 2(2-\beta) Q u_0 R^2 + 2\omega u_0 Q\, p_{0n}^2 \right\},$$

$$a_0 = \left\{ p_{0n}^2 Q \omega^2 + (2-\beta) Q R^2 \omega^2 \right\}$$

and

$$u_{i0} = u_{j0}$$

Eq (9) has been solved explicitly to find the values of wave number k. The phase velocities and instability factors for different values of k have been calculated and plotted in Figures 1(a)–1(d) for various values of radius (R) of the wave guide and non-thermal electron parameter (β). From Figures 1(a)–1(b), it is interesting to see that for forward going wave, the phase velocity (v_f) decreases with the increase of negative ion concentrations (n_{j0}). Moreover, v_f is higher for lower values of β. On the other hand, the absolute value of the phase velocity (v_b) of the reflected wave increases with the increase of n_{j0} and β. The variation of the instability factor (k_{im}) with negative ion

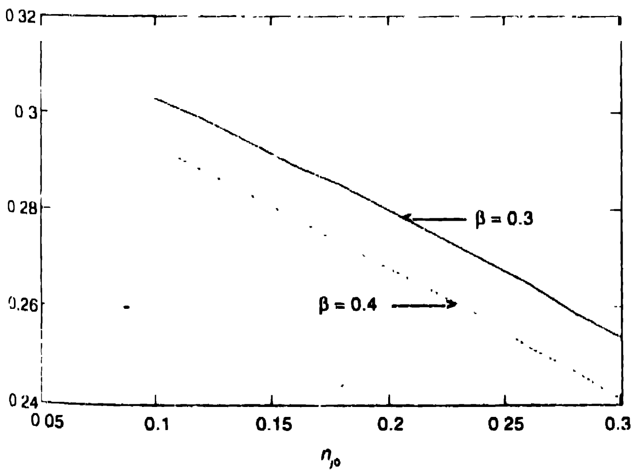

Figure 1(a). Variation of v_f, the phase velocity of the forward going wave with n_{j0} in (O^-, Ar^+) plasma, when β is parameter for $u_0 = 0.1$, $p_{0n} = 5.5$, $R = 1$, $\omega = 1$.

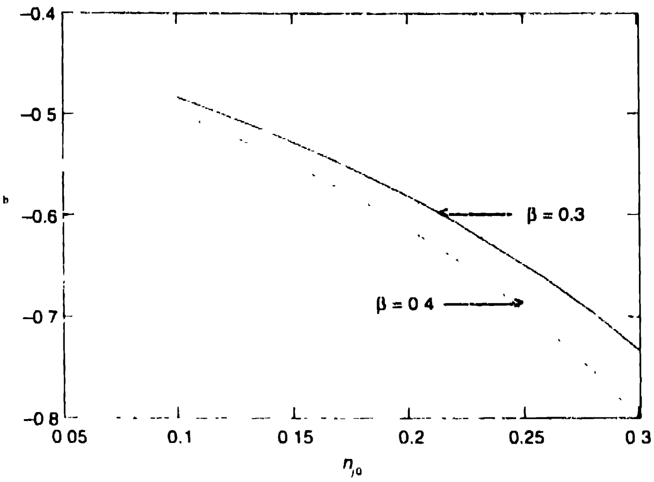

Figure 1(b). Plot of variation of v_b, the phase velocity of the backward going wave with n_{j0} in (O^-, Ar^+) plasma, when β is parameter for $u_0 = 0.1$, $p_{0n} = 5.5$, $R = 1$, $\omega = 1$

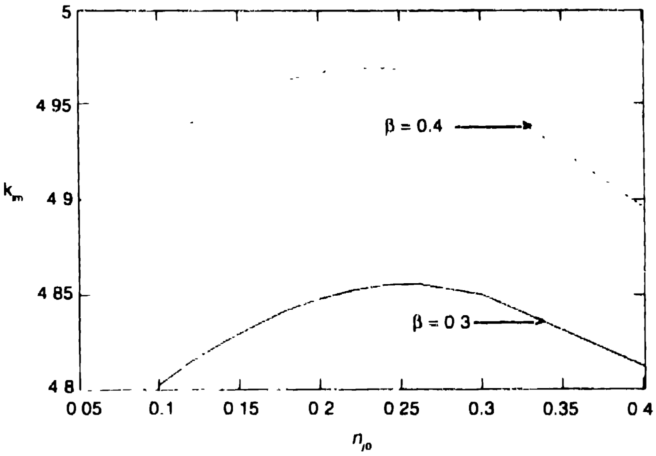

Figure 1(c). Dependence of k_{im}, the instability factor on n_{j0} in (O^-, Ar^+) plasma, for different values of β and other parameters $u_0 = 0.1$, $p_{0n} = 5.5$, $R = 1$, $\omega = 1$

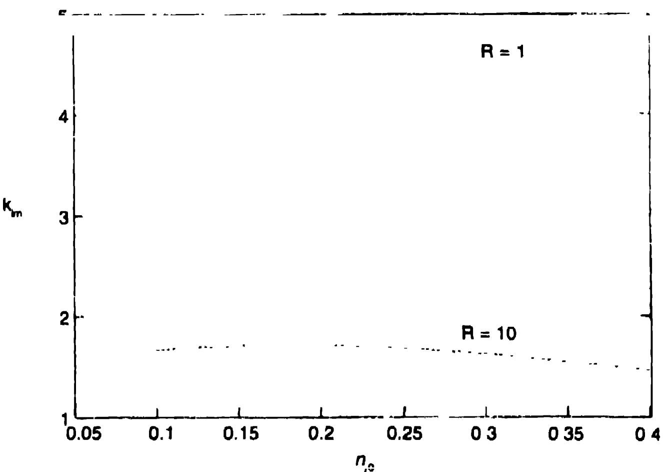

Figure 1(d). Dependence of k_{im}, on n_{j0} in (O^-, Ar^+) plasma, for different values of R and other parameters $\beta = 0.4$, $u_0 = 0.1$, $p_{0n} = 5.5$, $R = 1$, $\omega = 1$

concentration (n_{j0}) for different values of β and R are shown in Figure 1(c)–1(d). It is observed from Figure 1(c) that instability factor (k_{im}) have significant variation for a particular range of the values ($\approx$ 0.2–0.3) of negative ion concentration (n_{j0}). Moreover, the wave is more unstable for higher values of β and lower values of R.

4. Sagdeev potential

In order to derive the expression for Sagdeev potential, we use the perturbation expansions as adopted in the linear part but we do not linearise the basic equations. Consequently, we obtain a set of nonlinear equations. At this stage, we assume that the radial behaviour of the perturbations is described by the lowest order Bessel function. Following Mondal *et al* [34], we take the perturbed quantities to be of the form $J_0(k_\perp r)\,f(x,t)$, where $f(x,t) = N_s(x,t), U_s(x,t), ..$ and $k_\perp = p_{0n}/R$. Integrating the former nonlinear equations over r from 0 to R after multiplying with $r\,J_0(k_\perp r)$, we have

$$\frac{\partial N_s}{\partial t} + n_{s0}\frac{\partial U_s}{\partial x} + u_{s0}\frac{\partial N_s}{\partial x} + \alpha N_s \frac{\partial U_s}{\partial x} + \alpha U_s \frac{\partial N_s}{\partial x} = 0, \quad (10)$$

$$\frac{\partial U_s}{\partial t} + u_{s0}\frac{\partial U_s}{\partial x} + \alpha U_s \frac{\partial U_s}{\partial x} \quad \frac{\psi_s}{Q_s}\frac{\partial \phi}{\partial x} \quad (11)$$

and

$$\frac{\partial^2 \phi}{\partial x^2} - \phi = N_e - \sum \psi_s N_s , \quad (12)$$

where

$$\alpha = \frac{\int_0^R J_0^3 r dr}{\int_0^R J_0^2 r dr} .$$

To obtain the stationary solutions of the eqs. (10)–(12), we have to use the transformation $\xi = x - Mt$ where M is the velocity of the nonlinear structure. As a result, eqs. (10)–(12) yield

$$U_s = \frac{N_s(M - u_{s0})}{n_{s0} + \alpha N_s} \quad (13)$$

$$(M - u_{s0})U_s - \frac{\alpha U_s^2}{2} = \left(\frac{\psi_s}{Q_s}\right)\phi . \quad (14)$$

Combining relations (13) and (14), one gets

$$N_s = \frac{n_{s0}}{\alpha\left[-1 + \left\{1 - \frac{2\alpha\phi}{Q_s(M - u_{s0})^2}\right\}^{-1/2}\right]} \quad (15)$$

Therefore, Poisson equation (12) takes the form

$$\frac{\partial^2 \phi}{\partial \xi^2} = \phi + \left(1 - \beta\phi + \beta\phi^2\right)e^{\phi} - \sum \frac{\psi_s n_{s0}}{\alpha\left[-1 + \left\{1 - \frac{2\alpha\phi}{Q_s(M - u_{s0})^2}\right\}^{-1/2}\right]} = -\frac{\partial V}{\partial \phi} , \quad (16)$$

where V is the Sagdeev potential.

Numerical computations of Sagdeev potential V have been done for a model plasma consisting of negative ions and non-thermal electrons in a bounded system. In Figures 2(a)–2(c), we exhibit the form of the Sagdeev potential V as a function of ϕ for different values of β, n_{j0} and M. In Figure 2(a), variation of Sagdeev potential has been shown for different values of ϕ

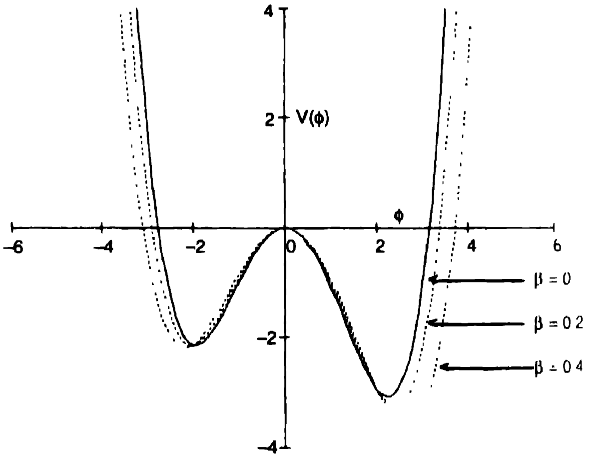

Figure 2(a). Variation of $V(\phi)$, the Sagdeev potential with ϕ (O^-, Ar^+) plasma, when β is parameter for n_{j0} = 0.13, α = –2, u_0 = 0, M = 1.63.

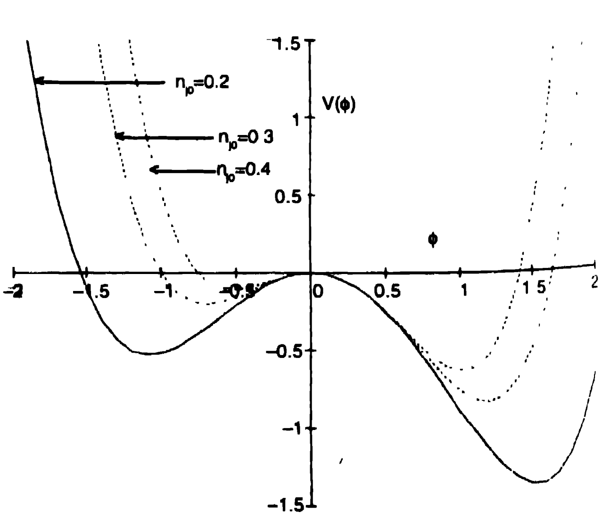

Figure 2(b). Variation of $V(\phi)$ with ϕ in (O^-, Ar^+) plasma, when n_{j0} parameter for β = 0.3, α = –2, u_0 = 0.1, M = 1.63.

and β and constant values of n_{j0}. The form of the Sagdeev potential shows that due to particle trapping, both the compressive and rarefactive soliton may be formed in presence

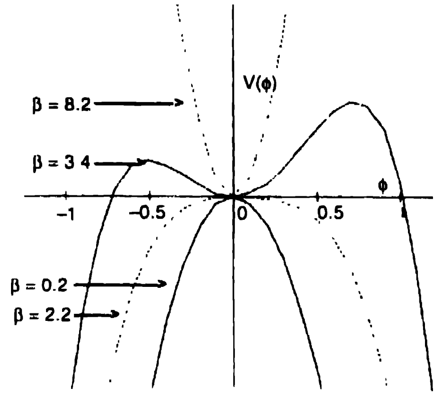

Figure 2(c). Plot of $V(\phi)$ with ϕ in bounded plasma for different values of β in the absence of negative ions, *i.e.* $n_{j0} = 0$, when $\alpha = -2$, $u_0 = 0.1$, $M = 1.63$

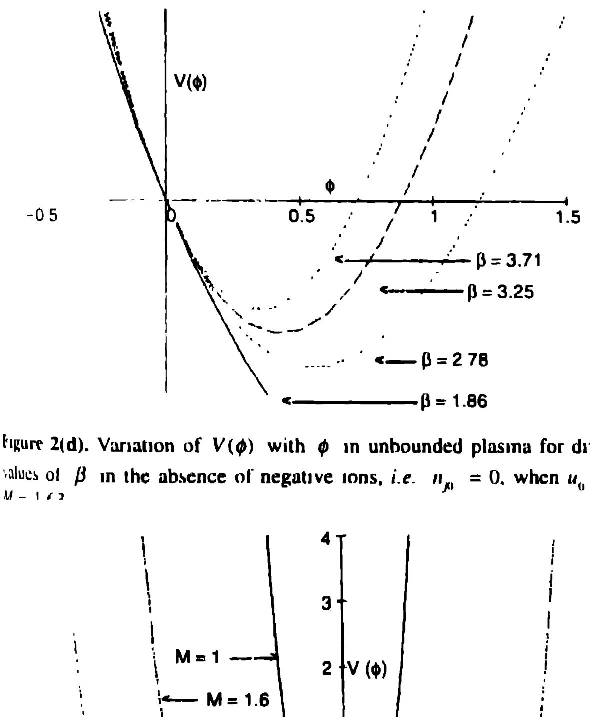

Figure 2(d). Variation of $V(\phi)$ with ϕ in unbounded plasma for different values of β in the absence of negative ions, *i.e.* $n_{j0} = 0$, when $u_0 = 0.1$, $M = 1.63$

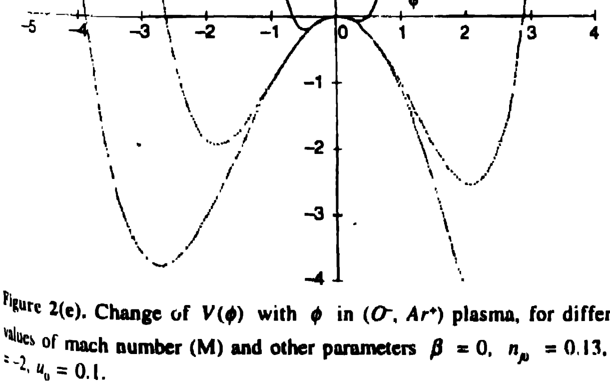

Figure 2(e). Change of $V(\phi)$ with ϕ in (O^-, Ar^+) plasma, for different values of mach number (M) and other parameters $\beta = 0$, $n_{j0} = 0.13$, $\alpha = -2$, $u_0 = 0.1$.

and absence of non-thermal electrons (β). Moreover, it is seen that Sagdeev potential is higher for higher values of β which indicates that particle trapping is high for large values of β. In Figure 2(b), variation of V has been shown for different values of ϕ and n_{j0} and constant values of β. It is observed that due to increase of negative ion concentration (n_{j0}), the amplitude of the Sagdeev potential decreases and it has a minimum value when the percentage of negative ion concentration (n_{j0}) is maximum. Moreover, Figure 2(b) shows that the existence of both the rarefactive and compressive solitary waves is possible in presence of negative ion. The variation of Sagdeev potentials for different values of ϕ and β in the absence of n_{j0} in bounded and unbounded plasma, have been shown in Figure 2(c) and 2(d) respectively. It is interesting to see that due to the presence of non-thermal electrons but in the absence of negative-ions, ion-acoustic solitary waves will not exist for any value of β in bounded plasma. However, the solitary waves are found to be present in an unbounded plasma in the absence of negative ions for large values of β of non-thermal electrons. In unbounded plasma, the same results are obtained by previous authors [30]. Mach number (M) also play an importnat role for the formation of solitary waves in a bounded plasma consisting of non-thermal electrons and negative ions. Variation of Sagdeev potential for different values of ϕ and M for some constant values of β and n_{j0} is shown in Figure 2(e). It is observed that V increases with the increase of M.

5. Solitary wave solution

In oder to study the small-amplitude acoustic solitary waves in a bounded plasma, we expand eq. (16) as a power series of ϕ. To obtain the first-order K-dV soliton we take the terms upto ϕ^2 from (16) and get

$$\frac{d^2\phi}{d\xi^2} = H_1\phi - H_2\phi^2 \tag{17}$$

where

$$H_1 = \left\{(2-\beta) + \frac{Qn_{i0} - n_{j0}}{Q(M-u_0)^2}\right.$$

$$H_2 = -\left. 1/2 + \frac{3\alpha\left(Q^2 n_{i0} - n_{j0}\right)}{Q^2(M-u_0)^2}\right\} \tag{18}$$

Eq. (17) has the soliton solution given by

$$\phi_1 = \phi_{01} \ \sec h^2(\theta), \tag{19}$$

where

$$\theta = (H_1/4)^{1/2}.\xi.$$

The amplitude (ϕ_{01}) and the width (δ_1) of the solitary waves are given by

$$\phi_{01} = 3H_1/2H_2, \; \delta_1 = - \qquad (20)$$

To see the effect of higher-order nonlinearity on the ion-acoustic solitary waves, we take the terms upto ϕ^3 of (16)

$$\frac{d^2\phi}{d\xi^2} = H_1\phi - H_2\phi^2 + H_3\phi^3, \qquad (21)$$

where

$$H_3 = \left\{(1+3\beta)/6 + \frac{5\alpha(n_{i0}Q^3 - n_{j0})}{Q^3(M-u_0)^6}\right.$$

$$\frac{}{d\xi} = L_1\phi^2 - L_2\phi^3 + L_3\phi^4, \qquad (22)$$

where $L_1 = H_1$, $L_2 = (2H_2/3)$ and $L_3 = -(H_3/2)$.

Integrating again, we get finally the second-order potential

$$\frac{2L_1}{\left(L_2^2 - 4L_1L_3\right)^{1/2}\left[2\cosh^2(\theta) - 1\right] + L_2} \qquad (23)$$

The width of the second-order soliton is

$$\delta_2 = \frac{2}{\sqrt{L_1}}\cosh^{-1}\left[\frac{0.6905\,L_2}{\left(L_2^2 - 4L_1L_3\right)^{1/2}} + 1.6905\right]^{1/2}. \qquad (24)$$

The structures of the solitary waves are shown in Figures 3(a)–3(b) for different values of Q (mass ratio of negative ion to positive ion) and β. Figure 3(a) has been drawn to see the effect of negative ions on the solitary waves in bounded plasma. We see that Q has dominant role on the formation of the solitary waves. For (O^-, Ar^+) plasma, first-order solitary wave is rarefactive; but second-order solitary wave is compressive in nature. For (Cl^-, H^+) plasma, both the first-order and second-order solitary waves are compressive, not rarefactive.

Figure 3(b) shows the solitary structure for different values of β. In (O^-, Ar^+) plasma rarefactive soliton exist for first-order but compressive soliton exist in second-order. It is important to see that the amplitude of second-order compressive soliton is higher than that of the first-order rarefactive soliton. Moreover, for the increase of β, amplitude of rarefactive soliton decreases but the amplitude increases for compressive soliton. For large values of β, compressive soliton becomes spiky. In profile of the solitary waves, the rarefactive soliton exist for 1st order and compressive soliton exist for higher order. The amplitude of the solitary wave increases with increasing β.

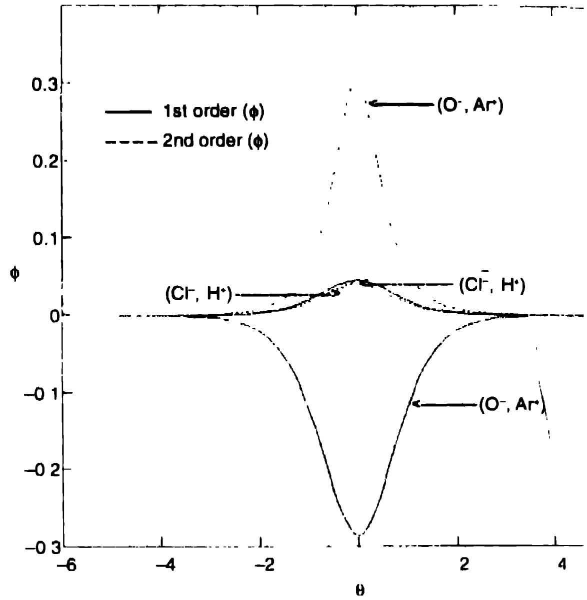

Figure 3(a). Structure of the solitary waves (both 1st-order and higher order) in plasma having (O^-, Ar^+), (Cl^-, H^+) ions and other parameters α = -2, u_0 = 0.1, n_{j0} = 0.13, M = 1.63

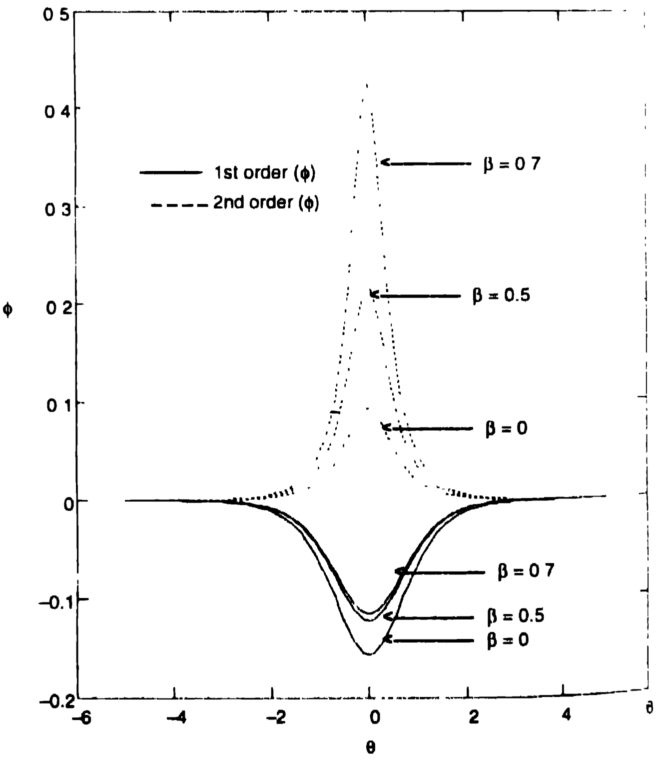

Figure 3(b). Structure of the solitary waves (both 1st-order and higher-order) in (O^-, Ar^+) plasma for different values of β and other parameters α = -2, u_0 = 0.1, n_{j0} = 0.13, M = 1.72.

The variation of width of soliton with β both for 1st-order and higher order and for different values of n_{j0} is shown in

Figures 4(a)–4(b). The width of the soliton increases with β and it has large values for higher values of n_{j0}.

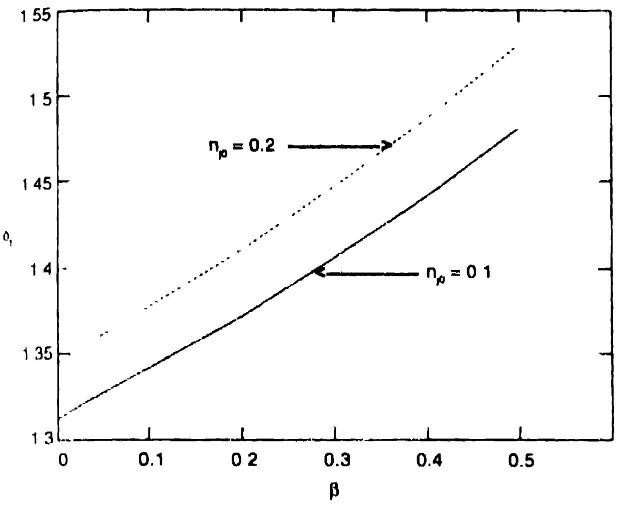

Figure 4(a). Variation of width of soliton (1st-order) with β in (O^-, Ar^+), plasma for different values of n_{j0} and other parameters α = −2, u_0 = 0 1, M = 1 72

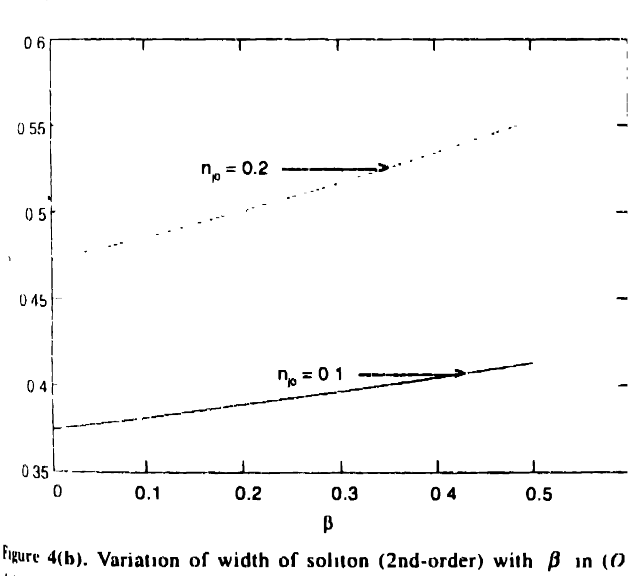

Figure 4(b). Variation of width of soliton (2nd-order) with β in (O^-, Ar^+) plasma for different values of n_{j0} and other parameters α = −2, u_0 = 0 1, M = 1 72.

6. Summary and concluding remarks

In the present paper, we have theoretically investigated the propagation of ion-acoustic wave in a bounded plasma consisting of positive ions, negative ions and non-thermal electrons. The role of negative ions, non-thermal electrons and radius of the cylinder on the phase velocity and instability of the waves have been graphically discussed. The dependence of Sagdeev potential as well as the amplitude of ion-acoustic solitary waves on the plasma parameters have also been critically discussed. We obtained some important results as follows :

For a forward going ion-acoustic wave, the phase velocity increases with the decrease of negative ion concentration and also for low values of non-thermal parameter. Both the non-thermal electrons and radius of the wave guide play significant roles on the phase velocities and instability of ion acoustic waves. Ion-acoustic wave becomes more unstable in a negative ions plasma having large values of β and low value of R.

From the behaviour of Sagdeev potential, it is seen that both compressive and rarefactive solitary waves may be formed in a bounded plasma consisting of negative ions and non-thermal electrons. In an unbounded plasma, earlier authors showed that both rarefactive and compressive solitary waves are formed in a presence of positive ions, negative ions and warm electrons and only compressive solitons exist in a plasma having positive ions. In the present paper, it is seen that compressive soliton will also exist in an unbounded plasma system in presence of non-thermal electrons for large values of β. Moreover, solitary waves will not exist at all, in the absence of negative ions in a bounded plasma having positive ions and non-thermal electrons.

Nature of solitary waves also depends on the types of negative ions and positive ions present in the plasma. In (O^-, Ar^+) plasma, the first-order solitary wave is rarefactive but second-order solitary wave is compressive. On the other hand in (Cl^-, H^+) plasma, both the first-order and second-order solitary waves are compressive, not rarefactive. For the increase of the values of non-thermal parameter β, the amplitude of the rarefactive soliton decreases but it is increased in compressive solitary wave. For large values of β, the compressive solitary waves become spiky.

However, in our present study, we have not considered the temperature of the ions and static magnetic field. Interesting results on the stability of ion-acoustic waves and formation of solitary waves will come out if both the effects of magnetic field and ionic temperature are considered. Ion-acoustic waves in bounded plasma considering the effect of inhomogeneity in the plasma will also give some interesting results.

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