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Self-focussing of a laser beam, incident normally on a plasma-free space interface

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Abstract : The authors have analyzed the propagation of the transmitted part of a laser beam with transverse Gaussian intensity profile incident normally from free space onto a plane plasma surface. The modification of irradiance profile of the beam, caused by the nonlinear dielectric constant of the plasma has been taken into account. It has been shown that in this case also there exist three regimes of propagation in the beam width-beam power plane; the regions are characterized by steady divergence, oscillatory divergence and self-focussing. The dependence of beam width on distance of propagation has been investigated for typical points in the three regions. It is seen that the transverse irradiance profile, the three regions in the beam power-beam width plane and the dependence of beam width parameter on distance of propagation get considerably modified by considering incidence on plasma-free space interface.

Keywords : Laser, plasma, self-focussing

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1. Introduction

Self-focussing of laser beams in plasmas and other nonlinear media, occupies a unique position in high field science on account of the fact that it considerably influences other nonlinear phenomena. The last forty years or so have been a period of intense scientific activity [1–17] in this field and an appreciation of the progress in this field can be made from the reviews by Sodha *et al* [18], Esarey *at al* [19], Umstadter [20] and others.

High intensity electromagnetic (EM) waves, on entering the plasma, accelerate the plasma electrons to quiver speeds comparable to the speed of light in vacuum. This produces a change in the plasma frequency and hence a field-dependent dielectric constant. The effect is instantaneous so that the laser light experiences refraction in the plasma with a nonlinear dielectric constant even if the pulse length is short. For longer pulse lengths, other nonlinearities namely, the ponderomotive and the thermal may be operative. The intensity profile of the laser beam which is most often generated in the fundamental TEM_{00} mode *viz*. the transverse Gaussian intensity profile, consequently gets significant modification after transmission across free space plasma interface. The change can be analyzed according to the usual laws of reflection and transmission on the interface between the plasma and free space (or air), after allowance is made for nonlinear dielectric constant of the plasma. Most theoretical studies, however, assume the laser intensity profiles to be the one obtained from the source. A solitary exception to this practice seems to be a study by Sodha *et al* [21], taking nonlinear refraction at the interface of the plasma and free space into account. However, this investigation was limited to obtaining the critical power curve relating the initial beam power and beam width of the laser light inside the plasma for uniform wave guide propagation in the plasma and evaluation of the dependence of the beam width parameter

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on the distance of propagation. The authors did point out that for points above the critical curve, self-focussing occurs but they did not explore the propagation characteristics of the laser beam for the entire range of beam power and beam width. This communication reports results of detailed investigation of the problem following Sharma *et al* [13] who considered the whole range of beam power and beam width. For consistency, the transmission coefficient has been expanded upto r^4 rather than r^2. It is seen that the positive quadrant of beam width beam power plane can be divided in three regions, corresponding to different modes of beam propagation as follows :

(i) Steady divergence : the beam width increases steadily.

(ii) Oscillatory divergence : the beam width oscillates between the original value and a higher one.

(iii) Self-focussing : the beam width oscillates between the original value and a lower one.

The study shows that the transverse irradiance profile of the beam and the dependence of the beam width on distance of propagation, get considerably modified by consideration of reflection by the interface; so are the critical and divider curves, which separate the three regions in the beam power-beam width plane, on account of the reflection coefficient having an irradiance and hence radial dependence of the dielectric constant.

2. Intensity profile of transmitted beam

A normally incident Gaussian laser beam on a plane plasma-free space interface (z = 0) may be represented as,

$$E = \hat{j}E_{00}\exp\left(\frac{-r^2}{2r_0^2}\right)\exp\left[i\omega\left(t-\frac{z}{c}\right)\right] \qquad (1)$$

where E_{00} is the amplitude of electric vector $\boldsymbol{E}$, r_0 the width of the beam, ω the wave frequency, c the speed of light in vacuum, $\hat{j}$ is a unit vector and a cylindrical system of coordinates is used.

The intensity profile of the transmitted beam at the interface (z = 0) is given by

$$E_{10}^2(r) = E_{00}^2(2/(n+1))^2\exp\left(-r^2/r_0^2\right), \qquad (2)$$

where $2/(n + 1)$ is the amplitude transmission coefficient, and $n = n(r)$ is the refractive index over the wave front (z = 0), whose dependence on E_{10}^2 can in general, be expressed as

$$n^2 = \epsilon = \epsilon_0 + \Phi(E_{10}^2)$$

$$= \epsilon_0 + \Phi\left\{(2/(n+1))^2 . E_{00}^2\exp\left(-r^2/r_0^2\right)\right\}. \qquad (3)$$

Eq. (3) is a relationship between n and $(r/r_0)^2$. For given numerical values of E_{10}^2, the nature of function Φ and other relevant parameters $n(r=0)$, $\left\{r_0^2\partial n/\partial r^2\right\}_{r=0}$ and $\left\{r_0^4\partial^2 n/\partial(r^2)^2\right\}_{r=0}$ can be evaluated. In the paraxial approximation, the amplitude of the transmission coefficient $[2/(n + 1)]$ may be expanded in powers of

Thus,

$$\{2/(n+1)\} = a + b\left(r^2/r_0^2\right) + c\left(r^2/r_0^2\right)^2, \qquad (4)$$

where $a = 2/\{1+n(r=0)\}$, (4a)

$$b = -\left\{\left(2/(1+n)^2\right).\left(r_0^2\partial n/\partial\left(r^2\right)\right)\right\}_{r=0} \qquad (4b)$$

and

$$c = \left[\left\{\left(4/(1+n)^3\right)\left(r_0^2\partial n/\partial\left(r^2\right)\right)^2 - \left(2/(1+n)^2\right)\left(r_0^4\partial^2 n/\partial\left(r^2\right)^2\right)\right\}\right]_{r=0}. \qquad (4c)$$

Knowing n, $\left(\partial n(r)/\partial\left(r^2/r_0^2\right)\right)$ and $\left(r_0^4\partial^2 n(r)/\partial\left(r^2\right)^2\right)$ at r = 0, the parameters a, b and c can be evaluated. Hence, the radial intensity profile of the transmitted beam at z = 0 is given by

$$E_1^2(r) = E_{00}^2\left[a + b\frac{r^2}{r_0^2} + c\frac{r^4}{r_0^4}\right]^2\exp\left(-\frac{r^2}{r_0^2}\right) \qquad (5)$$

where E_{10} is equal to the field intensity at z = 0 and the parameters a, b, c can be determined for given values of E_{10}^2 and other parameters with a knowledge of the functional nature of Φ.

3. Self-focussing of transmitted beam

The field E_1 in the plasma may be obtained by solving the wave equation,

$$\nabla^2 E_1 + \left(\omega^2/c^2\right)\epsilon(r,z)E_1 = 0, \qquad (6)$$

consistent with eq. (5). It will be useful to point out here that the wave equation contains second order derivatives of the field variables while the dielectric constant enters the equation as it is. Therefore, the paraxial approximation used with the wave equation will require the expansion of field variables in powers of r^2 up to r^4 while the evaluation of the dielectric constant (corresponding to an equal accuracy) requires expansion of field variable and other quantities only up to the first power in r^2. Thus for the determination of $\in(r,z)$, one can write in the paraxial approximation,

$$E_1E_1^*(r,z) = F_1(z) - r^2F_2(z) \tag{7}$$

and $$\in = \in_0 + \Phi(E_1E_1^*) = \in_1(z) - r^2 \in_2(z), \tag{8}$$

where $$\in_1(z) = \in_0 + \Phi\left(E_1E_1^*\right)_{E_1E_1^*=F_1(z)} \tag{8a}$$

and $$\in_2(z) = \left(d\Phi/dE_1E_1^*\right)_{E_1E_1^*=F_1(z)} \cdot F_2(z). \tag{8b}$$

Eq. (6) can be solved, following Akhamanov *et al* [22] and Sodha *et al* [23]. Thus, one considers the JWKB solution of the one dimensional equation

$$\left(d^2E_1/dz^2\right) + \left(\omega^2/c^2\right)\in_1(z)E_1 = 0$$

as $$E_1 \propto \left(\in_1(z)\right)^{-1/4} \exp\left\{\left(-i\omega/c\right)\int_0^z \sqrt{\in_1(z)}dz\right\}.$$

The above solution suggests a solution of eq. (6) of the form

$$E_1(r,z) = A(r,z)\exp\left\{\left(-i\omega/c\right)\left(\int_0^z \sqrt{\in_1(z)}dz + \sqrt{\in_1(z)}S(r,z)\right)\right\}, \tag{9}$$

where S is the eikonal. Substituting for E_1 from eq. (9) in eq. (6), and considering slowly converging/diverging fields, neglecting the term $\left(\partial^2A/\partial z^2\right)$ and equating the real and imaginary parts on both sides of the resulting equation, one obtains

$$2(\partial S/\partial z) + (\partial S/\partial r)^2 = \left(-r^2 \in_2(z)/\in_1(z)\right) + \left(c^2/\omega^2\right)(1/A)\left\{\left(\partial^2A/\partial r^2\right) + (\partial A/r\partial r)\right\}, \tag{10a}$$

$$\left(\partial A^2/\partial z\right) + (\partial S/\partial r)\left(\partial A^2/\partial r\right) + A^2\left(\left(\partial^2S/\partial r^2\right) + (\partial S/r\partial r)\right) = 0. \tag{10b}$$

Considering the wavefront to be in general spherical, one may write

$$S = \left(r^2/2\right)\beta(z) + \phi(z) \tag{11}$$

where $\beta(z)$ is obviously the curvature of the wave front. Substituting for S from eq. (11) in eq. (10b) and writing

$$\beta(z) = (f)^{-1} \cdot (df/dz), \tag{12}$$

one obtains (see Sodha *et al* [23] p.13–17)

$$A^2 = \left(\text{Const}/f^2\right)\zeta(r/r_0f), \tag{13}$$

where ζ is a function of (r/r_0f). Further, according to eq. (9),

$$E_1E_1^* = A^2. \tag{14}$$

For eq. (14) to match with $E_1E_1^*$ at $z = 0$ (eq. (5)), one must have

$$E_1(r,z)E_1^*(r,z) = E_{00}^2\left(a + b\left(r^2/r_0^2f^2\right) + c\left(r^4/r_0^4f^4\right)\right)^2 \exp\left(-r^2/r_0^2f^2\right). \tag{15}$$

From eq. (15), one can immediately write

$$\left.\begin{aligned} F_1(z) &= a^2E_{00}^2/f^2 \\ \text{and} \quad F_2(z) &= \left(1-(2b/a)\right)\left(a^2E_{00}^2/r_0^2f^4\right). \end{aligned}\right\} \tag{16}$$

With the solutions of S and A^2 (eqs. (11) and (13), respectively), eq. (10a) leads to a second order differential equation for the beam width parameter f namely,

$$\frac{\in_1(f)}{f}\frac{d^2f}{dz^2} = \left(c^2N/\omega^2r_0^4f^4\right) - \in_2(z), \tag{17}$$

where $N = \left(1 + (16c/a) - \left(4b^2/a^2\right) - (4b/a)\right)$.

On transforming the z-coordinate and the initial beam width r_0 to dimensionless parameters ξ and ρ_0,

$$\xi = cz/\omega r_0^2 \quad \text{and} \quad \rho_0 = r_0\omega/c.$$

Eq. (17) reduces to

$$\in_1(f)\frac{d^2f}{d\xi^2} = \left\{\left(N/f^3\right) - \rho_0^2r_0^2 \in_2(f)f\right\}. \tag{17a}$$

4. Specific nonlinearities

4.1. Ponderomotive nonlinearity :

In this case, $\phi(EE^*)$ is given as [18]

$$\phi\left(EE^*\right) = \Omega^2\left\{1 - \exp\left(-(3\alpha m/4M)EE^*\right)\right\},$$

where $\alpha = \left(e^2M/6k_BT_0m^2\omega\right)$.

e and m are respectively, the charge and rest mass of an electron, M is the mass of the heavier particle (neutral or ions), T_0 the temperature of plasma and k_B the Boltzmann constant.

Eq. (3) can be expressed as

$$n^2(r) = 1 - \Omega^2 \exp\left\{\left(-\frac{3\alpha m}{4M}\right)\left(\frac{2}{(1+n(r))}\right)^2 \times E_{00}^2 \exp\left(-r^2/r_0^2\right)\right\}.$$

This equation relates $n(r)$ with r^2/r_0^2 for chosen values of Ω, the incident beam axial amplitude and other constants; it can be used to obtain $n(0)$, $\left(\partial n(r)/\partial\left(r^2/r_0^2\right)\right)_0$ and $\left(\partial^2 n(r)/\partial\left(r^2/r_0^2\right)^2\right)_0$. These values can then be used in eqs. (4a), (4b) and (4c) to obtain the constants a, b and c.

Thus from eqs. (8), (8a) and (8b) one obtains

$$\in_1(z) = \in_1(f) = 1 - \Omega^2 \exp\left(\left(-3\alpha m/4M\right)F_1(z)\right)$$

$$= 1 - \Omega^2 \exp\left(\left(-3\alpha m/4M\right)\left(a^2 E_{00}^2/f^2\right)\right)$$

and

$$\in_2(z) = \in_2(f) = \Omega^2\left\{\exp\left(\left(-3\alpha m/4M\right)\left(a^2 E_{00}^2/f^2\right)\right)\right\} \times \left(1-(2b/a)\right)\left(-3\alpha m/4M\right)\left(a^2 E_{00}^2/f^2\right).$$

Denoting $\left(3\alpha m/4M\right)\left(a^2 E_{00}^2/f^2\right)$ (a quantity proportional to beam power) with p, one can express $\in_1(z)$ and $\in_2(z)$ as

$$\in_1(f) = 1 - \Omega^2 \exp(-p) \tag{18}$$

and

$$\in_2(f) = \Omega^2\left(1-(2b/a)\right)p.\exp(-p) \tag{19}$$

Substituting for $\in_2$ from eq. (19) in eq. (17a) one obtains the condition

$$\rho^2 = \left\{N \exp(p_c)/\Omega^2\left(1-(2b/a)\right)p_c\right\} \tag{20}$$

for $d^2f/d\xi^2$ to vanish, where ρ denotes the beam width $\rho_0 f$ at z (or ξ or f) and p_c represents a corresponding critical power for which $d^2f/d\xi^2$ vanishes. At $\xi = 0$ ($z = 0$ or $f = 1$) eq. (20) reduces to

$$\rho_0^2 = \left\{N \exp(p_{co})/\Omega^2\left(1-(2b/a)\right)p_{co}\right\} \tag{20a}$$

If a beam enters the plasma with width ρ_0 and power p equal to the critical power p_{co} it will have $d^2f/d\xi^2$ equal to zero at $\xi = 0$ ensuring $df/d\xi$ and f to retain their initial values (at $\xi = 0$) namely, $df/d\xi = 0$ and $f = 1$ throughout the passage of the beam. This mode of propagation is termed the uniform wave guide propagation. A ρ_0 *versus* p_{co} graph (shown in Figure 1 by p_c) is known as the critical power (or simply critical) curve. In case the point representing the initial beam width ρ_0 and the initial power p of the beam does not fall on the critical curve $d^2f/d\xi^2)_{\xi=0}$ will not vanish. However, as the beam propagates through the plasma, the beam width and beam power change and in suitable situations, may reach values that satisfy eq. (20) and hence lead to $d^2f/d\xi^2$ to vanish. This is a condition that corresponds to a point of inflexion in the f *versus* ξ graph of the beam and hence leads to an oscillatory convergence or divergence of the beam according as the initial value of $d^2f/d\xi^2)_{\xi=0}$ is negative or positive. Thus, the requirement for oscillatory convergence (self-focussing) or oscillatory divergence is that the beam at some value of ξ acquires a width $\rho = \rho_0 f$ and power $p = \left(p_0/f^2\right)$ (where ρ_0 and p_0 correspond to $\xi = 0$) so that (p_c,ρ) satisfies eq. (20). Now from eq. (18), one obtains

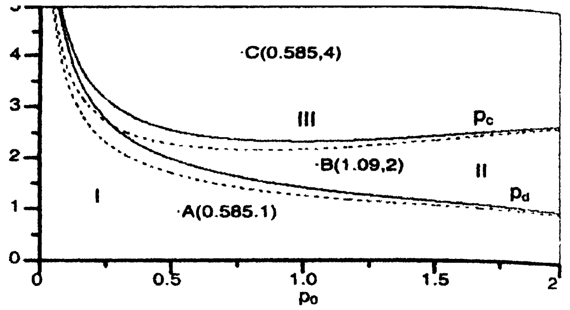

Figure 1. The critical curve (p_c) and divider curve (p_d) in the dimensionless initial beam width ($\rho_0 = r_0\omega/c$) *versus* dimensionless initial beam power $p_0 = (3\alpha m/4M)a^2E_{00}^2$ plane for ponderomotive nonlinearity. Dotted curves correspond to infinitely extended plasma. Regions I, II and III have been illustrated. The curves correspond to $\Omega^2 = 0.5$. The points A (0.585,1), B (1.09,2), C (0.585,4) refer to typical points for which f-ξ variation has been given in Figure 2.

$$\exp p = \left(\Omega^2/(1-\in_1(f))\right). \tag{21}$$

Hence eq. (20) can be put in the form

$$\rho^2 p = \left\{N/\left(1-(2b/a)\right)\left(1-\in_1(f)\right)\right\}. \tag{22}$$

Expressing ρ^2 and p in terms of their initial values one gets

$$\rho_0^2 p_0 = \left\{N/\left(1-(2b/a)\right)\left(1-\in_1(f)\right)\right\}. \tag{22a}$$

Eq. (22a) yields a real positive solution for $\in_1(f)$ ($0<\in_1(f)<1$) for all real values of the other parameter involved. However, eq. (18) puts a restriction on the value of f as one gets

$$p = \left(p_0/f^2\right) = \ln\left(\Omega^2/(1-\in_1(f))\right) \tag{23}$$

according to which f can have a real solution only for

$$\Omega^2 > (1 - \in_1(f)). \tag{24}$$

Using (23) with (22a), one obtains the condition existence of a point of inflexion in the f–ξ graph at a real value of f according to which the initial values of beam power and beam width satisfy the inequality

$$\rho_0^2 p_0 > \{N/\Omega^2(1-(2b/a))\}, \tag{25}$$

If one replaces the inequality sign by an equality

$$\rho_0^2 = N/[\Omega^2(1-(2b/a))p_0], \tag{25a}$$

one obtains a curve p_d as a ρ_0 *versus* p_0 graph which separates the region of steady divergence I from that of oscillatory divergence II. Points (p_0, ρ_0) that fall above the critical curve (region III) always lead to self-focussing. It may be pointed out here that the transmitted beam to be Gaussian, one must have $2/(1 + n) = a$, a constant. In this case, $b = c = 0$ and $N = 1$ which reduces eq. (20a) to the form valid for infinitely extending plasma. The critical and divider curves p_c and p_d for $\Omega^2 = 0.5$ and ponderomotive nonlinearity have been shown in Figure 1. To obtain these curves, we start with choosing a value for $(3am/4M)E_{00}^2 = \alpha'$ (say) over the range 0 to 2 at suitable intervals. For each value of α', we calculate a, b, c and hence N along the line prescribed earlier, p_0 is now obtained as $p_0 = a^2\alpha'$. Substitution of $p_0 = p_{c0}$ in eq. (20a), gives ρ_0 such that (p_0, ρ_0) lie on the critical curve. Similarly, ρ_0 for the divider curve can be obtained for each p_0 from eq. (25a) for the curve p_d. The curves p_c and p_d can be clearly distinguished from the two dotted curves which correspond to an infinitely extending plasma and have been included in the diagram for comparison. A comparison of the continuous and dotted graphs shows that reflection at the interface increases the area of the $p_0 - \rho_0$ corresponding space to steady divergence.

The two graphs p_c and p_d divide the entire positive quadrant of the (p_0, ρ_0) plane in regions I, II and III.

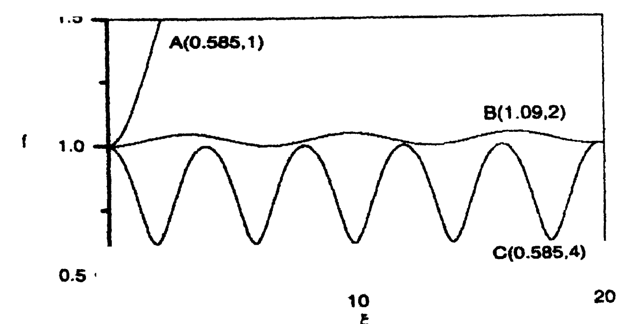

Figure 2. Variation of beam width parameter f, with dimensionless distance of propagation ξ, corresponding to initial points (p_0, ρ_0) *viz.* A, B and C in the three regions, as shown in Figure 1. The curves correspond to $\Omega^2 = 0.5$.

The f *versus* ξ graphs have been obtained for typical points A (0.585,1), B (1.09,2) and C (0.585,4) of Figure 1 and have been shown in Figure 2. The graphs show steady divergence for the point A in region I, oscillatory divergence for the point B in region II and oscillatory convergence (self-focussing) for the point C in region III.

4.2. Collisional nonlinearity :

For collisional plasma, $\Omega(EE^*)$ is given as [18]

$$\Phi(EE^*) = \Omega^2\left\{1-\left(1+(a/2)EE^*\right)^{(s/2)-1}\right\}, \tag{26}$$

s is a parameter characterizing the nature of collisions and α is the same as defined in the ponderomotive case. Proceeding exactly as in the ponderomotive case, one obtains the equation for vanishing $d^2f/d\xi^2$ as

$$\rho^2 p_c = \frac{N(1+p_c)^{(4-s)/2}}{(1-(s/2))(1-(2b/a))\Omega^2},$$

where

$$p = (a/2)(a^2E_{00}^2/f^2)$$

and p_c is its critical value ensuring $d^2f/d\xi^2$ to vanish. By going to $\xi = 0$, one obtains the critical power curve as

$$\rho_0^2 p_{co} = \frac{N(1+p_{co})^{(4-s)/2}}{(1-(s/2))(1-(2b/a))\Omega^2}. \tag{27}$$

Similarly, the divider curve is obtained as

$$\rho_0^2 p_0 = \{N/(1-(s/2))(1-(2b/a))\Omega^2\}. \tag{28}$$

For $s = 1$ (heavy particles being predominantly neutrals), these equations reduce to

$$\rho_0^2 p_{co} = 2N(1+p_{co})^{3/2}/(1-(2b/a))\Omega^2 \tag{29}$$

and

$$\rho_0^2 p_0 = \{2N/(1-(2b/a))\Omega^2\}. \tag{30}$$

These equations match with the equations for relativistic nonlinearity with a different constant used in defining p. The critical and the divider curves have been drawn for $\Omega^2 = 0.5$ selecting values for $\alpha' = aE_{00}^2/2$ in the range 0 to 2 and proceeding as in the case of the ponderomotive nonlinearity. The curves have been shown in Figure 3, for $s = 1$ and divide the entire positive quadrant of p_0, ρ_0 plane in three regions I, II and III.

The f *versus* ξ graphs for typical points in the three regions are depicted in Figure 4. The parameters a, b and c vary continuously with α' and can be evaluated as already explained. A sample of these calculated values for selected values of α' have been listed in Table 1. These values can be and have been used as check values in the computer calculations.

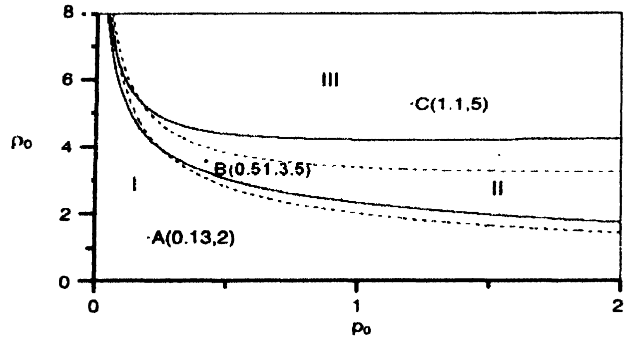

Figure 3. The critical curve (p_c) and divider curve (p_d) in the dimensionless initial beam width ($\rho_0 = r_0\omega/c$) *versus* dimensionless initial beam power $p_0 = (\alpha/2)a^2E_{00}^2$ plane for collisional nonlinearity (s = 1). Regions I, II and III have been illustrated. The curves correspond to $\Omega^2 = 0.5$. The points A (0.13, 2), B (0.51, 3.5), C (1.1, 5) refer to typical points for which f-ξ variation has been given in Figure 4.

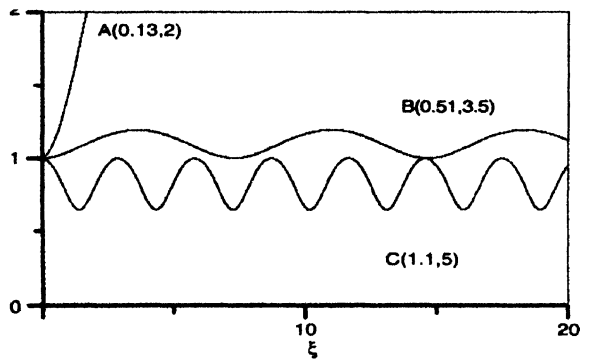

Figure 4. Variation of beam width parameter f, with dimensionless distance of propagation ξ, corresponding to initial points (p_0, ρ_0) *viz.* A, B and C in the three regions, as shown in Figure 3; the curves corresponds to $\Omega^2 = 0.5$.

Table 1. Values of the constants a, b, c for selected values of α' for ($\Omega^2 = 0.5$).

		α'	0	0.5	1	1.5	2
Collisionless		a	1.17	1.084	1.045	1.025	1.014
		b	0	0.044	0.0296	0.152	0.0068
		c	0	−0.0035	−0.0041	−0.0035	−0.0025
Collisional	$s = 1$	a	1.17	1.13	1.105	1.09	1.08
		b	0	0.027	0.033	0.03	0.032
		c	0	−0.031	−0044	−0.049	−0.05

4.3. Relativistic nonlinearity :

As pointed out in the introduction, relativistic nonlinearity is dominant in the initial duration of a high irradiance laser beam propagation. In this case [13,19] for a circularly polarized beam,

$$\Phi(EE^*) = \Omega^2\left\{1-\left(1+\left(e^2/m_0^2\omega^2c^2\right)EE^*\right)^{-1/2}\right\}. \qquad (31)$$

Eq. (31) and eq. (26) are identical when s = 1 and $\alpha/2 = e^2/m_0^2\omega^2c^2$.

Hence, the results for collisional plasmas with s = 1 and $\alpha/2 = e^2/m_0^2\omega^2c^2$ are applicable to the case of relativistic nonlinearity.

5. Conclusions

It is seen that after transmission through a nonlinear plasma-free space interface the transverse irradiance profile of an initially Gaussian beam gets significantly altered. A study of self-focussing of this beam in the JWKB – parabolic equation approximation, predicts three regions in the incident beam power – beam width plane, which correspond to self-focussing, oscillatory divergence and steady divergence. The dependence of beam width parameter on distance of propagation has been studied for typical points in the three regions. The critical and divider curves, demarcating the three regions are significantly different in the two cases and so are the f-ξ curves, characterizing the self-focussing/defocussing.

The transverse irradiance profile of the beam, the critical and divider curves and the dependence of beam width parameter on distance of propagation are considerably different from the case, when the plasma is of infinite extent and the source is located in the plasma, as is commonly assumed in similar studies.

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