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# Theory of ultrasonic attenuation in heavy fermion compounds

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tract . The mechanism of the ultrasonic attenuation in heavy fermion compounds is not yet studied clearly both experimentally and etically A microscopic theoretical model is proposed here to study the attenuation in the compounds like $UPt_3$, $CeCu_2Si_2$, $CeRu_2Si_2$ in their normal :s We consider the Periodic Anderson Model and incorporate phonon coupling to the hybridisation between the conduction electrons and ctrons as well as to the $f$-electrons alone The phonon Green's function is calculated by Zubarev technique The temperature dependence of the sonic attenuation coefficient ( $\alpha$ ) is calculated from the imaginary part of the phonon self-energy and the velocity of sound in the dynamic and wavelength limit The dependence of the parameters like the electron-phonon coupling parameters ($g$, $r$), hybridisation ($v$), position of $f$-level ($d$), lomb correlation ($u$), frequency ( $\omega$ ), is investigated numerically through plots

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**Introduction**

m after the discovery of heavy fermion superconductivity, rasonic data have revealed power law dependence for :nuation coefficients for $T < T_c$ and a maximum around ~0.5 K for $UPt_3$ is observed [1-3]. The attenuation maximum urs exactly where the elastic constant and the velocity of ind exhibit minima [4]. It is also observed that the pronounced asonic attenuation peak of $URu_2Si_2$ coincides with the middle the elastic constant step indicating that the maximum of $\alpha$ urs at $T_c \approx 1.2$ K.

The first attenuation measurements in zero magnetic field owing Kondo effect were done by Müller *et al* [5]. A peak in attenuation was seen at 10–12K [6]. Similar low temperature omalies were observed in the elastic constants and the ocity of sound for compounds $CeCu_6$ ($T^*$ = 4K), $CeCuSi_2$ ($T^*$ 0K), $URu_2Si_2$ ($T^*$ = 70K), $CeBe_{13}$ ($T^*$ = 340K) [7].

Thalmeier [8] has considered Grüneisen parameter coupling sound waves to the bands to explain the elastic anomalies in temperature quasi-particle resime. Others have considered strain-dependence of the hybridisation to investigate the v temperature anomaly in sound velocity of heavy fermion systems by using a mean field approach [9]. Rout and Das [10] have calculated velocity of sound of superconducting heavy fermion systems. In this paper, a microscopic theoretical model is proposed to investigate the ultrasonic attenuation in normal state heavy fermion systems. The attenuation Müller peak [5] can be described in terms of a narrow resonance peak with width of ~ 2 meV in the density of state lying above the Fermi surface resulting from the hybridisation of $f$-level and the conduction band.

## 2. Formalism

The system is described by the Hamiltonian

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{e-p} + \mathcal{H}_p , \quad (1)$$

$$\mathcal{H}_0 = \sum_{K,\sigma} \varepsilon_K c^{\dagger}_{K,\sigma} c_{K,\sigma} + \varepsilon_f \sum_{K,\sigma} f^{\dagger}_{K,\sigma} f_{K,\sigma}$$
$$+V \sum_{K,\sigma} \left(f^{\dagger}_{K,\sigma} c_{K,\sigma} + c^{\dagger}_{K,\sigma} f_{K,\sigma}\right)$$
$$+(U/2) \sum_{i,\sigma} n^f_{i,\sigma} n^f_{i,-\sigma} . \quad (2)$$

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$$\mathcal{H}_{e-p} = \sum_{K,q,\sigma}\Big[f_1(q)\left(c^{\dagger}_{K+q,\sigma} f_{K,\sigma} + f^{\dagger}_{K+q,\sigma} c_{K,\sigma}\right)$$

$$+f_2(q)\left(f^{\dagger}_{K+q,\sigma} f_{K,\sigma}\right)\Big]\left(b_q + b^{\dagger}_{-q}\right), \quad (3)$$

$$\mathcal{H}_p = \sum_q \omega_q b^{\dagger}_q b_q . \quad (4)$$

$\mathcal{H}_0$, $\mathcal{H}_{e-p}$ and $\mathcal{H}_p$ are the Hamiltonians representing Periodic Anderson Model, electron-phonon interaction and free phonon term. Here $c^{\dagger}_{K,\sigma}(c_{K,\sigma})$ $f^{\dagger}_{K,\sigma}(f_{K,\sigma})$, $b^{\dagger}_q(bq)$ are the creation (annihilation) operators of $d$- and $f$- electrons and phonons respectively. $\varepsilon_k$, $\varepsilon_f$ and $V$ are the conduction electron energy, position of $f$-level and hybridisation respectively. $U$ is the Coulomb interaction. $f_1(q)$ and $f_2(q)$ are the phonon coupling to hybridisation and $f$-electron respectively.

The double time Green's function of Zubarev type is defined as

$$D_{q,q}(t-t') = -i\Theta(t-t')\left\langle\left[A_q(t)\,;\,A_q(t')\right]\right\rangle . \quad (5)$$

Applying Dyson's approximation, the phonon Green function can be written as

$$D_{q,q}(\omega) = \left(\omega_q/\pi\right)\left[\omega^2 - \omega_q^2 - \Sigma_q(\omega)\right]^{-1}, \quad (6)$$

where phonon self energy is given by

$$\Sigma_q(\omega) = 4\pi\,\omega_q \chi_{q,q}(\omega),$$

$$\chi_{q,q}(\omega) = \Sigma_{K,\sigma}\Big[f_1^2(q)\,\Gamma_3 + f_1(q)\,f_2(q)\left(\Gamma_4 + \Gamma_5\right)$$

$$+f_2^2(q)\,\Gamma_6\Big]. \quad (7)$$

$\Gamma_i(k,q,\omega)$'s ($i$ = 3 to 6) represent the electron response functions. They are defined by dropping $k, q, \omega$ in terms of $\alpha$'s and $\beta$'s as

$$\Gamma_3 = \langle\langle\left(\alpha^a + \alpha^b\right);\left(\beta^a + \beta^b\right)\rangle\rangle_{\omega},$$

$$\Gamma_4 = \langle\langle\left(\alpha^a + \alpha^b\right);\beta^c\rangle\rangle_{\omega},$$

$$\Gamma_5 = \langle\langle\alpha^c\,;\left(\beta^a + \beta^b\right)\rangle\rangle_{\omega},$$

$$\Gamma_6 = \langle\langle\alpha^c\,;\beta^c\rangle\rangle_{\omega}. \quad (8)$$

These operators are written in abreviated form as

$$\alpha^a = c^{\dagger}_{k-q,\sigma} f_{k,\sigma}, \quad \alpha^b = f^{\dagger}_{k-q,\sigma} c_{k,\sigma},$$

$$\alpha^c = f^{\dagger}_{k-q,\sigma} f_{k,\sigma}, \quad \alpha^d = c^{\dagger}_{k-q,\sigma} c_{k,\sigma}.$$

$$\left(\beta^a + \beta^b\right) = c^{\dagger}_{k'-q',\sigma'} f_{k',\sigma'} + f^{\dagger}_{k'-q',\sigma'} c_{k',\sigma'},$$

$$\beta^c = f^{\dagger}_{k'-q',\sigma'} f_{k',\sigma'} .$$

In the long wavelength limit, $q \to 0$. One has $\omega =$ and $\omega_0 = v_0 q$ where $v_0$ and $v$ are the bare and renormal longitudinal sound velocities. The velocity of sound is gi by

$$v = v_0\left[1 + \frac{4\pi\chi(\omega,q)}{\omega_0}\right]^{1/2}$$

The ultrasonic attenuation coefficient $\alpha(\omega,T)$ for so waves of frequency $\omega$ and at a temperature $T$ is obtained fr the imaginary part of self energy through

$$\alpha(\omega,T) = -\frac{4\pi}{\ }\,\mathrm{Im}\,\chi(\omega,q), \quad ($$

where the -ve sign indicates absorption of energy. To st ultrasonic attenuation, the parameters are made dimension with respect to Debye frequency $\omega_D$.

$$\tilde{\omega} = \frac{\omega}{\omega_D}, c = \frac{\omega}{\omega_D}, p = \frac{\omega_0}{\omega_D}, e = \frac{\eta}{\omega_D}, d = \frac{\varepsilon_f}{\omega_D};$$

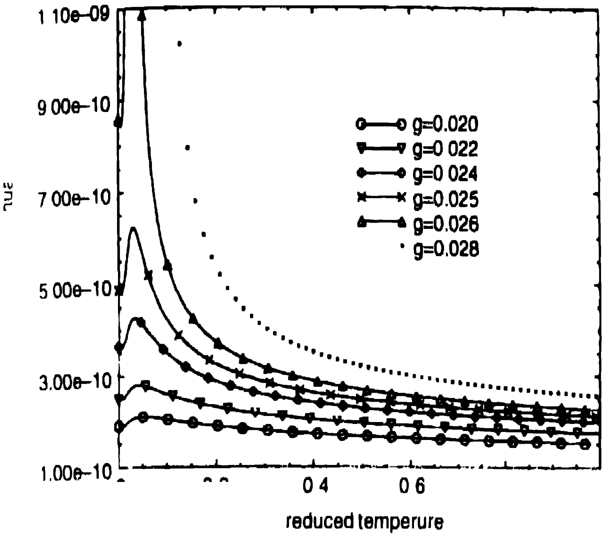
$$u = \frac{U}{\omega_D}, d_1 = d + un, V = \frac{V}{\omega_D}, g = \frac{N(0)\,f_1(0)^2}{\omega_D}$$

$$r = \frac{f_2(0)}{f_1(0)}, t = \frac{kT}{\omega_D}, \tilde{\omega}_l = \frac{\omega_l}{\omega_D}, x = \frac{\varepsilon_k}{\omega_D}, \tilde{W} = \frac{W}{\omega_D}$$

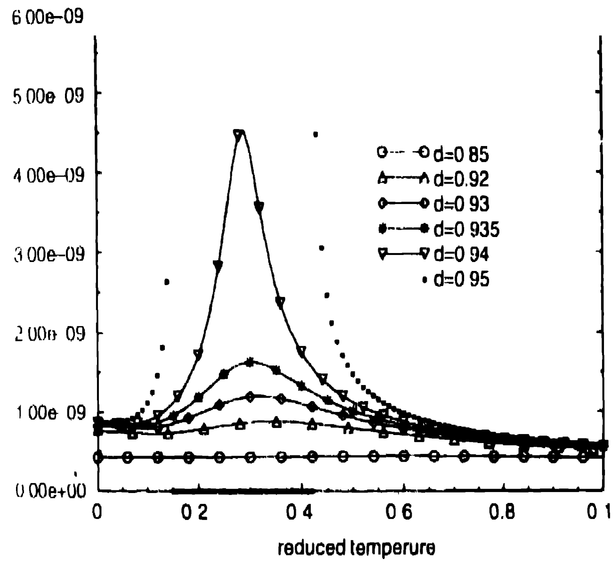
## 3. Results and discussion

The velocity of sound is evaluated numerically under half-fil band situation. The Fermi level is taken at the middle of band with $\varepsilon_F = 0$. The dimensionless parameters involve numerical calculations are the phonon coupling strength the ratio ($r$) between the phonon coupling to hybridisation the $f$-level, the position of the bare $f$-level ($d$), hybridisation Coulomb interaction ($u$). The temperature variation of ultrasonic attenuation of sound of the heavy fermion syste shows unusually high anomaly at low temperatures for cer physical parameters. The anomaly is discussed below.

Figure 1 shows the variation of ultrasonic attenuation temperature. As phonon coupling ($g$) to the hybridisat increases, the ultrasonic absorption increases and as a re peak height increases near temperature $t$ = 0.05. With De frequency $\omega_D \approx$ 200K, this corresponds to fluctuat temperature $T^* \approx$ 10K. The phonon coupling increases hybridisation between $f$-electron and conduction band results in increase of mixed valence behaviour. When effec evel lies below the Fermi level, the attenuation decreases with crease of phonon coupling ($r$) to the $f$-electron alone.




igure 1 The variation of ultrasonic attenuation with temperature for ifferent values of $g$ = 0 020, 0 022, 0 024, 0 025, 0.026, 0 028 and fixed alues of $r$ = 0, $d$ = -0 94, $v$ = 0 05, $u$ = 1 0, $e$ = 0 005, $p$ =0.12.




igure 2. The variation of ultrasonic attenuation with temperature for ifferent values of $d$ = - 0 85, -0 92, 0.93, -0.935, -0 94, 0 95 and fixed values of $g$ = 0 026, $r$ = 0, $v$ = 0 05, $u$ = 1 0, $e$ = 0.005, $p$ = 0 12

Figure 2 shows the variation of attenuation peak with temperature. As $f$-level moves from $d$ = – 0.85 to – 0.95 from above, towards the Fermi level $\varepsilon_F = 0$, the attenuation increases and attenuation maximum shifts to lower temperature. The shifting of $f$-level in the conduction band changes the hybridisation between $d$- and $f$-electrons. This leads to change in the density of states near Fermi level and changes the fluctuation temperature ($T^*$). Hence, the ultrasonic attenuation and softening elastic constant of several heavy fermion compounds can be explained.

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