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LMA MSW solution from the inverted hierarchical model of neutrinos

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Abstract : We examine whether the inverted hierarchical model of neutrinos can explain the large mixing angle (LMA) MSW solution of the solar neutrino problem or it is completely ruled out. The left-handed Majorana neutrino mass matrix for the inverted hierarchical model is generated through the seesaw mechanism using the diagonal form of the Dirac neutrino mass matrix and the non-diagonal texture of the right-handed Majorana mass matrix. In a model-independent way, we construct a specific form of the charged lepton mass matrix having a special structure in 1–2 block, which contribution to the leptonic mixing (MNS) matrix leads to the predictions $\sin^2 2\theta_{12} = 0.8517$, $\sin^2 2\theta_{23} = 0.9494$ and $|V_{e3}| = 0.159$ at the unification scale. These predictions are found to be consistent with the LMA MSW solution of the solar neutrino problem. The inverted hierarchical model having opposite signs of mass eigenvalues generally gives stability against the quantum radiative corrections in the MSSM. A numerical analysis of the renormalisation group equations (RGEs) in the MSSM shows a mild decrease of the mixing angles with the decrease of energy scale and the corresponding values of the neutrino mixings at the top-quark mass scale are $\sin^2 2\theta_{12} = 0.8472$, $\sin^2 2\theta_{23} = 0.9399$ and $|V_{e3}| = 0.1509$ respectively.

Keywords : Majorana neutrino mass, inverted hierarchical, mixing angles, mass eigenvalues, radiative correction.

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1. Introduction

Neutrino physics is one of the fast developing areas of particle physics. The recent Super-Kamiokande experimental results on both solar [1] and atmospheric [2] neutrino oscillations support the approximate bimaximal mixings. Though these results favour the large mixing angle (LMA) MSW solution with active neutrinos, such interpretation is not beyond doubt at this stage [3–5]. We also have little idea about the pattern of the neutrino mass spectrum whether it is hierachical or inverted hierarchical, and both possibilities are consistent with the neutrino oscillation explanations of the atmospheric and solar neutrino deficits [5,6]. The data from the long baseline experiment using a Neutrino factory will be able to confirm the actual pattern of the neutrino masses in the near future [7].

In the theoretical front, the hierarchical model of neutrino masses and its generation have been widely studied and found to be consistent with the explanation of the LMA MSW solar neutrino solution [8,9]. However, the inverted hierarchical model of neutrino masses generally predicts the maximal mixing angles θ^{ν}_{12} and θ^{ν}_{23} close to 45°, and are suitable for the explanation of the vacuum oscillation (VO) solution of the solar neutrino oscillation [6,10] and the atmospheric neutrino oscillation data. The presently available atmospheric data gives the lower bound on mixing parameter $\sin^2 2\theta_{23} \geq 0.88$ and the best-fit value of the mass-squared difference $\Delta m^2_{23} = 3\times 10^{-3}$ eV^2. It is quite obvious that the prediction from the inverted hierarchical model fails to explain the LMA MSW solution which has upper experimental limit [4,6] $\sin^2 2\theta_{12} \leq 0.988$ at 95% C.L.,

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and the best-fit values $\sin^2 2\theta_{12} = 0.8163$ and $\Delta m_{12}^2 = 4.2 \times 10^{-5}$ eV^2. Combining LMA MSW solution and atmospheric data, the best-fit value of the mass splitting parameter is obtained [6] as $\xi = \Delta m_{12}^2/\Delta m_{23}^2 = 0.014$. It has been argued [6,10] that the contribution from the diagonalisation of the charged lepton mass matrix cannot give a significant reduction to θ_{12}^{ν} needed for the explanation of the LMA MSW solution. On such ground, the inverted hierarchical model is not taken seriously for the explanation of the LMA MSW solution. An attempt was made to explain the LMA MSW solution from the inverted hierarchical model by considering two types of charged lepton mass matrices [11] and was partially successful. We are interested to make further investigations in this paper whether the inverted hierarchical model gives an acceptable LMA MSW solution when we include the contribution from the diagonalisation of the charged lepton mass matrix having special form in the 1–2 block, to the leptonic mixing matrix.

The paper is organised as follows. In Section 2, we outline the seesaw mechanism for generating the neutrino mass matrix which can lead to the inverted hierarchical mass pattern, and the construction of the charged lepton mass matrix suitable for the LMA MSW solution. In Section 3, we describe briefly the procedure for the analysis of the renormalisation group equations (RGEs) within the minimal supersymmetric standard model (MSSM). This is followed by a summary and discussion in Section 4.

2. Generation of the inverted hierarchical neutrino mass matrix and the charged lepton mass matrix

The inverted hierarchical model of neutrinos has its origin from the low energy non-seesaw models [12], *e.g.*, the Zee-type of model using a singly charged singlet scalar field and also the models with an approximate conserved $L_e - L_\mu - L_\tau$ lepton number. However, it is also possible to generate the inverted hierarchical model through the seesaw mechanism at high energy scale within the framework of the grand unified theories with a chiral $U(1)$ family symmetry [10,11]. In a model-independent way, we consider the Dirac neutrino mass matrix m_{LR} and the non-diagonal form of the right-handed Majorana mass matrix M_R in the seesaw formula [13] given by

$$m_{LL} = -m_{LR} M_R^{-1} m_{LR}^T, \tag{1}$$

where m_{LL} is the left-handed Majorana mass matrix. The leptonic mixing matrix known as the MNS mixing matrix [14] is defined by

$$V_{MNS} = V_{eL} V_{\nu L}^{\dagger}, \tag{2}$$

where V_{eL} and $V_{\nu L}$ are obtained from the diagonalisation of the charged lepton m_l and m_{LL} as

$$m_l^{\rm diag} = V_{eL} m_l V_{eR}^{\dagger} = {\rm Diag}(m_e, m_\mu, m_\tau),$$
$$m_{LL}^{\rm diag} = V_{\nu L} m_{LL} V_{\nu L}^T = {\rm Diag}(m_{\nu 1}, m_{\nu 2}, m_{\nu 3}). \tag{3}$$

If the charged lepton mass matrix is diagonal, the MNS matrix (2) is simply reduced to

$$V_{MNS} = V_{\nu L}. \tag{4}$$

Expressing m_{LL} in the basis where the charged lepton mass matrix is diagonal, we have

$$m'_{LL} = V_{eL} m_{LL} V_{eL}^T,$$
$$m'^{\rm diag}_{LL} = V'_{\nu L} m'_{LL} V'^T_{\nu L},$$
$$V_{MNS} = V'^{\dagger}_{\nu L}. \tag{5}$$

The neutrino flavour eigenstate ν_f is related to the mass eigenstate ν_i by the relation

$$\nu_f = V_{fi} \nu_i$$

and the MNS mixing matrix is given by V_{fi} where $f = \tau, \mu, e$ and $i = 1, 2, 3$. From the above expressions, we can calculate the following observed quantities :

(i) the mass-squared differences Δm_{12}^2, Δm_{23}^2 and their ratio

$$\xi = \frac{|\Delta m_{12}^2|}{|\Delta m_{23}^2|},$$

(ii) the atmospheric mixing angle,

$$S_{\rm at} = \sin^2 2\theta_{23} = 4|V_{\mu 3}|^2 (1 - |V_{\mu 3}|$$

(iii) the solar mixing angle,

$$S_{\rm sol} = \sin^2 2\theta_{12} = 4|V_{e2}|^2 |V_{e1}|^2,$$

(iv) the CHOOZ angle,

$$S_C = 4|V_{e3}|^2 \left(1 - |V_{e3}|^2\right) \text{ or simply } |V_{e3}|.$$

First, we consider the diagonal form of the charged lepton mass matrix m_l given by

$$m_l \begin{pmatrix} \lambda^6 & 0 & 0 \\ 0 & \lambda^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} m_\tau, \tag{6}$$

where the value of the Wolfenstein parameter [15] is $\lambda = 0.22$ and the ratios of the charged lepton masses are $m_\tau : m_\mu : m_e = 1 : \lambda^2 : \lambda^6$ respectively. From eq. (6), we have $V_{eL} = 1$ which leads to $V_{MNS} = V_{\nu L}^{\dagger}$. We consider the diagonal form of the Dirac neutrino mass matrix m_{LR} as the up-quark mass matrix [16]

$$m_{LR} = \begin{pmatrix} \lambda^8 & 0 & 0 \\ & \lambda^4 & 0 \\ & 0 & 1 \end{pmatrix} m_t, \qquad (7)$$

where the up-quark masses are in the ratios [17] $m_t : m_c : m_u = 1 : \lambda^4 : \lambda^8$. With the proper choice of the elements in M_R, we generate the inverted hierarchical neutrino mass matrix through the seesaw formula (1). We present here the following examples [18] :

Example (a) :

$$m_{LL} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & \lambda^3 & 0 \\ 1 & 0 & \lambda^3 \end{pmatrix} m_0, \qquad (8)$$

with the choice

$$M_R = \begin{pmatrix} -\lambda^{22} & \lambda^{15} & \lambda^{11} \\ \lambda^{15} & \lambda^8 & -\lambda^4 \\ \lambda^{11} & -\lambda^4 & 1 \end{pmatrix} v_R,$$

where $m_0 = \left(\frac{\lambda^{-3} m_t^2 10^9}{2 v_R} \right)$ eV.

The three right-handed Majorana neutrino masses are given by $M_R^{\rm diag} = (3.055 \times 10^{-12}, 2.4403 \times 10^{-8}, 1.0)\, v_R$. Eq (8) yields

$$V_{MNS} = V_{\nu L}^{\dagger} = \begin{pmatrix} -0.70577 & 0.70844 & -5.0\times10^{-11} \\ -0.50094 & -0.49906 & -0.70711 \\ -0.50094 & -0.49906 & 0.70711 \end{pmatrix} v_R \quad (9)$$

and the neutrino mass eigenvalues $m_i = (1.4195, -1.4089, 0.01065)\, m_0$, $i = 1, 2, 3$. With the input values $m_0 = 0.05$ and $m_t = 82.43$ GeV at GUT scale, the seesaw scale is predicted to be $v_R = 0.68 \times 10^{16}$ GeV. This gives the mass splitting parameter $\xi = \Delta m_{12}^2 / \Delta m_{23}^2 = 0.014$, and the mixing angles $\sin^2 2\theta_{12} = 0.9999$, $\sin^2 2\theta_{23} \approx 1.00$, $|V_{e3}| = 0.0$.

Example (b) :

$$m_{LL} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & -(\lambda^3 - \lambda^4)/2 & -(\lambda^3 + \lambda^4)/2 \\ 1 & -(\lambda^3 + \lambda^4)/2 & -(\lambda^3 - \lambda^4)/2 \end{pmatrix} m_0, \qquad (10)$$

with the choice

$$M_R = \begin{pmatrix} \lambda^{23} & \lambda^{16} & \lambda^{12} \\ \lambda^{16} & \lambda^8 & -\lambda^4 \\ \lambda^{12} & -\lambda^4 & 1 \end{pmatrix} v_R,$$

where $m_0 = \left(\frac{\lambda^{-4} m_t^2 10^9}{2 v_R} \right)$ eV.

The three right-handed Majorana neutrino masses are given by $M_R^{\rm diag} = (1.4701 \times 10^{-13}, 2.44 \times 10^{-8}, 1.0)\, v_R$. With the input values $m_0 = 0.05$ and $m_t = 82.43$ GeV at GUT scale, the seesaw scale is predicted to be $v_R = 2.89 \times 10^{16}$ GeV. The neutrino mass eigenvalues are $m_i = (1.4195, -1.4089, 0.00239)\, m_0$, and the other predictions are $\xi = 0.0151$, $\sin^2 2\theta_{12} = 0.9999$, $\sin^2 2\theta_{23} \approx 1.00$, $|V_{e3}| = 0.0$.

Example (c) :

$$m_{LL} = \begin{pmatrix} \lambda^3 & 1 & 1 \\ 1 & \lambda^4/2 & -\lambda^4/2 \\ 1 & -\lambda^4/2 & \lambda^4/2 \end{pmatrix} m_0, \qquad (11)$$

with the choice

$$M_R = \begin{pmatrix} 0 & \lambda^{16} & \lambda^{12} \\ \lambda^{16} & \lambda^8 & -(\lambda^4 + \lambda^{12}) \\ \lambda^{12} & -(\lambda^4 + \lambda^{12}) & 1 \end{pmatrix} v_R,$$

where m_0 and $M_R^{\rm diag}$ are the same with those given in example (b). The predictions are $m_i = (1.4195, -1.4089, 0.002343)\, m_0$, $\xi = 0.015$, $\sin^2 2\theta_{12} = 0.9999$, $\sin^2 2\theta_{23} \approx 1.00$, $|V_{e3}| = 0.0$.

In the avove results, the $V_{MNS} (= V_{\nu L}^{\dagger})$ obtained from the m_{LL} alone fails to explain the LMA MSW solution, and any small deviation in the texture of m_{LL} will hardly affect the maximal value of $\sin^2 2\theta_{12}$ [6,10]. The last hope is that there could be a significant contribution to θ_{12} from V_{eL} obtained from the diagonalisation of the charged lepton mass matrix m_l having special structure in 1–2 block [11]. We wish to examine here how $\theta_{\rm sol} = (\theta_{12}^{\nu} - \theta_{12}^{e})$ can resolve the LMA MSW solar neutrino mixing scenario [11].

We parametrise the charged lepton mixing V_{eL} defined in eq. (3), by the following three rotations [19,20] :

$$V_{eL} = \bar{R}_{23} \bar{R}_{13} \bar{R}_{12}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \bar{c}_{23} & \bar{s}_{23} \\ 0 & -\bar{s}_{23} & \bar{c}_{23} \end{pmatrix} \begin{pmatrix} \bar{c}_{13} & 0 & \bar{s}_{13} \\ 0 & 1 & 0 \\ -\bar{s}_{13} & 0 & \bar{c}_{13} \end{pmatrix} \begin{pmatrix} \bar{c}_{12} & \bar{s}_{12} & 0 \\ -\bar{s}_{12} & \bar{c}_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad (12)$$

where $\bar{s}_{ij} = \sin\theta_{ij}^{e}$ and $\bar{c}_{ij} = \cos\theta_{ij}^{e}$. Putting $\theta_{13}^{e} = \theta_{23}^{e} = 0$, eq. (12) reduces to

$$V_{eL} = \begin{pmatrix} \bar{c}_{12} & \bar{s}_{12} & 0 \\ -\bar{s}_{12} & \bar{c}_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \qquad (13)$$

This gives a special form in the 1–2 block. We then reconstruct [19] the symmetric charged lepton mass matrix using eq. (13) from the relation

$$m_l = V_{eL}^{\dagger} m_l^{\rm diag} V_{eR}$$

$$= \begin{pmatrix} \lambda^6\bar{c}_{12}^2+\lambda^2\bar{s}_{12}^2 & \lambda^6\bar{c}_{12}\bar{s}_{12}-\lambda^2\bar{c}_{12}\bar{s}_{12} & 0 \\ \lambda^6\bar{c}_{12}\bar{s}_{12}-\lambda^2\bar{c}_{12}\bar{s}_{12} & \lambda^6\bar{s}_{12}^2+\lambda^2\bar{c}_{12}^2 & \\ 0 & 0 & 1 \end{pmatrix} m_\tau, \quad (14)$$

where we have taken $V_{eL} = V_{eR}$ for symmetric matrix. For a specific choice of $\theta_{12}^e = 13°$, and $\lambda = 0.22$, eq. (14) leads to

$$m_l = \begin{pmatrix} 0.00256 & -0.01058 & 0 \\ -0.01058 & 0.04596 & 0 \\ 0 & 0 & 1 \end{pmatrix} m_\tau \quad (15)$$

which has a special form in the 1–2 block. The diagonalisation of m_l in eq. (15) gives

$$V_{eL} = \begin{pmatrix} 0.97439 & 0.22488 & 0 \\ -0.22488 & 0.97439 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (16)$$

which is now completely unitary. The corresponding eigenvalues of the charged lepton mass matrix are given by

$$m_l^{\rm diag} = \left(1.182\times10^{-4}, 4.8402\times10^{-2}, 1.0\right) m_\tau \quad (17)$$

which give almost correct physical mass ratios [17] $m_e : m_\mu : m_\tau = \lambda^6 : \lambda^2 : 1$. The MNS mixing matrix (2) is now calculated, using eqs. (9) and (16), as

$$V_{MNS} = V_{eL} V_{\nu L}^{\dagger} = \begin{pmatrix} -0.5750 & -0.8025 & 0.1590 \\ -0.6468 & 0.32696 & 0.6890 \\ -0.50094 & 0.49906 & 0.70711 \end{pmatrix}. \quad (18)$$

This leads to the mixing angles $\sin^2 2\theta_{12} = 0.8517$, $\sin^2 2\theta_{23} = 0.9494$, and $|V_{e3}| = 0.159$, and these predictions are consistent with the explanation of LMA MSW solution. The possible choice of $\theta_{12}^e = 14°$ in eq. (14) also leads to the predictions of $\sin^2 2\theta_{12} = 0.8298$, $\sin^2 2\theta_{23} = 0.9415$, and $|V_{e3}| = 0.1710$ while maintaining the good prediction of the ratios of the charged lepton masses. However, its value of $|V_{e3}|$ is above the CHOOZ and PALO VERDE experimental constraint [21] $|V_{e3}| \le 0.16$.

Taking the first result with $\theta_{12}^e = 13°$, 1the left-handed neutrino mass m'_{LL} in the basis where the charged lepton mass matrix is diagonal, is now expressed for our convenience, as

$$m'_{LL} = \begin{pmatrix} 0.437972 & -0.897698 & -0.973193 \\ -0.897698 & -0.443296 & -0.230068 \\ -0.973193 & -0.230068 & -0.005324 \end{pmatrix} m_0, \quad (19)$$

where $V_{MNS} = V_{\nu L}'^{\dagger}$ is the same as that given in eq. (18), and the neutrino mass eigenvalues are

$$m_i = \left(1.4196, -1.4089, 4.234\times10^{-7}\right) m_0; \quad i = 1, 2, 3$$

which give the mass splitting parameter $\xi = \Delta m_{12}^2 / \Delta m_{23}^2 = 0.014$, and the correct magnitudes of the masses with the choice $m_0 = 0.05$.

We further discuss some possible realisations of the texture of the charged lepton mass matrix m_l and its diagonalisation matrix V_{eL} given in eqs. (15) and (16). For simplicity, we express them in the following approximate forms

$$m_l \sim \begin{pmatrix} \lambda^4 & -\lambda^3 & 0 \\ -\lambda^3 & \lambda^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} m_\tau \quad (20)$$

and

$$V_{el} \sim \begin{pmatrix} 1 & \lambda & 0 \\ -\lambda & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (21)$$

It is interesting to note that the positions of the zeros in the mass matrix in eq. (20) have the same structure with those of lepton mass matrix obtained in the gauge theory of the standard model with an horizontal $U(1)$ gauge factor [22]

$$m_l = \begin{pmatrix} \varepsilon^{|2+6a-2b|} & \varepsilon^{|3a|} & 0 \\ \varepsilon^{|3a|} & \varepsilon^{|2(1-b)|} & 0 \\ 0 & 0 & 1 \end{pmatrix} m_\tau \quad (22)$$

and

$$V_{eL} = \begin{pmatrix} 1 & \delta_{e\mu} & 0 \\ -\delta_{e\mu} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (23)$$

where $b = 1/2$, $a = 1$, $\delta_{e\mu} = (m_e/m_\mu)^{1/2}$. Similar form of the texture of the charged lepton mass matrix is proposed in $SU(5)$ model [23], and also in a model based on $SUSY$ $SO(10) \times U(2)$ using a 126-dimensional Higgs [24].

We can see in the present analysis how the MNS mixing matrix differs from the CKM mixing matrix. The CKM matrix of the quark mixings defined by $V_{CKM} = V_{uL} V_{dL}^{\dagger}$, is usually parametrised by [15]

$$V_{CKM} \sim \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}, \quad (24)$$

where $\lambda = 0.22$ and $|A| = 0.90$. For our choice of the diagonal up-quark mass matrix in eq. (7), we have $V_{uL} = 1$

leading to $V_{CKM} = V_{dL}^{\dagger}$. Since $m_d = m_l^T$, we have $V_{eL} = V_{dL}^{\dagger} = V_{CKM}$. Neglecting higher power of λ in eq. (24), we have

$$V_{eL} \sim \begin{pmatrix} 1 & \lambda & 0 \\ -\lambda & & \\ 0 & & \end{pmatrix} \tag{25}$$

which is almost same as V_{eL} given in eq. (21). The positions of the zeros are the same. Thus, we can have the relation $V_{MNS} = V_{CKM} V_{\nu L}^{\dagger}$ which is true for the present example. Such linkage gives partial justification to our motivation for the choice of the charged lepton mass matrix given in eq. (15). This in turn saves the inverted hierarchical model of neutrino masses.

3. Renormalisation effects in MSSM

It is desirable to inspect how the values of $\sin^2 2\theta_{12}$, $\sin^2 2\theta_{23}$, $|V_{e3}|$ and ξ evaluated at the unification scale where the seesaw mechanism is operative, respond to the renormalisation group analysis on running from higher scale (M_u = 2 × 10^{16} GeV) down to the top quark mass scale ($\mu = m_t$) [25]. We consider the renormalisation group equations (RGEs) for the three gauge couplings (g_1, g_2, g_3) and the third family Yukawa couplings (h_t, h_b, h_τ) in the minimal supersymmetric standard model (MSSM) in the standard fashion [26]. At high scale $\mu = 2\times10^{16}$ GeV, we assume the unification of gauge couplings as well as third generation Yukawa couplings for large $\tan\beta$ [26]. We choose the input $\alpha_2 = (5/3)\alpha_1 = \alpha_3 = 1/24$, $h_t = h_b = h_\tau = 0.7$ and corresponding to large $\tan\beta = v_u/v_d$.

There are two main approaches for taking quantum radiative corrections of neutrino masses and mixings. One approach [27] deals directly with the mixing angles and neutrino mass eigenvalues, and this allows for easy quantitative discussion of the evolution of the masses and mixings. For the inverted hierarchical model with opposite sign of mass eigenvalues $m_{\nu 1} = -m_{\nu 2}$, the normalisation effects are generally weak, and hence the evolution of the mixing angles are very mild [27].

In this paper, we follow the alternative approach where the quantum correction is taken on all the elements of the neutrino mass matrix in the basis where the charged lepton mass matrix is diagonal. The diagonalisation of the neutrino mass matrix at a particular energy scale, leads to the physical neutrino masses as well as the mixing angles [28–30]. For simplicity, we neglect the threshold corrections of the heavy right-handed neutrino masses and see the maximum effect of the radiative corrections in running from GUT scale to top-quark mass scale.

Following the standard procedure, we express m_{LL} in terms of K, the coefficient of the dimension five neutrino mass operator [28–30] in a scale-dependent manner

$$m_{LL}(t) = v_u^2(t) K(t), \tag{26}$$

where $t = \ln(\mu)$ and $v_u(t)$ is the scale-dependent [30] vacuum expectation value (VEV) $v_u = v_0 \sin\beta$, v_0 = 174 GeV. In the basis where the charged lepton mass matrix is diagonal, we can write eq. (26) as [28,30]

$$m'_{LL}(t) = v_u^2(t) K'(t), \tag{27}$$

where $K'(t)$ is the coefficient of the dimension five neutrino mass operators in the basis where the charged lepton mass matrix is diagonal. The evolution equations are given by [30]

$$\frac{d}{dt} \ln v_u(t) = \frac{1}{16\pi^2}\left[\frac{3}{20} g_1^2 + \frac{3}{4} g_2^2 - 3h_t^2\right], \tag{28}$$

$$\frac{d}{dt} \ln K'(t) = -\frac{1}{16\pi^2} \times \left[\frac{6}{5} g_1^2 + 6g_2^2 - 6h_t^2 - h_\tau^2\delta_{i3} - h_\tau^2\delta_{3j}\right]. \tag{29}$$

The evolution equation of $m'_{LL}(t)$ in eq. (27) simplifies [30] to

$$\frac{d}{dt} \ln m'_{LL}(t) = \frac{1}{16\pi^2} \times \left[-\frac{9}{10} g_1^2 - \frac{9}{2} g_2^2 + h_\tau^2\delta_{i3} + h_\tau^2\delta_{3j}\right] \tag{30}$$

Upon integration from high scale ($t_u = \ln M_u$) to lower scale ($t_0 = \ln m_t$) where $t_0 \le t \le t_u$ and $t = \ln\mu$, we get [30]

$$m'_{LL}(t_0) = e^{\frac{9}{10} I_{g1}(t_0)} e^{\frac{9}{2} I_{g2}(t_0)} \begin{pmatrix} m'_{LL11}(t_u) & m'_{LL12}(t_u) & m'_{LL13}(t_u) e^{-I_\tau(t_0)} \\ m'_{LL21}(t_u) & m'_{LL22}(t_u) & m'_{LL23}(t_u) e^{-I_\tau(t_0)} \\ m'_{LL31}(t_u) e^{-I_\tau(t_0)} & m'_{LL32}(t_u) e^{-I_\tau(t_0)} & m'_{LL23}(t_u) e^{-2I_\tau(t_0)} \end{pmatrix} \tag{31}$$

where $I_{gi}(t_0) = \frac{1}{16\pi^2}\int_{t_0}^{t_u} g_i^2(t) dt, \quad i = 1, 2, 3;$

$$I_f(t_0) = \frac{1}{16\pi^2}\int_{t_0}^{t_u} h_f^2(t) dt, \quad f = t, b, \tau. \tag{32}$$

Using the numerical values of $I_{gi}(t)$ and $I_f(t)$ at different energy scales t, $t_0 \le t \le t_u$ the left-handed Majorana mass matrix $m'_{LL}(t)$ in eq. (31) is estimated at different energy scales from the value of $m'_{LL}(t_u)$ given in eq. (19). At each scale, the leptonic mixing matrix $V_{MNS}(t) = V'^{\dagger}_{\nu L}(t)$ is calculated and this is turn, gives mixing angles $\sin^2 2\theta_{12}$,

$\sin^2 2\theta_{23}$ and $|V_{e3}|$. For example, at the top-quark mass scale $t_0 = \ln m_t = 5.349$, we have calculated $I_\tau(t_0) = 0.100317$ and the leptonic mixing matrix

$$V_{MNS} = \begin{pmatrix} -0.56962 & -0.80795 & 0.15085 \\ -0.67061 & 0.35075 & -0.65364 \\ -0.4752 & 0.47349 & 0.74161 \end{pmatrix}, \tag{33}$$

which leads to the low-energy predictions : $\sin^2 2\theta_{12} = 0.8472$ and $\sin^2 2\theta_{23} = 0.9399$. There is a mild reduction from the values estimated at the high energy scale t_u. This feature shows the compatibility of the inverted hierarchical model with the explanation of the LMA MSW solution. The parameter $|V_{e3}| = 0.15085$ meets the CHOOZ constraint $|V_{e3}| \leq 0.16$ [21]. The neutrino mass eigevalues at low-energy scale are obtained as $m_i = (1.3533, -1.3436, 3.8376 \times 10^{-7}) m_0$. However, the mass splitting parameter $\xi = \Delta m_{12}^2 / \Delta m_{23}^2 = 0.01449$ remains almost constant. The running of the mixing angles $S_{sol} = \sin^2 2\theta_{12}$ and $S_{at} = \sin^2 2\theta_{23}$ is shown in Figure 1 by the solid-line and dotted-line respectively. Both parameters decrease with the decrease in energy scale t. This is a desirable feature for maintaining the stability condition of the inverted hierarchical model.

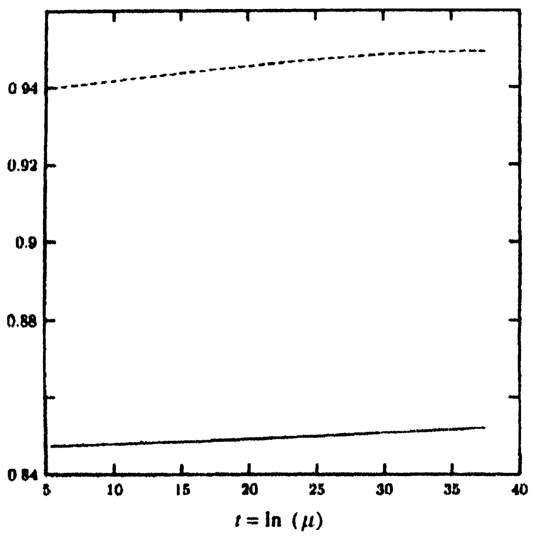

Figure 1. Variations of $S_{sol} = \sin^2 2\theta_{12}$ and $S_{at} = \sin^2 2\theta_{23}$ with energy scale $t = \ln(\mu)$ which are represented by solid-line and dotted-line, respectively.

This feature arises from the fact that the model gives opposite sign of mass eigenvalues, and in fact it is a realisation of the mechanism proposed by Baroieri *et al* [31]. The effect of the scale-dependence of the vacuum expectation value (vev) can only change the overall scale of the masses but not the mixings. The exponential term which depends on the top-quark Yukawa coupling gets cancelled with the inclusion of vev in eq. (31). It is particularly important for low $\tan\beta$ region where the stability of the overall magnitude of neutrino masses is maintained. As discussed before, if the CHOOZ constraint is relaxed to $|V_{e3}| \leq 0.2$, then we would be able to get even lower value of $\sin^2 2\theta_{12}$ suitable for the explanation of the best-fit value of the LMA MSW solution.

4. Summary and discussion

The left-handed Majorana neutrino mass matrix m_{LL} which explains the inverted hierarchical pattern of neutrino masses, has been generated from the seesaw mechanism using non-diagonal texture of the right-handed Majorana neutrino mass matrix M_R and diagonal from the Dirac neutrino mass matrix. We have explained the loptonic mixing matrix generated from the diagonalisation of m_{LL} of the inverted hierarachical model and the mixing angles so far obtained $\sin^2 2\theta_{12} \simeq 0.999$, is too large for the explanation of the LMA MSW solution. Such high value of $\sin^2 2\theta_{12}$ can be tonned down by considering the contribution from the charged lepton mass matrix having special structure in the 1–2 block. With such consideration, the predictions on the mixing angles at the high scale are $\sin^2 2\theta_{12} = 0.8517$, $\sin^2 2\theta_{23} = 0.9494$ and $|V_{e3}| = 0.159$ which are consistent with the LMA MSW solution.

The above results which are calculated at the high energy scale (say, $\mu = M_u = 2 \times 10^{16}$ GeV) where the seesaw mechanism operates, decrease with the decrease in energy scale, under the quantum radiative corrections within the framework of the MSSM. This is a good feature at least for $\sin^2 2\theta_{12}$ in this inverted hierarchical model as it gives the stability under quantum radiative corrections and shows complete consistency of the model with the explanation of the LMA MSW solution. Experimental data from a Neutrino factory may confirm the pattern of the neutrino masses in near future, and hence the sign of Δm_{23}^2. Such confirmation of the detailed pattern of neutrino masses and their mixing angles is very important as it may give a clue to the understanding of quark masses and their mixing angles within the framework of an all-encompassing theory [5].

Though we have constructed both m_{LL} and m_l in a model-independent way and have shown how the inverted hierarchical model of neutrinos can explain the present experimental data particularly LMA MSW solution, the present work is expected to be an important clue for building models from the grand unified theories with the chiral $U(1)$ symmetry.

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