Indian J. Phys. **79**(4), 393-399 (2005)

IJP AN INTERNATIONAL JOURNAL

Self-focussing of TEM_{10} mode laser beams in plasmas

Ashutosh Sharma, M P Verma and M S Sodha*
Institute of Advanced Studies in Education, University of Lucknow, Lucknow-226 007, Uttar Pradesh, India

V K Tripathi
Department of Physics, Indian Institute of Technology, New Delhi-110 016, India

E-mail : msodha@rediffmail.com

Received 13 July 2004, accepted 2 February 2005

Abstract : The nonlinear propagation of a laser beam in the TEM_{10} mode, through a plasma, has been studied by expanding the eikonal around the axis and an intensity maximum. Three kinds of nonlinearities *viz* ponderomotive, collisional and relativistic, have been considered. It has been observed that uneven refraction of the beam in x and y directions, rules out the possibility of a uniform waveguide mode propagation (*i.e.* without convergence or divergence). The beam width parameters characterizing convergence/divergence in x and y directions respectively, have been obtained by numerically integrating the corresponding differential equations. In the axial region, the beam suffers steady divergence, enhanced by nonlinearity. For regions close to irradiance maxima, two coupled differential equations for beam widths in x and y directions have been obtained. These have been solved for a set of typical parameters and a discussion of the results is presented.

Keywords : Laser, plasma, self-focussing

PACS Nos. : 52.38.Hb, 52.35.Mw

1. Introduction

The self-focussing of laser beams in plasmas has been extensively investigated in the last three decades [1-12]. Most of these investigations consider beams with Gaussian distribution of intensity along the wavefront implying that the laser is operated in the TEM_{00} mode. However, there are situations where higher order modes (*e.g.* the TEM_{10} mode) also become important. When lasers are operated with mirrors having annular output coupling apertures, the lowest order mode (TEM_{00}) is suppressed and higher order modes, such as the TEM_{10} mode are excited. Sodha *et al* [13] have studied the asymmetric focussing of this mode in a dielectric with quadratic nonlinearity, for regions around the axis and around the maximum of irradiance, the analysis was limited to the geometrical optics approximation. This treatment is valid for modest intensities as at higher intensities, the dielectric constant tends to saturate. The saturating nonlinearity modifies the nature of self-focussing of Gaussian beams. One expects similar effects in the case of non-Gaussian beams.

There are two regions of the TEM_{10} mode laser beam, which are of interest *viz.* (i) around the axis $x = 0$, $y = 0$ and (ii) around points of maximum irradiance. Since a solution applicable to the whole wave front of the beam is not obtainable, the authors have adopted an approach [13], similar to that adopted in the paraxial approximation [1–3]. In this approximation [1–3, 13], the nature of intensity distribution of the beam remains unchanged and beam width parameters f_1 and f_2 define the intensity distribution. It is well known that the results of paraxial approximation are not valid for points far from the axis, since the expansion is made as a series in r^2 (x^2 and y^2). Hence, the intensity distributions for the paraxial ($x = 0$, $y = 0$) region and regions around the maximum intensity are of different nature; the

*Corresponding Author

mathematical basis for this difference is the fact that the expansion is made in terms of x^2, y^2, and $(x-x_1/f_1)^2$, y^2 respectively. In the approximations used, the beam retains TEM$_{10}$ mode structure with different values of f_1 and f_2 in the two regions of interest.

In this paper, we study the self-focussing of a laser beam in the TEM$_{10}$ mode propagating in plasma. Three types of nonlinearities have been considered *viz.* ponderomotive, collisional and relativistic, which conform to the saturating nature of the dielectric constant ϵ with increasing EE^*. As expected, it is seen that the focussing of the beam is asymmetric. Case (i) corresponds to steady divergence, because $EE^* \to 0$ as x, y go to zero and consequently, the nonlinearity is negligible. However for case (ii), depending on the initial values of the beam power and beam width, (a) steady divergence, (b) oscillatory divergence and (c) self-focussing in x and y directions are predicted. This result is qualitatively similar to the results of the recent analysis, related to the Gaussian (TEM$_{00}$) mode [12].

2. Self-focussing

Consider the propagation of a TEM$_{10}$ mode laser beam in a plasma along the z-direction,

$$E(x,y,z) = A(x,y,z)\exp\left(-i(\omega t - kz)\right) .$$

At $z = 0$

$$AA^* = E_0^2\left(x^2/r_0^2\right)\exp\left(-\left(x^2+y^2\right)/r_0^2\right) \tag{1}$$

where E_0 and r_0 are constants; the intensity maxima in the $z = 0$ plane lie at $(\pm\, r_0, 0)$. Following Sharma *et al* [12] the dielectric constant ϵ of the plasma can be expressed as

$$\epsilon(x, y, z) = \epsilon_0(z) + \epsilon_1(x,y,z). \tag{2}$$

The JWKB solution of the one-dimensional wave equation suggests that for a slowly diverging or converging beam, one may write

$$E(x,y,z) = A_0(x,y,z).\exp$$

$$\times i\left(\omega t - \int^z \frac{\omega}{c}\sqrt{\epsilon_0(z)}dz - \frac{\omega}{c}\sqrt{\epsilon_0(z)}S(x,y,z)\right), \tag{3}$$

where the term $\left(\epsilon_0(0)/\epsilon_0(z)\right)^{1/4}$ is included in A_0 and S is an eikonal. Substituting for E into the wave equation and separating the real and imaginary parts of the resulting equation, one obtains

$$\frac{\partial A_0^2}{\partial z} + \frac{\partial S}{\partial x}\frac{\partial A_0^2}{\partial x} + \frac{\partial S}{\partial y}\frac{\partial A_0^2}{\partial y} + A_0^2\left(\frac{\partial^2 S}{\partial x^2} + \frac{\partial^2 S}{\partial y^2}\right) = 0 \tag{4}$$

and

$$\frac{2\partial S}{\partial z} + \left(\frac{\partial S}{\partial x}\right)^2 + \left(\frac{\partial S}{\partial y}\right.$$

$$= \frac{1}{\epsilon_0(z)}\epsilon_1(x,y,z) + \frac{c^2}{\omega^2}\cdot\frac{1}{A_0}\left(\frac{\partial^2 A_0}{\partial x^2} + \frac{\partial^2 A_0}{\partial y^2}\right\} . \tag{5}$$

We solve these equations in two different limits.

3. Axial beam propagation

For the portion of the beam close to the z-axis, $|x|$, $|y| << r_0$, eq. (4) is satisfied by

$$S(x,y,z) = \frac{1}{2}\beta_1(z)x^2 + \frac{1}{2}\beta_2(z)y^2 + \varphi(z) \tag{6}$$

and $$A_0^2(x,y,z) = \frac{E_0^2}{f_1 f_2}\frac{x^2}{r_0^2 f_1^2}\exp\left[-\frac{\;}{r_0^2 f_1^2} \quad \frac{\;}{r_0^2 f_2^2}\right. \tag{7}$$

where $\varphi(z)$ is an arbitrary function of z, which does not affect focussing, and

$$\beta_1(z) = \frac{1}{f_1}.\frac{df_1}{dz}\ , \quad \beta_2(z) = \frac{1}{f_2}.\frac{df_2}{dz} . \tag{8}$$

$\beta_1(z)$ and $\beta_2(z)$ represent the curvatures of the wavefront in the respective directions and vanish for a plane wave front.

The dielectric constant can in general, be expressed as [2]

$$\epsilon = \epsilon_0 + \phi(E\cdot E^*) = \epsilon_0 + \phi(A_0^2) . \tag{9}$$

The functional form of ϕ depends on the specific operative nonlinearity. For ponderomotive nonlinearity [2,3]

$$\phi(EE^*) = \Omega^2\left\{1 - \exp\left[-\frac{3}{4}a\frac{m}{M}EE^*\right.\right. \tag{10}$$

where $\Omega = \omega_p/\omega$, ω_p is the plasma frequency corresponding to unmodified electron density and electron rest mass m. M is the ion mass, and $a = e^2M/6m^2\omega^2 k_B T_0$, $-e$ is the electron charge, k_B is the Boltzmann constant and T_0 is the plasma temperature.

Substituting $EE^* = A_0^2$ from eq. (7) in eq. (10), expanding the function in powers of the coordinates and retaining terms upto squares of x and y, one finally obtains

$$=(1-\Omega^2)+\Omega^2\frac{3}{4}\alpha\frac{m}{M}\frac{E_0^2}{f_1f_2}\frac{x^2}{r_0^2f_1^2}.$$

The dimensionless quantity

$$p=\frac{3}{4}\alpha\frac{m}{M}\frac{E_0^2}{f_1f_2}=\frac{p_0}{f_1f_2} \tag{11}$$

is proportional to E_0^2 and represents dimensionless beam power, p_0 is the dimensionless beam power at z = 0. Thus,

$$\in(x,y,z)=(1-\Omega^2)+\frac{\Omega^2p}{r_0^2f_1^2}.x^2.$$

The lowest order term in y involves x^2y^2 and is therefore neglected. Comparison with eq. (2) gives

$$\in_0(z)=1-\Omega^2$$

and

$$\in_1(x,y,z)=\frac{\Omega^2p}{r_0^2f_1^2}$$

Substituting for S, A_0^2, $\in_0(z)$ and $\in_1(x, y, z)$ in eq. (5) and equating the coefficients of x^2 and y^2 separately to zero, one obtains

$$\left(1-\Omega^2\right)\frac{d^2f_1}{d\xi^2}=\frac{1}{f_1^3}\left\{1+\Omega^2\rho^2p\right\} \tag{12}$$

and
$$\left(1-\Omega^2\right)\frac{d^2f_2}{d\xi^2}=\frac{1}{f_2^3}, \tag{13}$$

where $\rho=\rho_0f_1$. The parameters $\xi=cz/r_0^2\omega$ and $\rho_0=r_0\omega/c$ represent dimensionless distance of propagation and dimensionless beam width at z = 0, respectively. The boundary conditions corresponding to a plane wave front at ξ = 0, are

$$\frac{df_1}{d\xi}=\frac{df_2}{d\xi}=0 \text{ and } f_1=f_2=1;\ \xi=0. \tag{14}$$

Since the terms on RHS of eqs. (12) and (13) are always positive f_1 and f_2 keep on increasing with increasing ξ implying steady divergence.

It is also instructive to visualize the physics behind the steady divergence. Since near x = 0, y = 0, the electric field increases with increasing x, the refractive index also increases with increasing x; this augments the diffraction divergence. For the case of collisional and relativistic nonlinearities, qualitatively similar results are obtained.

4. Propagation close to irradiance maxima

The propagation of the beam around the intensity maxima can be analyzed in a way similar to that in Section 3. One can use an approximation akin to the paraxial approximation by substituting $x = r_0f_1' + x_1$ and requiring $|x_1|<<r_0$. Eqs. (4) and (5) remain unchanged except that $\partial/\partial x$ is replaced by $\partial/\partial x_1$. The eikonal may be expressed as

$$S(x,y,z)=\frac{1}{2}\beta_1'(z)x_1^2+\frac{1}{2}\beta_2'(z)y^2+\varphi'(z). \tag{15}$$

Then A_0^2 satisfying eq. (4) can be written as

$$A_0^2(x,y,z)=\frac{E_0^2}{f_1'f_2'}\left(1+\frac{x_1}{r_0f_1'}\right.$$

$$\exp\left[-\left(1+\frac{x_1}{r_0f_1'}\right.\right.\quad \exp\left[-\frac{}{r_0^2f_2'^2}\right], \tag{16}$$

$$\text{where } \beta_1'(z)=\frac{1}{f_1'(z)}\frac{df_1'}{dz},\quad \beta_2'(z)=\frac{1}{f_2'(z)}.\frac{df_2'}{dz} \tag{17}$$

The eikonal expansion may have a term linear in x_1. However, it does not contribute to the curvature of the wavefront and hence, self-focussing.

4.1. *Ponderomotive nonlinearity (collisionless case) :*

The dielectric constant $\in(x,y,z)$ for the ponderomotive nonlinearity may be expressed up to terms in x_1^2 and y^2, as

$$\in(x,y,z) = \in_0(z) + \in_1(x,y,z),$$

$$\begin{aligned}&\text{where } \in_0(z)=1-\Omega^2\exp\left(-p.\exp(-1)\right)\\&\text{and } \in_1(x,y,z)=-\Omega^2\exp\left(-p.\exp(-1)\right)p\\&\qquad\times\exp(-1)\left[\frac{2x_1^2}{r_0^2f_1'^2}\quad\frac{}{r_0^2f_2'^2}\right.\end{aligned} \tag{18}$$

Substituting for $S(x, y, z)$, $A_0^2(x,y,z)$, $\in_0(z)$ and $\in_1(x,y,z)$ in eq. (5) and equating the coefficients of x_1^2 and y^2 separately to zero, one obtains

$$\frac{d^2f_1'}{d\xi^2}=\frac{1}{\in_0(z)f_1'^3}\left\{1\quad\frac{2\Omega^2\rho_1^2p.\exp(-1)}{\exp(p.\exp(-1))}\right\} \tag{19}$$

$$\frac{d^2f_2'}{d\xi^2}=\frac{1}{\in_0(z)f_2'^3}\left\{1-\frac{\Omega^2\rho_2^2p.\exp(-1)}{\exp(p.\exp(-1))}\right. \tag{20}$$

where $\rho_1 = \rho_0 f_1'$, $\rho_2 = \rho_0 f_2'$ and the other symbols are the same as in Section 3. At $z = 0(\xi = 0)$, one has $f_1' = f_2' = 1$ and if in addition, the wavefront is plane, $df_1'/d\xi)_{\xi=0} = df_2'/d\xi)_{\xi=0} = 0$. Thus, the boundary conditions for integrating eqs. (19) and (20) are $df_1'/d\xi)_{\xi=0} = df_2'/d\xi)_{\xi=0} = 0$, $f_1' = f_2' = 1$, $\xi = 0$.

4.2. *Collisional nonlinearity :*

In a collisional plasma, ohmic nonlinearity arises through the heating of electrons and subsequent redistribution of plasma density. In this case, the dielectric constant can be written as

$$\in = 1 - \Omega^2\left(1 + \frac{\alpha}{2} EE^*\right)^{\frac{s}{2}-1} \tag{21}$$

where α and Ω have the same meaning as in collisionless case, while s is a parameter characterizing the nature of collisions. Proceeding as before, one obtains for the portion of the beam around the intensity maxima, the split parts of the dielectric constant as

$$\in_0(z) = 1 - \Omega^2\left(1 + p.\exp(-1)\right)^{\frac{s}{2}-1} \tag{22}$$

and $$\in_1(x, y, z) = \left(\frac{s}{2} - 1\right)\frac{\Omega^2 p.\exp(-1)}{\left(1 + p.\exp(-1)\right)^{2-s/2}} \cdot \left(\frac{2x_1^2}{r_0^2 f_1'^2} , \frac{}{r_0^2 f_2'^2}\right), \tag{23}$$

where p represents the dimensionless quantity $\frac{\alpha}{2} E_0^2 / f_1' f_2'$. The differential equations for f_1' and f_2' can then be obtained in the form :

$$\frac{d^2 f_1'}{d\xi^2} = \frac{1}{\in_0(z) f_1'^3} - \frac{(2-s)\Omega^2 \rho_1^2 p.\exp(-1)}{\left(1 + p.\exp(-1)\right)^{2-\frac{s}{2}}} \tag{24}$$

$$\frac{d^2 f_2'}{d\xi^2} = \frac{1}{\in_0(z) f_2'^3} - \frac{(2-s)\,\Omega^2 \rho_2^2 p.\exp(-1)}{2\left(1 + p.\exp(-1)\right)^{2-\frac{s}{2}}} \tag{25}$$

s is equal to 1 or -3 corresponding to collisions with neutrals and ions respectively. The expression for the relativistic nonlinearity (as pointed out by Sharma *et al* [12]) can be obtained from the expressions given above by putting $s = 1$ and $\frac{\alpha}{2} = e^2/m^2\omega^2c^2$.

5. Discussion

The behaviour of the portion of the beam close to the axis is governed by eqs. (12) and (13). Eq. (13) can be directly integrated for initially plane wave front ($df_2/d\xi = 0$ and $f_2 = 1$ at $z = 0$) to get

$$f_2 = \sqrt{1 + \xi^2/(1 - \Omega^2)} \, .$$

Eq. (12) involves f_2 and can be integrated numerically after substituting this expression for f_2. The parameters f_1 and f_2 as functions of ξ have been graphically represented in Figure 1 for suitable values of parameters Ω, ρ_0 and p_0, (subscript zero indicates values at $\xi = 0$). The graphs are clearly as expected.

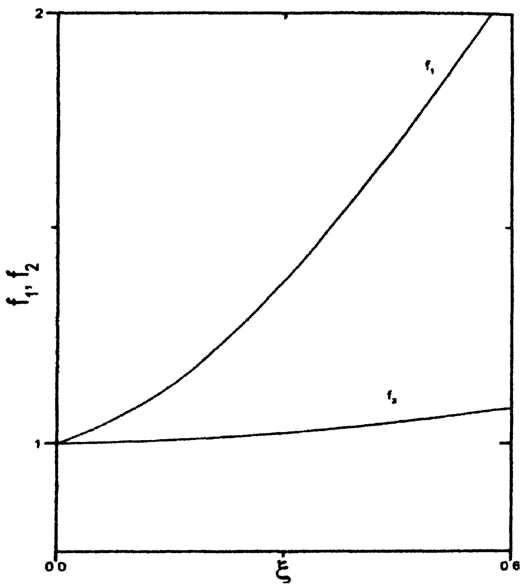

Figure 1. Beam width parameters f_1 and f_2 corresponding to x and y axes respectively, as functions of ξ for $\Omega^2 = 0.2$, $\rho_0 = 10$ and $p_0 = 1$ (axial beam propagation in collisionless plasma).

From eq. (19) (collisionless plasma), one observes that for $d^2f_1'/d\xi^2 = 0$, to vanish at $\xi = 0$, the initial beam width ρ_0 and initial beam power p_0 must satisfy

$$\rho_0^2 = \frac{\exp(p_0 \exp(-1))}{2\Omega^2 p_0 \exp(-1)} \, . \tag{26}$$

A corresponding relation for $d^2f_2'/d\xi^2$ to vanish at $\xi = 0$, is

$$\rho_0^2 = \frac{\exp(p_0 \exp(-1))}{\Omega^2 p_0 \exp(-1)} \, . \tag{27}$$

The two conditions are different and can not simultaneously be satisfied for any choice of initial beam width and beam power. Since the beam power p is equal to $p_0/f_1'f_2'$, even if one started with $d^2f_1'/d\xi^2 = 0$, f_1' will it remain unchanged because f_2' will be changing. One concludes that a uniform wave guide mode of propagation is just not possible for the TEM$_{10}$ mode. Eqs. (26) and (27) represent relations between the initial beam width and initial beam power shown by curves C(1) and C(2) respectively in Figure 2. These curves are not like the

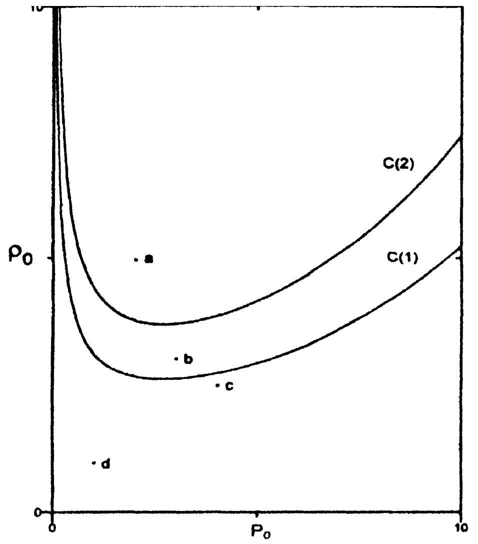

Figure 2. Initial beam width ρ_0 *versus* initial beam power p_0 near irradiance maxima for (collisionless plasma) with $\Omega^2 = 0.2$. $d^2f_1'/d\xi^2)_{\xi=0}$ indicated by (C(1)) and $d^2f_2'/d\xi^2)_{\xi=0}$.

critical power curve obtained for the Gaussian beam in so far as they are relevant only at $\xi = 0$; yet they are significant in as much as they determine the sign of $d^2f_1'/d\xi^2$ and $d^2f_2'/d\xi^2$ for any chosen initial values of beam width ρ_0 and beam power p_0, thus $d^2f_1'/d\xi^2$ at $\xi = 0$ will be positive, zero or negative according as the point (p_0,ρ_0) lies on the same side of the curve C(1) as the origin, on the curve and on the other side of the curve, respectively. Similar conclusion can be drawn for $d^2f_2'/d\xi^2$ at $\xi = 0$. The signs of these quantities indicate whether f_1' and f_2' will initially decrease or increase; negative sign indicates decrease *viz.* self-focussing, while the positive sign indicates initial divergence.

If now, the initial point (p_0,ρ_0) lies above the curve C(2) and hence also above C(1) (see Figure 2), then both $d^2f_1'/d\xi^2$ and $d^2f_2'/d\xi^2$ are initially ($\xi = 0$) negative. This implies that f_1' and f_2' both start falling below their initial values unity. In other words, the beam gets self-focussed in both directions in the early part of its propagation. Obviously, if the point (p_0,ρ_0) lies in between the two curves or below C(1) or much below C(1), the variation of f_1' and f_2' will be very different. These variations can be obtained by integrating eqs. (19) and (20) numerically. The two equations are not independent and must be integrated simultaneously. This has been done choosing different pairs of initial values (p_0,ρ_0) and the corresponding plots of f_1' and f_2' as functions of ξ are given in Figures 3 to 6. Figure 3 corresponds to the point (p_0,ρ_0) lying above C(2) in Figure 2.

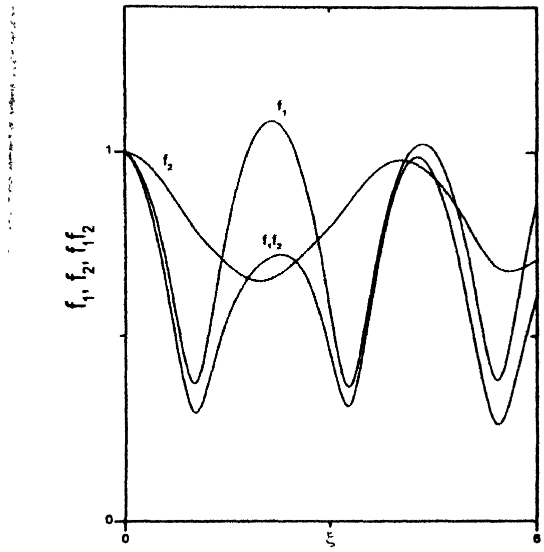

Figure 3. Beam width parameters f_1' and f_2' corresponding to x and y directions respectively, and their product $f_1'f_2'$ as functions of dimensionless distance of propagation ξ for the point a(2, 5) of Figure 2.

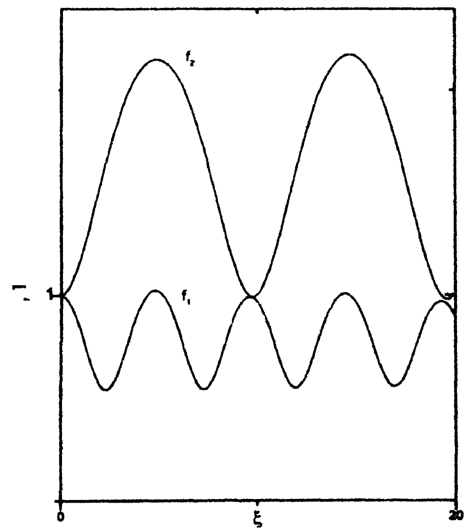

Figure 4. Beam width parameters f_1' and f_2' corresponding to the x and y directions respectively, as functions of dimensionless distance of propagation ξ for the point b(3, 3) of Figure 2.

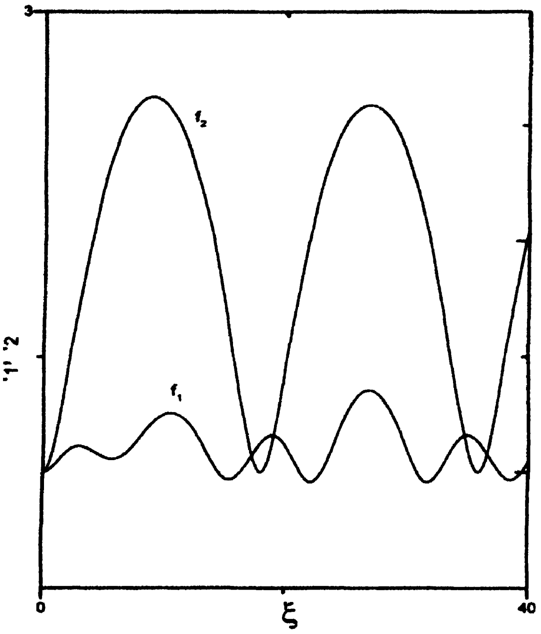

Figure 5. Beam width parameters f'_1 and f'_2 corresponding to the x and y directions respectively, as functions of dimensionless distance of propagation ξ for the point c(4, 2.5) of Figure 2.

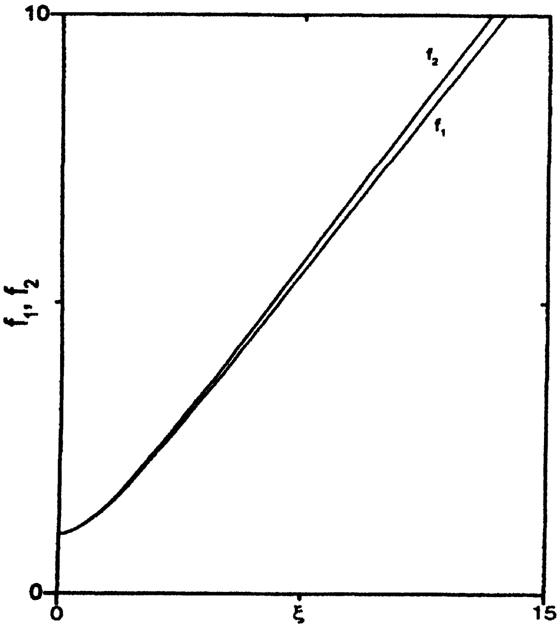

Figure 6. Beam width parameters f'_1 and f'_2 corresponding to the x and y directions respectively, as functions of dimensionless distance of propagation ξ for the point d(1, 1) of Figure 2.

This figure also shows the variation of $f_1'f_2'$, its reciprocal determining the irradiance of the beam at ξ. Figure 4 corresponds to a point lying between the curves C(1) and C(2) and shows that f_2' experiences oscillatory divergence while f_1' undergoes an oscillatory convergence, in other words, gets self-focussed. Figure 5 depicts the results for a point (p_0,ρ_0) lying below the curve C(1) and Figure 6 plots f_1' and f_2' for a point much below C(1).

Figure 5 corresponds to oscillatory divergence, while Figure 6 corresponds to steady divergence of the beam both in x and y directions.

The above analysis and calculations can be conducted for the collisional plasma (and relativistic plasma) starting with eqs. (24) and (25). The results so obtained are similar to those reported for the collisionless plasma discussed above.

6. Conclusions

Near the axis ($x = 0$, $y = 0$), the electric field and hence the refractive index increases with increasing x; thi augments the diffraction divergence of the beam. Fo finite values of x (near the axis), the field decreases wi increasing y; thus the nonlinearity tends to converge th beam in the y direction effectively reducing the diffractio divergence. This is obvious from the slower rise of f_2 i Figure 1 compared to that of f_1.

For regions close to the irradiance maxima, one obtain two coupled equations for beam widths f_1' and f_2' in and y directions. The critical curves for x and y direction are different and so are the curves for the two bear width parameters f_1' and f_2'. Specific types of nonlinearitie have been considered and dependence of f_1' and f_2' o distance of propagation, has been investigated for typic values of the parameters.

Acknowledgment

The authors are grateful to the Department of Scienc and Technology, Government of India for financi support.

References

[1] S A Akhmanov, A P Sukhorukov and R V Khokhlov *Sov. Phy. Usp.* **10** 609 (1968)

[2] M S Sodha, A K Ghatak and V K Tripathi *Self-focusing of Lase Beams in Dielectrics, Plasmas and Semiconductors* (Delhi : Tat McGraw Hill) (1974)

[3] M S Sodha, A K Ghatak and V K Tripathi *Prog. Opt.* **13** 169 (197

[4] E Esarey, P Sprangle, J Krall and A Ting *IEEE J. Quant. Electr* 3 1879 (1997)

[5] T M Antonsen (Jr.) and P Mora *Phys. Rev. Lett.* **69** 2204 (1992)

[6] P Sprangle, E Esarey, J Krall and G Joyee *Phys. Rev. Lett.* **69** 220 (1997)

[7] D Subbarao, R Uma and H Singh *Phys. Plasmas* **5** 3440 (1998)

[8] Piero Chessa, Patrick Mora and T M Antonsen (Jr.) *Phys. Plasmas* **5** 3451 (1998)

[9] Li-Ming Chen and Hai Lin *Phys. Plasmas* **8** 2974 (2001)

[10] B Hafizi, A Ting, R F Hubbard, P Sprangle and J R Penano *Phys. Plasmas* **10** 1483 (2003)

[11] E Garmire, R Y Chiao and C H Townes *Phys. Rev. Lett.* **16** 347 (1966)

[12] A Sharma, G Prakash, M P Verma and M S Sodha *Phys. Plasmas* **10** 4079 (2003)

[13] M S Sodha, V P Nayyar and V K Tripathi *J. Opt.Soc. Am.* **64** 941 (1974)