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# Sensitivity of piezoelectric transducers

A K Ghosh[1*] and T K Munshi[2]
[1]Department of Physics, Kalna College, Kalna, Burdwan-713 409, West Bengal, India
[2]Department of Physics, Kharagpur College, Kharagpur, Midnapore-721 305, West Bengal, India
E-mial : ghanjan@yahoo.com

**Abstract** : A theoretical analysis based on an established transducer model has revealed a mathematical expression for sensitivity (*e.g.*, volts. per unit signal force) of piezoelectric plate transducer operating in its thickness mode and air as its surrounding medium. The sensitivity is found to be a strong function of frequency, with the principal peak being at the plate's fundamental thickness resonance. Other peaks of sensitivity are found to be at the odd multiples of this fundamental resonance frequency, whereas at even multiples of this frequency, the sensitivity is practically zero.

**Keywords** : Sensitivity, piezoelectric transducers, Laplace transform, thickness mode, resonance.

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## 1. Introduction

Here, our object is to study the sensitivity of a piezo electric plate transducer operating in its thickness mode and air as its surrounding medium. For this purpose, we have derived the required mathematical expressions from an established transducer model. We have investigated theoretically the frequency dependence of sensitivity. The interesting feature is that the sensitivity is not only a strong function of frequency but also the sensitivity has peaks at the odd multiples of the plate's fundamental thickness resonance-frequency, whereas at even multiples of this frequency, the sensitivity is practically zero. In our model, we consider a piezoelectric plate transducer which is mechanically coupled to semi-infinite real media positioned at each its two faces. For the use of this device as a receiving one, let $F_1$ be the amplitude of the signal force incident on the front face of the transducer and $V_o$, the amplitude of the corresponding signal voltage developed across its electrodes. $Z_L$ is the electrical impedance of the load attached to it. Here, we assume that the transducer is subject to linear, planar and unidirectional wave motion in the thickness direction and there is no intrinsic loss within the transducer. The bar over a symbol denotes its Laplace transform and $S$ is the Laplace parameter.

*Corresponding Author

## 2. Theoretical model

First we describe the primary piezoelectric action as follows Let $T_F$ be the force transmission coefficient for the front face of the crystal. The characteristic acoustic impedance of a material $Z_{CA}$ is the product of its density $\rho$ and the longitudinal wave velocity $c$ [1]. $Z_C$ is $AZ_{CA}$, where $A$ = area of the transducer. If $h$ is the piezoelectric charge constant for the thickness direction, $h/sZ_C$ represents the conversion of signal force to signal voltage. $\overline{K}_F$ represents the effect of mechanical reverberation at the front face and $\overline{U}$ represents the influence of the external electrical load on primary piezoelectric action [2]. The polarization charge developed in piezo-material produces an electric field in the material in such a direction as to oppose the incident mechanical stress.

Now, we describe the secondary piezoelectric action by two feed back loops as follows :

Again, at this stage $\overline{K}_F$ represents the effect of mechanical reverberation at the front face of the transducer. $\overline{Z}_F$ represents a dimensionless transfer function for the influence of the external load on secondary piezoelectric action. $k$ is the laterally clamped electro-mechanical coupling coefficient and $T$ is the time taken for longitudinal pressure waves to cross the transducer thickness. $K^2/sT$ accounts for energy conversion in feed back. $T_F/2$ represents the effect of the front face transmission coefficient on secondary piezoelectric action. The second feed back loop operates in the same way as the first except for the fact that $\overline{K}_F$ is replaced by $\overline{K}_B$ for the back face and $T_F$ is replaced by $T_B$.

Now,

$$\frac{\overline{V}_o}{\overline{F}_1}=\frac{-hT_F\overline{K}_F\overline{U}/sZ_C}{1-\overline{Z}_F\left(\overline{K}_F\frac{T_F}{2}+\frac{\overline{K}_BT_B}{2}\right)\frac{k^2}{sT}}, \tag{1}$$

$$\overline{U}=\frac{sC_0\overline{Z}_L}{1+sC_0\overline{Z}_L}, \tag{2}$$

$$\overline{Z}_F=\frac{1}{sC_0\overline{Z}_L}. \tag{3}$$

Here, $C_0$ is the transducer's static capacitance. $\overline{U}$ ranges from zero to unity under short and open circuit conditions respectively. $Z_F$ is zero under open circuit condition. Again,

$$\overline{Z}_E=\frac{1}{sC_0}\left[1-\frac{k^2}{sT}\left(\frac{\overline{K}_FT_F}{2}+\frac{\overline{K}_BT_B}{2}\right)\right], \tag{4}$$

$Z_E$ is transducer's electrical impedance.

$$\overline{K}_F=\frac{\left(1-e^{-sT}\right)\left(1-R_Be^{-sT}\right)}{1-R_FR_Be^{-2sT}} \tag{5}$$

and

$$\overline{K}_B=\frac{\left(1-e^{-sT}\right)\left(1-R_Fe^{-sT}\right)}{1-R_FR_Be^{-2sT}}. \tag{6}$$

$$T_F=\frac{2Z_C}{Z_C+Z_1} \tag{7}$$

$$T_F=\frac{2Z_C}{Z_C+Z_2} \tag{8}$$

where $Z_1 = A\rho_1c_1$ and $Z_2 = A\rho_2c_2$. Here, $\rho_1$ is the density of the transducer's front face medium and $\rho_2$ is the density of its backing medium. Also $c_1$ and $c_2$ are the velocities of longitudinal pressure waves in the front and back face media respectively. $R_F$ and $R_B$ are the front and back face reflection coefficients respectively. These are given by

$$R_F=\frac{Z_C-Z_1}{Z_C+Z_1}, \tag{9}$$

and

$$R_B=\frac{Z_C-Z_L}{Z_C+Z_L}. \tag{10}$$

Eqs.(1) to (10) are found in Ref. [3].

Now, we consider the case of the air coupled transducer. For zero electrical and mechanical loan and with air at front and back faces of the transducer, omitting Laplace notation, we can write $Z_1 = Z_2$, $K_B = K_F$ and $R_B = R_F$. Therefore from eq.(5) we get

$$K_F=\frac{1-e^{-j\omega T}}{1+R_Fe^{-j\omega T}}. \tag{11}$$

In open circuit condition, U = 1, $Z_F$ = 0. [eqs.(2) and (3)]. So, eq. (1) gives

$$\frac{V_o}{F_1}=\frac{-hT_FK_F}{j\omega Z_C}. \tag{12}$$

This indicates that if the amplitude of the incident signal force $F_1$ is held constant w.r.t. frequency, then the signal voltage $V_o$ will have a frequency dependent characteristic that is purely dependent on $K_F$, which describes reverberation within the transducer. To hold $V_o$ constant w.r.t. frequency, it would be necessary to drive the transducer with a frequency-dependent signal force (as evident from eq. (12)). Using eqs. (11) and (12) one gets

$$\frac{V_0(0)}{F_1}=\frac{-hT}{Z_C}. \tag{13}$$

Let $G_0(\omega)=\dfrac{V_0(\omega)}{V_0(0)}$,

then from eqs. (12) and (13) we have

$$G_0(\omega)=\frac{T_FK_F}{j\omega T}. \tag{14}$$

For convenience, let us take $R = f/f_n = \omega T/\pi$ whence eqs. (11) and (14) give

$$G_0(R)=\frac{R_F+1}{j\pi R}\left(\frac{1-e^{-j\pi R}}{1+R_Fe^{-j\pi R}}\right), \tag{15}$$

$T_F = R_F+1$ from eqs. (7) and (9).

## 3. Results and discussion

Let $G_0(R) = Q$ for $R$ = 1. $e^{-j\pi} = -1$ . So eq. (15) gives

$$Q=\frac{2}{\pi}\left(\frac{1+R_F}{1-R_F}\right). \tag{16}$$

Therefore, $R_F = \dfrac{Q\pi - 2}{Q\pi + 2}$ (17)

Putting this in to eq. (15) we get after some manipulation,

$$G_o(R) = \frac{1}{jR}$$

$$\times \left[\frac{1}{Q} + \frac{\pi}{2}\left\{\frac{1+(\cos\pi R - j\sin\pi R)}{1-(\cos\pi R - j\sin\pi R)}\right\}\right] \quad (18)$$

Therefore,

$$G_o(R) = \frac{2Q}{}$$

$$\times \left[\frac{-\pi Q \sin\pi R - 2j(1-\cos\pi R)}{\pi^2 Q^2 \cos\pi R - 4\cos\pi R + \pi^2 Q^2 + 4}\right] \quad (19)$$

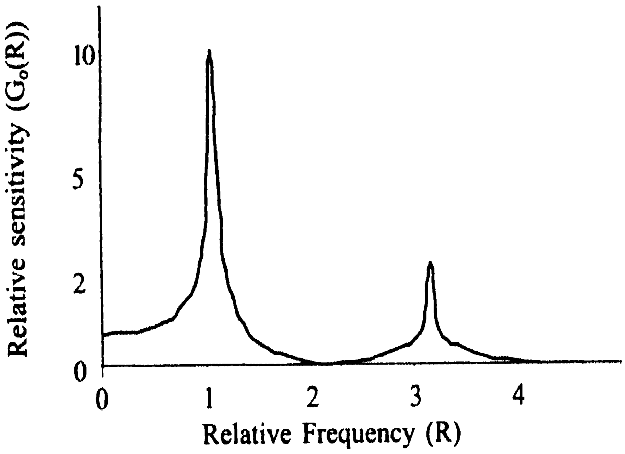


**Figure 1.** Plot of $G_o$ $(R)$ against $R$

Eq. (19) has been used to plot the magnitude of $G_0$ $(R)$ for values of $R$ in the following figure (Figure 1).

The graph obtained here shows clearly that the sensitivity of our transducer depends on the frequency in the way we mentioned from the very beginning.

## 4. Conclusion

This study gives an expression for sensitivity of piezoelectric plate transducer operating in its thickness mode and air as its surrounding medium. The sensitivity is found to be a strong function of frequency, with the principal peak being at the plate's fundamental thickness resonance. Other peaks of sensitivity are found to be at the odd multiples of this fundamental resonance frequency, whereas at even multiples of this frequency, the sensitivity is practically zero.

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