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A local potential with analytical distorted wave approximation model for elastic scattering of pions from a nucleus

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Abstract : The ordinary equivalent local form of Kisslinger [*Phys. Rev.* **98** 761 (1955)] type optical potential together with an analytical distorted wave approximation (ADWA) is explained to discuss the elastic scattering of pions with intermediate energies from a nucleus. Calculation of angular distribution of the scattering of negative pion on $^{208}_{82}$Pb using Born approximation gives an acceptable fit to the data. The possible values for the potential parameters and ADWA parameters are obtained by comparison of data with the theoretical formula.

Keywords : Nuclear scattering, angular distribution, analytical distorted wave approximation, optical potential.

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We have discussed previously the pion-nucleus elastic scattering for low energy region [1]. The study of such interaction with incident energies around the range of the first pion nucleon resonance Δ (1232) has been a interesting subject for many theoretical and experimental nuclear physicists over the past decade. Theoretical studies are mostly based upon phenomenological local potential models. The reason for the importance of this approach is that both *s* and *p* -wave components are present in pion-nucleus scattering. Here, we explain the pion-nucleus elastic scattering for the energy range where l=0,1 partial waves are in resonance by an equivalent local form [1,2] of a Kissliger optical potential (KOP) [3] using the method of Born approximation (BA). But pion-nucleon two body interaction becomes stronger at intermediate energies for larger distances from the center of the nucleus which causes pions not to penetrate deeper in to the nucleus. Therefore, the elastic scattering cross section would have a considerable increase which is not consistent with the values obtained from KOP. This needs to increase the strength of the potential. Regarding the spin and isospin degrees of freedom it is possible to relate the scattering amplitude to the complex phase shifts. The real part of the phase shifts is measured by an observer which give the elastic component of the scattering. On the other hand, the imaginary part is responsible for inelastic scattering. The imaginary part indicates that this potential behaves like a source or a well for incident flux. These all indicate that the KOP must be altered from the original form for medium energies. For these reasons, we use the distorted wave impulse approximation method (DWAM) [5] in our calculation and so, the approximated plane waves are used instead of the usual distorted wave functions. Therefore, a sort of analytic distorted wave approximation [6-8] should be applied which has been successfully used in calculation of other particles from nuclei. A brief review of the theoretical calculation of optical potential (OP) is described in Section 2. In Section 3, we present elastic scattering formalism using the ADWA model. Section 4 being devoted to the discussion of results and conclusions.

The actual KOP for pion scattering from a nucleus [1] is:

$$U_k(r) = \frac{(\hbar c)^2}{2\omega}\cdot(q(r) + \nabla\cdot\alpha(r)\nabla). \tag{1}$$

The local transformed form of this potential is calculated from a Schrödinger like equation which is obtained from a Klein Gordon equation by using the Krell-Ericson transformation [4] :

$$U_L = \frac{(\hbar c)^2}{2\omega}\left[\frac{q}{1-\alpha} - \frac{k^2\alpha}{1-\alpha} - \left[\frac{\nabla^2\alpha}{1-\alpha} + \frac{(\nabla\alpha)^2}{4(1-\alpha)^2}\right]\right] + \frac{\alpha V_c}{1-\alpha}. \tag{2}$$

The s and p-wave coefficients of the potential which are denoted by $\alpha(r)$ and $\beta(r)$ respectively, take the following equations :

$$q(r) = -4\pi P_1\left(b_0\rho(r) - e_\pi b_1 \Delta\rho(r)\right) + \Delta q(r), \tag{3}$$

$$\alpha(r) = \frac{\alpha_1(r)}{1 + \frac{1}{3}\varsigma\alpha_1(r)} + \alpha_2(r), \tag{4}$$

where

$$\alpha_1(r) = 4\pi \frac{\left(c_0\rho(r) - e_\pi c_1 \Delta\rho(r)\right)}{} \tag{5}$$

$$\alpha_2(r) = 4\pi \frac{\left(C_0\rho_{np}(r) - e_\pi C_1\rho(r)\Delta\rho(r)\right)}{P_2} \tag{6}$$

and

$$\rho(r) = \rho_n(r) + \rho_p(r), \tag{7}$$

$$\Delta\rho(r) = \rho_n(r) - \rho_p(r), \tag{8}$$

$$\rho_{np} = 4\rho_n(r)\rho_p(r). \tag{9}$$

Here, e_π takes +/– sign relative to the +/– charge state of pion and $\rho_p(r)$ and $\rho_n(r)$ are proton and neutron density distributions of target nucleus respectively. P_1 and P_2 are kinematics constants which depend on pion energy. The value of the potential parameters c_0, c_1, b_0, b_1, C_0 and C_1 are presented in Table 1. These parameters are energy-dependent and their values are obtained by fitting the theoretical angular distribution to the experimental data values for different energy values. In eq.(2), we have ignored the second-order s-wave contributions and take $\varsigma = 1$. The second order Coulomb interaction also has been omitted from these calculations. Here, we have used tw parameter Fermi distribution for proton and neutron:

$$\rho_i = \frac{\rho_{0_i}}{1 + \exp\left(r - \frac{c_i}{a_i}\right)}$$

Here, i stands for proton and neutron, ρ_{0_i} =0.0815 (fm) $c_n = 6.4$ fm , $c_p = 6.54$ fm, $a_p = a_n = 0.545$ fm. The RMS radii calculated by eq.(10) is 5.545 fm. To explain the necessary streng of the local potential and also to include the spin and isosp degrees of freedom, one can argue that KOP should have deformed shape as

$$V_{dop} = \frac{(\hbar c)^2}{2\omega}\left(a_0 q(r) + a_1\nabla\cdot\alpha\nabla\right),$$

where a_0, a_1 are r dependent parameters. It is straight forward show that these parameters are related to the phase shifts different spin-isospin states. They also vary with energy a scattering angle. To explain these features, we replace the incide wave function calculated from the OP with a distorted pla waves:

$$\Psi_i'^{d}(r) = N\exp\left(i(\alpha + i\beta)k.r\right). \tag{1}$$

Here, α and β take energy dependent values and N is functions of r and scattering angle which may take the followin form:

$$N = \left(A_1 r^{B_1} - A_2 r^{B_2} i\right)\sin\theta_{sc}.$$

These parameters α, β, A_i, and B_i should also be obtaine from a fit of the differential cross section data to the theoretic formula resulted from the scattering amplitude:

Table 1. Local Potential parameter values calculated in this work with $\varsigma = 1$ Note · In each case , the left number is the real and the right number is the imaginary part.

T_π(MeV)	b_0 (fm)	b_1 (fm)	c_0 (fm^3)	c_1 (fm^3)	C_0 (fm^6)	C_1 (fm^6)
162	0.179;0.117	-0.12;0.004	0.039;0.554	0.276;1.630	0.043.620	2.227;0.471
180	- 0.085;0.045	-0.124;0 007	0.146;0 639	0 082;0.159	0.030;2.138	0.172;4.514
291	-0.043;0 135	-0.118;0.020	-0.101;0.651	-0.056;0.121	0.728;0.469	-0.599;10183

Table 2. ADWA and kinematic parameter values calculated in this work.

T_π(MeV)	α	β	A_1			B_2	k (fm^{-1})	k_{eff}(fm^{-1})	P	
162	1.35	-0.02	4	0 08	0.7	0.6	1.353	1.8268	1.3247	1.1623
180	1.33	-0.03	4.5	0.08	0.08	0.6	1.454	1.9355	1.3440	1.1720
291	1.2	-0.185	6.2	0.08	0.7	0.6	2.060	2.5014	1.4632	1.2316

$$f(\theta_{sc}) = -\frac{\mu}{2\pi\hbar^2}\int \exp\left(-ik_f.r\right) V_{op}\Psi_i^d(r). \quad (14)$$

Real and imaginary parts of the Kisslinger optical potential for three energies are presented in Figure 1. The strength of the repulsive real part has a substantial decrease with increasing energy and the attractive imaginary part also becomes weaker when energy is increased. An attractive minimum in the real part is gradually produced with increasing energy. We have calculated the angular distribution for each energy by using eq. (14) numerically by taking 400 points of integration. The results are compared with data in Figure 2. These results show that although there is difference between the theoretical estimates and data, but altogether, using the local form of KOP with the ADWA provides a quite successful theory of elastic scattering of negatively charged pions from $^{208}_{82}$Pb target. This can be extended to the other target nuclei. Theoretical results are sensitive to the parameters of the optical potential which are obtained by comparing the theoretical and experimental differential cross section. In these case, the optimum values for the parameters of the analytic function of the ADWA are also calculated. We think that both of the α and β parameters need to show a smooth variations with the pion energy. A clear fact is obvious that the incident and emerging wave functions would provide an overlap function that resembles the overlap of the two actual distorted waves in the spatial region of integration. In doing so, it is clear that, these wave functions should provide a qualitatively good theoretical background for the study of inelastic scattering of pions from nuclei.

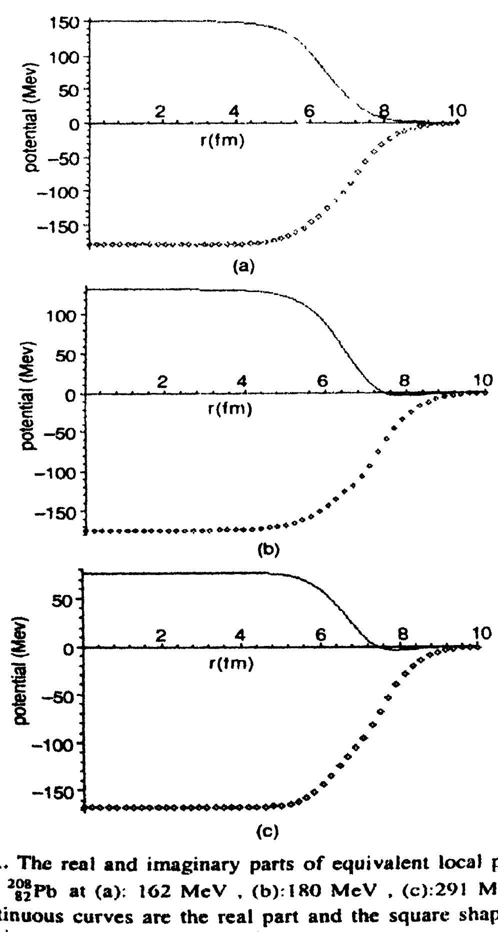

Figure 1. The real and imaginary parts of equivalent local potential, U_l, for $\pi^- + ^{208}_{82}$Pb at (a): 162 MeV , (b):180 MeV , (c):291 MeV energies. The continuous curves are the real part and the square shape curves are the imaginary part of the potential.

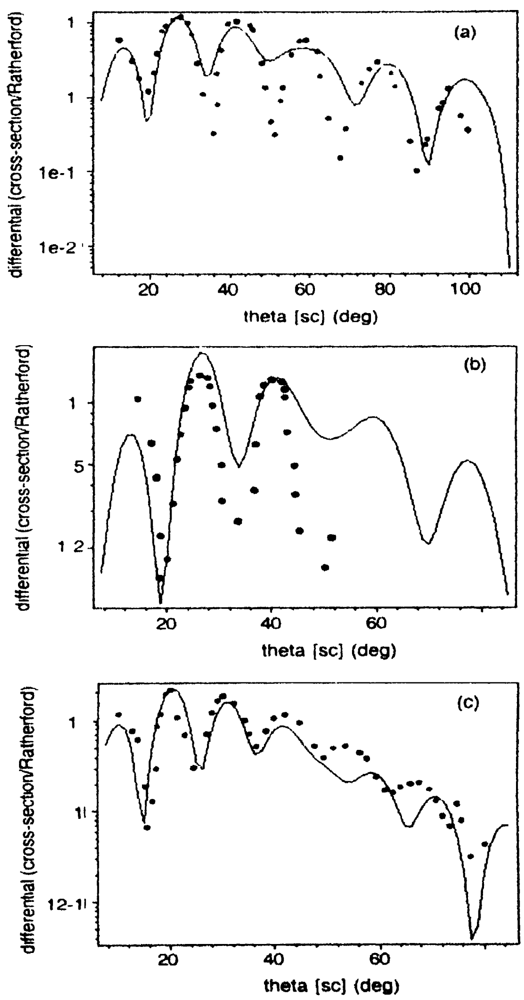

Figure 2. Elastic scattering differential cross section of negative pion, continuous curves, from $^{208}_{82}$Pb target using eq.(14) at (a) : 162 MeV, (b) . 180 Mev , (c) :291 MeV energies. Data points are from references [9], [10] and [11] respectively.

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